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Nonsplit Edge Geodetic Domination Number of a Graph

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Abstract

In this paper, we have defined an inventive parameter called the nonsplit edge geodetic domination number of a graph, and some of its general properties are studied. The nonsplit edge geodetic domination number of some standard graph is obtained. In this work, we also determine the realization results of the nonsplit edge geodetic domination number and the edge geodetic number of a graph.

Keywords: Geodetic domination number; Edge geodetic domination number; Nonsplit edge geodetic domination number

MSC 2010 No.: 05C69, 05C12

1. Introduction

The graph G under consideration is a simple graph. We use Buckley and Harary (1990), Harary (1969), and Hayness et al. (1998) for basic graph theoretic terminology. If a point v is a complete subgraph induced by its neighbors, then it is an extreme point.

The shortest path is the distance between any two points on a graph. This gives rise to the concept of geodetic set and the geodetic number of a graph. This concept was introduced in Buckley and Harary (1990) and further studied in Chartrand et al. (2002). The concept of edge geodetic number of a graph was introduced in Santhakumaran and John (2007) and further studied in Shobha and Goudar (2018b) and Anne Mary Leema et al. (2021). The concept of adjacent points gives rise to the concepts of dominating set and domination number of a graph. For the results regarding domination number of a graph, we refer to Hayness et al. (1998). The concept of geodetic domination number of G was introduced in Hansberg and Volkmann (2010). The idea of edge geodetic domination number of a graph was established by Arul Paul Sudhahar et al. (2016). Tejaswini and Goudar (2016) proposed the concept of nonsplit geodetic number of a graph. Furthermore, Arul Paul Sudhahar and Jeba Lisa (2021a) introduced the concept of split geodetic domination number of a graph and nonsplit geodetic domination number of a graph.

Throughout this paper, we use the notation GS set for geodetic set, EG set for edge geodetic set, GD set for geodetic dominating set, EGD set for edge geodetic dominating set, NGD set for nonsplit geodetic dominating set, NEGD set for nonsplit edge geodetic dominating set and min NEGD set for minimum nonsplit edge geodetic dominating set of G.

2. Preliminaries

If each edge of G is included in a geodesic connecting some pair of points in S, then $S \subseteq V(G)$ is an EG set, and its lowest cardinality is the edge geodetic number indicated by $g_1(G)$. See Santhakumaran and John (2007).

If each point of G is dominated by a point of S, the GS set S is considered a GD set, and its lowest cardinality is named the geodetic domination number $\gamma_g(G)$ of G. See Palani and Nagarajan (2011).

If S of $V(G)$ is both an EG set and the dominating set of G, then S is an EGD set of G. Its lowest cardinality is the edge geodetic domination number, represented as $\gamma_{1g}(G)$. See Arul Paul Sudhahar et al. (2016). Every edge geodetic cover of G includes all its simplicial points of G. In particular, every edge geodetic cover of G includes all its end points of G (Santhakumaran and John (2007)). Every EGD set of G includes all its simplicial points of G (Arul Paul Sudhahar et al. (2016)). Consider any two points $w = (w_1, w_2)$ and $k = (k_1, k_2)$ in $V = V_1 \times V_2$. Then w and k are adjacent in $G_1 \times G_2$ whenever $[w_1 = k_1$ and $w_2 adj k_2]$ or $[w_2 = k_2$ and $w_1 adj k_1]$ where $G_1 \times G_2$ is the product of the graphs G_1 and G_2 (Harary (1969)). The windmill graph $W_d(k, n)$ is made up of k copies of K_n with a point in common (Nagabhushana et al. (2017)).

Our Contribution

In Section 2, we define the nonsplit edge geodetic domination number of a graph. We have investigated some results regarding the nonsplit edge geodetic domination number of a graph. In addition, we have determined the nonsplit edge geodetic domination number of various graphs, such as path and Cartesian product graph. Finally, in Section 3 we discuss the realization results for the nonsplit edge geodetic domination number of graph G. The nonsplit edge geodetic domination number that we defined differs from the edge geodetic domination under the property of split.

