Chapter

Perspective Chapter: Viscoelastic Mechanical Equivalent Models

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Abstract

Today, we are living in a polymeric era where thousands of daily used products are manufactured from some polymeric materials with different tasks and under a wide range of ambient conditions, including time duration of loading and working condition temperature. This leads to focusing light spot on behavior of such specific materials and investigating the strain associated with the applied stress to understand both of creep and stress relaxation behavior of the loaded polymeric components. Hence, this chapter deals with the estimation of induced strain allied with the applied force on a polymeric material via establishing the so-called mechanical equivalent models starting from the simple elastic element (spring with a modulus of elasticity E), simple viscous element (damper or dashpot with fluid viscosity η), Maxwell model, Voigt model, modified Maxwell model, modified Voigt model, and Maxwell-Voigt model. The theoretical analysis was built on derivation of the prompted deformation, as a function of time in each of the employed models, as a result of the applied external load (force) and then by depending on Hook's law transforming the gained expressions into stress (σ) and strain (ε) notation, followed by comparing the obtained equation with the general formula of the Hook's law to find exact values of the constant and as coefficients of the stress and strain. Final theoretical analysis showed that Maxwell's modified model was the best describing behavior of a loaded polymeric material to some extent followed by the other models.

Keywords: polymers, mechanical equivalent models, Maxwell model, Voigt model, creep, stress relaxation

1. Introduction

Rheology is a branch of physical sciences concerned in the wide sense with the deformation and flow of materials. Whereas, theoretical rheology aims to establish the general laws of rising and development in time of deformation and investigate the general properties of processes on a strictly mathematical basis. Applied rheology establishes a bridge between the theoretical results and practical applications by introducing certain additional simplifying assumptions. The rheological properties of real materials are determined qualitatively and quantitatively by experimental rheology, which supplies the theory with new ideas and constitutes an ultimate basis of its verification [1]. Observation of the physical facts and their superficial description, without looking for deeper causes of observed phenomena, is essential to formulate a phenomenal approach in rheology. However, the final of rheology, as a science, is to establish the relation of physical causes given conditions of deformation and flow, to the known properties of the constituent particles of materials aspects of rheology based on the nature of inter-atomic forces and the structure of the matter. The concerns of rheology, with problems of flow and deformation of materials, its ranges of interest are conventional, thus any kind of physical effect, which form their definition is instantaneous (time-dependent) are considered from a rheological point of view as particular or limiting cases. Especially, this is concerned with elastic and plastic types of deformation. The classical theory of elasticity is founded on a linear dependence between stress and strain and its time independence. Thus, when a loaded elastic body exhibits an instantaneous response to the applied stress and if the physical causes are removed, then the strain is fully recoverable. The assumption about the smaller elastic strain allows us to apply the superposition principle for both mechanical variables, the stress and the strain [2]. However, in investigating the mechanical behavior of different conditions, behave themselves in accordance with the assumptions stated for elastic body. For example, when applying constant stress for an extended time intervals, the resulting strain increases in time. On the other hand, constant strain find a time-dependent decrease in stress. Thus, it is found that mechanical properties of certain groups of materials are variable with time [3]. Moreover, it states that there is no one-to-one correspondence between stress and strain as for an elastic body and at an arbitrary time-instant. The mechanical variables depend on the past history of straining and stressing, respectively.

2. Creep and stress relaxation

In particular, the viscoelastic materials have the ability to increase their deformation in time by constant stress is called creep process, and the property of stress drop-in time by a constant strain is the stress relaxation process, see **Figure 1.**

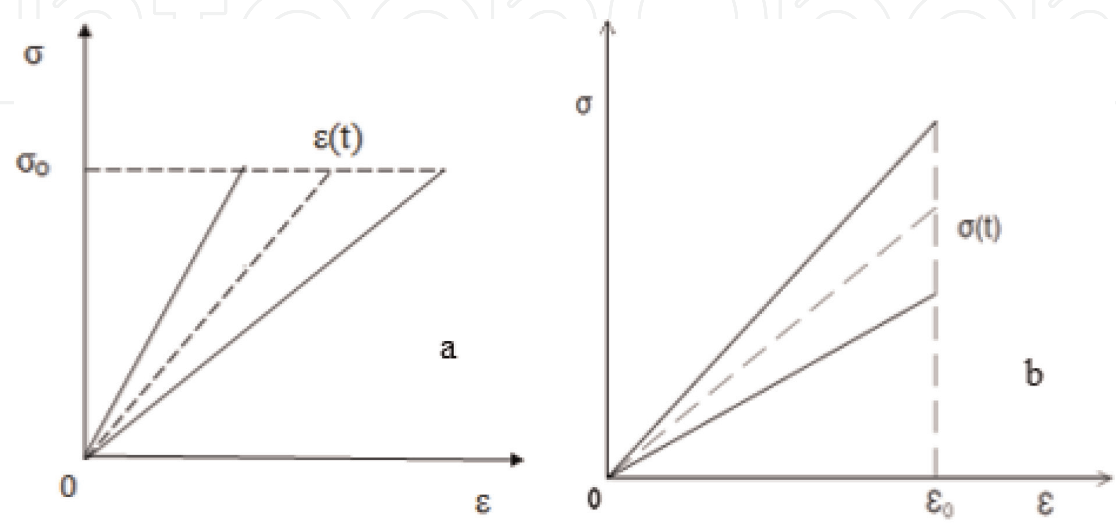


Figure 1.
(a) Creep and (b) stress relaxation mechanical behavior.

Both of the above definitions are used in a narrower sense rather to specify a two-time dependent physical function that describes the characteristic features of viscoelastic behavior relaxation and creep. In general, both phenomena occur at any variable stress and strain even simultaneously. In describing the rheological phenomena by means of mathematical formulation, we usually follow some general principles which are in accordance with our physical experience, on the other hand for particular materials we make some constitutive assumptions on an experimental basis [4]. What are restrictive conditions on the possible rheological processes? The physical relations obtained in such a way are so-called constitutive equations or equations of state in more or less general way stress and strain as the physical causes and the physical effort respectively. The aim of deriving the constitutive equations is to characterize and classify as adequately as possible the real material properties known from the experimental data. The constitutive equation must be in general in agreement with the two fundamental principles which secure its invariance. The first one is the principle of objectively material properties. It simply states that the material properties are objective and cannot be dependent on the observer and lies point of view, no matter how lies position is. Thus, according to the principle, if it is found that certain rheological processes are described by a constitutive equation, then every process equivalent to it is compatible with the same constitutive equation two equivalence of processes are stated on the basis of the transformation relation of space and time [5].

In order to determine the relation of stress to a rheological process, we state in agreement with physical experience the local character of stress. Thus, the stress at a material particle depends on what happens only in an arbitrary small vicinity of the particle. The distant parts of the body do not have a direct influence on the value of stress at the particle considered. Further, we can use causality principle. It expresses the fact that any physical process, at an arbitrary time-instant, may depend on what occurred in all past instants, that is, on the past history of happenings only. These two principles give rise to the concept of determinism for stress. According to this principle, the stress at a material particle in an arbitrary time is determined by the past history of the rheological process in an arbitrary small vicinity of the particle. In certain cases, we also assume some restrictive conditions on the possible motion during a rheological process. These conditions are constitutive restraints connected with the general features of geometry of the possible motion, for example the assumption of in compressibility of a media.

The condition of incompressibility implies that every possible motion is isochoric, that is, the deformational motion of rheology of a body occurs with a constancy of volume. Thus, the density of the body does not change during the process considered. Constitutive equations are presented in different mathematical formulation for instance in the form of differential and integral equations and as functional.

In general, the differential form of constitutive equation contains strain, strain rate, stress, stress rate, and higher derivatives of both strain stress with respect to time. It is also containing some explicit functions of time [3]. In the last case, the physical properties of the material are variable with time independently of existing stress state and must be given in advance. If there are some temperature changes, the influence of which should be taken into account in the constitutive equation may appear explicitly, temperature as a new variable [6]. Thus, in general, the differential form of a constitutive equation contains may be written in form

$$f(\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}, \dots \dots, \sigma, \dot{\sigma}, \ddot{\sigma}, \dots \dots \ \mathrm{t(time)}, T(\mathrm{temp.})) = 0 \tag{1}$$

If the temperature T does not appear in the equation, the rheological process is said to be isothermal; on the other hand, if the time-variable t disappears explicitly in the equation, then the differential equation with constant coefficients.

