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The effect of queue size on the throughput, in group failure mode, for the loaded transport channel

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The effect of queue size on the throughput, in group failure mode, for the loaded transport channel

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Abstract: The external data flow decreases the throughput of the transport connection. The indicator of this external load is the queue size in front of the protocol data. In this article, using a mathematical model in analytical and numerical forms, the relation between the throughput of the channel and the protocol parameters are presented including the queue size parameter. In this work the effect of the queue size on time-out duration has been shown, which is one of the important parameters and it's studied weakly in researches. Also, the relation between round-trip delay, the reliability of the transmission of the information segments with queue size are also shown.

1. Introduction

Transmission connection throughput has an extremely significant feature of computer networks. This indicator determines the quality of network services for subscribers and it's determined by the values of protocol parameters such as (window size and timeout duration), and the characteristics of the data transmission path (duration round trip delay, reliability of the transmission of the information segments for both directions of the transport connection). Also, the external data flow acts to determine the throughput of the selected transport connection. This external flow decreases the throughput of the channel even if they have one common path route. The main indicator for this external load on the transport connection is the queue size in front of the protocol block data in selected sections of the transit nodes. By studying this indicator, the distribution of queue lengths in transit nodes from external network streams for analyzed connection can be estimated, and then it is used to manipulative the active characteristics of the linking and the selection of protocol parameters for the communication time between given subscribers. TCP [1] is an important transport layer protocol. Modern computer networks with the TCP protocol have a significant role to cope with a huge number of today's network problems.

As accrued before, many problems were solved by adding new algorithms like Reno, New Reno, Tahoe [17]. These modifications can be considered long-standing and the research continues to improve the speed of the transport connection. Nowadays, the external loads on the shared network resources are not be considered by known models of asynchronous control procedures of a separate data link and transport protocol [2]–[8]don't consider the external loads on the shared network resources. These loads could be on any virtual connection along the path between subscribers, which have common nodes. In [9],[10], the study of the data conversion process of the load transmission connection is carried out under the condition that the protocol parameters and channel attributes are very limited. Many modern studies on transport connection channels show that only a few factors or parameters such as timeout, round-trip delay, or congestion, have affected the speed of transport connection. However, there are no clearly expressed analytical formulas or numerical analyzes [17]. In [16], to improve the speed of transport protocols a thorough presentation of modern modifications is carried out. The study clearly showed that modern studies consider only certain factors of congestions, timeout, and round-trip delay. But the work [15] does not consider the probability of data delivery reliability in the forward and reverse channels. While in our study, the conditions for the probability of data delivery reliability in the communication channel have been considered for both directions, analytical solutions were found and their confirmation, using numerical studies. In this paper, we present the mathematical model of the transmission connection under the transmission protocol connection in the group failure mode. This model is taking to account the factors such as losing data in forward and reverse directions, the mechanism of data retransmission, which counts time-out losing acknowledgments from receiving host, and another important factor "non-zero queue" from external interconnection. The significant improvement of this study compares to the study of [13] is the solution for all states of the Markov chain have been found except one case, which is not such important for the research. The mathematical model that is shown here is the same model, which was published in [12], but here the model has been shown for group failure mode, also the tasks and conditions are dissimilar.

2. Description of the mathematical model

Let's consider data exchange between subscribers, which are connected by a multi-link data path. We suppose that the following conditions are true [11],[12]: The connection between nodes is duplex and having the same speed in both forward and reverse directions. The path length, which is expressed by hop numbers, is equal to D_f. The reverse channel, in which acknowledgments are delivered to the sender, has a length D_o. The confirmation is a receipt that contains information about whether the segment data sequence was correctly sent to the recipient. The reliability of the transmission of the information segments in the path, from sender to receiver and vice versa (the forward and reverse channel), are given as F_f and F_o respectively. The time processing of segments is equal in all nodes. Besides, the interacting subscribers have an unlimited flow of segments for transition and the length of segments is equal. Acknowledgments from the receiver are sending with their counterflow. Also, we propose that segment retransmission is organized according to group failure mode [23]. We suppose that, the loss of segments is not happening of the lack of buffer memory. The function of probability $b_n, n = \overline{0, N}$ is given. This means each segment in the flow, that we are analyzing, will meet a queue with $n \leq N$, where N is the maximum length of the queue. The maximum length of queues N defined by the buffer pools capacity of the transition nodes. We call the time t, which is needed to take the segment into the output line as a cycle. The cycle is defined as a sum of time needed to take segment into the output line, channel propagation signal, and the time for processing segment for the node side. The timeout S is expressed as the length of t. It is launched before the beginning of the first transition segment in the queue and it will be fixed for all segments within a window size. We consider that the window size of the controlling protocol, is defined by W. $S > W$ sets the timeout duration. The sum of the length of forward and reverse path $D = D_f + D_o$ can be presented as round-trip delay in the unloaded channel, which was expressed in cycles t. After sending the next segments, the protocol will copy it into the queue of transmitted but unconfirmed segments, then it will launch timeout. As soon as the queue size becomes equal to the width of the window W, the control protocol will pause transmission while waiting for the acknowledgment or the expiration of timeout S for confirmation. If the acknowledgment is positive, the segments, which sent successfully, will be deleted from the queue. If the timeout, for determining segment, is reached, that segment will be retransmitted and the timeout of confirmation will be reset and launched again. Then the time of confirmation by the source of end-to-end acknowledgment is dispersed according to geometric law with F_o parameter and the duration of sampling cycle t. The usefulness of virtual association in a loaded multilink that is controlled by the transport protocol, data transmission path with segment queues before sending data or confirmations can be described by a markovized process of the dynamics of a transmitted queue but not confirmed segments, in which the queue size of the advancing or inverse data flow of the testing connection is an additional variable of

Markov process. In the Markov chain state (i,n), a sequence of size $(i-n)$ segments that have been sent from the source, in which one of the links in the process of transfer met a queue with the length of n segments. States of Markov chain $i=\overline{0,W+n}$, $n=\overline{0,N}$, corresponding to the size of the queue which is transmitted but not yet confirmed segments in the flow source. And the time from the beginning of the transmission of the sequence, while the states $i=\overline{W+n+1,S-1}, n=\overline{0,N}$ refer to the time, during which the sender is not active and is waiting for an acknowledgment of correct reception of the sent sequence of W segments.

