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On Regular Inverse Eccentric Fuzzy Graphs

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Abstract

Two new concepts of regular inverse eccentric fuzzy graphs and totally regular inverse eccentric fuzzy graphs are established in this article. By illustrations, these two graphs are compared and the results are derived. Equivalent condition for the existence of these two graphs are found. The exact values of Order and Size for some standard inverse eccentric graphs are also derived.

Keywords: Regular fuzzy graph; Totally regular fuzzy graph; Regular inverse eccentric fuzzy graph; Totally regular inverse eccentric fuzzy graph

MSC 2010 No.: 05C72, 05C75

1. Introduction

Euler introduced the concept of graph theory. The idea of Fuzzy sets was introduced by Zadeh (1965). In various areas of mathematics like Topology, Algebra, Number Theory and Optimization Techniques, these are being fuzzified. One such area is Fuzzy Graph Theory. During the year 1973, the definition of fuzzy graph was introduced in a systematic way by Haufmann. Later, Rosenfeld

(1975) elaborated the concept of Fuzzy Graphs. Fuzzy Graphs ware also introduced independently by Yeh and Bang during the same time. Properties of fuzzy graphs are studied by Harary (1988). These graphs are coined by imposing a relation using the membership function on both the vertex set and the edge set. Sunitha and Vijayakumar (1999) and Sharma et al. (2013) reviewed fuzzy trees. Nagoor Gani and Radha (2008) and Radha introduced regular fuzzy graphs, totally regular fuzzy graphs by defining the degree on both the vertex set and edge set using the membership function. Order and Size in Fuzzy Graph were studied by Nagoor Gani and Basheer Ahamed (2003). Meenal (2019) found a new graph called eccentric fuzzy graph based on the membership function defined by eccentricity and diameter that motivated the research scholar to explore Inverse Eccentric Fuzzy Graphs.

Simple models of relations are represented pictorially as a graph for better understanding. The graph considered in this paper are undirected, connected without self-loops and parallel edges. On a fuzzy subset, a symmetric binary fuzzy relation is coined and it is named as Fuzzy graph. In graph, vertices are represented by objects and edges are represented by relationships between objects. While describing the objects or in their relationship when there is an ambiguity a Fuzzy Graph Model to designed. Throughout this paper p and q represent, respectively, the number of vertices and the number of edges of a crisp graph.

For two arbitrary vertices in a crisp graph the distance will represent the length of the shortest path between them. Eccentricity is defined for a vertex as the farthest distance from that vertex to any other vertex whereas radius (and diameter) are defined for the crisp graph to be the minimum(and maximum) eccentricity of among all the vertices in a graph. The set of vertices adjacent to a vertex in vertex set is called the neighborhood of that vertex. The degree of a vertex is the number of edges incident with the vertex in crisp graph.

Defining the problem: Motivated by the research work of *On Defining Fuzzy Graph from Degree Sequence* by Kalpana et al. (2018), we define a fuzzy graph by giving membership function to the vertices and edges based on the eccentricity of the vertex and the radius of the graph. We name this fuzzy graph as inverse eccentric fuzzy graph, IEF(G) and analyze its properties.

2. Inverse Eccentric Fuzzy Graphs

Definition 2.1. (Notation)

$\lfloor \frac{a}{b} \rfloor$ denotes up to the first digit of the decimal, when a is divided by b. For example, $\lfloor \frac{3}{4} \rfloor = 0.7$.

Definition 2.2.

Let G^*:(V,E) be a graph with radius rad(G^*). An Inverse Eccentric Fuzzy Graph IEF(G):(σ_{ie}, μ_{ie}) is a set with a pair of Inverse eccentric membership functions, inverse eccentric fuzzy vertex set function $\sigma_{ie} : V(G^*) \rightarrow [0, 1]$ on the vertex set is defined as,

$$\sigma_{ie}(u) = \frac{rad(G^*)}{ecc(u)}, \text{ for all } u \in V(G^*),$$

and the inverse eccentric fuzzy edge set function $\mu_{ie} : E(G^*) \to [0,1]$ on the edge set is defined as

$$\mu_{ie}(uv) = min\{\sigma_{ie}(u), \sigma_{ie}(v)\}, \text{ for all } uv \in E(G^*).$$

That is, every edge is an effective edge.

