Applications and Applied Mathematics: An International Journal (AAM)

Volume 17
Issue 3 *Special Issue No. 10 (October 2022)*

Article 2

10-2022

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Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Special Issue No. 10 (October 2022), pp. 17 – 30

# Controllability Results for Nonlinear Impulsive Functional Neutral Integrodifferential Equations in $n$ - Dimensional Fuzzy Vector Space

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Received: November 20, 2021; Accepted: August 13, 2022

## Abstract

In this paper, we concentrated to study the controllability of fuzzy solution for nonlinear impulsive functional neutral integrodifferential equations with nonlocal condition in $n$ - dimensional vector space. Moreover, we obtained controllability of fuzzy result for the normal, convex, upper semi-continuous and compactly supported interval fuzzy number. Finally, an example was provided to reveal the application of the result.

**Keywords:** Neutral functional differential equation; Impulsive partial differential equation; Fuzzy sets; Fixed point

**MSC 2010 No.:** 34K40, 35R12, 94D05, 47H10

# 1. Introduction

In various fields of engineering and physics, many problems that are related to linear viscoelasticity, nonlinear elasticity had mathematical models and were described by the problems of differential or integral equations or integrodifferential equations. Fuzzy set had been introduced by Zadeh $(1965)$ as an extension of the classical notion of set. Classical set theory allowed the membership of the elements in the set in binary terms, a bivalent condition an element either belongs or did not belong to the set. Fuzzy set theory permits the gradual assessment of the membership of elements in a set, described with the aid of a membership function valued in the real unit interval $[0, 1]$. Balasubramaniam et al. $(2004)$ proved existence and uniqueness of fuzzy solution for semilinear fuzzy integrodifferential equations with nonlocal conditions. A differential and integral calculus for fuzzy - set - valued, shortly fuzzy - valued, mappings were developed in recent papers of Dubois et al. $(1982)$ and Puri et al. $(1986)$. Bede et al. (2007) first-order linear differential equations under generalized differentiability. Chang et al. $(2008)$ controllability of mixed volterra - fredholm - type integrodifferential inclusions in Banach spaces.

Hullermeier $(1997)$ provided an approach to modeling and simulation of uncertain dynamical systems. Hernandez et al. $(2007)$ proved the existence of solutions for impulsive partial neutral functional differential equations. Kaleva $(1987)$ established fuzzy differential equations, fuzzy sets and systems. A large number of existence and controllability had been established by several authors (Narayanamoorthy et al. $(2013)$, Radhakrishnan et al. $(2013)$ and Nagarajan et al. $(2022)$) who proved the existence results for the nonlinear first order fuzzy neutral integrodifferential equations. Wang et al. $(2002)$ established controllability of abstract neutral functional differential systems with infinite delay, dynamics of continuous, discrete and impulsive systems. Wang et al. $(2007)$ proved on fuzzy $n$ - cell numbers and $n$ - dimension fuzzy vectors, fuzzy sets and systems. Wan et al. $(2012)$ described approximate controllability for abstract measure differential systems. Kwun et al. $(2009)$ evaluated nonlocal controllability for the semilinear fuzzy integrodifferential equations in $n$ - dimensional fuzzy vector space.

# 2. Existence and Uniqueness of Fuzzy Solution

In this section, we could be frequently confronted with a differential equation, and we studied the controllability of fuzzy solution for nonlinear impulsive functional neutral integrodifferential equations with nonlocal condition in $n$ - dimensional vector space,

$$\begin{aligned}(x_i(t) - h(t, x_{it}))' &= \mathbb{A}_i x_i + \int_0^t k(t, s, x_{it}(s))ds, \\ &\quad + f(t, x_{it}) + z(t), \quad t \in \mathbb{J} = [0, b] \text{ on } (\mathbb{E}^i_{\mathbb{N}})^n, \quad (1)\\ x_i(t) + g(x_i) &= \phi_i(t), \quad (2)\\ \Delta(x_i(t_k)) &= I_k(x_i(t_k^-)), \quad (3)\end{aligned}$$

where $b > 0$, $\mathbb{A} : \mathbb{J} \to (\mathbb{E}^i_{\mathbb{N}})^n$ was a fuzzy coefficient, $(\mathbb{E}^i_{\mathbb{N}})^n$ was the set of all upper semicontinuous convex normal fuzzy set with bounded $\theta$ - level intervals, $f : J \times (\mathbb{E}^i_{\mathbb{N}})^n \to (\mathbb{E}^i_{\mathbb{N}})^n$ was a nonlinear continuous function, $g : J \times (\mathbb{E}^i_{\mathbb{N}})^n \to (\mathbb{E}^i_{\mathbb{N}})^n$ was a nonlinear continuous function, $h : \mathbb{J} \times (\mathbb{E}^i_{\mathbb{N}})^n \to (\mathbb{E}^i_{\mathbb{N}})^n$ was a nonlinear continuous function, $\Delta(x_i(t_k)) = x_i(t_k^+) - x_i(t_k^-), I_k \in C((\mathbb{E}^i_{\mathbb{N}})^n, (\mathbb{E}^i_{\mathbb{N}})^n)$ were continuous functions and $\phi_i : \mathbb{J} \to (\mathbb{E}^i_{\mathbb{N}})^n$. We consider the existence and uniqueness of fuzzy solutions for the nonlinear first order impulsive fuzzy functional neutral integrodifferential equations with nonlocal conditions $(1)-(3)$ $(u \equiv 0)$. We defined that

$$\begin{aligned} &\mathbb{A} = (\mathbb{A}_1, \mathbb{A}_2, ..., \mathbb{A}_n), \quad x = (x_1, x_2, ...x_n), \quad f = (f_1, f_2, ...f_n), \quad k = (k_1, k_2, ...k_n), \\ &u = (u_1, u_2, ...u_n), \quad g = (g_1, g_2, ...g_n), \quad h = (h_1, h_2, ..., h_n), \quad \mathbb{I}_k = (\mathbb{I}_1, \mathbb{I}_2, ..., \mathbb{I}_n), \\ &x_0 = (x_{0_1}, x_{0_2}, ...x_{0_n}). \end{aligned} \tag{4}$$