3. Nonsplit Edge Geodetic Domination Number of a Graph

In this section, we define the nonsplit edge geodetic domination number $\gamma_{1nsg}(G)$ of a graph and initiate a study of this parameter.

Definition 3.1.

An EGD set $S \subseteq V(G)$ is considered to be a NEGD set of G, if $< V - S >$ is connected and its smallest cardinality is named the nonsplit edge geodetic domination number of G, represented by $\gamma_{1nsg}(G)$.

Example 3.1.

Take a look at Figure 1's graph G. The min NEGD set of G is $S = \{d_1, d_2, d_8, d_5\}$. Because there are no NEGD sets with 2 and $3-$elements, $\gamma_{1nsg}(G) = 4$.

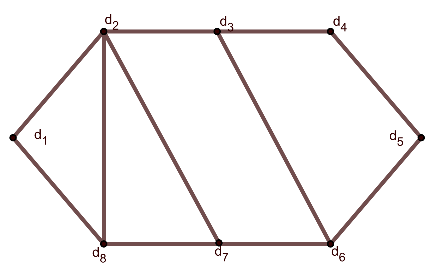

Figure 1. Graph G with $\gamma_{1nsg}(G) = 4$

Remark 3.1.

The set $S_1 = \{d_1, d_5, d_6\}$ is a minimum NGD set for the graph G shown in Figure 1. As a result, $\gamma_{nsg}(G)$ and $\gamma_{1nsg}(G)$ are not the same.

Theorem 3.1.

Let $G = P_n$ $(n \geq 3)$. Then,

$$\gamma_{1nsg}(P_n) = \begin{cases} 2, & \text{if } 3 \leq n \leq 4, \\ n-2, & \text{if } n \geq 5. \end{cases}$$

Proof:

Consider $V(P_n) = \{h_1, h_2, \ldots, h_n\}$. We have the following cases.

Case i: $n = 3, 4$.

Let S represent the collection of every end point of P_n and S belongs to every EGD set of P_n. Therefore, S is an EGD set and $< V - S >$ is connected. Hence, $\gamma_{1nsg}(P_n) = 2$.

Case ii: $n > 4$.

Let $S = \{h_1, h_n\}$. Then, S is an EG set of P_n. However, S is not a NEGD set of P_n. Let $S' = \{h_1, h_4, h_5, \ldots, h_n\}$. As a result, S' is an EGD set and $< V - S' >$ is connected. Thus, S' is a NEGD set. We demonstrate that S' is the min NEGD set. Let $T = S' - \{h_k\}$,$(5 \leq k \leq n - 1)$. Then, T isn't a NEGD set because $< V - T >$ is not connected. Let $T = S' - \{h_k\}, k = 1$ or n. Then, T isn't an EG set of P_n. If $T = S' - \{h_4\}$, then T is not a P_n dominating set. As a result, S' is the min NEGD set. Thus, $\gamma_{1nsg}(P_n) = n - 2$. ∎

Theorem 3.2.

Let $G = K_{1,n}$ with $n \geq 2$. Then, $\gamma_{1nsg}(G) = n$.

Proof:

Let $G = K_{1,n}$, where $n \geq 2$. Let $V(G) = \{h, d_1, d_2, d_3, \ldots, d_n\}$, where h is the support point of G and $deg(h) = n$. Let $S = \{h\}$. The set S is not an EGD set of G despite the fact that S is undoubtedly the dominating set. Let S represent the collection of every end point of G, and S belongs to every EGD set of G. Therefore, S is an EGD set and $< V - S >$ is connected. Hence, S is the min NEGD set of G. Consequently, $\gamma_{1nsg}(G) = n$. ∎

Theorem 3.3.

Every NEGD set of G includes its simplicial point and every NEGD set of G includes its end point.

Proof:

Since every NEGD set of G is a EGD set of G, we know that every EGD set of G includes all its simplicial points of G. Therefore, end points and simplicial points of G belongs to every nonsplit edge geodetic dominating set of G. ∎

Theorem 3.4.

Assume that G is a connected graph of order $n \geq 2$, then $\gamma_{1nsg}(G) = n - 1$ if and only if there is a point x in G that is adjacent to all other points in G and $G - x$ is the union of at least two complete graphs.