We define $P(i,n)$, $i=\overline{0,S-1}, n=\overline{0,N}$ as the probabilities of Markov chain states. Then the sequence of transmitted, but not confirmed data segments of considered virtual connection with a zero-length queue grows to the state of a Markov chain with coordinates $(D-1,0)$ with probability b_0. The further increasing size of this sequence occurs with a probability of $b_0(1-F_o)$. In the states (i,n), $i=\overline{D-1+n,S-1}, n=\overline{0,N}$, it is possible, that sender receives the acknowledgment, and depending on the acknowledgment results, the sender transmits new segments (with a positive acknowledgment), or retransmit distorted segments. Since the transmitted sequence of segments of the virtual connection, that we are analyzing, may encounter a queue of non-zero length at any moment of transferring process (on the path of the sequence to the addressee or when transferring confirmation to the sender of information flow), the transition from the state $(i,0)$, $i=\overline{0,S-2}$ to state $(i,n), i=\overline{0,S-2}, n=\overline{1,N}$ occurs with probability bn.

3. State probabilities for Markov chain

The transition probabilities of the Markov chain can be expressed as π_{in}^{jm}, where (i,n) is the coordinates of the initial state, and the resulting state (j,m) from the initial state into the chain. Then the dynamics of the process of transmitting information flow in the group failure mode for loaded data transmission channel can be set with the following values of transition probabilities:

$$\pi_{in}^{jm}=\begin{cases} b_0,\ j=i+1, m=0; i=\overline{0,D-2}, n=0;\\ b_0(1-F_0), j=i+1, m=0;\ i=\overline{D-1,S-2}, n=0;\\ b_m,\ j=i, m=\overline{1,N};\ i=\overline{0,S-2}, n=0;\\ b_0F_oF_f^{\,i-D+2},\ j=D-1, m=0, i=\overline{D-1,W-1}, n=0,;\\ b_0F_oF_f^{\,i-D+2}, j=W+D-2-i, m=0; i=\overline{W,W+D-3}, n=0,\\ b_0F_o(1-F_f^{\,i-D+2}),\ \ j=0, m=0;\ i=\overline{D-1,W+D-3}, n=0,\\ b_0F_o, j=0, m=0;\ i=\overline{W+D-2,S-2}, n=0,\\ 1,\ j=0, m=0;\ \ i=S-1, n=\overline{0,N},\\ 1, j=i+1, m=n;\ i=\overline{0,D-2+n}, n=\overline{1,N};\\ 1-F_0, j=i+1, m=n;\ i=\overline{D-1+n,S-2}, n=\overline{1,N},\\ F_oF_f^{\,i-D+2-n}, j=D-1, m=0;\ i=\overline{D-1+n,W-1+n}, n=\overline{1,N},\\ F_oF_f^{\,i-D+2-n}, j=W+D-2-i, m=0;\ i=\overline{W+n,W+D-3+n}, n=\overline{1,N},\\ F_o(1-F_f^{\,i-D+2-n}), j=0, m=0; i=\overline{D-1+n,W+D-3+n}, n=\overline{1,N}, \end{cases} \tag{1}$$

There are different solutions for the equilibrium system of Markov chain state probabilities, and can be determined by the relationship between $W, S, D,$ and N (window size, timeout, path length, and maximum queue length) The time-out length must be bigger or equal to the round trip-delay length ($S\geq D$) and must go beyond the width of window size, also time-out should be longer than the waiting time for the beginning data transiting in transmission nodes because of existed queues inside the virtual channel. Due to the wide variety of protocol parameter values, the system has several different solutions. For the analysis and research of the data transmission process of loading arbitrary values of protocol parameters in the channel, $b_0=0$ is an indispensable condition. In this paper, the study works to analyze

the data transmission process in the load channel with a non-zero queue length ($b_0 = 0$) and a protocol parameter with a common path and the maximum queue length $S \geq W + D + N - 1$.
For $W \geq D$, the solution of equilibrium system from system equation (1) can be written as the following:

$$P(0,0) = F_o \sum_{n=1}^{N} \sum_{i=D-1+n}^{W+D-3+n} (1 - {F_f}^{i-D+2-n}) P(i,n) + \sum_{n=1}^{N} P(S-1,n) + F_o \sum_{n=1}^{N} \sum_{i=D+W-2+n}^{S-2} P(i,n) \quad (2)$$

$$P(i,0) = F_o \sum_{n=1}^{N} {F_f}^{W-i} P(D+W-2-i+n,n)\,, i = \overline{1,D-2} \quad (3)$$

$$P(D-1,0) = F_o \sum_{n=1}^{N} \sum_{i=D-1+n}^{W-1+n} P(i,n)\, {F_f}^{i-D+2-n} \quad (4)$$

$$P(i,0) = 0, i = \overline{D,S-1} \quad (5)$$

$$P(0,n) = b_n P(0,0), n = \overline{1,N} \quad (6)$$

$$P(i,n) = P(i-1,n) + b_n P(i,0), i = \overline{1,D-1}, n = \overline{1,N} \quad (7)$$

$$P(i,n) = P(i-1,n), i = \overline{D,D-1+n}, n = \overline{1,N} \quad (8)$$

$$P(i,n) = (1-F_0) P(i-1,n), i = \overline{D+n,S-1}, n = \overline{1,N} \quad (9)$$

Let's start solving the equilibrium system. According to equation (8), we get : $P(i,n) = P(D-1,n), i = \overline{D,D-1+n}, n = \overline{1,N}$, and from equation (9) we have: $P(i,n) = P(D-1+n,n){\bar{F}_o}^{i-D-n+1}, i = \overline{D+n,S-1}, n = \overline{1,N}$, taking to account these relations and from equations (4) and (5) for $i = \overline{1,D-1}$ we find:

$$P(i,0) = \frac{F_o \Phi^{W-i}}{\bar{F}_o} \sum_{m=1}^{N} P(D-1,m)\,, i = \overline{1,D-2}, Where: \Phi = F_f(1-F_o), \bar{F}_o = 1 - F_o \quad (10)$$

$$P(D-1,0) = \frac{F_o F_f (1-\Phi^{W-D+1})}{1-\Phi} \sum_{m=1}^{N} P(D-1,m) \quad (11)$$

With equations (7), (6) and (10) we can find:

$$P(i,n) = b_n [P(0,0) + \frac{F_o \Phi^{W-i}}{\bar{F}_o} \sum_{m=1}^{N} P(D-1,m)], i = \overline{1,D-2}, n = \overline{1,N}$$

$$P(D-1,n) = b_n [P(0,0) + \frac{F_o F_f (1-\Phi^{W-1})}{(1-\Phi)} \sum_{m=1}^{N} P(D-1,m)] \quad (12)$$