Definition 2.3.

Let IEF(G) be an Inverse eccentric fuzzy graph on G^*:(V,E). The order p_{ie} of an inverse eccentric fuzzy graph IEF(G):(σ_{ie}, μ_{ie}) is defined as

$$p_{ie} = \sum_{v \in V(G^*)} \sigma_{ie}(v), \text{ where } v \in V(G^*).$$

The size q_{ie} of an inverse eccentric fuzzy graph IEF(G):(σ_{ie}, μ_{ie}) is defined as

$$q_{ie} = \sum_{uv \in E(G^*)} \mu_{ie}(uv), \text{ where } uv \in E(G^*).$$

Example 2.1.

Consider the graph G^*:(V,E) with the Vertex set V=$\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and Edge set $E = \{v_1v_2, v_2v_3, v_2v_4, v_3v_4, v_3v_5, v_4v_5, v_3v_6, v_4v_6, v_5v_6\}$.

Define $IEF(G) : (\sigma_{ie}, \mu_{ie})$ under crisp graph G^*. The eccentricity of a graph G^* is $ecc(v_1) = ecc(v_5) = ecc(v_6) = 3$; $ecc(v_2) = ecc(v_3) = ecc(v_4) = 2$. Thus, the radius is, $rad(G^*) = min\{ecc(v_i)/v_i \in V(G^*)\}$=2. The membership function of vertex set is $\sigma_{ie}(v_1) = \sigma_{ie}(v_5) = \sigma_{ie}(v_6) = \frac{rad(G^*)}{ecc(v_1)} = \frac{2}{3} = 0.6$; $\sigma_{ie}(v_2) = \sigma_{ie}(v_3) = \sigma_{ie}(v_4) = \frac{3}{3} = 1$. The membership function of edge set is $\mu_{ie}(v_2v_3) = min\{\sigma_{ie}(v_2), \sigma_{ie}(v_3)\} = 1 = \mu_{ie}(v_3v_4)$; $\mu_{ie}(v_1v_2) = 0.6 = \mu_{ie}(v_3v_6) = \mu_{ie}(v_4v_5) = \mu_{ie}(v_5v_6) = \mu_{ie}(v_3v_5) = \mu_{ie}(v_4v_6)$. The Order of IEF(G) is $p_{ie} = \sum_{v_i \in V(G^*)} \sigma_{ie}(v_i) = 0.6 + 1 + 1 + 1 + 0.6 + 0.6 = 4.8$. The Size of IEF(G) is $q_{ie} = \sum_{v_iv_j \in E(G^*)} \mu_{ie}(v_iv_j) = 0.6 + 1 + 1 + 1 + 0.6 + 0.6 + 0.6 + 0.6 + 0.6 = 6.6$.

Theorem 2.1.

Let IEF(G) be an Inverse eccentric fuzzy graph corresponding to the graph G^*. Then $\sigma_{ie}(u) \leq 1$, for all $u \in V(G^*)$.

Proof:

Let IEF(G) be an Inverse eccentric fuzzy graph corresponding to the graph G^*. Then, by Definition 2.2, ecc(u)≥rad(G^*), for all u∈ $V(G^*)$. $\sigma_{ie}(u)$=$\frac{rad(G^*)}{ecc(u)} \leq \frac{rad(G^*)}{rad(G^*)} \leq 1$. ■

Definition 2.4.

Let IEF(G) be an Inverse Eccentric Fuzzy graph corresponding to the graph G^*, then the Degree of an Inverse Eccentric Fuzzy Graph denoted by $deg_{ie}(u)$ is defined by

$$deg_{ie}(u) = \sum_{u \neq v} \mu_{ie}(uv), \text{ where } \mu_{ie}(uv) > 0; \text{ if } uv \in E(G^*) \text{ and } \mu_{ie} = 0; \text{ if } uv \notin E(G^*).$$

Example 2.2.