Proof:

Let G be a graph and $\gamma_{1nsg}(G) = n - 1$. Let S be a NEGD set such that $V(G) - S = \{x\}$. Assume that H is a component of $G - \{x\}$, and that H contains two non-adjacent neighbours, m and h of x. Let $v_1, v_2, \ldots, v_l$ be the shortest $m - h$ path in H with $v_1 = m$ and $v_l = h$. Then, $l \geq 3$ and we obtain the contradiction that $V(G)\backslash\{x, v_2\}$ is an EGD set. Thus, $N(x) \cap V(H)$ is a complete graph. If $G - \{x\}$ has only one component, then x is a simplicial point which is a contradiction in itself. Hence, $G - x$ is the disjoint union of $a \geq 2$ graphs $H_1, H_2, ..., H_a$.

We now prove that x is adjacent to every other point in G. Assume, however, x is not adjacent to some point in H_i, say in H_1. This implies that there is a $x - m$ path xhm with $h, m \in V(H_1)$ such that $mx \notin E(G)$. Because G is connected, H_2 has a y neighbour of x. Now $d(m, y) = 3$, and we have the contradiction that $V(G) - \{v, w\}$ is a NEGD set. If G has a point x where $d(x) = n - 1$ and $G - \{x\}$ is the union of (at least two) complete graphs, thus $\gamma_{1nsg}(G) = n - 1$. ■

Theorem 3.5.

Let T represent any tree and the collection of every pendant point of T dominates every internal point of T. Then $\gamma_{1nsg}(G) = |pend(G)|$.

Proof:

Let S represent the collection of pendant points of T. According to Theorem 2.3, each pendant point belongs to every NEGD set of T. Therefore, $\gamma_{1nsg}(G) = |pend(T)|$ ■

Theorem 3.6.

If G is a connected graph with $diam(G) \leq 3$, then the min NEGD set contains no cut point of G.

Proof:

Assume $u \in cut(G)$ and S is the min NEGD set of G.

Suppose $u \in S$. Let the components of $G - u$ be $G_1, G_2, \ldots, G_n (n \geq 2)$. Consider $S' = S - \{u\}$. We demonstrate that S' is a NEGD set of G.

Consider $x \in V - S'$. Since S is a NEGD set of G, x is on a geodesic P that connects a pair of points v and u of S , $< V - S >$ is connected and $x \in N[S]$. Assume that $v \in G_1$, since u is adjacent to at the very least one point of every $G_j, (1 \leq j \leq n)$. Consider u and z are adjacent in G_l, where $l \neq 1$ and $z \in S$. Thus, $z \neq u$. Therefore, x lies on the $v - z$ geodesic of S'.

We show that $x \in N[S']$. Suppose $x \notin N[S']$, then no elements of S' are adjacent to x. Because x is not an end point, two points $a, b \notin S'$ exist that are adjacent to x and lie on the $v - w$ path. It shows that $d(v, u) > 3$ and x is not dominated by S', which contradicts $diam(G) \leq 3$. Therefore, $x \in N[S']$. Thus, S' is enough to form a min NEGD set and S is not the min NEGD set. This proves that no cut point belongs to the min NEGD set with $diam(G) \leq 3$. ■

Theorem 3.7.

Let $W_n = K_1 + C_{n-1}$ with $n \geq 4$. Then $\gamma_{1nsg}(W_n) = n - 1$.

Proof:

Let $V(W_n) = \{x_1, x_2, \ldots, x_{n-1}, u\}$. Let $S = \{x_1, x_2, \ldots, x_{n-1}\}$. Then, S is a NEGD set. We demonstrate that S is the min NEGD set of W_n. Consider $S_1 = S - \{x_i\}$, where $1 \leq i \leq n-1$. Then, S_1 is not an EG set of W_n, so that S_1 is not a NEGD set of graph W_n. Therefore, we can conclude that S is the min NEGD set of W_n. Thus, $\gamma_{1nsg}(W_n) = n - 1$. ∎

Theorem 3.8.