3. Polymers

Polymers exist in nature in such forms as wood, rubber, jute, hemp, cotton, silk, wool, hair, horn, and flesh. In addition, there are countless man-made polymeric products, such as synthetic fibers, engineering plastics, and artificial rubber. In certain aspects, the deformation of polymeric solids bears a strong resemblance to that of metals and ceramics. Polymers become increasingly deformable with increasing temperature, as witnessed by the onset of additional flow mechanisms [7]. Also, the extent of polymer deformation is found to vary with time, temperature, tress, and microstructure, constituting parallel deformations for fully crystalline solids. Furthermore, time–temperature equivalence for polymeric deformation is indicated, which is strongly reminiscent of the time–temperature parametric relations that will be discussed the basic features of the polymeric structure that dominate flow and fracture properties will be also discussed.

4. Viscoelastic response of polymers

The deformation process of many materials depends to a varying degree on both time-dependent and time-independent processes. It is known that when the test temperature is sufficiently high, a test bar would creep with time under a given load. Likewise, were the same bar to have been stretched to a certain length and then held firmly, the necessary stress to maintain the stretch would gradually relax such response is said to be viscoelastic. Since the glass and melting temperatures and most of the polymeric materials are not much above ambient (and in fact may be lower as in the case of natural rubber) these materials exhibit viscoelastic creep and stress relaxation phenomena at room temperature (25°C). When the elastic strain and viscous flow rate are small (approximately 1 up to 2% and 0.1, respectively) the viscoelastic strain may be approximately by:

$$\varepsilon = \sigma f(t) \qquad \text{Linear Viscoelasticity} \tag{2}$$

That is, (stress/strain) ratio is a function of time only. This response is called linear viscoelasticity and involves a simple addition of linear elastic and linear viscous (Newtonian) flow components [8].

When the stress–strain ratio of a material varies with time and stress, then:

$$\varepsilon = g(\sigma, t) \text{ Nonlinear Viscoelasticity} \tag{3}$$

Stress–strain ratio is a function of time only = Linear viscoelasticity.

Stress–strain ratio is a function of time and stress = Nonlinear viscoelasticity.

The viscoelastic response is nonlinear. A comparison of creep behavior between metals and polymers is clearly shown in **Table 1**.

On the basis of a simple creep test, it is possible to define a creep modulus as in the next equation:

$$E_c(t) = \frac{\sigma_o}{\varepsilon(t)} \text{ Creep modulus} \tag{4}$$

Creep behavior	Metals	Polymers
Linear elastic	No	Sometimes
Recoverable	No	Partially
Temperature range	High temperature above 0.2 T_h	All temperatures above 200°C

Table 1.
Metals and polymers creep behavior comparison.

where:
$E_c(t)$: Creep modulus as a function of time.
σ_o: Constant applied stress.
$\varepsilon(t)$: Time-dependent strain.
Now, a relaxation modulus $E_r(t)$ is defined as:

$$E_r(t) = \frac{\sigma(t)}{\varepsilon_o} \text{ Relaxation modulus} \tag{5}$$

where:
$E_r(t)$: Relaxation modulus as a function of time.
$\sigma(t)$: Time-dependent stress.
ε_o: Constant induced strain.
Both $E_c(t)$ and $E_r(t)$ moduli are varying with time as a time-dependent deformation; thus, the designer of a plastic component must look beyond the basis tensile test data when coupling the deformation response of a polymeric material. For example, for $\varepsilon_{\text{critical}}$ there is a linear relationship $\sigma = E\varepsilon_{\text{critical}}$ but this material will creep, that is, the level $\varepsilon_{\text{critical}}$ will increase with time, so to account for this additional deformation, the designer makes use of isochrounus stress–strain curves derived from creep data.

5. Mechanical models analogy

There is a strong similarity between behavior of some specific mechanical components, including (spring—elastic element and damper—viscous element) and the so-called viscoelastic materials, including polymerics, so it is better to define the basic two elements:

Spring as an elastic element, with stiffness K as indicated in **Figure 2**.

Damper is representing a viscous element with damping constant C exactly as illustrated in **Figure 3** below.

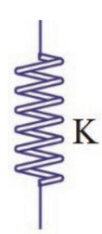


Figure 2.
Elastic element (linear spring) with stiffness constant K.

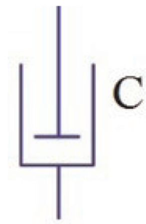


Figure 3.
Viscous element (damper) with damping coefficient C.

Based on alternative layout and assembly of the above two mentioned elements, there will be many equivalent models but the most popular equivalent mechanical models are listed below [9]:

1. Maxwell model
2. Voigt model
3. Modified Maxwell model
4. Modified Voigt model
5. Maxwell-Voigt model

6. Deformation process in polymeric solids

The following mathematical relationships are describing the mutual dependence of the main governing parameters, including the applied stress, the associated strain, and Young's modulus, in both cases of time-dependent and independent.

	Tensile test	Shear test	
Case (a)	$\varepsilon = \frac{\sigma}{E}$	$\gamma = \frac{\tau}{G}$	Time-independent behavior (6)
Case (b)	$\varepsilon = \frac{\sigma}{\eta}$	$\gamma = \frac{\tau}{\eta}$	Time-dependent behavior (7).

Where:
ε and γ: Tensile and shear strain rates.
σ and γ: Applied tensile and shear stresses.
η: Fluid viscosity in terms of stress-time.
It is better to use viscosity η instead of the damping coefficient C in the coming analysis. And the viscosity η is directly proportional to the ambient temperature T, ($\eta \propto T$) according to the Arrhenius-type relation as in the following equation:

$$\eta = \mathrm{A}e^{\Delta H \mathrm{RT}} \text{ Arrhenius equation} \tag{6}$$

Where:
ΔH: Viscous flow activation energy at a particular temperature.
T: Absolute temperature.
R: Universal gas constant.

A: Pre-exponential factor.

The viscosity depends on time, that is, at t = 0, the viscosity is extremely high, while at t goes to infinity, is small. In other words, deformation is purely viscous upon loading (t = 0) and is rigid to the dashpot consequently there is no strain associated with the same with time the viscous character of the dashpot element becomes evident as strain developed that is directly proportional to time. When the stress is removed, this strain remains. Now when the spring and dashpot are in series, as shown in **Figure 4** below, called Maxwell model, the mechanical response of the material possesses both elastic and viscous components [10], so the model is shown in the figure. Note that all the strains are recovered but the viscous strains arising from creep of the dashpot remain, since the elements are in series, the stress on each is the same and the total strain or strain rate is determined from the sum of the two components. Hence,

$$\frac{d\varepsilon}{\mathrm{dt}} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{\mathrm{dt}} \tag{7}$$

For stress relaxation conditions

$$\varepsilon = \varepsilon_o \text{ and } \frac{d\varepsilon}{\mathrm{dt}} = 0, \tag{8}$$

$$\frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{\mathrm{dt}} = 0 \tag{9}$$

$$\frac{1}{E}\frac{d\sigma}{\mathrm{dt}} = -\frac{\sigma}{\eta} \tag{10}$$

and by using the separation of variables procedure:

$$\frac{d\sigma}{\sigma} = -\frac{E}{\eta}\mathrm{dt} \tag{11}$$

$$\int_{\sigma_o}^{\sigma}\frac{d\sigma}{\sigma}\frac{d\sigma}{\sigma} = -\int_0^t \frac{E}{\eta}\mathrm{dt} \tag{12}$$

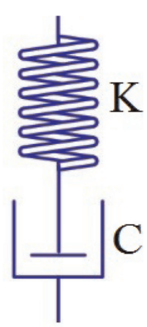


Figure 4.
Maxwell mechanical model (spring and dashpot in series).

$$\ln \sigma|_{\sigma_o}^{\sigma} = -\frac{E\,t}{\eta} \tag{13}$$

$$\ln \frac{\sigma}{\sigma_o} = -\frac{E\,t}{\eta} \tag{14}$$

$$\sigma(t) = \sigma_o\, e^{-\frac{Et}{\eta}} \tag{15}$$

$$\sigma(t) = \sigma_o\, e^{-\frac{t}{t}} \tag{16}$$

$$\varepsilon_{\text{Total}} = \varepsilon_S = \varepsilon_D \tag{17}$$

$$\sigma_{\text{Total}} = \sigma_S + \sigma_D \tag{18}$$

$$\sigma_{\text{Total}}(t) = E\varepsilon + \eta \frac{d\varepsilon}{\text{dt}} \tag{19}$$

Where t is the relaxation time defined by $\frac{\eta}{E}$.