Consider a path on 6 vertices. That is, $G^* \cong P_6$. Then, rad(P_6)=5. Let V(P_6)=$\{v_1, v_2, v_3, v_4, v_5, v_6\}$ where v_1 and v_6 are terminal vertices.

Also, $\sigma_{ie}(v_1) = \frac{rad(P_6)}{ecc(v_1)} = \frac{3}{5} = 0.6 = \sigma_{ie}(v_6)$; $\sigma_{ie}(v_2) = \frac{3}{4} = 0.7 = \sigma_{ie}(v_5)$; $\sigma_{ie}(v_3) = 1 = \sigma_{ie}(v_4)$. Hence, $deg_{ie}(v_1) = 0.6 = deg_{ie}(v_6)$; $deg_{ie}(v_2) = 0.6 + 0.7 = 1.3$; $deg_{ie}(v_3) = 0.7 + 1 = 1.7$; $deg_{ie}(v_4) = 1 + 0.7 = 1.7$; $deg_{ie}(v_5) = 0.7 + 0.6 = 1.3$.

3. Regular Inverse Eccentric Fuzzy Graph and Totally Regular Inverse Eccentric Fuzzy Graph

Definition 3.1.

Let IEF(G):(σ_{ie}, μ_{ie}) be an Inverse Eccentric Fuzzy Graph on an underlying crisp graph $G^* : (V, E)$. An inverse eccentric fuzzy graph IEF(G) is called k-regular inverse eccentric fuzzy graph or a k degree regular inverse eccentric fuzzy graph if the degree of every vertex is equal to a constant number (say k), that is, $deg_{ie}(v_i) = k$, for all $v_i \in V(G^*)$.

Example 3.1.

Any fuzzy graph which is connected with two vertices is a regular inverse eccentric fuzzy graph.

Definition 3.2.

A graph is said to be a complete inverse eccentric fuzzy graph if $deg_{ie}(u) = p_{ie} - 1$, for all $u \in V(G^*)$.

Example 3.2.

Let us consider a complete graph on 6 vertices K_6. Then, for all $u \in V(G^*)$,

$$deg_{ie}(u) = p - 1 = 6 - 1 = 5.$$

Also, the degree of each and every vertex is 5. Thus, K_6, is a regular inverse eccentric fuzzy graph. Therefore, a complete inverse eccentric fuzzy graph is a regular inverse eccentric fuzzy graph.

Definition 3.3.

The total degree of a vertex u_i in an inverse eccentric fuzzy graph is denoted by $tdeg_{ie}(u_i)$. It is defined as the sum of the membership values of the edges incident at the vertex u_i along with the membership value of the vertex u_i,

$$tdeg_{ie}(u_i) = deg_{ie}(u_i) + \sigma_{ie}(u_i), \text{ for all } u_i \in V(G^*).$$

If $tdeg_{ie}(u_i)$ is constant (say k), for each vertex $u_i \in V$, then IEF(G) is called k-totally regular inverse eccentric fuzzy graph.

Example 3.3.

Consider $G^* : (V, E)$ where the vertex set $V(G^*) = \{v_1, v_2, v_3, v_4, v_5\}$ and Edge set $E(G^*) = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_3v_4, v_4v_5\}$.

Define IEF(G):(σ_{ie}, μ_{ie}) by $\sigma_{ie}(v_1) = \sigma_{ie}(v_3) = \sigma_{ie}(v_4) = 1$; $\sigma_{ie}(v_2) = \sigma_{ie}(v_5) = 0.6$ and $\mu_{ie}(v_1v_2) = \mu_{ie}(v_4v_5) = \mu_{ie}(v_2v_3) = 0.6$; $\mu_{ie}(v_1v_3) = \mu_{ie}(v_3v_4) = \mu_{ie}(v_1v_4) = 1$. Then, $deg_{ie}(v_1) = deg_{ie}(v_3) = deg_{ie}(v_4) = 2.6$; $deg_{ie}(v_2) = 1.2$; $deg_{ie}(v_5) = 0.6$. Here, $deg_{ie}(v_1) \neq deg_{ie}(v_2)$. So, IEF(G) is not a regular inverse eccentric fuzzy graph. Also, $tdeg_{ie}(v_1) \neq tdeg_{ie}(v_2)$. So, IEF(G) is not a totally regular inverse eccentric fuzzy graph.

