Chapter

Games and Social Reality

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Abstract

In this chapter, I will show that social activities can be seen as activities in games. In Section 2, rested on a formal framework called Dynamic Belief-Desire-Obligation Logic, I propose a model of agents who can play games. In Section 3, based on this framework, a rule system of a game is interpreted as a normative system that determines the obligation space and the permission space of each player. In a game stage, each player fulfills her obligations and chooses an action type from their permission space and performs it. These actions change the state of the game and determine new obligation and permission spaces of all participating players. In Section 4, I interpret social actions as actions based on normative systems. Social normative systems determine obligation and permission spaces of members of a social organization and restrict their activities. In Appendix, formal systems mentioned in this chapter are described in detail.

Keywords: normative system, dynamic belief-desire-obligation logic, philosophy of action, social ontology, actions in games, speech acts, role-specific normative spaces

1. Introduction

This chapter deals with some problems in social ontology. I investigate the following questions. How can we characterize agents who constitute social organizations? How is the society structured? How is the social reality created? To the first question, I answer that agents who can play team games constitute social organizations. It is my proposal to the second question that role-specific normative requirements structure social organizations. The social reality is created by agents who perform actions respecting socially accepted normative systems. This is my answer to the third question. These activities of agents can be explained by analyzing agents who play team games. We can observe that many games are clearly structured. Thus, investigations on games may contribute to analysis of the social reality.

In philosophy of action, Donald Davidson proposed to reduce intention to desire and means-end-belief [1]. Michael Bratman opposed to this claim and interpreted intention as planning [2]. Furthermore, against reductionistic approach, John Searle pointed out that some actions are strongly influenced by deontic requirements [3]. To this problem, this chapter proposes a new alternative position. Namely, I suggest taking beliefs, desires, and normative beliefs as mental bases for planning and decision-making. Some actions are temporally extended and include substructural actions. A series of actions in a game is an example of such temporally extended actions. In this chapter, I will show how beliefs, desires, and normative beliefs are connected to realize the leading desire in the given constraints.

There are three major logical approaches to investigations on dynamic inferences, namely dynamic epistemic logic [4, 5], logic of belief revision [6, 7], and discourse representation theory [8, 9]. My formal approach in this chapter is mainly related to the last two approaches. The formal details of the framework in this chapter can be found in Appendix.

For the sake of space, I do not discuss collective actions in this chapter (for collective actions, see [10]). However, the model proposed in this chapter can be also applied to characterizations of collective actions.

2. Model of agents

A game is played by agents who perform actions keeping the rule system of the game. All participating agents respect the rule system of the game when they play it. Otherwise, the game will be broken down. Then, what kind of features are required for agents who can play games? This is the leading question of this section.

A rule system of a game can be interpreted as a normative system [11]. I proposed in some articles to represent a normative system by a pair ⟨BB, OB⟩ constituted of a belief base BB and an obligation base OB [11, 12]. A player masters this normative system and decides her action based on it. Furthermore, her decision of the next action is influenced by her desire of winning the game. In each game stage, a player will perform an action type that is considered as a promising move to win the game. Here, I take a desire base DB into consideration and define Belief-Desire-Obligation system ⟨BB, DB, OB⟩ as follows [13, 14].

2.1 (S2.1) Belief-Desire-Obligation system (BDO-system)

a. I assume that each of BB, DB, OB is a set of sentences, more precisely, a set of sentences in First-Order Logic (FO-Logic). Every sentence in BB expresses what is believed, every sentence in DB expresses what is desired, and every sentence in OB expresses what is believed to be obligated. We call BB, DB, OB belief base, desire base, and obligation base respectively.

b. A pair ⟨BB, DB⟩ is a Belief-Desire system (BD-system) ⇔ BB∪DB is consistent. Here, " ⇔" is used as an abbreviation for "if and only if."

c. A pair ⟨BB, OB⟩ is a Belief-Obligation system (BO-system) ⇔ BB∪OB is consistent.

d. A triple ⟨BB, DB, OB⟩ is a Belief-Desire-Obligation system (BDO-system) ⇔ ⟨BB, DB⟩ is a BD-system and ⟨BB, OB⟩ is a BO-system.

e. A BDO system for agent A is expressed as $\langle BB_A, DB_A, OB_A \rangle$.

f. We call socially accepted BO-system normative system.

Normative systems such as laws can be expressed as BO-systems [12]. In Section 3, we will see that a rule system of a game can be interpreted as a normative system.

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Note that the definition of BDO-system in (S2.1) allows conflicts between the BD-system and the BO-system. This feature reflects cases in which an agent has a desire that conflicts with her BO-system. When an agent prefers her desire even if this desire conflicts with an accepted normative system, a performance of a prohibited action becomes possible.

Now, we can introduce some logical operators and define a logical framework Belief-Desire-Obligation Logic (BDO-Logic) (for details, see Appendix (Ap.1)). For agent A, logical operators in BDO-Logic are B_A, M_A, O_A, F_A, P_A, and D_A. The intended meaning of sentences with these operators can be expressed as follows: A believes φ (B_A φ), A believes that possibly φ (M_A φ), A believes it is obligated φ (O_A φ), A believes it is forbidden φ (F_A φ), A believes it is permitted φ (P_A φ), and A believes it is desired φ (D_A φ). The following list describes some important features of these operators.

2.2 (S2.2) Operators in BDO-Logic

a. Epistemic operators B_A and M_A depend on only belief base BB_A, namely it holds: [B_A φ ⇔ φ follows from BB_A] and [M_A φ ⇔ φ is compatible with BB_A].

b. Normative operators O_A, F_A, and P_A depend on both belief base BB_A and obligation base OB_A. That means: [O_A φ ⇔ [not B_A φ and φ follows from $BB_A \cup OB_A$]], [F_A φ ⇔ $O_A \neg\varphi$], and [P_A φ ⇔ [not B_A φ and φ is compatible with $BB_A \cup OB_A$]].

c. Desire operators D_A is defined in analog to obligation operator O_A, namely: [D_A φ ⇔ [not B_A φ and φ follows from $BB_A \cup DB_A$]].

d. The inference system with B_A and M_A, which satisfies characterizations in (S2.2.a), is called B-Logic. The inference system with B_A, M_A, O_A, F_A, and P_A, which satisfies characterizations in (S2.2.a) and (S2.2.b), is called BO-Logic. The inference system with B_A, M_A, and D_A, which satisfies characterizations in (S2.2.a) and (S2.2.c), is called BD-Logic. Finally, the inference system with B_A, M_A, D_A, O_A, F_A, and P_A, which satisfies characterizations in (S2.2.a), (S2.2.b), and (S2.2.c), is called BDO-Logic.