Then, $\mathbb{A}, x, f, k, x_0, y, g, \mathbb{I}_k \in (\mathbb{E}^i_{\mathbb{N}})^n$. Instead of equations, we considered the following fuzzy integrodifferential equations in $(\mathbb{E}^i_{\mathbb{N}})^n$:

$$\frac{d}{dt}(x(t) - h(t, x(t))) = \mathbb{A}(t)\Big[x(t) + \int_0^t k(t-s)x(s)ds\Big] + f(t, x(t)) + u(t), \quad t \in J = [0, b], \tag{5}$$

$$x(0) + g(x) = \phi(t), \tag{6}$$

$$\Delta(x_i(t_k)) = I_k(x_i(t_k^-)), \tag{7}$$

where $\mathbb{A} : \mathbb{J} \to (\mathbb{E}^i_{\mathbb{N}})^n$ was fuzzy coefficient, $\mathbb{E}^i_{\mathbb{N}}$ was the fuzzy set of all upper semicontinuous, convex, normal fuzzy numbers on $\mathbb{R}$ with $\mathbb{E}^i_{\mathbb{N}} \neq \mathbb{E}^i_{\mathbb{N}}$, $f : \mathbb{J} \times (\mathbb{E}^i_{\mathbb{N}})^n \to (\mathbb{E}^i_{\mathbb{N}})^n$ , $h : \mathbb{J} \times (\mathbb{E}^i_{\mathbb{N}})^n \to (\mathbb{E}^i_{\mathbb{N}})^n$, $g : (\mathbb{E}^i_{\mathbb{N}})^n \to (\mathbb{E}^i_{\mathbb{N}})^n$ were all nonlinear functions and satisfied the global Lipschitz conditions,

(H1) $d_{\mathbb{H}}([g(x_t(.))]^\theta, [g(y_t(.))]^\theta) \le \Lambda_g d_{\mathbb{H}}([x_t(.)]^\theta, [y_t(.)]^\theta)$ ,
(H2) $d_{\mathbb{H}}([f(s, x_t(s))]^\theta, [f(s, y_t(s))]^\theta) \le \Lambda_f d_{\mathbb{H}}([x_t(.)]^\theta, [y_t(.)]^\theta)$ ,
(H3) $d_{\mathbb{H}}([h(s, x_t(.))]^\theta, [h_t(s, y_t(.))]^\theta) \le \Lambda_h d_{\mathbb{H}}([x_t(.)]^\theta, [y_t(.)]^\theta)$,
(H4) $d_{\mathbb{H}}([h(0, x_0 - g(x_t))]^\theta, [h(0, x_0 - g(y_t))]^\theta) \le \Lambda_h + \Lambda_{h_1}\Lambda_g d_{\mathbb{H}}([x_t(s))]^\alpha, [x_t(s))]^\theta)$ ,
(H5) $d_{\mathbb{H}}([\mathbb{I}_k(s, x_t(s))]^\theta, [f(s, y_t(s))]^\theta) \le \Lambda_{\mathbb{I}} d_{\mathbb{H}}([x_t(.)]^\theta, [y_t(.)]^\theta)$,

for all $x(.), y(.) \in (\mathbb{E}^i_{\mathbb{N}})^n$ and $\Lambda_g$,$\Lambda_h$,$\Lambda_{h_1}$, $\Lambda_f$, $\Lambda_k$ are positive.

The solution of the $(5)-(7)$ format is

$$x(\cdot) = \mathbb{S}(\cdot)[\phi(\cdot) - h(0, \phi(\cdot))] + h(t, x_s(\cdot)) + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s(\cdot))ds + \int_0^t \mathbb{S}(t-s)f(s, x_s(\cdot))ds + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-)). \tag{8}$$

**Theorem 2.1.**

Let $b > 0$. If hypothesis $(H1)-(H5)$ hold, then for every $\phi_i \in (\mathbb{E}^i_{\mathbb{N}})^n$, $(10)$ have a unique fuzzy solution $x \in \mathbb{C}([0, b] : (\mathbb{E}^i_{\mathbb{N}})^n)$.

***Proof:***

For each $x_t(t) \in (\mathbb{E}^i_{\mathbb{N}})^n$ and $t \in \mathbb{J} = [0, b]$, define $\psi x_t(t) \in (\mathbb{E}^i_{\mathbb{N}})^n$ by

$$(\psi x)(t) = \mathbb{S}(t)[(\phi - g(x)) - h(0, \phi - g(x))] + h(t, x_s) + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s)ds + \int_0^t \mathbb{S}(t-s)f(s, x_s)ds + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-)), \quad t \in \mathbb{J} = [0, b].$$

Thus, $\psi x : [0, b] \to (\mathbb{E}^i_{\mathbb{N}})^n$ is continuous, so $\psi$ is a mapping from $\mathbb{C}([0, b] : (\mathbb{E}^i_{\mathbb{N}})^n)$ into itself. Properties of $d_{\mathbb{H}}$, and inequalities, we have following inequalities. For $x, y \in \mathbb{C}([0, b] : (\mathbb{E}^i_{\mathbb{N}})^n)$. Let $D = d_{\mathbb{H}}([(\psi x)(t)]^\theta, [(\psi y)(t)]^\theta)$,