The extent of stress relaxation for a given material will depend on the relationship between and, so when >>, the material behaves elastically such that, in other words, when the spring and dashpot elements are combined in parallel as shown in **Figure 5**. (Voigt mechanical model), this unit predicts a different time-dependent deformation response. First, the strains in the two elements are equal and the total stress on the pair is given by the sum of the two components.

For creep test $\sigma_{\text{Total}}(t) = \sigma_o$ and after intefration, yields:

$$\varepsilon(t) = \frac{\sigma_o}{E}\left(1 - e^{-tt}\right) \tag{20}$$

The strain experienced by the Voigt model is shown in **Figure 6**. The absence of any instantaneous strain is predicted from this equation $\varepsilon(t) = \frac{\sigma_o}{E}\left(1 - e^{-tt}\right)$ and is related in physical sense to the infinite stiffness of the dashpot at t = 0. The creep strain seems to rise quickly thereafter but reaches a limiting value of $\frac{\sigma_o}{E}$ associated with the full extension of the spring under that stress. Upon unloading, the spring remains extended but now exerts negative stress on the dashpot. In other manner, the viscous strains are reversed, and in the limit when both spring and dashpot are unstressed, all the strains have been reversed. Consequently, the Voigt and the Maxwell models describe different types of viscoelastic responses [8].

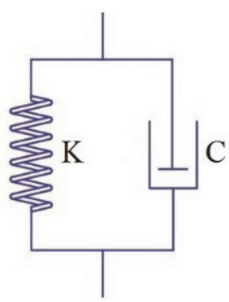


Figure 5.
Voigt mechanical model (spring and dashpot in parallel).

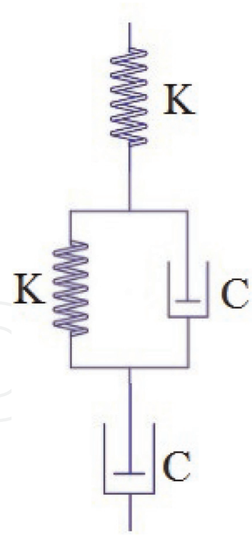


Figure 6.
Maxwell-Voigt mechanical model.

A some more realistic description of polymer behavior is obtained with a four elements model consisting of Maxwell and Voigt models in series precisely as shown in **Figure 6**.

By combining the above three last equations, it can be seen that the total strain experienced by this model may be given by:

$$\varepsilon(t) = \frac{\sigma}{E_1} + \frac{\sigma}{E_2}\left(1 - e^{-tt}\right) + \frac{\sigma}{\eta}t, \tag{21}$$

which takes account of elastic, viscoelastic, and viscous strain components, respectively.

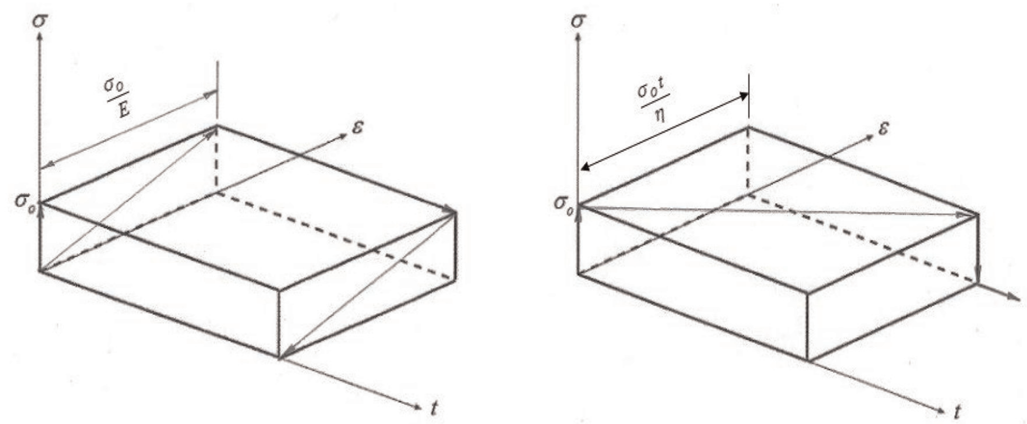
The temperature dependence of the mechanical response can be modeled by appropriate adjustment in dashpot and spring values, that is, [11], lower spring stiffness and dashpot viscosity levels for higher temperature and vice versa for lower temperature conditions. **Figure 7** illustrates in detail the induced strain as a function of the applied stress and loading time duration for the four models [11].

7. Equivalent mechanical models analysis

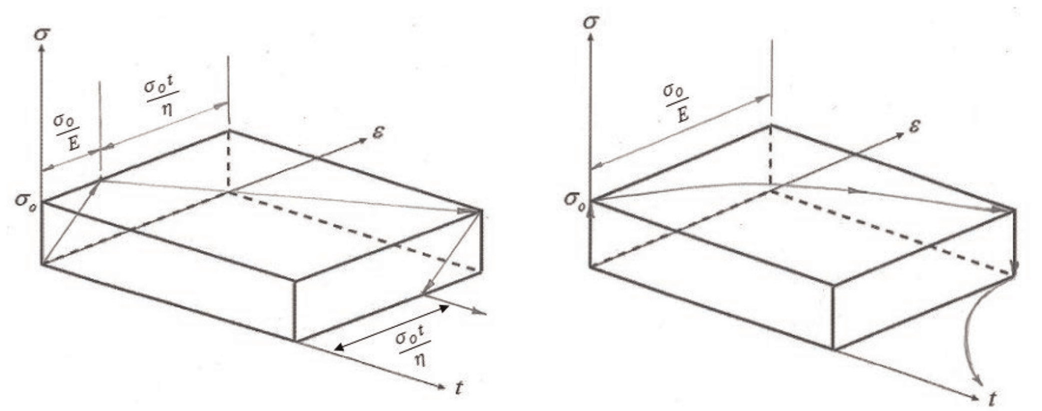
Simply, each mechanical component under direct stress, such as tensile or compressive stresses, will exhibit either elastic (temporary), Plastic (permanent), or recoverable (viscoelastic) deformation, so the following paragraphs will shed spotlight on the analogy (similarity) between behavior of the five equivalent mechanical models and an actual case of a polymeric (viscoelastic) component under direct load as illustrated below, so let us start with first one:

7.1 Maxwell model

Figure 8 shows a schematic representation for Maxwell model which contains two elements spring (E and Young's modulus) and dashpot (fluid viscosity in terms of stress- time) in series under effect of external force F, applied in two inline opposite ends, and for analysis, there are three identifying points A, B, and C.



(a) Simple Spring Behaviour **(b)** Simple Dashpot Behaviour



(c) Maxwell Model Behaviour **(d)** Voigt Model Behaviour

Figure 7.
Stress-strain time diagram for mechanical analogs.

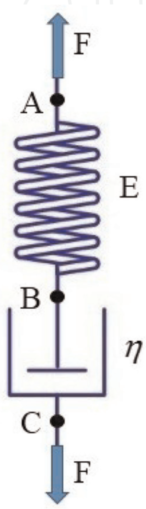


Figure 8.
Simple Maxwell model.

Hence, the required parameter is the total deformation or extension δ that occurs in the whole length of this model, so

$$\delta = \delta_{\mathrm{AB}} + \delta_{\mathrm{BC}} \tag{22}$$

Where:

F: Applied force to a linear spring and a dashpot in series to form a Maxwell model.

E: Modulus of the spring $(K = \mathrm{EL})$.

η: Dynamic viscosity of the dashpot

δ: Extension.