Let G be a connected graph of $n \geq 3$ points. Then, $2 \leq g_1(G) \leq \gamma_{1nsg}(G) \leq n - 1$.

Proof:

An EG set needs at the very least 2 points of G, so that $2 \leq g_1(G)$. Since all the NEGD set is an EG set, we have $g_1(G) \leq \gamma_{1nsg}(G)$. Let $|S| = n - 1$. Then, $< V - S >$ is connected. Thus, $\gamma_{1nsg}(G) \leq n - 1$. ∎

Theorem 3.9.

Let $G = W_d(k, n)$. Then, $\gamma_{1nsg}(W_d(k, n)) = k(n - 1)$.

Proof:

Let G represent the windmill graph. Let $V(W_d(k, n)) = \{v_1, v_2, \ldots, v_{k(n-1)}, v_l\}$, where v_l is the common point. Consider $S = V(W_d(k, n)) - \{v_l\}$. Then, S covers all the edges of $W_d(k, n)$ and dominates all the points of $W_d(k, n)$ and also $< V - S >$ is connected. Therefore, S is the min NEGD set of $W_d(k, n)$ and $|S| = k(n - 1)$. ∎

Theorem 3.10.

Let $G = K_2 \times K_{1,n}$. Then,

$$\gamma_{1nsg}(K_2 \times K_{1,n}) = \begin{cases} 3, & \text{if } n = 1, \\ n, & \text{if } n > 1. \end{cases}$$

Proof:

Case i: $n = 1$.

Consider $K_2 \times K_{1,1}$. Thus, $\gamma_{1nsg}(K_2 \times K_{1,1}) = 3$.

Case ii: $n > 1$.

Let $G = K_2 \times K_{1,n}$ be the graph and $V(K_2 \times K_{1,n}) = \{l_1, l_2, \ldots, l_{n+1}, h_1, h_2, \ldots, h_{n+1}\}$ with $2(1 + n)$ points. Let $S = \{l_2, h_3, h_4, \ldots, h_{n+1}\}$. Then, S is a NEGD set. Because S covers all of G's edges, S dominates every point of G and $< V - S >$ is connected.

We demonstrate that S is the min NEGD set of G. Consider $S_1 = S - \{l_2\}$ or $S_2 = S - \{h_i\}$, where $3 \leq i \leq n + 1$. Then, S_1 or S_2 is not an EG set. Therefore, S is the min NEGD set of G.

Therefore, $\gamma_{1nsg}(K_2 \times K_{1,n}) = n$. ■

Theorem 3.11.

Let $G = K_2 \times P_n$ with $(n \geq 2)$. Then

$$\gamma_{1nsg}(K_2 \times P_n) = \begin{cases} 3, & \text{for } n = 2, \\ \left\lceil \frac{n+1}{2} \right\rceil, & \text{for } n \text{ is even}, \\ \left\lceil \frac{n}{2} \right\rceil, & \text{for } n \text{ is odd}. \end{cases}$$

Proof:

Let $G = K_2 \times P_n$. Let $V(K_2 \times P_n) = \{h_1, h_2, \ldots, h_n, w_1, w_2, \ldots, w_n\}$ be the point of G. The following cases are considered.

Case i: Let n be an even integer.

We consider the following 2 subcases.

Subcase i : Let $n = 4, 8, 12, \ldots, n + 4$.

Let $S = \{h_1, w_3, h_5, w_7, \ldots, w_{n-1}, w_n\}$. As a result, S covers every edge of G, $< V - S >$ is connected and S dominates every point of G . Therefore, S is the min NEGD set of G and S has $\left\lceil \frac{n+1}{2} \right\rceil$ points. Hence, $|S| = \left\lceil \frac{n+1}{2} \right\rceil$.

Subcase ii: Let $n = 6, 10, 14, \ldots, n + 4$.

The set $S = \{h_1, w_3, h_5, w_7, \ldots, h_{n-1}, h_n\}$ is an EGD set of G and $< V - S >= \{h_2, h_3, h_4, w_1, w_2, w_4, \ldots\}$ is connected. Therefore, S is the min NEGD set of G and S has $\left\lceil \frac{n+1}{2} \right\rceil$ points. Hence, $|S| = \left\lceil \frac{n+1}{2} \right\rceil$.