$$\begin{aligned}
D = {} & d_{\mathbb{H}}([\mathbb{S}(t)[(\phi - g(x)) - h(0, \phi - g(x))] + h(t, x_s) \\
& + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s)ds + \int_0^t \mathbb{S}(t-s)f(s, x_s)ds \\
& + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-))]^\theta, [\mathbb{S}(t)[(\phi - g(y)) - h(0, \phi - g(y))] + h(t, y_s(t)) \\
& + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, y_s(s))ds + \int_0^t \mathbb{S}(t-s)f(s, y_s)ds + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(y(t_k^-))]^\theta) \\
\leq {} & d_{\mathbb{H}}([\mathbb{S}(t)((\phi(t) - g(x)) - h(0, \phi(t) - g(x))) \\
& + h(t, x_s(t))]^\theta, [\mathbb{S}(t)(\phi(t) - g(y) - h(0, \phi(t) - g(y))) + h(t, y_s(t))]^\theta) \\
& + d_{\mathbb{H}}([\int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s(s))ds]^\theta, [\int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, y_s(s))ds]^\theta) \\
& + d_{\mathbb{H}}([\int_0^t \mathbb{S}(t-s)f(s, x_s(s))ds]^\theta, [\int_0^t \mathbb{S}(t-s)f(s, y_s(s))ds]^\theta) \\
& + d_{\mathbb{H}}([\sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-))]^\theta, [\sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(y(t_k^-))]^\theta), \\
\leq {} & \Lambda_s \max_{1\leq i\leq n} \left\{ |(g^\theta_{il}(x) - g^\theta_{il}(y))|, |(g^\theta_{ir}(x) - g^\theta_{ir}(y))| \right\} \\
& + \Lambda_s \max_{1\leq i\leq n} \left\{ \begin{pmatrix} |(h^\theta_{il}(0, \phi(t) - g^\theta_{il}(x_s)) - h^\theta_{il}(0, \phi(t) - g^\theta_{il}(y_s)))|, |(g^\theta_{ir}(x) - g^\theta_{ir}(y))|, \\ |(h^\theta_{ir}(0, \phi(t) - g^\theta_{ir}(x_s)) - h^\theta_{ir}(0, \phi(t) - g^\theta_{ir}(y_s)))| \end{pmatrix} \right\} \\
& + \Lambda_s \max_{1\leq i\leq n} \left\{ |(h^\theta_{il}(t, x_s) - h^\theta_{il}(t, y_s))|, |(h^\theta_{ir}(t, x_s) - h^\theta_{ir}(t, y_s))| \right\} \\
& + \Lambda_s \max_{1\leq i\leq n} \left\{ \begin{pmatrix} |(h^\theta_{il}(0, \phi(t) - g^\theta_{il}(x_s)) - h^\theta_{il}(0, \phi(t) - g^\theta_{il}(y_s)))|, |(g^\theta_{ir}(x) - g^\theta_{ir}(y))|, \\ |(h^\theta_{ir}(0, \phi(t) - g^\theta_{ir}(x_s)) - h^\theta_{ir}(0, \phi(t) - g^\theta_{ir}(y_s)))| \end{pmatrix} \right\} \\
& + M_A \Lambda_s \max_{1\leq i\leq n} \left\{ |h^\theta_{il}(s, x_s) - h^\theta_{il}(s, y_s)|, |h^\theta_{ir}(s, x_s) - h^\theta_{ir}(s, y_s)| \right\} ds \\
& + \Lambda_s \int_0^t \max_{1\leq i\leq n} \left\{ |f^\theta_{il}(s, x_s) - f^\theta_{il}(s, y_s)|, |f^\theta_{ir}(s, x_s) - f^\theta_{ir}(s, y_s)| \right\} ds \\
& + \Lambda_s \max_{1\leq i\leq n} \left\{ \begin{pmatrix} \sum_{0<t_k<t} |(\mathbb{I}_k)^\theta_{il}(x(t_k^-)) - (\mathbb{I}_k)^\theta_{il}(y(t_k^-))|, \\ \sum_{0<t_k<t} |(\mathbb{I}_k)^\theta_{ir}(x(t_k^-)) - (\mathbb{I}_k)^\theta_{ir}(y(t_k^-))| \end{pmatrix} \right\} ds
\end{aligned}$$

$$\begin{aligned}
&\leq (\Lambda_s(\Lambda_g + \Lambda_(h_1) + \Lambda_h\Lambda_g) + \Lambda_h)d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) + \Lambda_s M_A \Lambda_h \int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds \\
&\quad + \Lambda_s\Lambda_f \int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds + \Lambda_s\Lambda_{\mathbb{I}} d_{\mathbb{H}}[x_s]^\theta, [y_s]^\theta \\
&\leq (\Lambda_s(\Lambda_g + \Lambda_(h_1) + \Lambda_h\Lambda_g + \Lambda_{\mathbb{I}}) + \Lambda_h)d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) \\
&\quad + \Lambda_s(M_A\Lambda_h + \Lambda_f)\int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds \\
&= \Omega_1 d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) + \Omega_2 \int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds,
\end{aligned}$$

where $\Omega_1 = \Lambda_s(\Lambda_g + \Lambda_(h_1) + \Lambda_h\Lambda_g + \Lambda_{\mathbb{I}}) + \Lambda_h$ and $\Omega_2 = \Lambda_s(M_A\Lambda_h + \Lambda_f)$.

Therefore,

$$\begin{aligned}
d_{\mathbb{H}}(\psi x(t), \psi y(t)) &= \sup_{\theta\in[0,1]} d_{\mathbb{H}}([x_s(t)]^\theta, [y_s(t)]^\theta) \\
&\leq \Omega_1 \sup_{\theta\in[0,1]} d_{\mathbb{H}}([x_s(t)]^\theta, [y_s(t)]^\theta) + \Omega_2 \sup_{\theta\in[0,1]} \int_0^t d_{\mathbb{H}}([x_s(t)]^\theta, [y_s(t)]^\theta) \\
&\leq \Omega_1 d_{\mathbb{H}}(x_s(t), y_s(t)) + \Omega_2 \int_0^t d_{\mathbb{H}}(x_s(t), y_s(t))ds.
\end{aligned}$$