Now, by differentiating the above equation with respect to time, using D-operator method yields:

$$D\delta = D\delta_{\mathrm{AB}} + D\delta_{\mathrm{BC}} \tag{23}$$

For springs, Hooks law applies as follows:

$$F = K\delta_{\mathrm{AB}} \tag{24}$$

$$F = \mathrm{EL}\delta_{\mathrm{AB}} \tag{25}$$

$$\mathrm{DF} = \mathrm{EL}\, D\delta_{\mathrm{AB}} \tag{26}$$

$$D\delta_{\mathrm{AB}} = \frac{\mathrm{DF}}{\mathrm{EL}} \tag{27}$$

For viscous element or dashpot, Newton's law of viscosity applies as follows:

$$F = \eta\, L\, D\delta_{\mathrm{BC}} \tag{28}$$

By employing D-operator method, yields:

$$D\delta = \frac{\mathrm{DF}}{\mathrm{EL}} + \frac{F}{\eta L} \tag{29}$$

If (A) denotes some characteristic cross-sectional dimension, where the force F is applied and L_o is the original length of the Maxwell unit, then:

$$\sigma = \frac{F}{A} \tag{30}$$

$$\sigma = \frac{F}{L^2} \tag{31}$$

and

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{\delta}}{\boldsymbol{L_o}} \tag{32}$$

$$\varepsilon \simeq \frac{\delta}{L} \tag{33}$$

$$\sigma + \left(\frac{\eta}{E}\right)\sigma = \eta\varepsilon \tag{34}$$

$$\left[\varepsilon L = \frac{\sigma\, L^2}{\mathrm{EL}} + \frac{\sigma L^2}{\eta L}\right] * \frac{\eta}{L} \tag{35}$$

$$F = \sigma\, L^2 \Rightarrow \mathrm{DF} = \sigma L^2 \tag{36}$$

$$\delta = \varepsilon\, L \Rightarrow D\delta = \varepsilon L \tag{37}$$

$$\eta\varepsilon = \frac{\eta}{E}\sigma + \sigma \tag{38}$$

Here, the dot () notation has been used in place of the D-operator. A comparison of the above first and last equations shows that the latter expression for Hook's law.

Where:

$a_o = 1$, $a_1 = \frac{\eta}{E}$, $b_o = 0$, $b_1 = \eta$

and the remaining constants are zeros, so

$$\left[a_o + a_1\frac{\partial}{\partial t} + \dots \dots\right]\sigma = \left[b_o + b_1\frac{\partial}{\partial t} + \dots\right]\varepsilon \tag{39}$$

$$a_o\sigma + a_1\frac{\partial\sigma}{\partial t} + \dots \dots = b_o\varepsilon + b_1\frac{\partial\varepsilon}{\partial t} + \dots \tag{40}$$

$$a_o\sigma + a_1\sigma + \dots \dots = b_o\varepsilon + b_1\varepsilon + \dots \tag{41}$$

$$1\sigma + \frac{\eta}{E}\sigma + \dots \dots = 0\varepsilon + \eta\varepsilon + \dots \tag{42}$$

now by comparing the above two equations, yields:

$$a_o = 1, a_1 = \frac{\eta}{E}, b_o = 0, b_1 = \eta$$

and the remaining constants are zeros.

7.1.1 Maxwell boundary conditions

It is noted that, for modulus of elasticity approaches infinity ($E \Rightarrow \infty$), the general equation for Maxwell model will be reduced to a simple dashpot only as indicated in the following set of equations:

$$D\delta = \frac{\mathrm{DF}}{\mathrm{EL}} + \frac{F}{\eta L} \tag{43}$$

$$D\delta = \frac{F}{\eta L} \qquad \text{for a dashpot only} \tag{44}$$

Whereas, for fluid viscosity approaches infinity ($\eta \Rightarrow \infty$), in the general equation of Maxwell model [12], this specific equation will be reduced to a simple spring only exactly as illustrated in the following mathematical equations:

$$D\delta = \frac{\mathrm{DF}}{\mathrm{EL}} + \frac{F}{\eta L} \tag{45}$$

$$D\delta = \frac{\mathrm{DF}}{\mathrm{EL}} \tag{46}$$

$$\delta = \frac{F}{\mathrm{EL}} \qquad \textbf{for a spring only} \tag{47}$$

7.2 Voigt model

This model has two elements, elastic element (spring) and viscous element (dashpot), connected in parallel, as shown in **Figure 9**.

Spring (E) and dashpot (η) in parallel, so

$$F = F_S + F_D \tag{48}$$

$$F_S = \mathrm{EL}\delta \tag{49}$$

$$F_D = \eta \mathrm{LD}\delta \tag{50}$$

where δ = total extension of either element of Voigt model, thus the above equation of the applied force will be in the following form:

$$F = \mathrm{EL}\delta + \eta \mathrm{LD}\delta \tag{51}$$

This equation is representing the characteristic equation of the Voigt model, and now, by analogy, it may be rewritten as follows:

$$F = \mathrm{EL}\delta + \eta \mathrm{LD}\delta \tag{52}$$

$$\sigma = E\varepsilon + \eta\varepsilon \tag{53}$$

$$\sigma = \frac{F}{L^2}, \quad \delta = \varepsilon\, L \quad \text{and} \quad D\delta = \varepsilon L \tag{54}$$

$$\frac{F}{L^2} = \frac{\mathrm{EL}}{L^2} * \varepsilon\, L + \frac{\eta L}{L^2} \varepsilon L \tag{55}$$

$$\sigma = E\varepsilon + \eta\varepsilon \tag{56}$$

Now, by comparing the obtained equation with the standard formula, yields:

$$a_o\sigma + a_1\sigma + \dots \dots = b_o\varepsilon + b_1\varepsilon + \dots \tag{57}$$

$$1\,\sigma + 0\sigma + \dots \dots = E\varepsilon + \eta\varepsilon + \dots \tag{58}$$

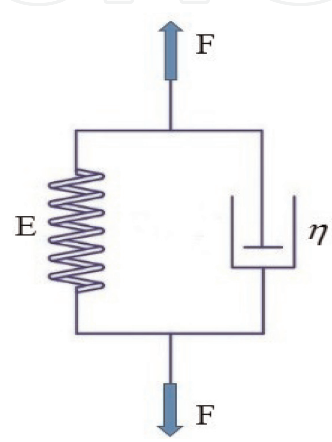


Figure 9.
Simple Voigt model.

$$a_o = 1, a_1 = 0, b_o = E, b_1 = \eta$$

and the remaining constants are zeros.

7.2.1 Voigt boundary conditions

Similarly, in Voigt model, as the modulus of elasticity approaches zero ($E \Rightarrow 0$), the governing equation will be in the following form:

$$F = \mathrm{EL}\delta + \eta\mathrm{LD}\delta \tag{59}$$

$$F = \eta\mathrm{LD}\delta \textbf{ for a dashpot only} \tag{60}$$

and on the other side if the fluid viscosity is zero ($\eta \Rightarrow 0$), then the resulting equation is:

$$F = \mathrm{EL}\delta + \eta\mathrm{LD}\delta \tag{61}$$

$$F = \mathrm{EL}\delta \textbf{ for a spring only} \tag{62}$$

In these cases, the Maxwell and Voigt models are degenerating into the simplest elements. Thus, it is required to establish a nondegenerate model which is a little more complex than the two previous models considered. The coming sections will deal with a modified model for both Maxwell and Voigt.

7.3 Modified Maxwell model

This modified model is shown in **Figure 10** below and it consists of three elements; they are:

- Elastic element (spring with modulus of elasticity)
- Elastic element (spring with modulus of elasticity)
- Viscous element (dashpot with fluid viscosity)

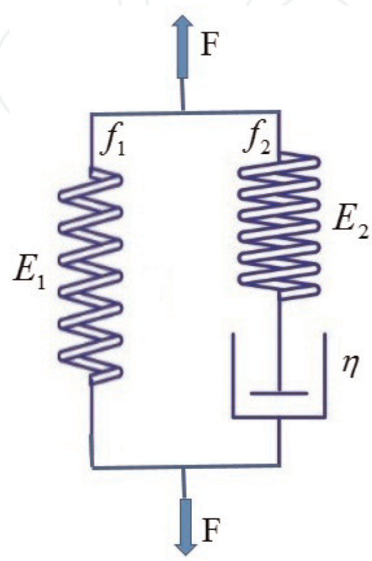


Figure 10.
Modified Maxwell model.