Case ii: Let n be an odd integer.

Let $S = \{h_1, w_3, h_5, w_7, h_9, w_{11}, \ldots\}$. We know that, S is an EGD set and $< V - S >= \{w_1, w_2, h_2, h_3, h_4, w_4, w_5, w_6, \ldots\}$ is connected. Therefore, S is a NEGD set. If we remove one point from S, then S is not an EGD set of G. Therefore, S is the min NEGD set and S has $\left\lceil \frac{n}{2} \right\rceil$ points. Thus, $\gamma_{1nsg}(K_2 \times P_n) = \left\lceil \frac{n}{2} \right\rceil$. ■

4. Realization Results

This section shows the realization results of nonsplit edge geodetic domination number and edge geodetic number of a graph.

Theorem 4.1.

For a connected graph G with order $n = h + 3$, we have $\gamma_{1nsg}(G) = h$ with $2 \leq k \leq h \leq n - 1$.

Proof:

Let $P_k : a_1, a_2, a_3, \ldots, a_k$. Let the graph G be obtained from P_k by including $h-k+3$ new points $d_1, d_2, \ldots, d_{h-k+1}, u_1, u_2$ and connecting each $d_i (1 \leq i \leq h-k+1)$ to both u_1 and u_2. Also, add the edges of u_1a_1 and u_2a_1. Figure 2 represents the resulting graph G.

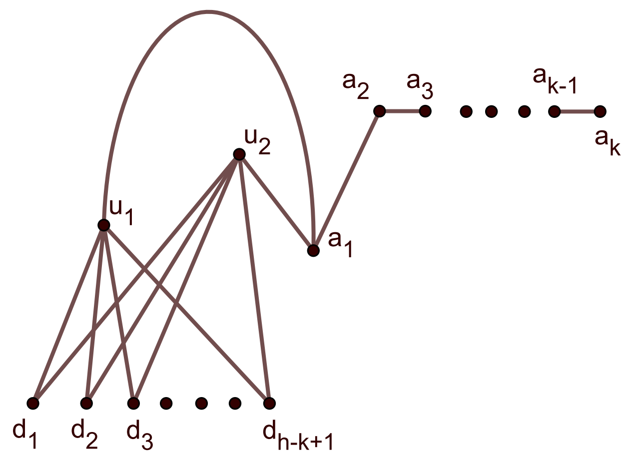

Figure 2. Graph G with $\gamma_{1nsg}(G) = h$

Let $S = \{d_1, d_2, \ldots, d_{h-k+1}, a_2, a_3, \ldots, a_k\}$. Then, the set S is an EGD set and $< V - S >$ is connected. As a result, S is the min NEGD set of G. Thus, $\gamma_{1nsg}(G) = h$. ■

Example 4.1.

Take a look at the graph given in Figure 3. The set $S = \{d_1, d_4\}$ is a minimum nonsplit edge geodetic dominating set of G in Figure 3a. As a result of this, $\gamma_{1nsg}(G) = 2$. The set $S = \{t_1, t_3, t_5, t_6, t_7\}$ in Figure 3b is a minimum nonsplit edge geodetic dominating set of G. Therefore, $\gamma_{1nsg}(G) > 2$.

Figure 3a: Graph G with $2 = h = \gamma_{1nsg}(G)$

Figure 3b: Graph G with $2 < h = \gamma_{1nsg}(G)$

Figure 3. Graph G with $\gamma_{1nsg}(G) = h$

Theorem 4.2.

For a connected graph G with $2 \leq h \leq l$, we have $g_1(G) = h$ and $\gamma_{1nsg}(G) = l$.

Proof:

This theorem can be proved by considering the following 3 cases.

Case i: $h < l$.