Hence,

$$\begin{aligned}
H_1(\psi x(t), \psi y(t)) &= \sup_{t\in\mathbb{J}=[0,b]} d_{\mathbb{H}}(\psi x(t), \psi y(t)) \\
&\leq \Omega_1 \sup_{t\in\mathbb{J}=[0,b]} d_{\mathbb{H}}(x_s(t), y_s(t)) + \Omega_2 \int_0^t \sup_{t\in\mathbb{J}=[0,b]} d_{\mathbb{H}}(x_s(t), y_s(t))dt \\
&\leq \Omega_1 H_1(x_s(t), y_s(t)) + \Omega_2 b\mathbb{H}_1(x_s(t), y_s(t)) \\
&\leq (\Omega_1 + \Omega_2 b)\mathbb{H}_1(x_s(t), y_s(t)).
\end{aligned}$$

Then, by assumption $\psi$ is a contraction mapping. Therefore, Equations $(5) - (7)$ have a unique fixed point $x \in \mathbb{C}(\mathbb{J} : \mathbb{E}_{\mathbb{N}}^i)^n$. ■

## 3. Controllability of fuzzy solution

During this segment, we were showed for the controller term in Equation $(1) - (3)$ in the control system, the solution of the form

$$\begin{aligned}
x(t) &= \mathbb{S}(t)[\phi - g(x) - h(0, \phi - g(x))] + h(t, x_s) + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s)ds \\
&\quad + \int_0^t \mathbb{S}(t-s)f(s, x_s)ds + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-)) + \int_0^t \mathbb{S}(t-s)z(s)ds. \qquad (9)
\end{aligned}$$

**Theorem 3.1.**

Equation (11) is controllable if there exist $z(t)$ such that the fuzzy solution $x(t)$ of $(11)$ satisfied $x(b) = \phi^1 - g(x)$, that is $[x(b)]^\theta = [\phi^1 - g(x)]^\theta$, where $\phi^1$ is target set.

***Proof:***

We presuppose that the control system with respect to the nonlinear control system $(11)$ was non-local controllable.

Then,

$$x(b) = \mathbb{S}(b)[\phi(t) - g(x)] + \int_0^t \mathbb{S}(b-s)z(s)ds = \phi^1 - g(x)$$

$$\begin{aligned}[x(b)]^\theta &= [\mathbb{S}(b)[\phi(t) - g(x)] + \int_0^b \mathbb{S}(b-s)z(s)ds]^\theta \\ &= \mathbb{S}_l^\theta(b)[\phi_l^\theta(t) - g_l^\theta(x)] + \int_0^b \mathbb{S}_l^\theta(b-s) \\ &\quad \times z_l^\theta(s)ds, [\mathbb{S}_r^\theta(b)[\phi_r^\theta(t) - g_r^\theta(x)] \\ &\quad + \int_0^b \mathbb{S}_r^\theta(b-s)z_r^\theta(s)ds] \\ \left[(\phi^1)\right]_l^\theta - g_l^\theta(x), (\phi^1)_r^\theta - g_r^\theta(x)] &= [\phi^1 - g(x)]^\theta.\end{aligned}$$

Define the fuzzy mapping $\zeta : \mathbb{P}(\mathbb{R}^n) \to (\mathbb{E}_\mathbb{N}^i)^n$ by

$$\zeta^\theta(v) = \begin{Bmatrix} \int_0^t \mathbb{S}(t-s)v(s)ds, v \subset \overline{\Gamma}_z \\ 0, \text{otherwise} \end{Bmatrix},$$

where $\overline{\Gamma}_z$ is closure of support $z$. Then, there exist $\zeta_i : \mathbb{P}(\mathbb{R}^n) \to (\mathbb{E}_\mathbb{N}^i)^n, i = 1, 2, \dots, n.$

$\zeta_i^\theta(v_i) = \left\{ \int_0^t \mathbb{S}(t-s)v_i(s)ds,\ v_i \subset \overline{\Gamma}_z \right\}$. Then, $\zeta_i^\theta(i = l, r)$ such that

$$\zeta_{il}^\theta(v_{il}) = \int_0^t \mathbb{S}_{il}^\theta(t-s)v_{il}(s)ds, v_{il} \in [z_{il}^\theta, z_i^1],$$
$$\zeta_{ir}^\theta(v_{ir}) = \int_0^t \mathbb{S}_{ir}^\theta(t-s)v_{ir}(s)ds, v_{ir} \in [z_i^1, z_{ir}^\theta].$$

We presuppose that $\zeta_i^{\theta}$'s are bijective mappings. Hence, the $\theta-$ set of $z(s)$ are

$$\begin{aligned}
[z(s)]^{\theta} &= \Pi_{i=1}^{n}[z_i(s)]^{\theta} = \Pi_{i=1}^{n}[z_{il}^{\theta}(s), z_{ir}^{\theta}(s)] \\
&= (\zeta_{il}^{\theta})^{-1}((\phi^1)_{il}^{\theta}) - g_{il}^{\theta}(x) - \mathbb{S}_{il}^{\theta}(b)[\phi_{il}^{\theta}(t) - g_{il}^{\theta}(x) - h_{il}^{\theta}(0, \phi(t) - g(x))] \\
&\quad -h_{il}^{\theta}(t, x_s(t)) - \int_0^t \mathbb{S}_{il}^{\theta}(t-s)\mathbb{A}_{il}^{\theta}h_{il}^{\theta}(s, x_s)ds - \int_0^t \mathbb{S}_{il}^{\theta}(t-s)f_{il}^{\theta}(s, x_s)ds \\
&\quad - \sum_{0<t_k<t} \mathbb{S}_{il}^{\theta}(t-t_k)(\mathbb{I}_k)_{il}^{\theta}(x(t_k^-)), (\zeta_{il}^{\theta})^{-1}((\phi^1)_{ir}^{\theta}) - g_{ir}^{\theta}(x) - \mathbb{S}_{ir}^{\theta}(b)[\phi_{ir}^{\theta}(t) - g_{ir}^{\theta}(x) \\
&\quad -h_{ir}^{\theta}(0, \phi(t) - g(x))] - h_{ir}^{\theta}(t, x_s(t)) - \int_0^t \mathbb{S}_{ir}^{\theta}(t-s)\mathbb{A}_{ir}^{\theta}h_{ir}^{\theta}(s, x_s)ds \\
&\quad - \int_0^t \mathbb{S}_{ir}^{\theta}(t-s)f_{ir}^{\theta}(s, x_s)ds - \sum_{0<t_k<t} \mathbb{S}_{ir}^{\theta}(t-t_k)(\mathbb{I}_k)_{ir}^{\theta}(x(t_k^-)).
\end{aligned}$$