The applied force on the outer terminals of this model is F, but this overall force will be divided into two sub-forces f_1 and f_2.

$$F = f_1 + f_2 \tag{63}$$

$$f_1 = E_1 L\delta \text{ and } \mathrm{Df}_2 = E_1 \mathrm{LD}\delta \tag{64}$$

and in return to the general governing equation for Maxwell model, which is exactly similar to the right-hand side of the Maxwell modified model, as shown in **Figure 11.**

$$D\delta = \frac{\mathrm{DF}_2}{E_2 L} + \frac{F_2}{\eta L} \tag{65}$$

Now, it is better to eliminate both of f_1 and f_2 in order to get an analysis of the induced stresses and the associated strains as follows:

$$f_1 = E_1 L\delta \tag{66}$$

$$\mathrm{Df}_1 = E_1 \mathrm{LD}\delta \tag{67}$$

$$\mathrm{Df}_2 = E_2 \mathrm{LD}\delta - \left(\frac{E_2}{\eta}\right) f_2 \tag{68}$$

$$F = f_1 + f_2 \tag{69}$$

$$\mathrm{DF} = \mathrm{Df}_1 + \mathrm{Df}_2 \tag{70}$$

$$\mathrm{DF} = E_1 \mathrm{LD}\delta + E_2 \mathrm{LD}\delta - \left(\frac{E_2}{\eta}\right) f_2 \tag{71}$$

$$\mathrm{DF} = E_1 \mathrm{LD}\delta + E_2 \mathrm{LD}\delta - \left(\frac{E_2}{\eta}\right)(F - E_1 L\delta) \tag{72}$$

$$\mathrm{DF} = (E_1 + E_2)\mathrm{LD}\delta - \left(\frac{E_2}{\eta}\right) F + \frac{E_1 E_2 L}{\eta}\delta \tag{73}$$

$$\mathrm{DF} + \left(\frac{E_2}{\eta}\right) F = (E_1 + E_2)\mathrm{LD}\delta + \frac{E_1 E_2 L}{\eta}\delta \tag{74}$$

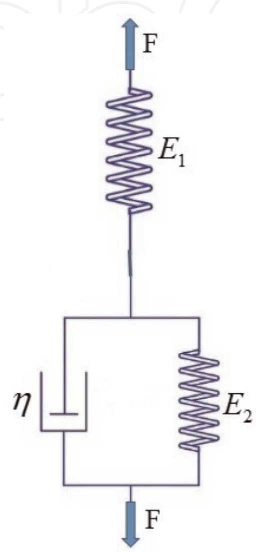


Figure 11.
Modified Voigt model.

$$\left[\mathrm{DF} + \left(\frac{E_2}{\eta}\right)F = (E_1 + E_2)L^2\varepsilon + \frac{E_1E_2L^2}{\eta}\varepsilon\right]\frac{\eta}{L^2E_2} \tag{75}$$

$$\sigma = \frac{F}{L^2} \quad \Rightarrow \sigma = \frac{\mathrm{DF}}{L^2} \tag{76}$$

$$\varepsilon = \frac{\delta}{L} \Rightarrow \quad \delta = \varepsilon\, L \quad \Rightarrow D\delta = \varepsilon\, L \tag{77}$$

$$f_2 = F - f_1 \tag{78}$$

$$f_2 = F - E_1L\delta \tag{79}$$

$$\left(\frac{\eta}{E_2}\right)\sigma + \sigma = \left(\frac{E_1 + E_2}{E_2}\right)\eta\varepsilon + E_1\varepsilon \tag{80}$$

$$\sigma + \left(\frac{\eta}{E_2}\right)\sigma = E_1\varepsilon + \left(\frac{E_1 + E_2}{E_2}\right)\eta\varepsilon \tag{81}$$

$$a_o\sigma + a_1\sigma + \ \dots\ \dots \ = b_o\varepsilon + b_1\varepsilon + \ \dots \tag{82}$$

$$1\,\sigma + \left(\frac{\eta}{E_2}\right)\sigma = E_1\varepsilon + \left(\frac{E_1 + E_2}{E_2}\right)\eta\varepsilon \tag{83}$$

By comparing the last two main equations, yields:

$$a_o = 1, a_1 = \left(\frac{\eta}{E_2}\right), b_o = E_1, b_1 = \left(\frac{E_1 + E_2}{E_2}\right)\eta \tag{84}$$

and the other constants and terms with higher orders are zeros.

The modified Maxwell model has the advantage over time period and is avoided in the separate spring element E_1, whereas it is permitted to occur in the left-hand branch of the sketch in **Figure 10**. Such limited relaxation behavior is typical of polymers and elastomers subjected to a long-duration stress environment, so that the modified Maxwell model (although still highly simplistic in terms of actual material performance) depicts the nature of viscoelastic behavior in a much more realistic manner [13].

7.4 Modified Voigt model

The full detail sketch for the modified Voigt model is shown in **Figure 11.** Where this model is consisting of the following elements:

- Elastic element (spring with modulus of elasticity E_1)
- Elastic element (spring with modulus of elasticity E_2)
- Viscous element (dashpot with fluid viscosity η).

And the external applied force is denoted by F.

Based on the given layout, the total deformation along the external terminals is δ and may be expressed in terms of the sub-deformation components as shown in the coming mathematical equation:

$$\delta = \delta_1 + \delta_2 \tag{85}$$

also, for the spring element E_1, there is:

$$F = E_1 L \delta_1 \tag{86}$$

and for simple Voigt model:

$$F = E_2 L \delta_2 + \eta \mathrm{LD} \delta_2 \tag{87}$$

Now, by eliminating δ_1 and δ_2 from the above equation, yields:

$$\delta = \delta_1 + \delta_2 \tag{88}$$

$$D\delta = D\delta_1 + D\delta_2 \tag{89}$$

$$\delta_2 = \delta + \delta_2 \tag{90}$$

$$\delta_2 = \delta + \frac{F}{E_1 L} \tag{91}$$

$$F = E_1 L \delta_1 \Rightarrow \mathrm{DF} = E_1 \mathrm{LD} \delta_1 \Rightarrow D\delta_1 = \frac{\mathrm{DF}}{E_1 L} \tag{92}$$

$$F = E_2 L \delta_2 + \eta\ \mathrm{LD} \delta_2 \tag{93}$$

$$\eta\ \mathrm{LD}\delta_2 = F - E_2 L \delta_2 \Rightarrow \quad D\delta_2 = \frac{F}{\eta L} - \frac{E_2}{\eta} \delta_2 \tag{94}$$

$$D\delta = \frac{\mathrm{DF}}{E_1 L} + \frac{F}{\eta\, L} - \frac{E_2}{\eta} \delta_2 \tag{95}$$

$$D\delta = \frac{\mathrm{DF}}{E_1 L} + \frac{F}{\eta\, L} - \frac{E_2}{\eta} \left(\delta - \frac{F}{E_1 L} \right) \tag{96}$$

$$\left[D\delta = \frac{\mathrm{DF}}{E_1 L} + \frac{F}{\eta\, L} - \frac{E_2}{\eta} \delta + \frac{E_2}{E_1 \eta L} F \right] * E_1 L \tag{97}$$

$$E_1 \mathrm{LD} \delta = \mathrm{DF} + \frac{E_1}{\eta} F - \frac{E_1 E_2 L}{\eta} \delta + \frac{E_2}{\eta} F \tag{98}$$

$$E_1 \mathrm{LD} \delta = \mathrm{DF} + \left(\frac{E_1 + E_2}{\eta} \right) F - \left(\frac{E_1 E_2 L}{\eta} \right) \delta \tag{99}$$