B_A φ roughly means that A believes that it must be the case φ and M_A φ roughly means that A believes that it may be the case φ. Stipulation (S2.2.a) expresses a standard logical characterization of doxastic state. Characterization (S2.2.b) expresses the idea that a normative belief is formed with a presupposition of a belief base (see **Figure 1**). For example, an obligation of payment of a ticket in a museum presupposes the belief that there is such a museum. (S2.2.b) also claims that what is believed

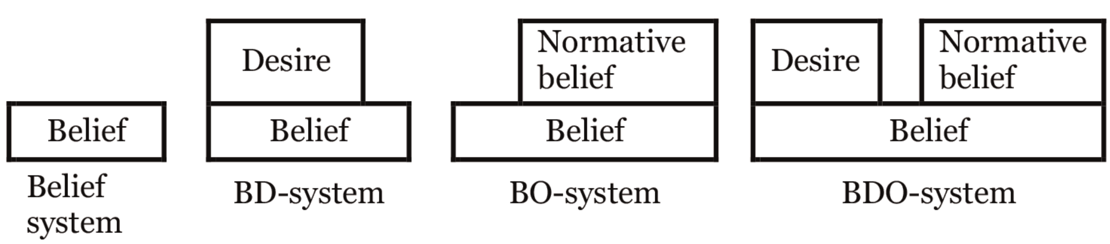

Figure 1.
Mental states of an agent (BDO-system).

cannot be normatively believed. Thus, an agent who once recognized that the required obligation was fulfilled is released from this obligation. For example, if you have already paid for a ticket, then you are released from obligation of the payment. Conforming with the standard view, (S2.2.b) characterizes permission as a deontological possibility. According to (S2.2.c), an agent who once recognized that the original desire was fulfilled will no longer try to fulfill this desire. For example, if you have already bought a book you wanted to read and recognized this result, then your desire of buying the book would disappear.

Based on BDO-Logic, I characterize intention as decision-making.

2.3 (S2.3) Characterization of four kinds of spaces for decision-making

An agent chooses an action type from a set of action sentences. In BDO-Logic, there are four kinds of such sets of action sentences (for details, see Appendix (Ap.1.i)). Here, we assume: $BDO_A = \langle BB_A, DB_A, OB_A \rangle$.

a. The desire space of agent A with BDO_A, Desire-Space(A, BDO_A), is the set of action sentences, which A desires to fulfill.

b. The obligation space of agent A with BDO_A, Obligation-Space(A, BDO_A), is the set of action sentences, which A believes being obligated to fulfill.

c. The prohibition space of agent A with BDO_A, Prohibition-Space(A, BDO_A), is the set of action sentences, which A believes being forbidden to fulfill.

d. The permission space of agent A with BDO_A, Permission-Space(A, BDO_A), is the set of action sentences, which A believes being permitted to fulfill.

e. The obligation space of agent A with BDO_A, the prohibition space of A with BDO_A, and the permission space of A with BDO_A are normative spaces of A with BDO_A.

Desire and obligation may create a direct motivation for actions, but permission has not this kind of motivational power. Rather, permission needs a support from desire. Related with normative spaces, there are three kinds of decision-making.

2.4 (S2.4) Three types of decision-making

a. [Desire-based Decision] Agent A chooses an action sentence from Desire-Space(A, BDO_A) and performs the chosen action. I call this kind of decision desire-based decision.

b. [Obligation-based Decision] Agent A chooses an action sentence from Obligation-Space(A, BDO_A) and performs the chosen action. I call this kind of decision obligation-based decision.

c. [Permission-based Decision] Agent A chooses an action sentence from Permission-Space(A, BDO_A) with help of her desire and performs the chosen action. I call this kind of decision permission-based decision.

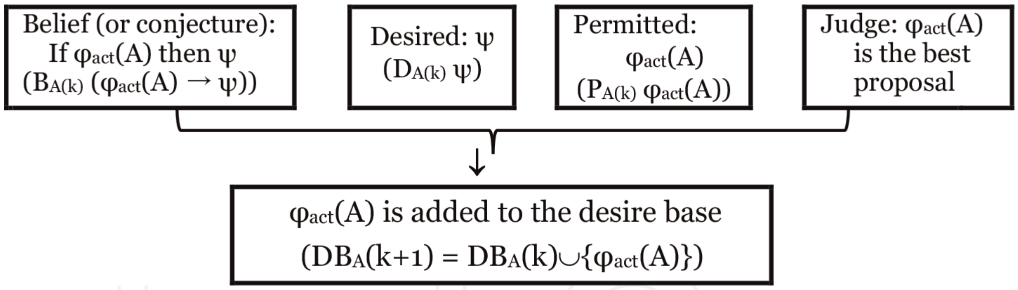

Figure 2.
Abduction-like update of desire base.

As Bratman pointed out, there might be several conflicting future options each of which is, in light of our desires and beliefs, equally attractive [15]. Characterizations in (S2.3) and (S2.4) take these phenomena into consideration. A BD-system or a BO-system provides some attractive options for decision-making. Thus. to perform an action, we need to choose one of action type from these options. Characterization (S2.4) shows that there are three types of decision-making. An agent can prefer desired-based decision or obligation-based decision or permission-based decision. In praxis, a permission-based decision is often supported by a desire-based decision (see abduction-like update of desire base in **Figure 2**).

Many philosophers in action theory focused their investigations on desire-based decisions. Searle criticized this preconception and pointed out that there are also obligation-based decisions [3]. Supplementing these preceding investigations, I will show that permission-based decisions play an important role in our everyday life.

To clarify the relationship between BDO-system and normative system, I define the notion of respect for a normative system.

2.5 (S2.5) Respect for a normative system

Here, we assume that agent A has BDO-system $\langle BB_A, DB_A, OB_A \rangle$.

a. $\langle BB_A, OB_A \rangle$ includes $\langle B_{NS}, OB_{NS} \rangle$ ⇔ (BB_{NS} is a subset of BB_A & OB_{NS} is a subset of OB_A).

b. [Compatible with a normative system] A decision that A makes is compatible with normative system $\langle BB_{NS}, OB_{NS} \rangle$ ⇔ ($\langle BB_A, OB_A \rangle$ includes $\langle B_{NS}, OB_{NS} \rangle$ & A's decision is obligation-based or permission-based).

c. [Respect] Agent A respects normative system $\langle BB_{NS}, OB_{NS} \rangle$ ⇔ any decision that A makes is compatible with $\langle BB_{NS}, OB_{NS} \rangle$.

According to (S2.5), an agent who respects a normative system obeys any obligation in it and she chooses only action types that are permitted in it. For example, a player of chess respects the normative system of chess and she plays chess keeping out of violation of it. Each time, this player tries to choose a promising move that is permitted in chess so that her desire to win will be finally fulfilled.

We can update a BDO-system $\langle BB, DB, OB \rangle$ by updating BB or DB or OB. We call the framework that allows this kind of updates Dynamic BDO-Logic (see Appendix (Ap.2)). A BDO-system in Dynamic BDO-Logic contains information about its stage.

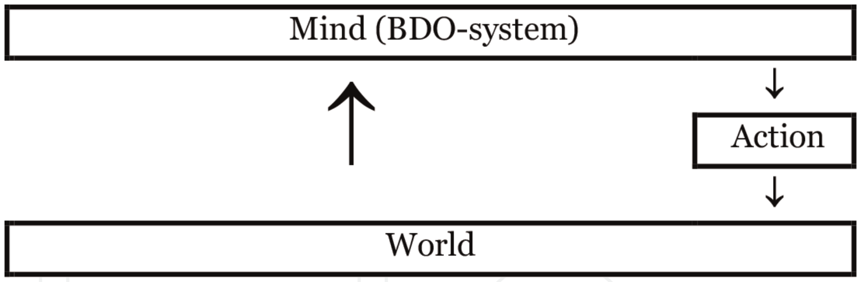

Figure 3.
Interaction between an agent and the world.

We write a BDO-system of Dynamic BDO-Logic as follows: BDO(k) = ⟨BB(k), DB(k), OB(k)⟩. A play of standard two-man games can be described in Dynamic BO-Logic that is a subsystem of Dynamic BDO-Logic [11, 13, 14]. Activities of an agent in the world can be described by using Dynamic BDO-Logic. An agent performs an action based on her current mental states. This action changes the world. When this agent recognizes this change, her mental states are updated, and she performs new action based on the new mental states. This kind of interaction between effects of action and updates of mental states goes on while an agent performs actions in the world (**Figure 3**).