Then, substituting this appearance into equation $(4.1)$ yields $\theta$-level set of $x(b)$ for each $i = 1, 2, \ldots, n$,

$$\begin{aligned}
[x_i(b)]^{\theta} &= \mathbb{S}_{il}^{\theta}(b)[\phi_{il}^{\theta}(t) - g_{il}^{\theta}(x) - h_{il}^{\theta}(0, \phi_{il}^{\theta}(t) - g_{il}^{\theta}(x))] + h_{il}^{\theta}(b, x_s(b)) \\
&\quad + \int_0^b \mathbb{S}_{il}^{\theta}(b-s)\mathbb{A}_{il}^{\theta}h_{il}^{\theta}(s, x_s(s))ds + \int_0^b \mathbb{S}_{il}^{\theta}(b-s)f_{il}^{\theta}(s, x_s(s))ds \\
&\quad + \sum_{0<b_k<b} \mathbb{S}_{il}^{\theta}(b-b_k)(\mathbb{I}_k)_{ir}^{\theta}(x(b_k^-)) + \zeta_{il}^{\theta}(\zeta_{il})^{-1}((\phi^1)_{il}^{\theta}) - \mathbb{S}_{il}^{\theta}(b)[\phi_{il}^{\theta}(t) - g_{il}^{\theta}(x) \\
&\quad -h_{il}^{\theta}(0, \phi(t) - g(x))] - h_{il}^{\theta}(b, x_s(b)) - \int_0^b \mathbb{S}_{il}^{\theta}(b-s)\mathbb{A}_{il}^{\theta}h_{il}^{\theta}(s, x_s(s))ds \\
&\quad - \int_0^b \mathbb{S}_{il}^{\theta}(b-s)f_{il}^{\theta}(s, x_s(s))ds - \sum_{0<b_k<b} \mathbb{S}_{il}^{\theta}(b-b_k)(\mathbb{I}_k)_{il}^{\theta}(x(b_k^-)), \\
&\quad \mathbb{S}_{ir}^{\theta}(b)[\phi_{ir}^{\theta}(t) - g_{ir}^{\theta}(x) - h_{ir}^{\theta}(0, \phi(t) - g(x))] + h_{ir}^{\theta}(b, x_s(b)) \\
&\quad + \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)\mathbb{A}_{ir}^{\theta}h_{ir}^{\theta}(s, x_s(s))ds + \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)f_{ir}^{\theta}(s, x_s(s))ds \\
&\quad + \sum_{0<b_k<b} \mathbb{S}_{ir}^{\theta}(b-b_k)(\mathbb{I}_k)_{ir}^{\theta}(x(b_k^-)) + \zeta_{ir}^{\theta}(\zeta_{ir})^{-1}\phi_{ir}^{\theta}(t) - g_{ir}^{\theta}(x) \\
&\quad -\mathbb{S}_{ir}^{\theta}(b)[\phi_{ir}^{\theta}(t) - g_{ir}^{\theta}(x) - h_{ir}^{\theta}(0, \phi_{ir}^{\theta}(t) - g_{ir}^{\theta}(x_s))] - h_{ir}^{\theta}(b, x_s(b)) \\
&\quad - \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)\mathbb{A}_{ir}^{\theta}h_{ir}^{\theta}(s, x_s(s))ds - \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)f_{ir}^{\theta}(s, x_s(s))ds \\
&\quad - \sum_{0<b_k<b} \mathbb{S}_{ir}^{\theta}(b-b_k)(\mathbb{I}_k)_{ir}^{\theta}(x(b_k^-)) \\
&= [(\phi^1)_{il}^{\theta} - (g)_{il}^{\theta}(x), (\phi^1)_{ir}^{\theta} - (g)_{ir}^{\theta}(x)] \\
&= [(\phi^1 - g(x))_i]^{\theta}.
\end{aligned}$$

Therefore, $[x(b)]^{\theta} = \Pi_{i=1}^{n}[x_i(b)]^{\theta} = \Pi_{i=1}^{n}[(\phi^1 - g(x))_i]^{\theta} = [\phi^1 - g(x)]^{\theta}$.

We now set

$$\xi x(t) = \mathbb{S}(t)[\phi - g(x) - h(0, \phi - g(x))] + h(t, x_s) + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s)ds + \int_0^t \mathbb{S}(t-s)$$
$$\times f(s, x_s)ds + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-)) + \int_0^t \mathbb{S}(t-s)\zeta^{-1}((\phi^1) - g(x) - \mathbb{S}(b)[\phi$$
$$-h(0, \phi - g(x))] - h(t, x_s(t)) - \int_0^b \mathbb{S}(b-s)\mathbb{A}h(s, x_s(s))ds - \int_0^b \mathbb{S}(b-s)$$
$$\times f(s, x_s(s))ds - \sum_{0<t_k<b} S(b-t_k)\mathbb{I}_k(x(t_k^-)),$$

where the fuzzy mapping $\zeta^{-1}$ satisfied above declaration. Now notice that $\xi x(b) = \phi^1 - g(x)$, which means that the control $z(s)$ steers Equation $(11)$ from the derivation to $\phi^1$ in the time b provided we could obtained a fixed point of the nonlinear operator. Assume the hypothesis $(H5)$. The method $(11)$ was linear $z \equiv 0$ was controllable. ■

**Theorem 3.2.**

Supposed that the hypothesis $(H1) - (H5)$ were satisfied. Then, the equation $(11)$ is a nonlocal controllable.