Now, by using separation of variables, yields:

$$\mathrm{DF} + \left(\frac{E_1 + E_2}{\eta} \right) F = E_1 \mathrm{LD} \delta + \left(\frac{E_1 E_2 L}{\eta} \right) \delta \tag{100}$$

$$\delta = \varepsilon\, L \qquad \Rightarrow D\delta = \varepsilon\, L \tag{101}$$

$$\mathrm{DF} + \left(\frac{E_1 + E_2}{\eta} \right) F = E_1 L^2 \varepsilon + \left(\frac{E_1 E_2 L^2}{\eta} \right) \varepsilon \tag{102}$$

$$\left[\mathrm{DF} + \left(\frac{E_1 + E_2}{\eta} \right) F = E_1 L^2 \varepsilon + \left(\frac{E_1 E_2 L^2}{\eta} \right) \varepsilon \right] * \frac{\eta}{(E_1 + E_2) L^2} \tag{103}$$

$$\frac{F}{L^2} + \left(\frac{\eta}{E_1 + E_2} \right) \frac{\mathrm{DF}}{L^2} = \left(\frac{E_1 E_2}{E_1 + E_2} \right) \varepsilon + \left(\frac{E_1 \eta}{E_1 + E_2} \right) \varepsilon \tag{104}$$

$$\sigma + \left(\frac{\eta}{E_1 + E_2}\right)\sigma = \left(\frac{E_1 E_2}{E_1 + E_2}\right)\varepsilon + \left(\frac{E_1 \eta}{E_1 + E_2}\right)\varepsilon \tag{105}$$

$$a_o\sigma + a_1\sigma + \dots\dots = b_o\varepsilon + b_1\varepsilon + \dots \tag{106}$$

Now, by comparing the last two equations, yields:

$$a_o = 1, a_1 = \left(\frac{\eta}{E_1 + E_2}\right), b_o = \left(\frac{E_1 E_2}{E_1 + E_2}\right), b_1 = \left(\frac{E_1\, \eta}{E_1 + E_2}\right) \tag{107}$$

And the other higher order terms are zeros.
In return to these two equations,
The first one is from the modified Maxwell model analysis:

$$\mathrm{DF} + \left(\frac{E_2}{\eta}\right)F = (E_1 + E_2)\mathrm{LD}\delta + \frac{E_1 E_2 L}{\eta}\delta \tag{108}$$

and the second one is from the modified Voigt model analysis:

$$\mathrm{DF} + \left(\frac{\mathrm{E}_1 + \mathrm{E}_2}{\eta}\right)\mathrm{F} = \mathrm{E}_1\mathrm{LD}\delta + \left(\frac{\mathrm{E}_1\mathrm{E}_2\mathrm{L}}{\eta}\right)\delta \tag{109}$$

These two last equations are having essentially the same form, although the constants vary depending on which model is finally selected, so the general form of these equations is:

$$\mathrm{DF} + P_o F = q_1\, D\delta + q_o\delta \tag{110}$$

These representations of viscoelastic behavior by either the modified Maxwell or modified Voigt model are identical. Both modified models are having two springs and one viscous element. Alternatively, it is possible to use one spring and two viscous elements, which would yield the following form of force deflection equation as shown in the following equation:

$$\mathrm{DF} + P_o F = q_2\, D^2\delta + q_1 D\delta \tag{111}$$

7.5 Maxwell-Voigt model

This specific model consists of Maxwell model connected in series with Voigt model as illustrated in **Figure 12.** Where the main components are:
Elastic element (spring with modulus of elasticity E_1)

- Elastic element (spring with modulus of elasticity E_2)
- Viscous element (dashpot with fluid viscosity η_1)
- Viscous element (dashpot with fluid viscosity η_2)

Let δ_1 and δ_2 represent the induced deformation in the Maxwell and Voigt elements, respectively, as a result of applying the external force F to both ends of the compound model [14], **Figure 12**, then:

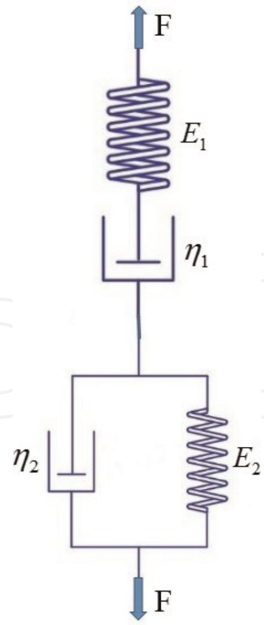


Figure 12.
Maxwell-Voigt model.

$$\delta = \delta_1 + \delta_2 \tag{112}$$

where the δ is the total elongation with length dimension. It is required to analyze this compound model by considering each model alone, as indicated in **Figure 13**.

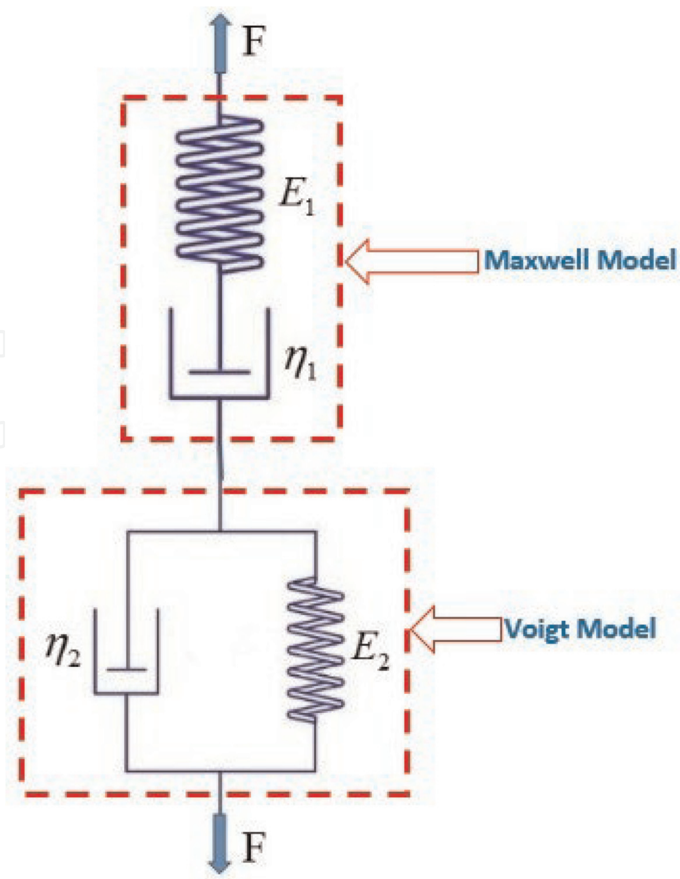


Figure 13.
Maxwell model in series with Voigt model.

Now, for Maxwell element, it is already stated that:

$$D\delta_1 = \frac{\mathrm{DF}}{E_1 L} + \frac{F}{\eta_1 L} \tag{113}$$

and for Voigt model:

$$F = F_S + F_D \tag{114}$$

which also may be written as:

$$F = E_2 L\delta_2 + \eta_2 \mathrm{LD}\delta_2 \tag{115}$$

It is possible to eliminate δ_1 and δ_2 that the final result will be in the following form:

$$D^2F + \left[\frac{E_1}{\eta_1} + \frac{E_1}{\eta_2} + \frac{E_2}{\eta_2}\right]\mathrm{DF} + \left[\frac{E_1E_2}{\eta_1\eta_2}\right]F = E_1\mathrm{LD}^2\delta + \left[\frac{E_1E_2L}{\eta_2}\right]D\delta \tag{116}$$

This essential equation may be rewritten in terms of stress and strain as follows:

$$\sigma + \left[\frac{E_1}{\eta_1} + \frac{E_1}{\eta_2} + \frac{E_2}{\eta_2}\right]\left(\frac{\eta_1\eta_2}{E_1E_2}\right)\sigma + \left(\frac{\eta_1\eta_2}{E_1E_2}\right)\overset{\dot{=}\eta_1\varepsilon+\left(\frac{\eta_1\eta_2}{E_2}\right)\dot{\varepsilon}}{\sigma} \tag{117}$$

and according to the general formula in the following equation:

$$a_o\sigma + a_1\sigma + a_2 \overset{\therefore\ \ldots = b_o\varepsilon+b_1\varepsilon+b_2\overset{+}{\ddot{\varepsilon}}\cdots}{\sigma} \tag{118}$$

$$a_o = 1, a_1 = \left(\frac{E_1}{\eta_1} + \frac{E_1}{\eta_2} + \frac{E_2}{\eta_2}\right)\left(\frac{\eta_1\eta_2}{E_1E_2}\right), a_2 = \left(\frac{\eta_1\eta_2}{E_1E_2}\right) \tag{119}$$