Let $C_6 : q_1, q_2, q_3, q_4, q_5, q_6, q_1$. Let the graph H_1 be obtained from C_6 by including h new points $q_7, m_i(1 \leq i \leq h-1)$ and connecting each $m_i, (1 \leq i \leq h-1)$ to the point q_1. Let the graph H_2 be obtained from H_1 by including the edges q_6q_7,q_2q_7 and q_4q_7. Let H_3 be the $(l-h-1)$ copies of the path $P_i : d_i, b_i, (1 \leq i \leq l-h-1)$. Let the graph G be obtained from H_2 and H_3 by joining every $b_i, (1 \leq i \leq l-h-1)$ to the point q_4 and connecting every $d_i, (1 \leq i \leq l-h-1)$ to the point q_1. Figure 4 represents the resulting graph G.

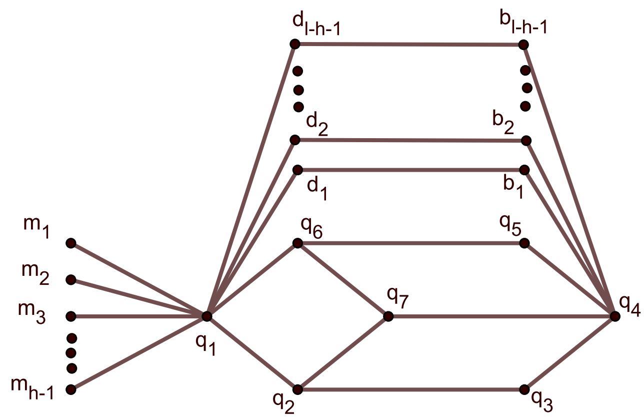

Figure 4. Graph G with $g_1(G) = h < \gamma_{1nsg}(G) = l$

Assume that $S = \{m_1, m_2, \ldots, m_{h-1}\}$ is a collection of all simplicial points of G. As we are aware, every EG set includes S, but S isn't an EG set of G. Let $S_1 = S \cup \{q_4\}$. Evidently, S_1 is the minimum EG set and $g_1(G) = h$. Let $S_2 = S_1 \cup \{q_7, b_1, b_2, \ldots, b_{l-h-1}\}$. Then, S_2 is the min NEGD set of G. Hence, $\gamma_{1nsg}(G) = l$.

Case ii: $h + 1 = l$.

Let $C_6 : q_1, q_2, q_3, q_4, q_5, q_6, q_1$. Let the graph G be obtained from C_6 by including $h-1$ new points $t_j, (1 \leq j \leq h-1)$ and joining every $t_j, (1 \leq j \leq h-1)$ to q_1. Also, add the edge q_2q_5. Figure 5 represents the resulting graph G.

Let $S = \{t_1, t_2, \ldots, t_{h-1}\}$ denote the collection of all simplicial points of G. According to Theorem 2.3, every NEGD set includes S, but S isn't an EG set of G. Assume that, $S_1 = S \cup \{q_4\}$. Then S_1 is an EG set and $g_1(G) = h$. Let $S_2 = S_1 \cup \{q_1\}$. Therefore, S_2 is the minimum EGD set of G and $< V - S_2 >$ is connected. As a result, S_2 is the min NEGD set. Hence, $\gamma_{1nsg}(G) = h + 1 = l$.

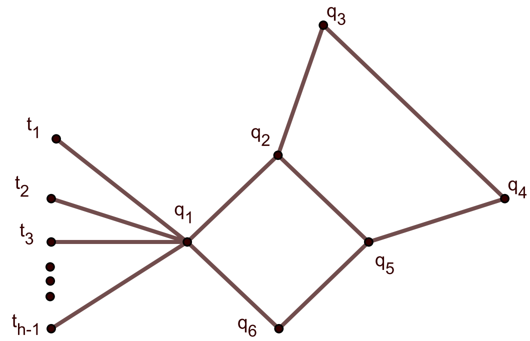

Figure 5. Graph G with $g_1(G) = h + 1 = \gamma_{1nsg}(G) = l$

Case iii: $h = l$.

Let $P_3 : v_1, v_2, v_3$. Let H be the $(h - 1)$ copies of the path $P_i : f_i, j_i$. Let the graph G be obtained from P_3 and H by connecting every f_k, $(1 \leq k \leq h - 1)$ to the point v_1 and joining every j_k , $(1 \leq k \leq h - 1)$ to the point v_2. Figure 6 represents the resulting graph G.