***Proof:***

We could simply verify that $\xi$ was continuous from $C([0, b] : (\mathbb{E}_{\mathbb{N}}^i)^n)$ to itself. For any $x, y \in \mathbb{C}([0, b] : (\mathbb{E}_{\mathbb{N}}^i)^n)$. Let $D_1 = d_{\mathbb{H}}([\xi x(t)]^\theta, [\xi y(t)]^\theta)$,

$$D_1 \leq d_{\mathbb{H}}([\mathbb{S}(t)[g(x) + h(0, \phi - g(x))] + h(t, x_s) + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, x_s)ds$$
$$+ \int_0^t \mathbb{S}(t-s)f(s, x_s)ds + \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{I}_k(x(t_k^-))]^\theta, [\mathbb{S}(t)[g(y)$$
$$-h(0, \phi - g(y))] + h(t, y_s) + \int_0^t \mathbb{S}(t-s)\mathbb{A}h(s, y_s)ds + \int_0^t \mathbb{S}(t-s)f(s, y_s)ds$$
$$+ \sum_{0<t_k<t} \mathbb{S}(t-t_k)\mathbb{S}_k(y(t_k^-))]^\theta) + d_{\mathbb{H}}([\int_0^t \mathbb{S}(t-s)\zeta^{-1}(\phi^1 - g(x)$$
$$-\mathbb{S}(b)[g(x) + h(0, \phi - g(x))] - h(s, x_s) - \int_0^b \mathbb{S}(t-s)\mathbb{A}h(s, x_s)ds$$
$$- \int_0^b \mathbb{S}(b-s)f(s, x_s) - \sum_{0<t_k<t} \mathbb{S}(b-t_k)\mathbb{I}_k(x(t_k^-))]^\theta, [\int_0^t \mathbb{S}(t-s)\zeta^{-1}((\phi^1)$$
$$-g(y) - \mathbb{S}(b)[g(y) + h(0, \phi - g(y))] - h(s, y_s) - \int_0^b \mathbb{S}(b-s)\mathbb{A}h(s, y_s)ds$$

$$
\begin{aligned}
&-\int_0^b \mathbb{S}(b-s)f(s,y_s)ds - \sum_{0<t_k<t} \mathbb{S}(b-t_k)\mathbb{I}_k(y(t_k^-))]^\theta) \\
D_1 \leq\ & (\Lambda_s(\Lambda_g + \Lambda_(h_1) + \Lambda_h\Lambda_g + \Lambda_{\mathbb{I}}) + \Lambda_h)d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) \\
&+\Lambda_s(M_A\Lambda_h + \Lambda_f)\int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds \\
&+\Lambda_h d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) + \Lambda_s(\Lambda_s(\Lambda_g + \Lambda_(h_1) + \Lambda_h\Lambda_g + \Lambda_{\mathbb{I}}) + \Lambda_h)\int_0^b d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds \\
&+\Lambda_s \int_0^t \left\{ \Lambda_s(M_A\Lambda_h + \Lambda_f)\int_0^t d_{\mathbb{H}}([x_s(t)]^\theta, [y_s(t)]^\theta)ds \right\} ds,
\end{aligned}
$$

where $\Omega_1 = \Lambda_s(\Lambda_g + \Lambda_(h_1) + \Lambda_h\Lambda_g + \Lambda_{\mathbb{I}}) + \Lambda_h \; and \; \Omega_2 = \Lambda_s(M_A\Lambda_h + \Lambda_f)$.

Then, we have

$$
\begin{aligned}
D_1 \leq\ & (\Omega_1 + \Lambda_h)d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) + \Omega_2(\int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds) \\
&+ \int_0^t \Lambda_s \left\{ \int_0^b d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds \right\} ds + \Lambda_s\Omega_1 \int_0^b d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
d_{\mathbb{H}}(\xi x_s, \xi y_s) &= \sup_{\theta\in[0,1]} d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) \\
&\leq (\Omega_1 + \Lambda_h)d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta) + \Lambda_s\Omega_1 \int_0^b d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds \\
&\quad +\Omega_2(\int_0^t d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)ds + \int_0^t \Lambda_s \int_0^b d_{\mathbb{H}}([x_s]^\theta, [y_s]^\theta)dsds).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\mathbb{H}_1(\xi x_s(t), \xi y_s(t)) &= \sup_{\theta\in[0,1]} d_{\mathbb{H}}([x_s(t)]^\theta, [y_s(t)]^\theta \\
&\leq (\Lambda_h + \Omega_1(1 + \Lambda_s b) + \Omega_2(b)(1 + \Lambda_s b))H_1(x_s(t), y_s(t)) \\
&= (\Lambda_h + (\Omega_1 + \Omega_2(b))(1 + \Lambda_s b))\mathbb{H}_1(x_s(t), y_s(t)).
\end{aligned}
$$

By the hypothesis $(H7)$, we took sufficiently small $b, \xi$ was a contraction mapping. Hence, the Banach fixed point theorem $(4.1)$ had a unique fixed point $x \in C([0,b] : (\mathbb{E}_{\mathbb{N}}^i)^n)$. ∎

## 4. Example

Consider the fuzzy solution of the nonlinear impulsive fuzzy integrodifferential equation of the form:

$$[x_i(t) - 2tx_i(t+h)^2]' = 2x_i(t) + 2tx_i(t+h)^2 + z(t),\ t \in \mathbb{J} = [0, b], \tag{10}$$

$$x_i(t) + \sum_{k=1}^{p} (c_k)_i x_i(t_k) = \phi_i(t) = 9 \in (\mathbb{E}^i_{\mathbb{N}})^n, \tag{11}$$

$$\Delta x_i(t_k) = \mathbb{I}_k x_i(t_k) = \frac{1}{(\mathbb{I}_k x_i(t_k))}. \tag{12}$$