Example 3.4.

Consider $G^* : (V, E)$ where the vertex set $V(G^*) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and Edge set $E(G^*) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$.

Define IEF(G):(σ_{ie}, μ_{ie}) by, $\sigma_{ie}(v_i) = 1$, for all $v_i \in V(G^*)$. $\mu_{ie}(v_iv_j) = 1$, for all $v_iv_j \in E(G^*)$. Then, $deg_{ie}(v_i)2$, forall $v_i \in V(G^*)$. This implies degree of every vertex is 2. So, IEF(G) is regular Inverse eccentric fuzzy graph. Now, $tdeg_{ie}(v_i) = 3$, for all $v_i \in V(G^*)$. This implies that the total degree of every vertex is 3. Also, IEF(G) is a totally regular inverse eccentric fuzzy graph.

Remark 3.1.

Hence we arrive at a relation between regular inverse eccentric fuzzy graph and totally regular inverse eccentric fuzzy graph. That is,

$$\begin{pmatrix}\text{regular inverse}\\ \text{eccentric fuzzy graph}\end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix}\text{totally regular inverse}\\ \text{eccentric fuzzy graph}\end{pmatrix}.$$

Theorem 3.1.

For an inverse eccentric fuzzy graph $IEF(G) : (\sigma_{ie}, \mu_{ie})$ on a graph $G^* : (V, E)$, the following conditions are equivalent:

(1) σ_{ie} is a constant function.
(2)(a) IEF(G) is regular inverse eccentric fuzzy graph, and
(b) IEF(G) is totally regular inverse eccentric fuzzy graph.

Proof:

Part I: (1)$\Rightarrow$ 2(a).
Assume that the condition (1) holds. Then, for all $u_i \in V(G^*)$, $\sigma_{ie}(u_i)$=c. This implies that the radius and diameter of the inverse eccentric fuzzy graph are equal. Hence, the degree of all the vertices will be equal to the number of edges incident on that vertex. Thus, $deg_{ie}(u_i) = k_1$. This implies total degree, of the vertex u_i for the inverse eccentric fuzzy graph will be $k_1 + 1$ (a constant value). Thus condition (1) implies condition 2(a).

Part II: (1)⇒ 2(b).
On the contrary, suppose 2(a) does not hold. Then, the total degree will not be the same for all the vertices in the inverse eccentric fuzzy graph. This implies that the degree of all the vertices will not be the same for all the vertices, which shows that σ_{ie} will not be a constant function, a contradiction to the condition (1). Hence, the result follows.

Part III: (2)⇒(1).
Conversely, IEF(G) is regular inverse eccentric fuzzy graph if and only if IEF(G) is totally regular inverse eccentric fuzzy graph. Suppose $\sigma(u_i) \neq k$, for some $u_i \in V(G^*)$. Then, $\sigma_{ie}(u_i) \neq \sigma_{ie}(u_j)$ for any pair of vertices $u_i, u_j \in V(G^*)$.

Case (i): 2(a)⇒1.
Consider IEF(G) to be a k-regular inverse eccentric fuzzy graph. Then, the degree of all vertices in an inverse eccentric fuzzy graph should be constant, which is taken as k. Therefore, total degree of all vertices in an inverse eccentric fuzzy graph is a sum of the constant value k and the membership values of a vertex u_i in an inverse eccentric fuzzy graph. Also, we know that $\sigma_{ie}(u_i)$ does not possess a constant value, for all $u_i \in V(G^*)$. So, the total degree of all the vertices must be different. This implies that IEF(G) is not totally regular inverse eccentric fuzzy graph. This contradicts the assumption.