Accordingly, from equation (12) we can find the state probabilities $P(D-1,n)$, $m = \overline{n+1,N}$ for arbitrary $n = \overline{1,N}$ through $P(D-1,m)$ and we get:

$$P(D-1,n) = \frac{b_n}{1-\Phi-F_o F_f (1-\Phi^{W-1}) \sum_{m=1}^{n} b_m} \left[P(0,0)(1-\Phi) + F_o F_f (1-\Phi^{W-1}) \sum_{m=n+1}^{N} P(D-1,m)\right], n = \overline{1,N} \quad (13)$$

When $n = N$, we get: $P(D-1,N) = b_N \frac{P(0,0)(1-\Phi)}{1-\Phi-F_o F_f (1-\Phi^{W-1})}$, substituting this relation into equation (13) for values $n = N-1$ to 1, recursively, we can find the functional expression for state probabilities $(D-1,n)$ via $P(0,0)$: $P(D-1,n) = b_n \frac{P(0,0)(1-\Phi)}{1-F_f+F_o F_f \Phi^{W-1}}, n = \overline{1,N}$. From here, up to the probability of the initial state, we obtain the probability distribution of states of the Markov chain:

$$P(i,0) = \frac{P(0,0) F_o F_f \Phi^{W-1}(1-\Phi)}{1-F_f+F_o F_f \Phi^{W-1}} \Phi^{-i}, i = \overline{1,D-2}$$

$$(D-1,0) = \frac{P(0,0) F_o F_f (1-\Phi^{W-D+1})}{1-F_f+F_o F_f \Phi^{W-1}}$$

$$P(i,n) = P(0,0) b_n \frac{1-F_f+F_o F_f \Phi^{W-1-i}}{1-F_f+F_o F_f \Phi^{W-1}}, i = \overline{0,D-2}, n = \overline{1,N}$$

$$P(i,n) = b_n \frac{P(0,0)(1-\Phi)}{1-F_f+F_o F_f \Phi^{W-1}}, i = \overline{D-1,D-1+n}, n = \overline{1,N}$$

$$P(i,n) = b_n \frac{P(0,0)(1-\Phi)}{1-F_f+F_o F_f \Phi^{W-1}} {\bar{F}_o}^{i-D-n+1}, i = \overline{D+n,S-1}, n = \overline{1,N}$$

By using the normalization condition, finally we can get the relation for initial state $P(0,0)$:

$P(0,0) = \frac{(1-F_f+F_oF_f\Phi^{W-1})(1-\Phi)F_o}{F_o(1-\Phi)[F_oF_f+D(1-F_f)+(1-\Phi)(1+\bar{N})]+F_o{}^2F_f(\Phi^{W-D+1}-\Phi^W)+(1-\Phi)^2\left(\bar{F}_o-\bar{F}_o{}^{S-D+1}\sum_{n=1}^{N}\frac{b_n}{(1-F_o)^n}\right)}$, Where $\bar{N} = \sum_{n=1}^{N} nb_n$, is the average length of the queue. Note: if we put the value of $F_f = 1$, we can get the same results for selective failure mode, which are represented in [12].

Let's consider the case when the size of the window does not exceed the duration of round-trip delay ($W < D$) and the duration of time-out is the same $S \geq W + D + N - 1$. According to equation (1) the system of equilibrium will change as the followings:

The equations of (2) (6) (9) will remain the same. Equation (3) is true for $i = \overline{1, W-1}$. Equation (7) is true for $i = \overline{1, W-1}, n = \overline{1, N}$. Equation (4) will change to $P(D-1,0) = 0$. Equation (8) will be changed to $i = \overline{W, D-1+n}, n = \overline{1, N}$. The solutions of the equilibrium system will be as the following for $W < D$:

$$P(i,0) = \frac{P(0,0)(1-\Phi)F_oF_f\Phi^{W-1-i}}{1-F_f+F_oF_f\Phi^{W-1}}, i = \overline{1, W-1}$$

$$P(i,n) = b_nP(0,0)[\frac{1-F_f+F_oF_f\Phi^{W-1-j}}{1-F_f+F_oF_f\Phi^{W-1}}], i = \overline{0, W-1}, n = \overline{1, N}$$

$$P(W-1,0) = \frac{P(0,0)(1-\Phi)F_oF_f}{1-F_f+F_oF_f\Phi^{W-1}}$$

$$P(i,n) = b_n\frac{P(0,0)(1-\Phi)}{1-F_f+F_oF_f\Phi^{W-1}}, i = \overline{W-1, D-1+n}, n = \overline{1, N}$$

$$P(i,n) = b_n\frac{P(0,0)(1-\Phi)}{1-F_f+F_oF_f\Phi^{W-1}}\bar{F}_o{}^{i-D-n+1}, i = \overline{D+n, S-1}, n = \overline{1, N}$$

As the way before we can find the initial state using normalization condition:

$$P(0,0) = \frac{(1-F_\text{п}+F_oF_\text{п}\Phi^{W-1})(1-\Phi)F_o}{F_o[F_oF_\text{п}(\Phi-\Phi^W)+F_oF_\text{п}(1-W)(1-\Phi)+(1-\Phi)^2(D+1+\bar{N})]+(1-\Phi)^2\left(\bar{F}_o-\bar{F}_o{}^{S-D+1}\sum_{n=1}^{N}\frac{b_n}{(1-F_o)^n}\right)}$$

Then, the transmission process with interval restrictions has been analyzed on the duration of the timeout. Let's present already found solutions. With restrictions $W \geq D, W + D - 1 \leq S \leq W + D + N - 1, 1 \leq N \leq D - 2$, the equations of the original equilibrium system (2-9) are transformed into:

$$P(0,0) = F_o\sum_{n=1}^{N}\sum_{i=D-1+n}^{W+D-3+n}(1 - F_f{}^{i-D+2-n})P(i,n) + \sum_{n=1}^{N}P(S-1,n)$$
$$+F_o\sum_{n=1}^{S-(W+D-1)}\sum_{i=D+W+n-2}^{S-2}P(i,n)$$

$$P(i,0) = F_o\sum_{n=1}^{Vi}F_f{}^{W-i}P(D+W-2-i+n,n), i = \overline{1, D+W+N-1-S}, Vi = S - D - W + i$$

$$P(i,0) = F_o\sum_{n=1}^{N}F_f{}^{W-i}P(D+W-2-i+n,n), i = \overline{D+W+N-S, D-2}$$