Then, $\theta-$ level set of fuzzy number $9$ and $2$ are

$$[9]^\theta = [\theta + 8, 10 - \theta]\ , [2]^\theta = [\theta + 1, 3 - \theta] \ \text{ for } \ \theta \in [0, 1].$$

Let

$$\begin{aligned} \int_0^t k(t, s, x_{it}(t))ds &= 9tx_i(t+h)^2, f(t, x_{it}(t)) = 9tx(t+h)^2, \\ g(x) &= (g_1(x_1), g_2(x_2)) = (\sum_{k=1}^{p} (c_k)_1 x_1(t_k), \sum_{k=1}^{p} (c_k)_2 x_2(t_k)), \\ x(t) + g(t) &= (x_1(t) + g_1(t), x_1(t) + g_1(t)), \quad x(t) = (x_{t_1}, x_{t_2}), \\ \mathbb{S}(t-s) &= (e^{-(t-s)}, e^{-(t-s)}). \end{aligned}$$

Then, $\theta-$level set of $\int_0^t k(t, s, x_{it}(t))ds = 9tx_i(t+h)^2$ is

$$\begin{aligned} \left[\int_0^t k(t, s, x_t(t))ds\right]^\theta &= [9tx_i(t+h)^2]^\theta \\ &= t[9]^\theta [x_i(t+h)^2]^\theta = t[\theta + 8, 10 - \theta][(x^\theta_{il}(9))^2, (x^\theta_{ir}(9))^2] \\ &= t[(\theta + 8)x^\theta_{il}(t+h))^2, (10 - \theta)(x^\theta_{ir}(t+h))^2]. \end{aligned}$$

Thus,

$$\begin{aligned} d_{\mathbb{H}}([\mathbb{I}(x_i(t_k))]^\theta, [\mathbb{I}(x_i(t_k))]^\theta) &\leq \max_k \left\{ \begin{pmatrix} |\frac{((u^\theta_{il}(t_k)) - (y^\theta_{il}(t_k)))}{((1 + I(x^\theta_{il}(t_k)))(1 + \mathbb{I}(y^\theta_{il}(t_k))))}|, \\ |\frac{((x^\theta_{ir}(t_k)) - (v^\theta_{ir}(t_k)))}{((1 + \mathbb{I}(u^\theta_{ir}(t_k)))(1 + \mathbb{I}(y^\theta_{ir}(t_k))))}| \end{pmatrix} \right\} \\ &\leq \max_k \left\{ \begin{pmatrix} \frac{(|(x^\theta_{il}(t_k)) - (y^\theta_{il}(t_k))|)}{((1 + |(x^\theta_{il}(t_k))|)(1 + |(y^\theta_{il}(t_k))|))}, \\ \frac{(|(x^\theta_{ir}(t_k)) - (v^\theta_{ir}(t_k))|)}{((1 + |(u^\theta_{ir}(t_k))|)(1 + |y^\theta_{ir}(t_k))|))} \end{pmatrix} \right\} \\ &= \Lambda_{\mathbb{I}} \max_k \left\{ |(x^\theta_{il}) - (y^\theta_{il})|, |(x^\theta_{ir}) - (y^\theta_{ir})| \right\} \\ &= \Lambda_{\mathbb{I}} d_{\mathbb{H}}([x_i]^\theta, [x_i]^\theta), \end{aligned}$$

where

$$\Lambda_{\mathbb{I}} = \frac{1}{((1 + x_{ir}^{\theta}(t_k))(1 + y_{ir}^{\theta}(t_k)))}$$

$$d_{\mathbb{H}}([f(t, x_{it}(t))]^{\theta}, [f(, x_{it}(t)]^{\theta}))$$

$$\begin{aligned} &\begin{pmatrix} \leq (10-\theta)b|x_{ir}^{\theta}(t+h) + y_{ir}^{\theta}(t+h)| \max\{|x_{il}^{\theta}(t+h) - y_{il}^{\theta}(t+h)|, \\ |x_{ir}^{\theta}(t+h) - y_{ir}^{\theta}(t+h)|\} \end{pmatrix} \\ &\begin{pmatrix} \leq 10b|x_{ir}^{\theta}(t+h) + y_{ir}^{\theta}(t+h)| \max\{|x_{il}^{\theta}(t+h) - y_{il}^{\theta}(t+h)|, \\ |x_{ir}^{\theta}(t+h) - y_{ir}^{\theta}(t+h)|\} \end{pmatrix} \\ &= \Lambda_f d_{\mathbb{H}}([x_t(t+h)]^{\theta}, [y_t(t+h)]^{\theta}), \\ &\text{where } \Lambda_f = 10b|x_{ir}^{\theta}(t+h), y_{ir}^{\theta}(t+h)| \end{aligned}$$

$$\begin{pmatrix} d_{\mathbb{H}}([\int_0^t k(t,s,x_t(s))ds]^{\theta}, \\ [\int_0^t k(t,s,y_t(s))ds]^{\theta}) \end{pmatrix}$$

$$\begin{aligned} &\leq (10-\theta)b\begin{pmatrix} |x_{ir}^{\theta}(t+h) + y_{ir}^{\theta}(t)| \max\{|x_{il}^{\theta}(t+h) - y_{il}^{\theta}(t+h)|, \\ |x_{ir}^{\theta}(t+h) - y_{ir}^{\theta}(t+h)|\} \end{pmatrix} \\ &\leq 10b\begin{pmatrix} |x_{ir}^{\theta}(t+h) + y_{ir}^{\theta}(t+h)| \max\{|x_{il}^{\theta}(t+h) - y_{il}^{\theta}(t+h)|, \\ |x_{ir}^{\theta}(t+h) - y_{ir}^{\theta}(t+h)|\} \end{pmatrix} \\ &= \Lambda_k b d_{\mathbb{H}}([x(t+h)]^{\theta}, [y(t+h)]^{\theta}), \\ &\text{where } \Lambda_k = 10b|x_{ir}^{\theta}(t+h), y_{ir}^{\theta}(t+h)|. \end{aligned}$$