Case (ii): 2(b)⇒(1).
Let IEF(G) be a totally regular inverse eccentric fuzzy graph. Therefore, the total degree of all vertices in an inverse eccentric fuzzy graph are equal. Consider the total degree of any two vertices in an inverse eccentric fuzzy graph. Then, $deg_{ie}(u_i) - deg_{ie}(u_j) = \sigma_{ie}(u_j) - \sigma_{ie}(u_i) \neq 0$. This implies the degree of any two vertices are not equal. This shows that IEF(G) is not regular, contradicting our assumption. Therefore, $\sigma_{ie}(u_i)$ has same membership values for all vertices in an inverse eccentric fuzzy graph. ■

Example 3.5.

Consider G^* where vertex set $V(G^*) = \{v_1, v_2, v_3, v_4\}$ and Edge set $E(G^*) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$.

Define inverse eccentric fuzzy graph IEF(G):(σ_{ie}, μ_{ie}) by $\sigma_{ie}(v_i) = 1$, for all $v_i \in V(G^*)$. So, $\mu_{ie}(v_iv_j) = min\{\sigma_{ie}(v_i), \sigma_{ie}(v_j)\} = 1$, for all $v_iv_j \in E(G^*)$. From Theorem 3.9, $\sigma_{ie}(v_i)$ is a constant function. And $deg_{ie}(v_1) = deg_{ie}(v_2) = deg_{ie}(v_3) = deg_{ie}(v_4) = 1$. Here, the degree of all vertices are equal and so, it is a regular inverse eccentric fuzzy graph. Similarly, $tdeg_{ie}(v_1) = tdeg_{ie}(v_2) = tdeg_{ie}(v_3) = tdeg_{ie}(v_4) = 2$. Here the total degree of all vertices are equal and so, it is a totally regular inverse eccentric fuzzy graph. Therefore, we can conclude that the constant function of the inverse eccentric fuzzy graph implies a regular inverse fuzzy graph and totally regular inverse eccentric fuzzy graph.

4. Order and Size of IEF(G) for some Standard Graphs

Theorem 4.1.

Let IEF(W_{p+1}) be an inverse eccentric fuzzy wheel graph of order p_{ie} and size q_{ie} corresponding to the graph W_{p+1}. Then $p_{ie} = p(0.5) + 1$ and $q_{ie} = p$, where p and q are the order and size of the graph G^*.

Proof:

Let $V(G^*)$ be the vertex set of W_{p+1}. There are p vertices on the rim and one central vertex, where $p \geq 4$. If $U(G^*)$ is the set of rim vertices, then $\mid U(G^*) \mid$=p implies that $ecc(u_i)$=2, where $1 \leq i \leq p$. If v_{p+1} is a central vertex, then $\mid \{v_{p+1}\} \mid = 1$ implies that $ecc(v_{p+1}) = 1$. Also, $rad(W_{p+1}) = 1$ and $V(G^*) = U(G^*) \cup \{v_{p+1}\}$.

The inverse eccentric fuzzy graph vertex set function is given by $\sigma_{ie}(v_i) = \frac{rad(G^*)}{ecc(v_i)}$. For the vertices on $U(G^*)$, $ecc(v_i) = 2$; $\sigma_{ie}(v_i) = \frac{1}{2} = 0.5$: $1 \leq i \leq p$. For the central vertex v_{p+1}, $ecc(v_{p+1}) = 1$; $\sigma_{ie}(v_{p+1}) = \frac{2}{2} = 1$. Order of the inverse eccentric fuzzy graph is given by

$$p_{ie} = \sum_{v_i \in V(G^*)} \sigma_{ie}(v_i) = \sum_{v_i \in V(G^*)} \sigma_{ie}(v_i) + \sigma_{ie}(v_{p+1}) = (0.5)p + 1.$$

Let $E(G^*)$ be the edge set of W_{p+1} and $E(G^*) = E(G_1^*) \cup E(G_2^*)$. That is,

$$E(G_1^*) = \{(v_i v_{i+1}) \cup (v_1 v_p)/1 \leq i \leq p-1\} \implies \mid E(G_1^*) \mid = p - 1 + 1 = p \text{ edges},$$

$$E(G_2^*) = \{(v_i v_{p+1})/1 \leq i \leq p\} \implies \mid E(G_2^*) \mid = p \, \text{e}dges.$$