$$P(D-1,0) = F_o\sum_{n=1}^{N}\sum_{i=D-1+n}^{W-1+n}P(i,n)\,F_f{}^{i-D+2-n}$$

$$P(0,n) = b_nP(0,0), n = \overline{1, N}$$

$$P(i,n) = P(i-1,n) + b_nP(i,0), i = \overline{1, D-1}, n = \overline{1, N}$$

$$P(i,n) = P(i-1,n), i = \overline{D, D-1+n}, n = \overline{1, N}$$

$$P(i,n) = (1-F_0)P(i-1,n), i = \overline{D+n, S-1}, n = \overline{1, N}$$

Similarly, we solve the equations and as a result we obtain the probabilities of the states of the Markov chain:

$$P(i,0) = \frac{P(0,0)(1-\Phi)F_oF_f\Phi^{W-i-1}\sum_{m=1}^{S-D-W+i}b_m}{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]}, i = \overline{1, D+W+N-1-S}$$

$$P(i,0) = \frac{P(0,0)(1-\Phi)F_oF_f\Phi^{W-i-1}}{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]}, i = \overline{D+W+N-S, D-2}$$

$$P(D-1,0) = \frac{P(0,0)F_oF_f(1-\Phi^{W-D+1})}{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]}$$

$$P(i,n)=\frac{b_nP(0,0)\left\{1-F_f+F_oF_f\left[\Phi^{W-1-i}\sum_{m=1}^{S-D-W+i}b_m+\sum_{m=S-D-W+1+i}^{N}b_m\Phi^{S-D-m}\right]\right\}}{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]},i=\overline{1,D+W+N-S-1}$$

$$,n=\overline{1,N}$$

$$P(i,n)=\frac{b_nP(0,0)[1-F_f+F_oF_{nf}\Phi^{W-i-1}]}{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]},i=\overline{D+W+N-S,D-2},n=\overline{1,N}$$

$$P(i,n)=\frac{b_nP(0,0)(1-\Phi)}{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]},i=\overline{D-1,D-1+n},n=\overline{1,N}$$

$$P(i,n)=\frac{b_nP(0,0)(1-\Phi)\bar{F}_o{}^{i-D-n+1}}{1-F_n+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\Phi^{S-D-m}\right]},i=\overline{D+n,S-1},n=\overline{1,N}$$

As before we can find the initial state $P(0,0)$ using normalization condition:

$$P(0,0)=F_o(1-\Phi)\{1-F_f+F_oF_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\,\Phi^{S-D-m}\right]\}\{F_o{}^2F_f[\Phi^{W-D+1}+1-\Phi-\Phi^{W}\sum_{m=1}^{S-D-W}b_m-\sum_{m=S-D-W+1}^{N}b_m\,\Phi^{S-D-m+1}+(1-\Phi)\sum_{m=S-D-W+2}^{N}b_m\,\Phi^{S-D-m}(m-S+D+W-1)]+(1-\Phi)[F_o\left(1-F_f\right)D+F_o(1-\Phi)(\bar{N}+1)+(1-\Phi)\left(\bar{F}_o-\sum_{n=1}^{N}b_n\,\bar{F}_o{}^{S-D-n+1}\right)]\}^{-1}$$

Consider analyzing the process of information transfer for window size less than the time-trip delay ($W\le D$) and interval restrictions on the timeout duration $D+W-1\le S\le D+W+N-1$ and the maximum of queue size $1\le N\le W-2$, then the equilibrium equations from (2-9) take the form:

$$P(0,0)=\sum_{n=1}^{N}P(S-1,n)+F_o\sum_{n=1}^{V}\sum_{i=D+W+n-2}^{S-2}P(i,n)\,,V=S-D-W+1$$

$$\mathrm{P}(\mathrm{i},0)=F_o\sum_{n=1}^{Vi}F_f{}^{W-i}P(D+W+n-2-i,n)\,,i=\overline{1,D+W+N-1-S},Vi=S-D-W+i$$

$$\mathrm{P}(\mathrm{i},0)=F_o\sum_{n=1}^{N}F_f{}^{W-i}P(D+W+n-2-i,n)\,,i=\overline{D+W+N-S,W-1}$$

$$\mathrm{P}(0,\mathrm{n})=b_n\mathrm{P}(0,0),n=\overline{1,N}$$

$$P(i,n)=P(i-1,n)+b_nP(i,0),i=\overline{1,W-1},n=\overline{1,N}$$

$$P(i,n)=P(i-1,n),i=\overline{W,D-1+n},n=\overline{1,N}$$

$$P(i,n)=P(D-1,n)=P(W-1,n)$$

$$P(i,n)=P(i-1,n)(1-F_0),i=\overline{D+n,S-1},n=\overline{1,N}$$

$$P(i,n)=P(D-1,n)\bar{F}_o{}^{i-D-n+1}=P(W-1,n)\bar{F}_o{}^{i-D-n+1}$$

Similarly, when solving equations, the probabilities of states of the Markov chain can be obtained:

$$P(i,0)=\frac{P(0,0)F_oF_f(1-\phi)\phi^{W-i-1}\sum_{m=1}^{S-D-W+i}b_m}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},i=\overline{1,D+W+N-1-S}$$

$$P(i,0)=\frac{P(0,0)F_oF_f(1-\phi)\phi^{W-i-1}}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},i=\overline{D+W+N-S,W-1}$$

$$P(i,n)=\frac{b_nP(0,0)\{1-F_f+F_oF_n[\phi^{W-1-i}\sum_{m=1}^{S-D-W+i}b_m+\sum_{m=S-D-W+i+1}^{N}b_m\phi^{S-D-m}]\}}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},i=\overline{1,D+W+N-1-S}$$

$$,n=\overline{1,N}$$

$$P(i,n)=\frac{b_nP(0,0)[1-F_f+F_oF_f\phi^{W-1-i}]}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},i=\overline{D+W+N-S,W-1},n=\overline{1,N}$$

$$P(i,n)=\frac{b_n(1-\phi)P(0,0)}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},n=\overline{W-1,D-1+n},n=\overline{1,N}$$

$$P(i,n)=\frac{b_n(1-\phi)P(0,0)}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]}\bar{F}_o{}^{i-D-n+1},n=\overline{D+n,S-1},n=\overline{1,N}$$

Next the initial state $P(0,0)$ has been found.

$$P(0,0)=\{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\,\phi^{S-D-m}]\}F_o(1-\phi)\{F_o{}^2F_f\,[1-\phi^{W}\sum_{m=1}^{S-D-W}b_m-\sum_{m=S-D-W+1}^{N}b_m\,\phi^{S-D-m+1}+(1$$

$$-\phi)\sum_{m=S-D-W+2}^{N} b_m\,\phi^{S-D-m}(m-S+D+W-1)]+(1-\phi)^2[1+F_o(D-W+\bar{N})\quad +F_oW\left(1-F_f\right)-\sum_{n=1}^{N} b_n\,\bar{F}_o^{\,S-D-n+1}]\}^{-1}$$