According to Searle, beliefs have mind-to-world direction of fit, whereas desires and intentions have world-to-mind direction of fit [16]. This roughly means that beliefs should be formed to match the world and that the world state should be changed to match desires or intentions. This Searle's view is not false but static. When we apply the dynamic view described in **Figure 3** to analysis of a goal-directed action, we see that a belief change caused by an observation of her own first action may affect her desire and decision-making for the next action to achieve the goal.

Now, we show that rules can be analyzed by using BO-systems. According to Searle, there are two types of rules, namely regulative and constitutive rules [17]. Regulative rules regulate a pre-existing activity, an activity whose existence is logically independent of the rules. Regulative rules characteristically take the form of or can be paraphrased as imperatives, e.g., "Officers must wear ties at dinner". Constitutive rules constitute an activity the existence of which is logically dependent on the rules. Constitutive rules can be paraphrased as "X counts as Y in context C". A typical example is an introduction of a term used in a game, e.g., "A checkmate is made when the king is attacked in such a way that no move will leave it unattacked" [17]. Both rules can be expressed in BO-Logic (here, I use tr as a translation function from English into a language of FO-Logic). In the normative system for officers ⟨$BB_{officer}$, $OB_{officer}$⟩, tr(Officers wear ties at dinner) is a member of $OB_{officer}$. Similarly, in the normative system of chess ⟨BB_{chess}, OB_{chess}⟩, tr(A checkmate is made if and only if the king is attacked in such a way that no move will leave it unattacked) is a member of BB_{chess}.

3. Description of games

What is a game? I require the following two conditions for games.

(S3.1.a) There is a normative system for the game that all participants of the game accept and respect.

(S3.1.b) The start and the end condition of the game are clearly characterized in the normative system of the game.

These requirements are not demanding. The first requirement (S3.1.a) is essential for my enterprise. Following this requirement, I interpret a rule system of a game as a normative system. The start and the end condition in the second requirement (S3.1.b) are required to temporally demarcate a game.

A play of standard two-man games can be described in Dynamic BDO-Logic. In Subsection 3.1 and Appendix (Ap.3)-(Ap.5), by using example of Tic-Tac-Toe, I demonstrate how to express its rule system as a normative system. More complex games and team games can be also expressed as normative systems [11]. In Subsection 3.3, I demonstrate how to represent team games by using Dynamic BDO-Logic.

3.1 Description of simple games

Tic-Tac-Toe is a paper-and-pencil game for two players who take turns marking the spaces in a 3×3 grid. The player who succeeds in placing three of her marks in a horizontal, vertical, or diagonal row wins the game (the discussion in this subsection is largely based on [13]). There are two types of propositions in Tic-Tac-Toe, namely action types and state types (for state types, see Appendix (Ap.3)). There is only one action type in Tic-Tac-Toe, namely: X places her mark in position s in stage k. The normative system for Tic-Tac-Toe is $\langle BB_{ttt}, OB_{ttt} \rangle$. To play Tic-Tac-Toe, every player must master this normative system. Contents of BB_{ttt} and OB_{ttt} can be explained as follows (for details, see Appendix (Ap.3) and (Ap.4)).

(S3.2) Belief base BB_{ttt} consists of the following elementary theory for Tic-Tac-Toe.

a. There are exactly two players who are opponents each other.

b. We use O and $\times$ as two marks of players.

c. If one player wins in Tic-Tac-Toe, then her opponent loses.

d. When it is turn of a player, it is not turn of her opponent.

e. If position s is occupied by mark m in stage k and $k \leq n$, then s is still occupied by m in stage n.

f. Position s is vacant in stage k $\Leftrightarrow$ s in not occupied by any mark in stage k.

g. The player who succeeds in placing three of her marks in a horizontal, vertical, or diagonal row wins the game.

h. The game ends $\Leftrightarrow$ a player wins or there is no vacant position in the 3×3 grid.

i. If a player places his mark m in position s in stage k, then s is occupied by m in k + 1 and the turn is alternated in k + 1.

(S3.3) Obligation base OB_{ttt} consists of the following sentences.

a. If the game is not yet ended and it is X's turn in stage k, then X places her own mark into a vacant position in k.

b. Any player does not place her mark into a non-vacant position in any stage.

c. Any player does not place her mark when it is not her turn.

d. Any player does not write her mark when the game ends.

In Tic-Tac-Toe, there are two players X and Y. In many two-person games, their rule systems and the game developments are shared by players. Let X + Y be the group consisting of X and Y. Then, a shared BDO-system can be defined as follows: $\langle BB_{X+Y}(k), DB_{X+Y}(k), OB_{ttt}\rangle$ with $BB_{X+Y}(k) = BB_X(k) \cap BB_Y(k)$ and $DB_{X+Y}(k) = DB_X(k) \cap DB_Y(k)$. Thus, $BB_{X+Y}(k)$ is the shared belief base of X + Y in stage k, $DB_{X+Y}(k)$ is the shared desire base of X + Y in stage k, and OB_{ttt} is the shared obligation base of X + Y. Initial-state is the set of sentences that describe the initial state and contains sentences such as "Every position (in 3×3 grid) is vacant in stage 0". We assume that $BB_{X+Y}(0)$ is BB_{ttt} supplemented by Initial-state (see Appendix (Ap.4.b) and (Ap.4.c)). As an abbreviation, we write: $BDO_{X+Y}(k) = \langle BB_{X+Y}(k), DB_{X+Y}(k), OB_{ttt}\rangle$. During the play of the game, $BB_{X+Y}(k)$ is updated by adding acquired new information to the previous belief base $BB_{X+Y}(k - 1)$ (see (S3.4.a) and Appendix (Ap.4.f1)).

Note that (S3.3.a) and (S3.3.c) play an important role for the calculation of obligation space. By the way, it holds in BDO-Logic: $(O(\varphi \rightarrow \psi)\ \&\ B(\varphi)) \Rightarrow O(\psi)$ (see [11] and Appendix (Ap.1.h)). Then: $O_{X+Y}(k)\ (\varphi \rightarrow \psi) \Rightarrow (B_{X+Y}(k)\ \varphi \Rightarrow O_{X+Y}(k)\ \psi)$. Now, it holds because of (S3.3.a): Players X + Y believe in stage k that it is obligated that [if the game is not yet ended and it is X's turn in k, then X places her own mark into a vacant position in k]. Thus, it holds: When players X + Y know that the game is not yet ended and it is X's turn in stage k, they believe that it is obligated that X places her own mark into a vacant position in k (this sentence has the following form: $B_{X+Y}(k)\ \varphi \Rightarrow O_{X+Y}(k)\ \psi$). Thus, when X + Y know that the game is not yet ended and it is X's turn in k, Obligation-Space(X, $BDO_{X+Y}(k)$) is the set consisting of tr(X places her mark into a vacant position in stage k). Similarly, because of (S3.3.c), when X + Y know that it is not Y's turn in stage k, then Obligation-Space(Y, $BDO_{X+Y}(k)$) is the set consisting of tr(Y does not place her mark in any position in stage k).

To play Tic-Tac-Toe seriously, it is required that players desire to play it and to win (thus, for every player X, DB_X = {play(X, Tic-Tac-Toe), ∃k won(X, k)}). Because a player desires to win a game, she tries to find good strategies for the victory.

There are two types of stages, namely, continuing stage and terminal stage. The game process of Tic-Tac-Toe depends on these two qualifications of stages. Here, we describe two schemas for updates of a BDO-system.