Next, we prove the controllability parts. Let us take the target set $\phi^1 = 2$,

$$\begin{aligned} [2]^{\theta} &= [\theta+1, 3-\theta] \text{ for } \theta \in [0,1] \\ [z(s)]^{\theta} &= [z_{il}^{\theta}, z_{ir}^{\theta}] \\ &= [(\zeta_{il})^{-1}((1+\theta) - \int_{k=1}^{n} c_k x(t_k) - t(\theta+1)(x_{sil}^{\theta})^2 - \int_0^b S_{il}^{\theta}(b-s)t(\theta+2) \\ &\quad \times (x_{sil}^{\theta})^2(s)ds - \int_0^b \mathbb{S}_{il}^{\theta}(b-s)t(\theta+1)(x_{sil})^{\theta})^2(s)ds, (\zeta_{ir})^{-1}((1+\theta) \\ &\quad - \sum_{k=1}^{n} c_k x(t_k) - t(\theta+1)(x_{sil})^{\theta})^2 - \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)t(\theta+2)(x_{sil})^{\theta})^2(s)ds \\ &\quad - \int_0^b S_{ir}^{\theta}(b-s)t(\theta+1)(x_{sil})^{\theta})^2(s)ds)]. \end{aligned}$$

Afterward, substituting this appearance into the integral method with respect to $(12)-(14)$ yields $\theta$ – level place of $x(b)$.

$$
\begin{aligned}
[x(b)]^{\theta} = [\mathbb{S}_{il}^{\theta}](b)[(\theta+1) - \sum_{k=1}^{n} c_k x(t_k) + t(\theta+1)(x_(s_{il})^{\theta})^2(t) \\
+ \int_0^b \mathbb{S}_l^{\theta}(t-s)t(\theta+1)(x_{sil})^{\theta})^2(t)ds \\
+ \int_0^b S_{il}^{\theta}(t-s)t(\theta+1)(x_{sil})^{\theta})^2(t)ds + \int_0^b \mathbb{S}_{il}^{\theta}(t-s)t(\theta+1)(x_{sil})^{\theta})^2(t)ds)
\end{aligned}
$$

$$
\begin{aligned}
&+ \int_0^b \mathbb{S}_{il}^{\theta}(b-s)(\zeta_{il})^{-1}((\theta+1) - t(\theta+1)(x_{sil}^{\theta})^2(t) - \int_0^b \mathbb{S}_{il}^{\theta}(b-s)t(\theta+1)(x_{sil}^{\theta})^2(t)ds) \\
&- \int_0^t \mathbb{S}_{il}^{\theta}(t-s)t(\theta+1)(x_{sil}^{\theta})^2(s)ds + (\int_0^t \mathbb{S}_{il}^{\theta}(t-s)t(\theta+1)(x_{sil}^{\theta})^2(s)ds)ds) \\
&\mathbb{S}_{ir}^{\theta}(b)(3-\theta) - \sum_{k=1}^{n} c_k x(t_k) + t(\theta+1)(x_{sir}^{\theta})^2(t) - \int_0^b \mathbb{S}_{ir}^{\theta}(t-s)t(3-\theta)(x_{sir}^{\theta})^2(t)ds \\
&+ \int_0^b \mathbb{S}_{ir}^{\theta}(t-s)t(3-\theta)(x_{sir}^{\theta})^2(t)ds + \int_0^b \mathbb{S}_{ir}^{\theta}(t-s)t(3-\theta)(x_{sir}^{\theta})^2(t)ds) \\
&+ \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)(\zeta_{ir})^{-1}((3-\theta) - t(\theta+1)(x_{sir}^{\theta})^2(t) - \int_0^b \mathbb{S}_{il}^{\theta}(b-s)t(3-\theta)(x_{sir}^{\theta})^2(t)ds) \\
&- \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)t(3-\theta)(x_{sil})^{\theta})^2(s)ds - \int_0^b \mathbb{S}_{ir}^{\theta}(b-s)t(3-\theta)(x_{sir}^{\theta})^2(s)dsds
\end{aligned}
$$

$$
\begin{aligned}
&= (\theta+1) - \sum_{k=1}^{n}(c_k)_1 x_{sil}^{\theta}(t_k), (3-\theta) - \sum_{k=1}^{n}(c_k)_1 x_{sir})^{\theta}(t_k) \\
&= [2 - \sum_{k=1}^{n}(c_k)_1(x_{i1}(t_k)]^{\theta} = [\phi_1^1 - g(x_s)]^{\theta}.
\end{aligned}
$$

Similarly,

$$[x_2(b)]^{\theta} = [x_{2l}^{\theta}(b), x_{2r}^{\theta}(b)] = [3 - \sum_{k=1}^{n}(c_k)_2(x_i)_2(t_k)]^{\theta} = [\phi_2^1 - g(x_s)]^{\theta}.$$

Therefore,

$$x(b) = [x_1(b), x_2(b)] = (\phi_1^1 - g(x_s), \phi_2^1 - g(x_s)) = \phi^1 - g(x_s).$$

After that, all section stated in theorem $4.1$ were contented, so the system $(12)-(14)$ was nonlocal controllable on $[0, b]$.

## 5. Conclusion

This paper contains some controllability of fuzzy solution for nonlinear impulsive functional neutral integrodifferential equations with nonlocal condition in $n$ - dimensional vector space proved by using the Banach fixed point theorem approach and the fuzzy number. Under some hypotheses, the controllability of first order nonlinear impulsive fuzzy integrodifferential equation was proved.


## *Acknowledgment:*

*The authors would like to thank the reviewers for their valuable suggestions and comments to improve the presentation of this paper.*


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