Consider another variety of interval restrictions on protocol parameters and the maximum queue size of the form $W \geq D, D+W-1 \leq S \leq W+N+1, D-2 \leq N \leq W-2$. Under these constraints, the equations of the original equilibrium system (2-9) will change in the following:

$$P(0,0)=F_o\sum_{n=1}^{N}\sum_{i=D-1+n}^{W+D-3+n}(1-F_f^{\,i-D+2-n})P(i,n)+\sum_{n=1}^{N}P(S-1,n)+F_o\sum_{n=1}^{N}\sum_{i=D+W-2+n}^{S-2}P(i,n)$$

$$P(i,0)=F_o\sum_{n=1}^{V_1}F_f^{\,W-i}P(D+W-2-i+n,n)\,,i=\overline{1,D-2},V_1=S-2-(D+W-2-i)$$

$$P(D-1,0)=F_o\left[\sum_{n=1}^{V_2}\sum_{i=D-1+n}^{W-1+n}P(i,n)\,F_f^{\,i-D+2-n}+\sum_{n=V_2+1}^{N}\sum_{i=D-1+n}^{S-2}P(i,n)\,F_f^{\,i-D+2-n}\right],V_2=S-W-1$$

$$P(i,0)=0,i=\overline{D,S-1}$$

$$P(0,n)=b_nP(0,0),n=\overline{1,N}$$

$$P(i,n)=P(i-1,n)+b_nP(i,0),i=\overline{1,D-1},n=\overline{1,N}$$

$$P(i,n)=P(i-1,n),i=\overline{D,D-1+n},n=\overline{1,N}$$

$$P(i,n)=\left(1-F_0\right)P(i-1,n),i=\overline{D+n,S-1},n=\overline{1,N}$$

By solving the equations, the following results achieved the probabilities of the states of the Markov chain:

$$P(i,0)=\frac{P(0,0)(1-\phi)F_oF_f\phi^{W-i-1}\sum_{m=1}^{S-D-W+i}b_m}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},i=\overline{1,D-2}$$

$$P(D-1,0)=\frac{P(0,0)F_oF_f[1-\phi^{W-D+1}\sum_{m=1}^{S-W-1}b_m-\sum_{m=S-W}^{N}\phi^{S-D-m}b_m]}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]}$$

$$P(i,n)=b_nP(0,0)[\frac{1-F_f+F_oF_f\left[\phi^{W-1-i}\sum_{m=1}^{S-D-W+i}b_m+\sum_{m=S-D-W+i+1}^{N}b_m\phi^{S-D-m}\right]}{1-F_f+F_oF_f\left[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}\right]}],i=\overline{0,D-2},n=\overline{1,N}$$

$$P(D-1,n)=\frac{b_nP(0,0)(1-\phi)}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},\ n=\overline{1,N}$$

$$P(i,n)=\frac{b_nP(0,0)(1-\phi)}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},\ i=\overline{D-1,D-1+n},n=\overline{1,N}$$

$$P(i,n)=\frac{b_nP(0,0)\left(1-\phi\right)\bar{F}_o^{\,i-D+1-n}}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\phi^{S-D-m}]},\ i=\overline{D+n,S-1},n=\overline{1,N}$$

From the normalization condition, we obtain the initial state P (0,0):

$$P(0,0)=\{1-F_f+F_oF_f\left[\phi^{W-1}\sum_{m=1}^{S-D-W}b_m+\sum_{m=S-D-W+1}^{N}b_m\,\phi^{S-D-m}\right]\}(1-\phi)\{(1-\phi)\left(1-F_f\right)F_oD+F_o^{\,2}F_f[\phi^{W-1}(2-2F_f+F_oF_f\phi\left(1+2\phi^{1-D}-\phi^{2-D}\right))\sum_{m=1}^{S-D-W}b_m$$
$$+\left(2-\phi\right)F_oF_п\sum_{m=S-D-W+1}^{S-W-2}b_m\left(\phi^{W-D+1}-\phi^{S-D-m}\right)+(1-\phi)\sum_{m=S-D-W+1}^{S-W-1}b_m\,\phi^{S-D-m}(m-S+D+W+1)+(D-1)(1-\phi)\sum_{m=S-W}^{N}b_m\,\phi^{S-D-m}+(1-\phi)-(1-\phi)\phi^{W-D+1}\sum_{m=1}^{S-W-1}b_m]+(1-\phi)^2(F_o\bar{N}$$
$$+1-\sum_{m=1}^{N}b_m\,\bar{F}_o^{\,S-D-m+1})\}^{-1}$$

Consider to present one more solution of the system of equilibrium equations for the parameters $W \geq D, D+W-1 \leq S \leq W+N+1,\ W-2 \leq N$. Under these constraints, the equations of the original equilibrium system (2-9) will change as the following:

$$P(i,0)=F_o\sum_{n=1}^{V_i}F_f^{\,W-i}P(D+W-2-i+n,n)\,,i=\overline{1,D-2}$$

$$P(D-1,0) = F_o\left[\sum_{n=1}^{E}\sum_{i=D-1+n}^{W-1+n} P(i,n)\, {F_f}^{i-D+2-n} + \sum_{n=E+1}^{X}\sum_{i=D-1+n}^{S-2} P(i,n)\, {F_f}^{i-D+2-n}\right]$$
$$P(0,n) = b_n P(0,0), n = \overline{1,N}$$
$$P(i,n) = P(i-1,n) + b_n P(i,0), i = \overline{1,D-1}, n = \overline{1,N}$$
$$P(i,n) = P(i-1,n), i = \overline{D,D-1+n}, n = \overline{1,X}$$
$$P(i,n) = P(i-1,n), i = \overline{D,S-1}, n = \overline{X+1,N}$$
$$P(i,n) = (1-F_0)P(i-1,n), i = \overline{D+n,S-1}, n = \overline{1,X}$$

Where $V_i = S-2-(D+W-2-i)$, $E = S-2-(W-1)$, $X = S-2-(D-1)$.