(S3.4.a) [Continuing stage] In stage k, the following beliefs are shared by all players: The game is not ended in k, it is X's turn in k, and Y is X's opponent. Then, it follows that Obligation-Space(X, $BDO_{X+Y}(k)$) = {tr(X places her mark in a vacant position in k)}, Obligation-Space(Y, $BDO_{X+Y}(k)$) = {tr(Y does not place her mark in any position in k)}, Permission-Space(X, $BDO_{X+Y}(k)$) is the set of sentences that express permissible placing actions in k, and Permission-Space(Y, $BDO_{X+Y}(k)$) is empty. Here, X wants to win the game and X chooses a placing action type from

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Permission-Space(X, $BDO_{X+Y}(k)$) to fulfill her obligation and to approach her leading desire. Now, suppose that X forms the desire to perform a placing action expressed by $\varphi_1(X)$. Thus, $DB_{X(k)}$ is obtained by adding $\varphi_1(X)$ to $DB_{X(k-2)}$. After X's performance of this action, both players confirm it and update their belief base so that it holds: $BB_{X+Y}(k+1)$ is the union of $BB_{X+Y}(k)$ and $\{\varphi_1(X)\}$. During the play, the number of vacant positions in the grid diminishes, and the permission spaces of both players decreases. (In detail, see Appendix (Ap.4.f1))

(S3.4.b) [Terminal stage] In stage k, all players believe that the game ended in k. In this case: Permission-Space of X and Y in k are empty, so that they stop to play. (In detail, see Appendix (Ap.4.f2))

Here, let us give an example of a play of Tic-Tac-Toe. In this example, each of A and B wants to play Tic-Tac-Toe and wants to win.

Suppose that A starts with placing his mark O in position [1,1] of the grid (formally: placing(A, O, [1,1], 0)). Both players try to win and make a promising move in each game stage. The following sequence of quadruples describes one possible game development: ⟨(A, O, [1,1], 0), (B, ×, [2,2], 1), (A, O, [3,3], 2), (B, ×, [1,3], 3), (A, O, [3,1], 4), (B, ×, [3,2], 5), (A, O, [2,1], 6)⟩. After A placed O into position [2,1] in stage 6, A and B recognize in stage 7 that three of O are in a vertical raw and that A won the game. Results of this action sequence is partly demonstrated in **Figure 4**. Here, we can see that the terminal stage of this game has been achieved in stage 7.

Now, let us closely investigate the relation between desire and permission in this game process. In the initial stage, A desires to win the game and A believes that A is permitted to place O in any position in 3 × 3 grid. Winning this game is the final desire of A in this game. In each stage, when it is A's turn, A forms a desire to perform a placing action that is permitted and will lead to fulfillment of her final desire. Update of desire during a game can be interpreted as an abduction-like reasoning demonstrated in **Figure 2**, where $\varphi_{act}(A)$ expresses A's performance of a placing action (an abduction (inference to the best explanation) can be demonstrated as **Figure 5**). In stage 0, A desires to place O into [1,1], because A conjectures that it can lead to A's victory. In stage 6, A believes that placing O into [2,1] is the best action to A's victory. Based on this consideration, A desires to perform this action and performs it. A's intention to play Tic-Tac-Toe corresponds to the prior intention and A's intention to perform the chosen placing action corresponds to intention-in-action in the literature [16].

3.2 Speech acts and creation of social facts

Many team games are controlled by referees, and statements made by them play an important role in game processes. In team games, referees have authority, and their

[1,1]	[1,2]	[1,3]
[2,1]	[2,2]	[2,3]
[3,1]	[3,2]	[3,3]

Stage 0

…

O	[1,2]	[1,3]
[2,1]	×	[2,3]
[3,1]	[3,2]	O

Stage 3

…

O	[1,2]	×
O	×	[2,3]
O	×	O

Stage 7

Figure 4.
An example of game progresses in Tic-Tac-Toe.

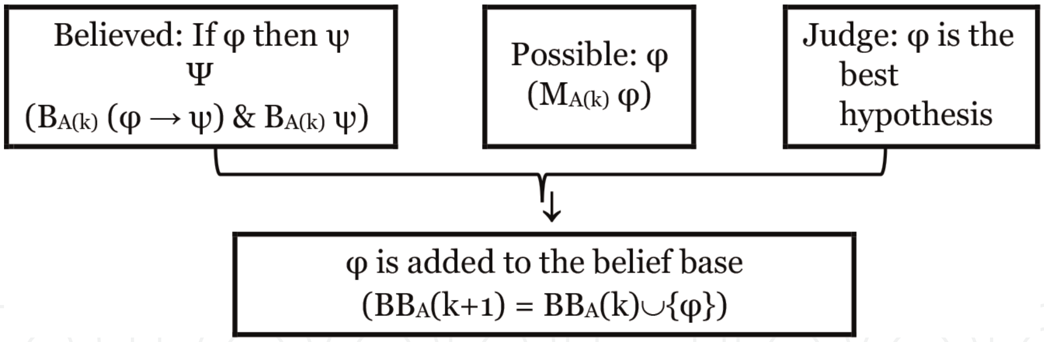

Figure 5.
Schema for abduction.

judgments are accepted by players and audiences. A judgment by a referee can be characterized as a declaration that is a type of illocutionary acts. In this subsection, to characterize declarations, we investigate speech acts based on Dynamic BDO-Logic. For this purpose, let us start with Searle's characterizations of five types of illocutionary acts. They are Assertives, Directives, Commissives, Expressives, and Declarations, which Searle characterizes as follows [18].

(S3.5.a) [Assertives] The point or purpose of the members of the assertive class is to commit the speaker (in varying degrees) to something's being the case, to the truth of the expressed proposition. Examples for assertives are assertion, complaint, conclusion, deduction, and so on.

(S3.5.b) [Directives] The illocutionary point of these consists in the fact that they are attempts (of varying degrees, and hence, more precisely, they are determiners of the determinable, which includes attempting) by the speaker to get the hearer to do something. Examples for directives are command, request, permission, advise, invitation, and so on.

(S3.5.c) [Commissives] Commissives are those illocutionary acts whose point is to commit the speaker (again in varying degrees) to some future course of action. Examples for commissives are promise, pledge, and so on.

(S3.5.d) [Expressives] The illocutionary point of expressives is to express the psychological state specified in the sincerity condition about a state of affairs specified in the propositional content. Examples for expressives are gratitude, apology, congratulation, and so on.

(S3.5.e) [Declarations] It is the defining characteristic of declarations that the successful performance of one of its members brings about the correspondence between the propositional content and reality, successful performance guarantee that the propositional content corresponds to the world. Examples for declarations are appointment, naming, and so on.

I propose to interpret these illocutionary acts as actions that are aimed to update belief base or obligation base.

(S3.6) Updates of BDO-bases by performances of illocutionary acts.