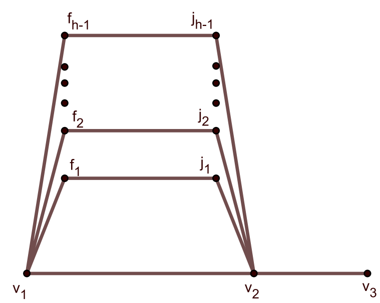

Figure 6. Graph G with $g_1(G) = h = \gamma_{1nsg}(G) = l$

Consider $S = \{f_1, f_2, \ldots, f_{h-1}, v_3\}$. Then, S is both an EG set and the NEGD set of G. Hence, $g_1(G) = \gamma_{1nsg}(G) = h$. ■

Example 4.2.

Take a look at the graph given in the Figure 7. The set $S = \{u_1, u_3, u_4\}$ is a minimum edge geodetic set of G and the set $S' = \{v_1, v_2, v_4, v_5, v_6, v_7\}$ is a minimum nonsplit edge geodetic dominating set of G in Figure 7a. As a result, $g_1(G) = 3 < \gamma_{1nsg}(G) = 6$. The set $S = \{v_1, v_5\}$ and $S' = \{v_1, v_5, v_8\}$ in Figure 7b is a minimum edge geodetic set of G and a minimum nonsplit edge geodetic dominating set of G respectively. Therefore, $g_1(G) = 2$ and $\gamma_{1nsg}(G) = 2 + 1 = 3$. In Figure 7c, $S = \{w_4, w_8, w_9\}$ is both the minimum geodetic and nonsplit geodetic dominating set of G. Thus, $g_1(G) = 3 = \gamma_{1nsg}(G)$.

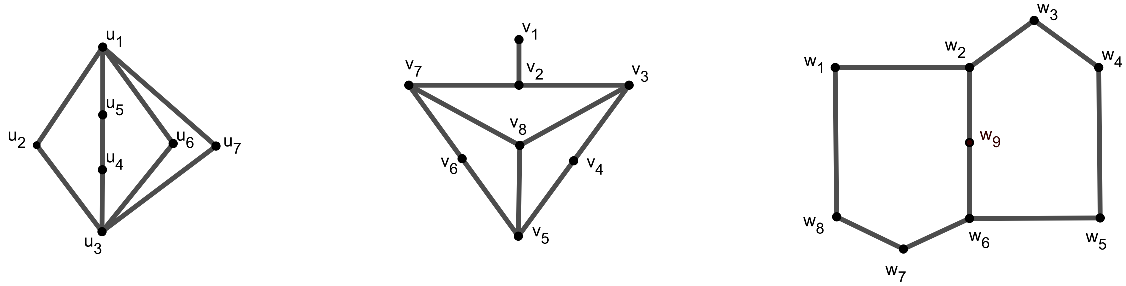

Figure 7a: Graph G with $g_1(G) = h < \gamma_{1nsg}(G) = l$

Figure 7b: Graph G with $g_1(G) = h = \gamma_{1nsg}(G) = h + 1 = l$

Figure 7c: Graph G with $g_1(G) = h = \gamma_{1nsg}(G) = l$

Figure 7. Graph G with $g_1(G) = h$ and $\gamma_{1nsg}(G) = l$

5. Conclusion

This work initiates the study of the nonsplit edge geodetic domination number of a graph. It says that nonsplit edge geodetic domination number doesn't exist if all the points of graph G are simplicial points. Also, nonsplit edge geodetic domination number of a graph is always greater than or equal to edge geodetic domination number of a graph. Therefore, nonsplit edge geodetic domination is more useful than edge geodetic domination. The idea of nonsplit edge geodetic domination number of a graph can be extended to study minimal nonsplit geodetic number of a graph, the minimal nonsplit edge geodetic number of a graph and the forcing nonsplit edge geodetic domination number of a graph. These results can be used to study in the improvement of town planning and communication network designing.

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