By solving the equations, the following results are achieved:

$$P(i,0) = \frac{(1-\phi)P(0,0)F_oF_f\phi^{W-i-1}\sum_{m=1}^{S-D-W+i} b_m}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}, i = \overline{1,D-2}$$

$$P(D-1,0) = \frac{F_oF_fP(0,0)\left[\sum_{m=1}^{S-D-1} b_m - \phi^{W-D+1}\sum_{m=1}^{S-W-1} b_m - \sum_{m=S-W}^{S-D-1} b_m\phi^{S-D-m}\right]}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}$$

$$P(i,n) = \frac{b_nP(0,0)\{1-\phi-F_oF_f\left[\sum_{m=1}^{S-D-1} b_m - \phi^{W-i-1}\sum_{m=1}^{S-D-W+i} b_m - \sum_{m=S-D-W+1+i}^{S-D-1} b_m\phi^{S-D-m}\right]\}}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}, i = \overline{0,D-2}, n = \overline{1,N}$$

$$P(D-1,n) = \frac{b_n(1-\phi)P(0,0)}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}, \overline{1,S-D-1}$$

$$P(i,n) = \frac{(1-\phi)P(0,0)b_n}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}, i = \overline{D,D+n-1}, n = \overline{1,X}$$

$$P(i,n) = \frac{(1-\phi)P(0,0)b_n}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}, i = \overline{D,S-1}, n = \overline{X+1,N}$$

$$P(i,n) = \frac{P(0,0)(1-\phi)b_n{\bar{F}_o}^{i-D-n+1}}{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]}, i = \overline{D+n,S-1}, n = \overline{1,N}$$

From here we can get $P(0,0)$:

$$P(0,0) = \{1-\phi-F_oF_f[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m(1-\phi^{S-D-m})]\}F_o(1-\phi)\{(1-\phi)^2F_oD - (1-\phi){F_o}^2F_f[(D-1)\sum_{m=1}^{S-D-1} b_m - \sum_{m=S-D-W+1}^{S-D-1} b_m\,\phi^{S-D-m} + \sum_{m=S-D-W+1}^{S-W-1} b_m\,\phi^{S-D-m} + \phi^{W-1}\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-W-1} b_m\,\phi^{S-D-m}(m-S+D+W)] - {F_o}^2F_f[\phi^{W-D+1}\sum_{m=1}^{S-W-2} b_m - \phi^{W-1}\sum_{m=1}^{S-D-W} b_m - \sum_{m=S-D-W+1}^{S-W-2} b_m\,\phi^{S-D-m}] + (1-\phi)^2[F_o\sum_{m=1}^{S-D-1}(mb_m+1) + F_o\sum_{m=S-D}^{N} b_m(S-D) + \bar{F}_o - {\bar{F}_o}^{S-D+1}\sum_{m=1}^{N} b_m{\bar{F}_o}^{-m}]\}^{-1}$$

The last variety of interval restriction on protocol parameters and the maximum queue size has form of $W \ge D, D < S < D+N+1$, when $F_o = 1$, the equilibrium equations are like the following:

$$P(0,0) = (1-F_f)\sum_{n=1}^{V} P(D-1+n,n) + \sum_{n=V+1}^{N} P(S-1,n), \text{ where } V = S-2-(D-1)$$
$$P(i,0) = 0, i = \overline{1,D-2}$$
$$P(D-1,0) = F_f\sum_{n=1}^{S-D-1} P(D-1+n,n)$$
$$P(0,n) = b_nP(0,0), n = \overline{1,N}$$
$$P(i,n) = P(i-1,n), i = \overline{1,D-2}, n = \overline{1,N}$$
$$P(D-1,n) = P(D-2,n) + b_nP(D-1,0), n = \overline{1,N}$$
$$P(i,n) = P(i-1,n), i = \overline{D,D-1+n}, n = \overline{1,N}$$

The initial state is obtained by solving all the equilibrium equations and from the normalization condition:

$$P(0,0) = \frac{1-F_f\sum_{m=1}^{V} b_m}{1+D-(D-1)F_f\sum_{m=1}^{V} b_m + \sum_{m=1}^{V} b_m m + (S-D)\sum_{m=V+1}^{N} b_m}, \quad \text{where } V = S-D-1$$

Research in a multidimensional area (S, W, D, F_o, F_f) is a difficult task. The solution for this problem is the dimension reduction of the attribute space. Effective options for this reduction are absolute reliable reverse data transmission path $(F_o = 1)$ and the case of a uniform forward and reverse data transmission path $(F_o, = F_f = F)$ with conditions of unlimited window size $(W \to \infty)$ and, consequently, timeout duration $(S \to \infty)$. The initial probability case for $F_o = 1$ looks as the following:

$$P(0,0) = \frac{1-F_f}{1+D+\bar{N}+F_f(1-D)} \text{ for } W \geq D.$$

$$P(0,0) = \frac{1-F_f}{1+D+\bar{N}+F_f(1-W)} \text{ for } W < D.$$

For the case of a uniform forward and reverse data transmission path $(F_o, = F_f = F)$ with conditions of unlimited window size $(\mathrm{W} \to \infty)$ and, consequently, time-out duration $(S \to \infty)$, the probability of initial states of Markov chain gets this form:

$$P(0,0) = \frac{(1-F)F}{D(1-F)F+(1-F+F^2)F(1+\bar{N})+1-2F+2F^2} \text{ for } W \geq D.$$

$$\mathrm{P}(0,0) = \frac{(1-\mathrm{F}+\mathrm{F}^2\Phi^{\mathrm{W}-1})(1-\Phi)\mathrm{F}}{\mathrm{F}[\mathrm{F}^2(\Phi-\Phi^{\mathrm{W}})+\mathrm{F}^2(1-\mathrm{W})(1-\Phi)+(1-\Phi)^2(\mathrm{D}+1+\bar{\mathrm{N}})]+(1-\Phi)^2\left((1-\mathrm{F})-(1-\mathrm{F})^{\mathrm{S}-\mathrm{D}+1}\sum_{\mathrm{m}=1}^{\mathrm{N}}\frac{\mathrm{b_m}}{(1-\mathrm{F})^{\mathrm{m}}}\right)} \text{ for } W < D.$$

after we found all the cases, now we can find the throughput for the selected loaded channel, which we study and to show how the throughput is behaving.