We assume: S is a speaker, H is a hearer, S + H is the group consisting of S and H, and G(S) is a group including S as a member. Let $\varphi_{act}(X)$ be an action sentence with X as the agent of this action. Furthermore, let $BB_{S + H}$(now) be the current belief base of S + H, $OB_{S + H}$(now) be the current obligation base of S + H, and $BB_{G(S)}$(now) be the current belief base of group G(S). Here, I propose that a successful performance of an illocutionary act effects the following updates of BDO-bases.

a. [Assertives] If S successfully performs an assertive illocutionary act with content expressed by φ addressing to H, then φ is added to $BB_{S + H}$(now).

b. [Directives] If S successfully performs a directive illocutionary act with content expressed by $\varphi_{act}(H)$ addressing to H, then $\varphi_{act}(H)$ is added to $OB_{S + H}$(now).

c. [Commissives] If S successfully performs a commissive illocutionary act with content expressed by $\varphi_{act}(S)$ addressing to H, then $\varphi_{act}(S)$ is added to $OB_{S + H}$(now).

d. [Expressives] If S successfully performs an expressive illocutionary act with content expressed by φ addressing to H, then φ is added to $BB_{S + H}$(now).

e. [Declarations] If S successfully performs a declaration with content expressed by φ addressing to G(S), then φ is added to $BB_{G(S)}$(now).

f. [Basic social facts] A sentence φ expresses a basic social fact for group G ⇔ φ is true because G has accepted φ (Thus, φ would not be true, if G had not accepted φ).

g. [Social facts] A sentence φ expresses a social fact for group G ⇔ φ is true because there is a basic social fact for G that supports the truth of φ. Thus, a basic social fact is also a social fact.

Speech act theory of John Austin and Searle investigates only singular sentences. This marks a limitation of this theory. Normally, utterances are made as a part of a comprehensive action. For example, in a discussion in a group, many assertions and many questions are made to reach an agreement. This kind of discussions can be considered as a game. The game starts with an identification of the goal of the discussion, and it ends when this goal is achieved. A contribution to the discussion can be considered as a move in a game. This game successfully ends when participants achieved an acceptable conclusion, otherwise it unsuccessfully ends. According to (S3.6.f) and (S3.6.g), when the discussion group G reaches an agreement expressed by φ, then the fact expressed by φ becomes a social fact in G. From the same reason, a successful declaration introduces a new social fact. Later, the so created social facts may become bases for individual and collective actions of members of G.

3.3 Description of team games

Team games can be also described in Dynamic BDO-Logic or in its subsystem [11]. For this description, expressions of roles are crucial. For there are many obligations that are imposed only to agents who have certain roles. This role-specific obligation can be expressed as an element of obligation base as follows: If x has $role_1$, then x performs $action_1$. For example, a goalkeeper in soccer has the following obligation: If agent x is a goalkeeper, then x tries to stop the ball from going into x's team's goal. There are also goalkeeper-specific permissions. For example, it is permitted only for a goalkeeper to hold the ball with her hands within team's penalty box. In other words, it is forbidden for all players other than goalkeepers to use her hands and it is forbidden for a goalkeeper to hold the ball with her hands outside of team's penalty box. As these examples show, roles in games can be formally captured by determining role-specific obligation space and permission space. The rule system for soccer can be expressed as a normative system $\langle BB_{soccer}, OB_{soccer} \rangle$, which determines role-specific normative spaces.

Let me explain these role-specific normative spaces by using an example of baseball games. In baseball, there are two main normative systems, namely a normative system for players and a normative system for umpires, represented as $\langle BB_{bb\text{-}players}, OB_{bb\text{-}players} \rangle$ and $\langle BB_{bb\text{-}umpires}, OB_{bb\text{-}umpires} \rangle$ respectively. Pitcher, catcher, first baseman, and so on belong to player-roles for defense, whereas there is only one player-role for offense, namely batter.

All players in a team desire their team's victory. Thus, it holds that all members of team G_1 have desire to win against team G_2. Thus, for every member X of G_1, X's desire base contains tr(G_1 win the game). This desire for a victory of own team is for its members the leading desire during the game. To fulfill this goal, players perform their actions respecting their normative spaces. When a player keeps permission space, she automatically keeps out of prohibition space. In general, given normative system $\langle BB_{NS}, OB_{NS} \rangle$, three kinds of normative spaces can be characterized as follows (for details, see Appendix (Ap.6)).

(S3.7.a) Obligation-Space*(A, $role_1$, G, t, $\langle BB_{NS}, OB_{NS} \rangle$) is the set of action sentences, which $\langle BB_{NS}, OB_{NS} \rangle$ obligates agent A to fulfill when A has $role_1$ in group G at time t.

(S3.7.b) Prohibition-Space*(A, $role_1$, G, t, $\langle BB_{NS}, OB_{NS} \rangle$) is the set of action sentences from the realization of which $\langle BB_{NS}, OB_{NS} \rangle$ prohibits agent A when A has $role_1$ in group G at time t.

(S3.7.c) Permission-Space*(A, $role_1$, G, t, $\langle BB_{NS}, OB_{NS} \rangle$) is the set of action sentences, which $\langle BB_{NS}, OB_{NS} \rangle$ allows agent A to fulfill only when A has $role_1$ in group G at time t.

(S3.7.d) (S3.7.a), (S3.7.b), and (S3.7.c) characterize role-specific obligation, prohibition, and permission spaces. In this chapter, we call these spaces roll-specific normative spaces.

Rules of baseball can be divided into declarative and normative sentences. The FO-translations of declarative sentences are assembled into $BB_{bb\text{-}players}$. The normative sentences are reformed in obligation sentences, and their FO-translations are assembled into $OB_{bb\text{-}players}$ (see [11]). Then, based on (S3.7), we can calculate role-specific obligation and permission spaces for all players. For example, the sentence tr(A stations herself directly back of the plate) belongs to Obligation-Space*(A, catcher, G_1, t, $\langle BB_{bb\text{-}players}, OB_{bb\text{-}players} \rangle$), when A is a catcher in team G_1 and t belongs to the defense time for G_1. A baseball game will be played without any problems, when all players respect $\langle BB_{bb\text{-}players}, OB_{bb\text{-}players} \rangle$ and try to fulfill their role-specific normative requirements.

There is also a normative system for umpires. For example, if the chief umpire of a baseball game believes in time t that the ball that the pitcher has thrown is a strike, then tr(A declares the thrown ball as a strike) belongs to Obligation-Space*(A, chief umpire, $G_{umpires}$, t, $\langle BB_{bb\text{-}umpires}, OB_{bb\text{-}umpires}\rangle$).

Let $G_{audience}$ denote the group of audiences of this game and assume that G_{all} consists of $G_{audiences}$, $G_{players}$, and $G_{umpires}$. In a baseball game, G_{all} accepts every declaration that umpires make in this game. For example, if an umpire declares φ, then φ is added to the belief base of G_{all} so that φ becomes a sentence that expresses a social fact for G_{all}. The fact expressed by φ is never doubted during the game and it is shared by all members of G_{all}. The BO-system for G_{all} at t can be expressed as $\langle BB_{G(all)}(t), OB_{G(all)}(t)\rangle$, where $BB_{G(all)}(t)$ contains both of $BB_{bb\text{-}players}$ and descriptions of all events in the game unitl t. For players and the manager of a team, knowledge of social facts during the game provides a basis for planning of next actions in the game. In other words, they plan next actions respecting role-specific normative spaces at the given situation.

In the philosophical literature, many philosophers have required mutual beliefs as a basis for collective actions [19, 20]. However, this requirement is too strong, as some philosophers pointed out [3, 21]. Therefore, I propose the following tic-for-tac-like strategy as an alternative, and I call this strategy "acceptance strategy".

> (S3.8.a) At the start of a game, you assume that all players accept and respect the normative system of the game.

> (S3.8.b) You respect the normative system of the game so long as you do not find any violation of the normative system by any player.

This strategy is strong enough for assuring the play, and it is far weaker and more natural than the requirement of mutual beliefs among players.

4. Games and society

Max Weber, a famous sociologist, stated that the interpretative understanding of social actions is an aim of sociology [22]. In this section, I propose that social actions should be understood in a wide sense. For not only collective actions but also individual actions can be considered as social actions when they presuppose socially accepted normative systems. For example, a violation of a criminal law is only possible when a socially accepted criminal law exists in that society. In this sense, a criminal action by an individual agent can be considered as a social action insofar she is aware that her action violates a criminal law. In other words, each citizen has a prohibition space that forbids criminal actions, while a criminal person intentionally ignores this space.