Consider the edges on $E(G_1^*)$, for all $1 \leq i \leq p-1$,

$$\mu_{ie}(v_i v_{i+1}) = min\{\sigma_{ie}(v_i), \sigma_{ie}(v_{i+1})\} = min\{0.5, 0.5\} = 0.5,$$

$$\mu_{ie}(v_1 v_p) = min\{\sigma_{ie}(v_1), \sigma_{ie}(v_p)\} = min\{0.5, 0.5\} = 0.5,$$

$$\mu_{ie}(v_i v_{p+1}) = min\{\sigma_{ie}(v_i), \sigma_{ie}(v_{p+1})\} = min\{1, 0.5\} = 0.5.$$

The Size of the inverse eccentric fuzzy graph is given by,

$$q_{ie} = \sum_{v_i v_j \in E(G^*)} \mu_{ie}(v_i v_j) = \sum_{v_i v_j \in E(G_1^*)} \mu_{ie}(v_i v_j) + \sum_{v_i v_{p+1} \in E(G_2^*)} \mu_{ie}(v_i v_{p+1}),$$

$$q_{ie} = 0.5(p) + 0.5(p) = p.$$

■

Remark 4.1.

The inverse eccentric fuzzy wheel graph W_{3+1} is of order $p_{ie} = p$ and size $q_{ie} = q$.

Example 4.1.

Consider the Wheel graph W_{7+1} of the vertex set $V(G^*) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, \}$ and Edge set $E(G^*) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_1, v_1v_8, v_2v_8, v_3v_8, v_4v_8, v_5v_8, v_6v_8, v_7v_8\}$.

The eccentricity and radius of W_{7+1} are

$$ecc(v_1) = ecc(v_2) = ecc(v_3) = ecc(v_4) = ecc(v_5) = ecc(v_6) = ecc(v_7) = 2; \;\; ecc(v_8) = 1.$$

Here $G^* \cong W_{8+1}$; $rad(G^*) = 1$. The membership function of vertex set is defined as

$$\sigma_{ie}(v_1) = \sigma_{ie}(v_2) = \sigma_{ie}(v_3) = \sigma_{ie}(v_4)\sigma_{ie}(v_5) = \sigma_{ie}(v_6) = \sigma_{ie}(v_7) = 0.5; \quad \sigma_{ie}(v_8) = 1.$$

The Order of W_{7+1} is $p_{ie} = 7(0.5) + 1 = 5 = p(0.5) + 1$. The membership function of edge set are defined as $\mu_{ie}(v_i v_j) = 0.5$, for all $v_i v_j \in E(G^*)$. The Size of W_{7+1} is, $q_{ie} = 14(0.5) = 7 = p$.

Theorem 4.2.

Let $IEF(P_m + K_2)$ be an inverse eccentric fuzzy fan graph of order p_{ie} and size q_{ie} corresponding to the graph $P_m + K_2$. Then p_{ie}=m(0.5)+1.5 and q_{ie}=m. Here, P_m is a path on m vertices.

Proof:

Let $V(G^*)$ be the vertex set of $P_m + K_2$. There are m vertices on P_m and 2 vertices on K_2. $U(G^*)$ is the set of m vertices in P_m. Then $\mid U(G^*) \mid = m$ implies that $ecc(v_i) = 2$. If u_1 is a vertex on K_2, then $\mid u_1 \mid = 1$ implies that $ecc(u_1) = 2$. If u_2 is a another vertex on K_2, then $\mid u_2 \mid$=1 implies that $ecc(u_2) = 1$. Also, the radius of the graph is given by $rad(P_m + K_2) = 1$. $V(G^*) = U(G^*) \cup \{u_1\} \cup \{u_2\}$.