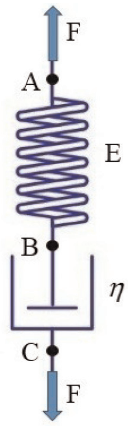
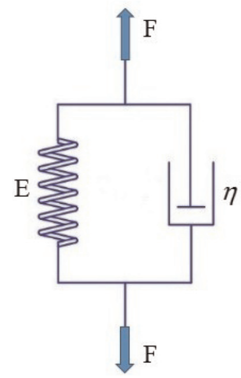
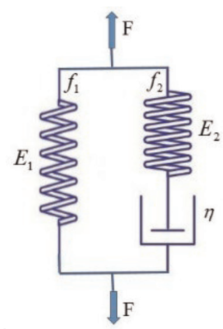
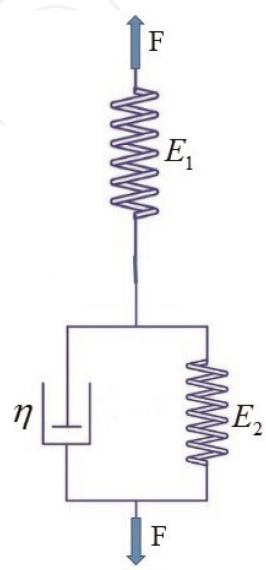
$$b_o = 0, b_1 = \eta_1, b_2 = \left(\frac{\eta_1\eta_2}{E_2}\right) \tag{120}$$

The relationships derived in the previous mechanical models are specific forms of the generalized Hook's law relating stress to strain, in accordance with the following equation [15]:

$$\left[a_o + a_1\frac{\partial}{\partial t} + a_2\frac{\partial^2}{\partial t^2} + \ \ldots .. \right]\sigma = \left[b_o + b_1\frac{\partial}{\partial t} + b_2\frac{\partial^2}{\partial t^2} + \ \ldots .. \right]\varepsilon \tag{121}$$

However, there has been no limitation on the time history of the applied force F or the induced deformation δ in other words these equations are representing the main basis of behavior of a viscoelastic material suffering from creep and stress relaxation. So to analyze any viscoelastic materials, means studying the relationship between the applied know force and the associated viscoelastic strain, it is so essential to know the numerical values of the included mechanical components (elastic element, spring and viscous element, and dashpot), so **Table 2** summarize these constants in detail based on the type of the proposed equivalent model.

By applying a constant force to previously unloaded models, the extension when measured as a function of time over a long period, is called creep motion or creep [16].

b_2	b_1	b_o	a_2	a_1	a_o	Schematic	Model
0	0	0	0	$\frac{\eta}{E}$	1		Maxwell
0	η	E	0	0	1		Voigt
0	$\left(\frac{E_1+E_2}{E_2}\right)\eta$	E_1	0	$\frac{\eta}{E_2}$	1		Modified Maxwell
0	$\left(\frac{E_1}{E_1+E_2}\right)\eta$	$\left(\frac{E_1E_2}{E_1+E_2}\right)$	0	$\frac{\eta}{E_1+E_2}$	1		Modified Voigt

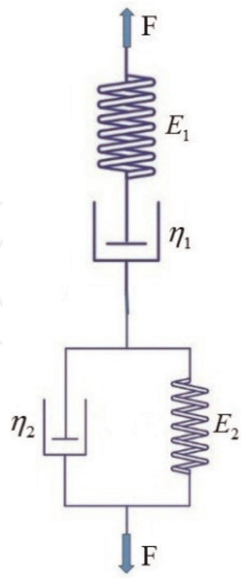
b_2	b_1	b_o	a_2	a_1	a_o	Schematic	Model
$\frac{\eta_1\eta_2}{E_2}$	η_1	0	$\left(\frac{\eta_1\eta_2}{E_1E_2}\right)$	$\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_2}\right)\left(\frac{\eta_1\eta_2}{E_1E_2}\right)$	1		Maxwell Voigt

Table 2.
Values of the constants in the generalized Hook's law for the proposed mechanical models.

Conversely, the application of a constant extension to previously unloaded models require a time-dependent force that can be measured. This force decreases in time accordingly, as a relaxation effect occurs within the model.

7.6 Creep response

Given a force, F, according to the relationship:

$$F = \mathrm{CH}(t) \tag{122}$$

where C is an arbitrary constant, and $H(t)$ is the **Heaviside function** of time and has the following values:

$$H(t) = 0 \text{ for } t \leq 0$$
$$H(t) = 1 \text{ for } t \geq 1$$

The time derivative of the function $H(t)$ is the well-known **Dirac function** (Δ), thus:

$$\Delta(t) = \mathrm{DH}(t) \tag{123}$$

Both $H(t)$ and $\Delta(t)$ are shown as a function of time, see **Figure 14**.

So, it is required to find the resulting extension $\delta(t)$ for the entire model, for a simple spring element with spring modulus E:

$$\delta(t) = \frac{F}{\mathrm{EL}} = \frac{C}{\mathrm{EL}}H(t) \tag{124}$$

And for the viscous element with viscosity η:

$$D\delta(t) = \frac{C}{\eta L}H(t) \tag{125}$$

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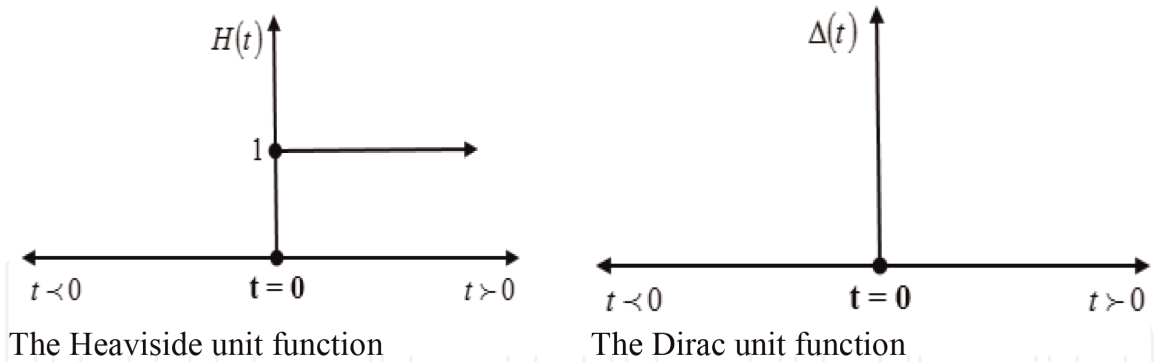

Figure 14.
Heaviside and Dirac unit functions. The Heaviside unit function. The Dirac unit function.

This differential equation can be integrated to give:

$$\delta(t) = \frac{C}{\eta\, L}\, t\, H(t) \tag{126}$$

The constant of integration in the last equation is zero since $\delta(t) = 0$ at $t = 0$. For Maxwell element, the application of the above last equations gives:

$$D\delta(t) = \frac{\mathrm{DF}}{\mathrm{EL}} + \frac{F}{\eta\, L} \tag{127}$$

$$D\delta(t) = \frac{C}{\mathrm{EL}}\Delta(t) + \frac{C}{\eta\, L}H(t) \tag{128}$$

which after integration shows the following time-dependent response:

$$\delta(t) = \frac{C}{\mathrm{EL}}H(t) + \frac{C\, t}{\eta\, L}H(t) \tag{129}$$

$$\delta(t) = \frac{C}{L}\left(\frac{1}{E} + \frac{t}{\eta}\right) H(t) \tag{130}$$

This last equation is representing the overall extension (deformation) for Maxwell model.