In the society, there are many social organizations, such as states, armed forces, companies, universities, hospitals, and so on. Each of these social organizations has its own normative system and most of its members respect it. Some social organizations such as states have a set of complex normative systems, and its members only partially know about them. At any rate, mature members of a social organization share some parts of its normative system.

Here, I distinguish two types of shared BO-systems, namely individually shared and socially shared BO-system.

> (S4.1.a) [Shared BO-system] We assume that A has BO-system $\langle BB_A, OB_A\rangle$ and that B has BO-system $\langle BB_B, OB_B\rangle$. Then, A and B share BO-system $\langle BB, OB\rangle \Leftrightarrow [\langle BB_A, OB_A\rangle$ includes $\langle BB, OB\rangle$ & $\langle BB_B, OB_B\rangle$ includes $\langle BB, OB\rangle]$.

(S4.1.b) [Socially shared BO-system] A and B socially share ⟨BB, OB⟩ ⇔ There is a group G such that [G is greater than A + B & members of G accept ⟨BB, OB⟩ as a normative system & A and B share ⟨BB, OB⟩].

(S4.1.c) [Individually shared BO-system] A and B individually share ⟨BB, OB⟩ ⇔ [A and B share ⟨BB, OB⟩ & A and B do not socially share ⟨BB, OB⟩].

In many social organizations, certain roles are assigned to their members, and they perform actions within their role-specific normative spaces to achieve a shared goal or an individual goal. In such cases, socially shared normative systems construct bases for role-specific normative spaces (see (S3.7.a)-(S3.7.d) and Appendix (Ap.6)). These socially shared normative systems are not originally founded by isolated individuals. Contrarily, individuals acquire them as already socially accepted systems. The reality of a social organization is founded on this socially shared normative systems and role-specific normative spaces that motivate or restrict actions of their members.

There are many interconnected normative systems in a society [11]. There are also fundamental normative systems, such as a monetary system, that support many social activities. For example, economic activities are only possible in a society where a monetary system is accepted and functioning. This monetary system is a normative system that can be described as $\langle BB_{ms}, OB_{ms}\rangle$. Many mature agents in this society have BO-systems, which include $\langle BB_{ms}, OB_{ms}\rangle$. Most members of the society accept this monetary system, restrict their desires by it, and make their decision within the normative spaces determined by it. The civil law and the corporation law contain many texts that are related to a monetary system. These laws in a country should be consistent as normative systems, where the consistency of normative systems is defined as follows.

(S4.2) Normative systems $\langle BB_1, OB_1\rangle, \ldots, \langle BB_k, OB_k\rangle$ are consistent ⇔ $\langle BB_{all}, OB_{all}\rangle$ is consistent in sense of (Ap.1.a) in Appendix, where BB_{all} is the union of $BB_1, \ldots, BB_k$ and OB_{all} is the union of $OB_1, \ldots, OB_k$.

An agent might be simultaneously associated with different roles in different organizations. In such a case, she has many different role-specific normative spaces. In other words, there are at least two groups G_1, G_2, and normative systems $\langle BB_{NS1}, OB_{NS1}\rangle$, $\langle BB_{NS2}, OB_{NS2}\rangle$ such that A has, in time t, two different roll-specific obligation spaces, namely Obligation-Space*(A, $role_1$, G_1, t, $\langle BB_{NS1}, OB_{NS1}\rangle$) and Obligation-Space*(A, $role_2$, G_2, t, $\langle BB_{NS2}, OB_{NS2}\rangle$). For example, the President of Liberal Democratic Party (LDP) in Japan is often the same person with the Prime Minister in Japan. Such a person A simultaneously has two different role-specific normative spaces, namely Obligation-Space*(A, president, LDP in Japan, t, $\langle BB_{NS(LDP)}, OB_{NS(LDP)}\rangle$) and Obligation-Space*(A, prime minister, Japan, t, $\langle BB_{Japanese(Laws)}, OB_{Japanese(Laws)}\rangle$).

Many social actions can be interpreted as moves in a game because they presuppose certain normative systems. When these actions constitute parts of a comprehensive action that has a start stage and an end stage, they can be interpreted as moves in a game in sense of (S3.1.a) and (S3.1.b). A construction of a building, an execution of a project, and so on are such comprehensive actions that can be interpreted as team games that involve division of labor.

Social fact and division of labor are two kye concepts of sociologist Émil Durkheim [23]. I have already given a characterization of social facts in (S3.6.g). Here, I characterize notion of division of labor. The division of labor is used for construction of a social organization. For example, a university is divided into many departments through division of labor. Role-specific normative spaces play an important role in the division of labor. As an example, let us describe the structure of the Cabinet of Japan. The Cabinet of Japan consists of a Prime Minister and up to nineteen Ministers of States due to the Constitution of Japan. Under the Constitution, the Prime Minister has distinguished powers. For example, the Prime Minister can decide the dissolution of the House of Representatives. This power can be expressed as follows: tr(A decides the dissolution of the House of Representatives in t) belongs to Permission-Space*(A, prime minister, Japan, t, $\langle BB_{constitution(Japan)}, OB_{constitution(Japan)} \rangle$), and for any person B who is not the Prime Minister, tr(B decides the dissolution of the House of Representatives in t) belongs to Prohibition-Space*(B, not(prime minister), humans, t, $\langle BB_{constitution(Japan)}, OB_{constitution(Japan)} \rangle$). Naturally, there are also many obligations that are only imposed on the Prime Minister. The sentences that express these obligations belong to Obligation-Space*(A, prime minister, Japan, t, $\langle BB_{constitution(Japan)}, OB_{constitution(Japan)} \rangle$).

What does it mean that a social norm is accepted in a society? Some philosophers, like David Lewis, Bratman, and Raimo Tuomela, require mutual beliefs among members for collective acceptance and agreement [19, 20, 24]. However, it is often criticized that this requirement is too strong [3, 21], and I agree with this criticism. Thus, in analog to acceptance strategy for normative systems of games (see (S3.8.a) and (S3.8.b) in Subsection 3.3), I propose the following slightly weekend acceptance strategy for social normative systems.

(S4.3.a) At the start, you assume that most of members in the organization respect the social normative system.

(S4.3.b) You respect the normative system so long as you do not find many violations of it by other members.

A social normative system loses its effectivity in a social organization when many members of the organization perform actions that it prohibits. They violate the social normative system and choose actions that lie outside of its permission space. This sometimes happens in a social crisis such as in a financial panic or in a political revolution. The acceptance strategy explains why this kind of social collapses is possible. In a social crisis, many members of the society prefer their own life maintenance to sustainability of the society. If many members of the society follow a normative system based on the acceptance strategy (S4.3.b), then they will cease to respect it when they see that many members violate it.

5. Conclusions

In this chapter, I introduced a formal framework, Dynamic Belief-Desire-Obligation Logic (Dynamic BDO-Logic), and I demonstrated how to describe two-player games and team games within this framework. Team games provide concrete examples for study of agent's individual and collective actions under acceptance of a normative system.

Many social activities of agents can be interpreted as moves in a game. The social reality is created by agents who plan their actions based on social normative systems and perform them. The structure of the society is constituted by acceptance of roles.

An agent in a social organization accepts her role in it and performs actions within role-specific normative spaces. These kinds of activities of agents support the persistence of a society.