4. Throughput of the loaded channel

The capacity of the transport connection in the conditions of competition flows of various subscribers for transmission channel throughput is defined as the ratio of the average amount of data, transmitted between two consecutive acknowledgements, to the average time of getting the acknowledgements [7], [8]. The states of Markov chain, for which it is possible to get receipts, have a contribution to the throughput of the virtual connection.
The following equations can calculate the bandwidth for the upper constraint $S \geq W + D + N - 1$:

$$Zg = P(0,0)F_f\{F_o{}^2F_f(1-\Phi^{W-D+1}) + [(1+\Phi^{W+1}-\Phi^W)\sum_{n=1}^{N}\frac{b_n}{n+1} + \frac{\bar{F}_o{}^{S-D+1}(F_f{}^W - F_f{}^{W+1}\bar{F}_o - 1 + \Phi)}{1-F_f}\sum_{n=1}^{N}\frac{b_n}{(n+1)\bar{F}_o{}^n}]\}\{1-F_f+F_oF_f\Phi^{W-1}\}^{-1} \text{for } W \geq D.$$

$$Zg = \frac{P(0,0)F_f}{1-F_f+F_oF_f\Phi^{W-1}}\sum_{n=1}^{N}\frac{b_n}{n+1}\{(1-\Phi^W) + \frac{(1-\Phi)(F_f{}^W-1)\bar{F}_o{}^{S-D-n+1}}{1-F_f}\} \text{ for } W < D.$$

For the cases of absolute reliable reverse data transmission path $(F_o = 1)$, for $W < D$ and the throughput depend on the closeness of the value of windows size with the value of round-trip duration, but for $W \geq D$, is invariant to D. For calculating we get the following formulas for $S \geq W + D + N - 1$:

$$Zg = \frac{F_f(F_f+\sum_{n=1}^{N}\frac{b_n}{n+1})}{1+D+\bar{N}+F_f(1-D)} \text{ for } W \geq D.$$

$$Zg = \frac{F_f\sum_{n=1}^{N}\frac{b_n}{n+1}}{1+D+\bar{N}+F_f(1-W)} \text{ for } W < D.$$

For the case of a uniform forward and reverse data transmission path $(F_o = F_f = F)$ with conditions of unlimited window size and time-out duration $(W \to \infty, S \to \infty)$, the throughput is as the followings:

$$Zg(W=\infty, S=\infty) = \frac{P(0,0)F}{1-F}(F^3 + \sum_{n=1}^{N}\frac{b_n}{n+1}) \text{ for } W \geq D$$

$$Zg(W, S=\infty) = \frac{P(0,0)F(1-\Phi^W)}{1-F}\sum_{n=1}^{N}\frac{b_n}{(n+1)} \quad \text{for } W < D$$

Consider the following solution for the throughput for the interval restrictions on the duration of the timeout $W + D - 1 \leq S \leq W + D + N - 1$ and the size $1 \leq N \leq D - 2$ of the opponents' queues when window size is greater than round-trip delay $W \geq D$:

$$Zg = \frac{F_{\text{п}}P(0,0)}{Y(1-F_f)}[F_o{}^2F_f(1-\Phi^{W-D+1})(1-F_f) + \sum_{n=1}^{S-D-W+1}\frac{b_n}{n+1}[(1-F_f)(1-\Phi^W) + (1-F_f{}^W)(1 - F_f)(\Phi-1)\bar{F}_o{}^{S-D-n+1}] + \sum_{n=S-D-W+2}^{N}\frac{b_n}{n+1}(1-\bar{F}_o{}^{S-D-n+1})]$$

Where $Y = 1 - F_f + F_o F_f\left[\Phi^{W-1}\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{N} b_m\,\Phi^{S-D-m}\right]$

For reverse data transmission path ($F_o = 1$), it takes the form:

$$Zg = \frac{F_f\{F_f(1-F_f)+(1-F_f)\sum_{n=1}^{S-D-W+1}\frac{b_n}{n+1}+\sum_{n=S-D-W+2}^{N}\frac{b_n}{n+1}\}}{(1-F_f)\{D(1-F_f)+F_f+1+\bar{N}\}}$$

For the case of a uniform forward and reverse data transmission path ($F_o, = F_f = F$):

$$Zg(W=\infty, S=\infty) = \frac{F^5}{1+F^3+\bar{N}F(1-\Phi)+\Phi(D-1)}$$

For queue size of $D-2 \le N \le W-2$ we get:

$$Zg = \frac{P(0,0)F_f}{Y}\{F_o{}^2F_f\left[1-\phi^{W-D+1}\sum_{m=1}^{S-W-1} b_m - \sum_{m=S-W}^{N}\phi^{S-D-m}\,b_m\right] + \frac{1-\phi}{1-F_f}\left[\sum_{m=1}^{S-D-W+1}\frac{b_m}{m+1}((1 -\phi^W)(1-F_f) - \bar{F}_o{}^{S-D+1-m}(1-F_f{}^W)) + \sum_{m=S-D-W+2}^{N}\frac{b_m}{m+1}(1-\bar{F}_o{}^{S-D-m+1} -F_f(1-\phi^{S-D-m+1}))\right]\}^{-1}$$

And for $N \ge W-2$ the throughput can be calculated as the followings:

$$Zg = \frac{P(0,0)F_f}{U}\left[F_o{}^2F_f\left(\sum_{m=1}^{S-D-1} b_m - \phi^{W-D+1}\sum_{m=1}^{S-W-1} b_m\right) + \frac{F_f(1-\phi)}{1-F_f}\sum_{m=1}^{S-W-1}\frac{b_m}{m+1}\left[(1-\phi^W)(1-F_f) -(1-F_f{}^W)\bar{F}_o{}^{S-D-n+1}\right]\right]$$

Where $U = 1-\phi-F_oF_f\left[(1-\phi^{W-1})\sum_{m=1}^{S-D-W} b_m + \sum_{m=S-D-W+1}^{S-D-1} b_m\left(1-\phi^{S-D-m}\right)\right]$.

For reverse data transmission path ($F_o = 1$), it takes the form:

$$Zg = \frac{P(0,0)F_f}{1-F_f\sum_{m=1}^{S-D-1} b_m}\{F_f\sum_{m=1}^{S-D-1} b_m + \sum_{n=1}^{S-W-1}\frac{b_m}{m+1}\},$$

For the case of a uniform forward and reverse data transmission path ($F_o, = F_f = F$):

$$Zg = \frac{P(0,0)F}{U}\left[F^3\left(\sum_{m=1}^{S-D-1} b_m - \phi^{W-D+1}\sum_{m=1}^{S-W-1} b_m\right) + \frac{F(1-\phi)}{1-F}\sum_{n=1}^{S-W-1}\frac{b_m}{m+1}\left[(1-\phi^W)(1-F) -(1-F^W)(1-F)^{S-D-m+1}\right]\right]$$

For the interval of $W+D-1 \le S \le W+D+N-1, 1 \le N \le W-2, W \le D$ the following is obtained:

$$Zg = \frac{P(0,0)F_f}{1-F_f+F_oF_f[\phi^{W-1}\sum_{m=1}^{S-D-W} b_m+\sum_{m=S-D-W+1}^{N} b_m\phi^{S-D-m}]}\left[\sum_{n=1}^{N}\frac{b_n}{n+1}(1-\Phi^W) + \frac{(1-\Phi)(F_f{}^W-1)\bar{F}_o{}^{S-D-n+1}}{1-F_f}\right]$$