For Voigt model the governing equation is as follows:

$$F = \mathrm{EL}\delta + \eta \mathrm{LD}\delta \tag{131}$$

$$\mathrm{CH}(t) = \mathrm{EL}\delta(t) + \eta \mathrm{LD}\delta(t) \tag{132}$$

In order to solve the above differential equation, it is so essential to multiply each side of this specific equation by an integrating factor:

Integrating Factor $= e^{\left(\frac{E}{\eta}\right)t}$, so that:

$$\mathrm{C}e^{\left(\frac{E}{\eta}\right)t}H(t) = \mathrm{EL}\delta(t)e^{\left(\frac{E}{\eta}\right)t} + \eta \mathrm{LD}\delta(t)e^{\left(\frac{E}{\eta}\right)t} \tag{133}$$

$$\mathrm{C}e^{\left(\frac{E}{\eta}\right)t}H(t) = \eta \mathrm{LD}\left[\delta(t)e^{\left(\frac{E}{\eta}\right)t}\right] \tag{134}$$

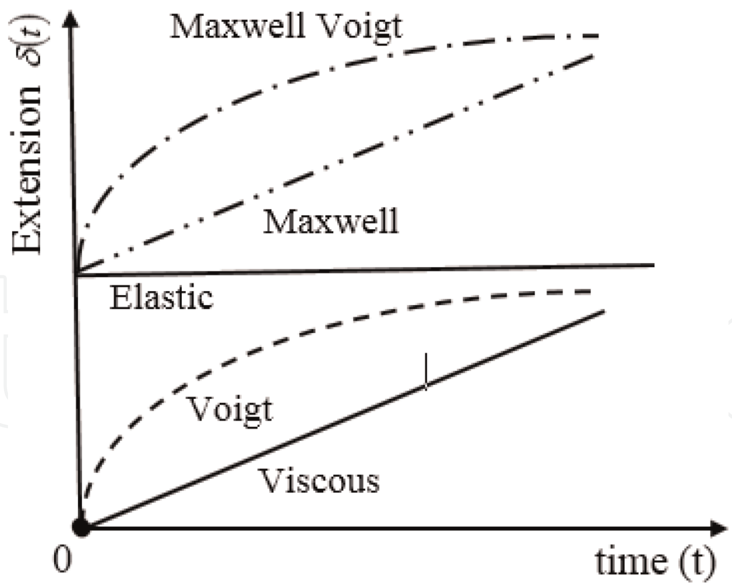

Figure 15.
Extension response (creep behavior) of the proposed mechanical models.

after integration yields:

$$\delta(t) = \frac{C}{\text{EL}}\left[1 - e^{-\left(\frac{E}{\eta}\right)t}\right]H(t) \tag{135}$$

This equation is representing the induced response of Voigt model. The extension response (creep behavior) of mechanical models, including the Maxwell-Voigt, modified Maxwell, modified Voigt, and simple elements models [17], is shown in **Figure 15**.

It is to be noted that the Maxwell element has a response equal to sum of the responses for the viscous and elastic elements, because it consists of these elements in series. Also, the Maxwell Voigt model consists of Maxwell and Voigt models in series, so that its response is given the following equation:

$$\delta(t) = C\left[\frac{1}{E_1 L} + \frac{t}{\eta L}\right]H(t) + \frac{C}{E_2 L}\left[1 - e^{-\left(\frac{E}{\eta}\right)}t\right]H(t) \tag{136}$$

8. Discussion

In most cases of nominating a material for manufacturing an industrial component, and sustaining the applied load, usually either tensile, compressive, or shear stress, it is so essential to seek the best mechanical properties, including Young's modulus (modulus of elasticity) and the associated strain(s). So, this theoretical analysis via creating a mechanical model for describing a polymeric material under stress and how such specific material will behave or exhibit a resistance during loading phase and what is the expected result—deformation style as a function of both loading time duration and the ambient temperature, so the first assumption was depending on a simple linear spring (elastic element) but after removing the applied load the spring will utterly return to its original dimensions that if the applied load within the elastic limit of this spring. Hence, this spring will not well cover the actual behavior of the loaded polymeric component, so the next proposal was considering only viscous element —dashpot but unfortunately this element also was not able to describe the gained behavior, these two results shed light on the combined models, so the next models was Maxwell, Voigt, modified Maxwell, modified Voigt, and Maxwell-Voigt models, the above theoretical equations supported by the finding shown in **Figure 15** gives a graduate interpretation of the expected behavior of the polymeric material under direct load and under constant room temperature, but it is very accurate to say that Maxwell-Voigt model is well telling or drawing the gained path of the residual strain in the polymeric material, so this is invitation to extend ideas for more models and more analysis for reaching the best behavior analogy between the mechanical models and polymeric materials as well.

9. Conclusions

As a result of this theoretical study, it is essential to conclude that all of the analyzed models are giving an interpretation of the behavior of some loaded polymeric materials but with different approximation but it looks like that each model is complementing the other models, or in other words, starting from simple spring alone or viscous dashpot alone is giving high error rate and this error starts decreases via adding another element either in series or in parallel to form Maxwell and Voigt models to reach optimum verification, respectively, so the modified Maxwell model is giving relatively the best fit with the actual behavior of the polymeric component under direct longitudinal load, but that is not meaning neglecting the other models results, especially both of Voigt and Maxwell-Voigt models. It is preferable to include a conclusion(s) section, which will summarize the content of the book chapter.

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Nomenclature

σ	Stress
σ_0	initial applied stress
$\sigma(t)$	stress as a function of time (t)
σ	first derivative of stress with respect to time
$\dot{\sigma}$	second derivative of stress with respect to time
ε	Strain
ε_0	induced strain
$\varepsilon(t)$	strain as a function of time (t)
ε	first derivative of strain with respect to time
$\dot{\varepsilon}$	second derivative of strain with respect to time
$\varepsilon_{\text{critical}}$	critical strain
γ	shear strain
γ	shear-strain rate

δ	deformation (longitudinal extension)
$\delta(t)$	deformation as a function of time
η	fluid viscosity of the employed dashpot
E	Young's modulus (modulus of elasticity)
G	shear modulus
$E_c(t)$	creep modulus as a function of time
$E_r(t)$	relaxation modulus as a function of time
F	applied external force
t	time
t	relaxation time
T	Temperature
K	spring stiffness
C	damping constant
ΔH	viscous flow activation energy at a particular temperature
R	universal gas constant
A	pre-exponent factor

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References

[1] Shenoy AV, Saini DR. Thermoplastic Melt Rheology and Processing. Madison Avenue, New York; 1996. p. 10016

[2] Marques P, Creus J. Computational Viscoelasticity. Springer Science & Business Media; 2012

[3] Dulgerova P. Biophysical modelling of viscoelastic deformations of cattle skin in vivo. Trakia Journal of Sciences. 2004;**2**(4):1-3

[4] Faith A. Understanding Rheology. Morrison: Oxford University Press; 2001

[5] Hussein E, A Modified Version of the Prosthetic Foot [thesis]. Nahrain University; 2011

[6] Meral F. Fractional calculus in viscoelasticity: An experimental study. Communication Nonlinear Science Numerical Simulation. 2010;**15**:939. DOI: 10.1016/j.cnsns.2009.05.004

[7] Korcz H, Polybutylene polymers for hot melt adhesives, Adhesive Age, 1984; 19-23.

[8] Brinson F, & Brinson L. Polymer engineering science and viscoelasticity. An introduction, 2008;99-157.

[9] Hussein EK. Theoretical and Applied Rheology [Internet], 2017. Available from: https://www.researchgate.net/publication/307882579

[10] Adams M, Briscoe B, Johnson S. Friction and lubrication of human skin. Tribology Letters. 2007;**26**(3):239-253

[11] Soldatenkov IA. Calculation of the Deformation Component of the Force of Friction for a Standard Elastoviscous Base. Journal of Friction and Wear. 2008;**29**(1):7-14

[12] Moore DF. The Friction and Lubrication of Elastomers. Oxford: Pergamon Press; 1972

[13] Masen MA. A systems based experimental approach to tactile friction. Journal of the Mechanical Behavior of Biomedical Materials. 2011;**4**:1620-1626

[14] Sinha SK, Chong M, Lim SC. Scratching of polymers – Modelling abrasive wear. Wear. 2007;**262**: 1038-1047

[15] Kussay AS, Tudor A, Hussein EK, Wahad HS. The adhesion and hysteresis effect in friction skin with artificial materials. In: 13th International Conference on Tribology, ROTRIB'16, IOP Conference Series: Materials Science and Engineering. Vol. 174. 2017. p. 012018

[16] Hussein EK, Subhi KA, Gaaz TS. Effect of Stick-Slip Phenomena between Human Skin and UHMW Polyethylene. Pertanika. Journal of Science & Technology. 2021;**29**(3)

[17] Cua AB, Wilhelm KP, Maibach HI. Elastic properties of human skin: relation to age, sex, and anatomical region. Archives of Dermatological Research. 1990;**282**(5):283-288