A human is born in a society that is already structured by activities of its current members and forerunners. She can choose her future actions, but her choice is always restricted by existing social normative systems. It will be an illusion if one tries to construct a society solely from an agreement among totally free isolated agents.

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Appendix

In this appendix, I precisely describe the definitions and the theorems mentioned in the main text.

(Ap.1) Definition and Characterization of BDO-Logic

We use not, &, or, $\Rightarrow$, and $\Leftrightarrow$ as meta-language expressions of logical connectives. Let Cn(X) be an abbreviation for the deductive closure of X, where X is a set of FO-sentences. Thus, Cn(X) contains every FO-sentence that can be logically deduced from Cn(X). Furthermore, for set X of FO-sentences, we define the consistency of X as follows: consistent(X) $\Leftrightarrow$ there is no FO-sentence φ such that $\varphi \in Cn(X)$ and $\neg\varphi \in Cn(X)$. Here, we assume that each of BB, DB, and OB is a set of FO-sentences.

a. [BO-system] Pair $\langle BB, OB \rangle$ is a BO-system $\Leftrightarrow$ consistent($BB \cup OB$).

b. [BD-system] Pair $\langle BB, DB \rangle$ is a BD-system $\Leftrightarrow$ consistent($BB \cup DB$).

c. [BDO-system] Triple $\langle BB, DB, OB \rangle$ is a BDO-system $\Leftrightarrow$ [$\langle BB, DB \rangle$ is a BD-system & $\langle BB, OB \rangle$ is a BO-system].

d. In the following definitions, we assume that agent A has BDO-system BDO_A, where $BDO_A = \langle BB_A, DB_A, OB_A \rangle$. A sentence in BDO-Logic is recursively defined as follows.

   (d1) If φ is a FO-sentence and BDO_A is a BDO-system, then each of $B_A\,\varphi$, $M_A\,\varphi$, $O_A\,\varphi$, $F_A\,\varphi$, $P_A\,\varphi$, and $D_A\,\varphi$ is a sentence in BDO-Logic.
   (d2) If each of Φ and Ψ is a sentence in BDO-Logic, then each of not Φ, Φ & Ψ, Φ or Ψ, $\Phi \Rightarrow \Psi$, and $\Phi \Leftrightarrow \Psi$ is a sentence in BDO-Logic.
   (d3) Every sentence in BDO-Logic satisfies (Ap.1.d1) or (Ap.1.d2).

e. Now, we define operators B_A, M_A, O_A, F_A, P_A, and D_A as abbreviations in meta-language.

   (e1) [Belief] $B_A\varphi \Leftrightarrow \varphi \in Cn(BB_A)$.
   (e2) [Possibility] $M_A\varphi \Leftrightarrow$ consistent($BB_A \cup \{\varphi\}$).
   (e3) [Obligation] $O_A\varphi \Leftrightarrow [\varphi \in Cn(BB_A \cup OB_A)$ & not $(\varphi \in Cn(BB_A))]$.

(e4) [Prohibition] $F_A\varphi \Leftrightarrow O_A\neg\,\varphi$.
(e5) [Permission] $P_A\varphi \Leftrightarrow$ [consistent($BB_A \cup OB_A \cup \{\varphi\}$)& not ($\varphi \in Cn(BB_A)$)].
(e6) [Desire] $D_A\varphi \Leftrightarrow$ [$\varphi \in Cn(BB_A \cup DB_A)$ & not ($\varphi \in Cn(BB_A)$)].

f. [Truth] Sentence Φ in BDO-Logic is true in $BDO_A \Leftrightarrow \Phi$ follows from (Ap.1.e1)-(Ap.1.e6).

g. [Validity] Sentence Φ in BDO-Logic is valid $\Leftrightarrow \Phi$ is true in all BDO-systems.

h. [Theorem] There are many sentences in BDO-Logic that are valid [11]. For example, $(O_A(\varphi \to \psi)\ \&\ B_A\,\varphi) \Rightarrow O_A\,\psi$ is a valid sentence in BDO-Logic. The proof is straightforward. We assume $O_A(\varphi \to \psi)\ \&\ B_A\,\varphi$. Then, because of (Ap.1.e1) and (Ap.1.e3), $\psi \in Cn(BB_A \cup OB_A)$. If $\psi \in Cn(BB_A)$, then $(\varphi \to \psi) \in Cn(BB_A)$, which contradicts to $O_A(\varphi \to \psi)$. Thus, not ($\psi \in Cn(BB_A)$). Then, because of (Ap.1.e3), $O_A\,\psi$. Q.E.D.

i. An agent chooses an action type from a set of action sentences. In BDO-Logic, there are four kinds of such sets of action sentences. Here, we assume that $\varphi_{act}(A)$ is a FO-sentence that expresses that A performs certain action.

(i1) Desire-Space(A, BDO_A) = $\{\varphi_{act}(A)\colon D_A\,\varphi_{act}(A)\}$.
(i2) Obligation-Space(A, BDO_A) = $\{\varphi_{act}(A)\colon O_A\,\varphi_{act}(A)\}$.
(i3) Prohibition-Space(A, BDO_A) = $\{\varphi_{act}(A)\colon F_A\,\varphi_{act}(A)\}$.
(i4) Permission-Space(A, BDO_A) = $\{\varphi_{act}(A)\colon P_A\,\varphi_{act}(A)\ \}$.

(Ap.2) Dynamic Belief-Desire-Obligation Logic (Dynamic BDO-Logic)

We can update a BDO-system ⟨BB, DB, OB⟩ by updating BB or DB or OB. We call the framework that allows this kind of updates Dynamic BDO-Logic. A BDO-system in Dynamic BDO-Logic contains information about its stage. We write a BDO-system of Dynamic BDO-Logic as follows: BDO(k) = ⟨BB(k), DB(k), OB(k)⟩.

(Ap.3) Formalization of Games (Example: Tic-Tac-Toe)

Here, by using example of Tic-Tac-Toe, I demonstrate how Dynamic BDO-Logic can be applied to descriptions of game processes. The normative system for Tic-Tac-Toe is a BO-system $\langle BB_{ttt}, OB_{ttt}\rangle$. There are two types of propositions, namely action tapes and state types:

Action type: placing(X, mark(X), s, k).
State type: occupied(s, m, k), turn(X, s), vacant(s, k), opponent(X, Y), end(k), won(X, k), play(X, g).

Here, X and Y are used as variables of players, s, s_1, … are used as variables of positions, m is used as a variable of marks, k and n are used as variables of game stages, and g is a variable for games. For the sake of readability, I use Many-Sorted Logic instead of FO-Logic. However, all formulas of Many-sorted Logic can be translated into formulas of FO-Logic. I use a function mark(X), where there are two values for this function, namely O and $\times$.

a. Belief base BB_{ttt} consists of the following elementary theory for Tic-Tac-Toe: BB_{ttt} = {(ET1), (ET2), (ET3), (ET4), (ET5), (ET6)}.