The inverse eccentric fuzzy vertex set function is given by $\sigma_{ie}(v_i) = \frac{rad(G^*)}{ecc(v_i)}$. For the vertices on $U(G^*)$, $ecc(v_i) = 2$; $\sigma_{ie}(v_i) = \frac{1}{2} = 0.5$. For the vertex on K_2, $ecc(u_1) = 2$; $\sigma_{ie}(u_1) = \frac{1}{2} = 0.5$. For the another vertex on K_2, $ecc(u_2) = \frac{2}{2} = 1$; $\sigma_{ie}(u_2) = \frac{2}{2} = 1$. The Order of the inverse eccentric fuzzy graph is given by

$$p_{ie} = \sum_{u \in V(G^*)} \sigma_{ie}(u) = \sum_{v_i \in V(G^*)} \sigma_{ie}(v_i) + \sigma_{ie}(u_1) + \sigma_{ie}(u_2),$$

$$p_{ie} = m(0.5) + 1 + 0.5 = m(0.5) + 1.5.$$

Let $E(G^*)$ be the edge set of $P_m + K_2$, $E(G^*) = E(G_1^*) \cup E(G_2^*)$. That is,

$$E(G_1^*) = \{(v_i v_{i+1}) 1 \leq i \leq m-1\} \implies \mid E(G_1^*) \mid = \text{m-1 edges},$$

$$E(G_2^*) = \{(u_1 u_2) \cup (u_2 v_i) 1 \leq i \leq m\} \implies \mid E(G_2^*) \mid = \text{m+1 edges}.$$

Consider the edges on $E(G_1^*)$, for all $1 \leq i \leq m-1$,

$$\mu_{ie}(v_i v_{i+1}) = min\{\sigma_{ie}(v_i), \sigma_{ie}(v_{i+1})\} = min\{0.5, 0.5\} = 0.5.$$

Consider the edges on $E(G_2^*)$, for all $1 \leq i \leq m$,

$$\mu_{ie}(u_2 v_i) = min\{\sigma_{ie}(u_2), \sigma_{ie}(v_i)\} = min\{1, 0.5\} = 0.5,$$

$$\mu_{ie}(u_1 u_2) = min\{\sigma_{ie}(u_1), \sigma_{ie}(u_2)\} = min\{0.5, 1\} = 0.5.$$

The Size of the inverse eccentric fuzzy graph is given by

$$q_{ie} = \sum_{uv \in E(G^*)} \mu_{ie}(uv) = \sum_{uv \in E(G_1^*)} \mu_{ie}(uv) + \sum_{uv \in E(G_2^*)} \mu_{ie}(uv),$$

$$q_{ie} = (0.5)m + (0.5)m = m.$$

■

Example 4.2.

Consider a fan graph $P_3 + K_2$ with vertex set $V(G^*) = \{v_1, v_2, v_3, u_1, u_2\}$ and Edge set $E(G^*) = \{v_1v_2, v_2v_3, u_1u_2, v_1u_2, v_2u_2, v_3u_2\}$.

The eccentricity of $P_3 + K_2$ are $ecc(v_1) = ecc(v_2) = ecc(v_3) = ecc(u_1) = 2$; $ecc(u_2) = 1$. Thus, radius of $P_3 + K_2$ is rad($P_3 + K_2$)=$min\{ecc(v_i)/v_i \in V(G^*)\} = 1$. The membership function of vertex set is defined as $\sigma_{ie}(v_1) = \sigma_{ie}(v_2) = \sigma_{ie}(v_3) = \sigma_{ie}(u_1) = 0.5$; $\sigma_{ie}(u_2) = 1$. The Order of a graph $P_3 + K_2$ is $p_{ie} = 0.5 + 0.5 + 0.5 + 0.5 + 1 = m(0.5) + 1.5$. The membership function of edge set is defined as $\mu_{ie}(v_iv_j) = 0.5$, for all $v_iv_j \in E(G^*)$. The Size of a graph $P_3 + K_2$ is $q_{ie} = 0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.5 = 3 = m$.

5. Conclusion

In this paper, we discuss regular inverse eccentric fuzzy graphs and totally regular inverse eccentric fuzzy graphs under some conditions and illustrated them through examples. Also, the exact values of p_{ie} and q_{ie} for some standard graphs are obtained. To characterize the regular inverse eccentric fuzzy graphs on cycles, paths are considered as open problems. Defining some operations like union, join and finding Cartesian product, corona, etc., on inverse eccentric fuzzy graphs are suggested future direction for research.

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