(ET1) There are some trivial stipulations that can be expressed in FO-Logic. For the sake of understandability, we verbally summarize these conditions as follows: (1) There are exactly two players who are opponents each other. (2) We use O and × as two marks of players (∀X (mark(X) = O ∨ mark(X) = ×). (3) If one player wins in Tic-Tac-Toe, then her opponent loses. (4) When it is turn of a player, it is not turn of her opponent.
(ET2) [Persistence] ∀k ∀n ∀s ∀m (occupied(s, m, k) ∧ k ≤ n → occupied(s, m, n)).
(ET3) ∀k ∀s (vacant(s, k)↔ ¬ ∃ m occupied(s, m, k)).
(ET4) The definition of victory in Tic-Tac-Toe can be given in FO-Logic. However, for the sake of understanderbility, we verbally express the condition for victory: The player who succeeds in placing three of their marks in a horizontal, vertical, or diagonal row wins the game.
(ET5) ∀k (end(k) ↔ (∃X won(X, k) ∨¬ ∃ s vacant(s, k)).
(ET6) [Effect] ∀k ∀X ∀Y ∀s ((placing(X, mark(X), s, k) ∧ opponent(X, Y)) → (occupied(s, mark(X), k + 1) ∧ turn(Y, k + 1))).

b. Obligation base OB_{ttt} is defined as follows: OB_{ttt} = {(OB1), (OB2), (OB3), (OB4)}.

(OB1) ∀X ∀k (¬ end(k) ∧ turn(X, k) → $\exists^{=1}$s (placing(X, mark(X), s, k) ∧ vacant (s, k))).
(OB2) ∀X ∀k ¬∃ s (placing(X, mark(X), s, k) ∧¬ vacant(s, k)).
(OB3) ∀X ∀k (¬ turn(X k) → ¬ ∃ s placing(X, mark(X), s, k)).
(OB4) ∀X ∀k (end(k) → ¬ ∃ s placing(X, mark(X), s, k)).

c. The normative system for Tic-Tac-Toe is $\langle BB_{ttt}, OB_{ttt}\rangle$.

(Ap.4) By using Dynamic BDO-Logic, game processes of Tic-Tac-Toe can be described. Here, we describe game processes of Tic-Tac-Toe in general.

a. [Abbreviation 1] PLACING(X, mark(X), {s_1, ..., s_n}, k) = {placing(X, mark(X), s_1, k), ..., placing(X, mark(X), s_n, k)}.

b. Initial-state = {∀s vacant(s, 0), List(o) = {[1,1], [1,2], [1,3], [2,1], [2,2], [2,3], [3,1], [3,2], [3,3]}, mark(X) = O, mark(Y) = ×, turn(X, 0)}.

c. $BB_X(o) = BB_{ttt}\cup$Initial-state & DB_X = {play(X, Tic-Tac-Toe), ∃k won(X, k)} for X∈{A, B}.

d. [BDO-systems] $BDO_X(k) = \langle BB_X(k), DB_X(k), OB_{ttt}\rangle$ & $BDO_{X+Y}(k) = \langle BB_X(k)\cap BB_Y(k), DB_X(k)\cap DB_Y(k), OB_X\cap OB_Y\rangle$. Thus, it holds: $OB_X\cap OB_Y = OB_{ttt}$.

e. Obligation space and permission space.

(e1) Obligation-Space(X, $BDO_{X+Y}(k)$) = {$\exists^{=1}$s (placing(X, mark(X), s, k) ∧ vacant(s, k)): $B_{X(k)}$ (¬end(k) ∧ turn(X, k))}. This can be proved based on (Ap.1.i2) and (OB1).
(e2) Obligation-Space(X, $BDO_{X+Y}(k)$) = {¬∃s (placing(X, mark(X), s, k): $B_{X(k)}$¬ turn(X, k)}. This can be proved based on (Ap.1.i2) and (OB3).

(e3) Permission-Space(X, BDO_{X+Y}(k)) = {placing(X, mark(X), s, k): $P_{X(k)}$ placing(X, mark(X), s, k)}.

f. Schema for dynamic development of Tic-Tac-Toe

(f1) [Action in a continuing stage] In case: $B_{X+Y(k)}\neg$ end(k) & $B_{X+Y(k)}$ (turn(X, k) ∧ opponent(X, Y)). From (Ap.4.e) follows: Obligation-Space(X, k) = {$\exists^{=1}$s (placing(X, mark(X), s, k) ∧ vacant(s, k))} & Obligation-Space(Y, k) = {¬∃s (placing(Y, mark(Y), s, k)} & Permission-Space(X, k) = PLACING(X, mark(X), List(k), k) & Permission-Space(Y, k) = Ø. We assume here that X desires to place mark(X) in position s_1. Thus, DB_X(k) = DB_X(k - 2) ∪ {placing(X, mark(X), s_1, k)}. As special cases, we define: $DB_A(-2) = DB_A$ & $DB_B(-1) = DB_B$. Then, X decides to perform this action and both X and Y confirm that this action is performed. Thus, we set: BB_{x+Y}(k + 1) = BB_{X+Y}(k) ∪ {placing(X, mark(X), s_1, k)} & List(k + 1) = List(k) – {s_1}. Then, based on (ET6), we can infer: $B_{X+Y(k+1)}$ (occupied(s_1, mark(X), k) ∧ turn(Y, k + 1)).
(f2) [Action in the terminal stage] In case: $B_{X+Y(k)}$ end(k). The game ends here. Because of (OB4), it holds: Permission-Space(X, k) = Ø & Permission-Space(Y, k) = Ø.

g. [Translation into Propositional Logic] Each FO-sentence in BBttt and in OBttt is reducible to a propositional sentence, because the ranges of all variables in them are finite. For example, ∀x φ(x) can be translated into $\varphi(a_1) \wedge \ldots \wedge \varphi(a_k)$ and ∃x φ(x) can be translared into $\varphi(a_1) \vee \ldots \vee \varphi(a_k)$, where $a_1, \ldots, a_k$ are names for objects whose set is a range of variable x.

(Ap.5) [Description of game developments] The following sequence of formulas describes the game development discussed in Subsection 3.1: ⟨placing(A, O, [1,1], 0), placing(B, ×, [2,2], 1), placing(A, O, [3,3], 2), placing(B, ×, [1,3], 3), placing (A, O, [3,1], 4), placing(B, ×, [3,2], 5), placing(A, O, [2,1], 6)⟩. I call this sequence of actions "the record of a game". In general, a game of Tic-Tac-Toe can be described by a structure in form: ⟨⟨BB_G(0), OB_G⟩, ⟨BB_G(1), OB_G⟩, ... , ⟨BB_G(k), OB_G⟩⟩. I call this structure "the game history". In general, this game history can be constructed for any game that is describable by Dynamic BDO-Logic. For example, a game history can be constructed for a baseball game and a soccer game.

(Ap.6) [Role-specific normative spaces] For any agent A in group G, role-specific obligation, prohibition, and permission space are characterized as follows (here, we assume that φ_{act}(A) is a sentence, which expresses that A performs certain action).

a. Obligation-Space*(A, $role_1$, G, t, ⟨BB_{NS}, OB_{NS}⟩) = {φ_{act}(A): $O_{NS}\forall$ x ($role_1$(x, G, t) → φ_{act}(x)) & $B_{A(t)}$ $role_1$(A, G, t) & $O_{A(t)}$ φ_{act}(A)}.

b. Prohibition-Space*(A, not($role_1$), G, t, ⟨BB_{NS}, OB_{NS}⟩) = {φ_{act}(A): $F_{NS}\exists$ x (¬$role_1$(x, G, t)∧ φ_{act}(x)) & $B_{A(t)}\neg$ $role_1$(A, G, t) & $F_{A(t)}$ φ_{act}(A)}.

c. Permission-Space*(A, $role_1$, G, t, ⟨BB_{NS}, OB_{NS}⟩) = {φ_{act}(A): $P_{NS}\forall$ x ($role_1$(x, G, t) → φ_{act}(x)) & $F_{NS}\exists$ x (¬ $role_1$(x, G, t)∧ φ_{act}(x)) & $B_{A(t)}$ $role_1$(A, G, t) & $P_{A(t)}$ φ_{act}(A)}.

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