For reverse data transmission path ($F_o = 1$) we get the followings:

$$Zg = \frac{F_f\sum_{n=1}^{N}\frac{b_n}{n+1}}{1+D+\bar{N}-F_f(W-1)}$$

For the case of a uniform forward and reverse data transmission path ($F_o = F_f = F$):

$$Zg(W, S=\infty) = \frac{F^2(1-\phi)(1-\Phi^W)\sum_{n=1}^{N}\frac{b_n}{(n+1)}}{F^3+(1-\phi)^2[1+F(D-W+\bar{N})+W\phi]}$$

The ultimate throughput solution for $W \ge D, D < S < D+N+1,\ F_o = 1$ has the following form:

$$Zg = \frac{F_f[F_f\sum_{m=1}^{S-D-1} b_m+\sum_{m=1}^{S-W-1}\frac{b_m}{m+1}]}{1+D-(D-1)F_f\sum_{m=1}^{S-D-1} b_m+\sum_{m=1}^{S-D-1} m b_m+(S-D)\sum_{m=S-D}^{N} b_m}$$

As a result, numerical analyzes show that by increasing the queue number of rivals, the throughput of the transport channel decreases. You can easily see in (Figure 1,Figure 2), with a length of N = 8, the throughput is lower than with N = 6,4,2.

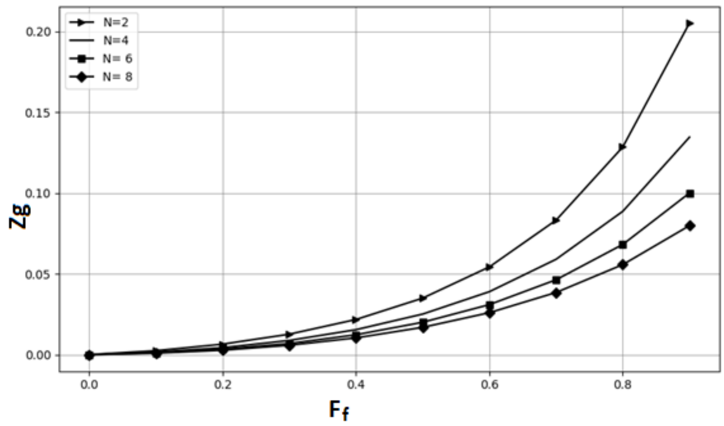

Figure 1. Dependency of the throughput on different queue sizes, for $D = 18, W = 19, b = 5, S \geq D + W + N - 1, F_o = 1$.

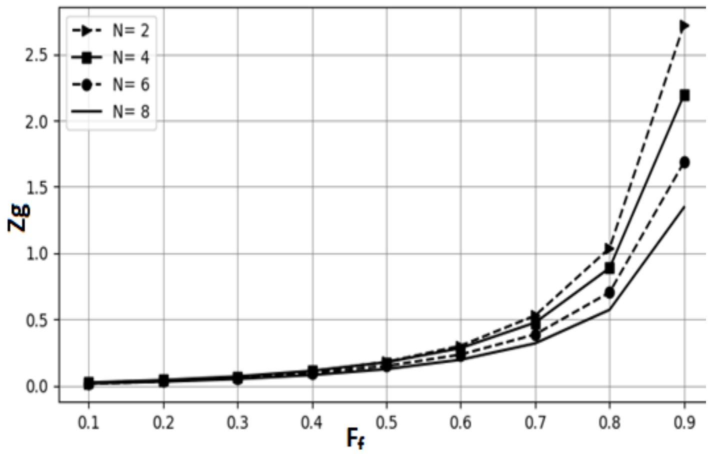

Figure 2. Dependency of the throughput on the reliability of data transmission in forwarding channel, for $D = 12, W = 15, b = 5, 1 \leq N \leq D - 2, D + W - 1 \leq S \leq D + W + N - 1, F_o = 1$.

Figure 3 shows how the throughput of transport connection changes by moving the round-trip delay with fixed values of the queue length and the geometric distribution of the queue length. a parabolic dependence is observed.

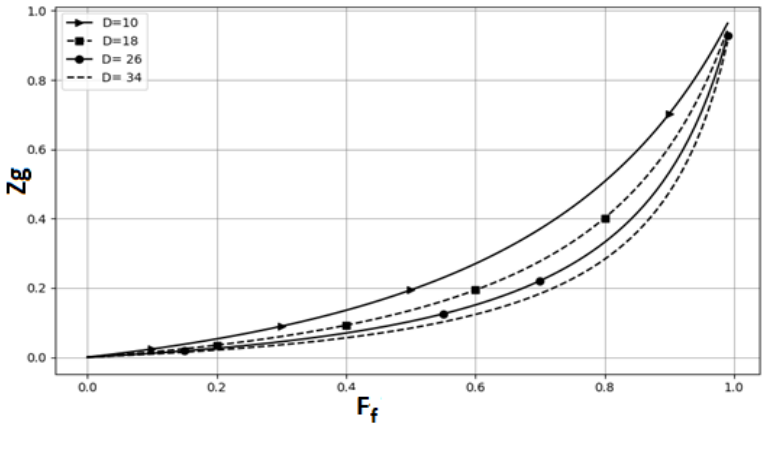

Figure 3. Dependency of the throughput on the reliability of data transmission in forwarding channel for different round-trip delay and with parameters of $N = 2, b \in [2..15]\ with\ step =\ 3, S \leq D + W + N - 1, F_o = 1$.

5. Conclusion
In this paper, the mathematical model, in analytical and numerical forms has been proposed, and the relationship between the parameters of the loaded transmission channels of different subscribers is found. Also, the throughput of the virtual channel is described with which, we can calculate throughput of the transport connections. The analysis of equations shows that the throughput of $W \geq D$ is unchanged for the round-trip duration, but for $W < D$, the throughput of the channel depends on how close the window size value is to the round-trip duration value. From the equations, we can see that as the competition between subscribers in the transmission connection intensifies, for $W < D$, the throughput of the channel decreases. According to the results of the numerical analysis, the throughput dependency on the reliability of transmission data in the forward channel has a parabolic dependence and the form of the throughput dependency on the queue size, and on the round-trip delay is hyperbolic. Consequently, the longer the queue size, the lower the throughput. In [11], the efficiency of the FEC (Forward Error Correction) model was presented, without considering the load on the transport channel. Formerly, after the completion of this work, the research will be continued. As a next step of improving this study, the method of forward error correction considering the queue size on the transport connection will be applied. Then, identify those cases in which the FEC method has positive results in comparison with the classical transport channel (without using FEC) in conditions of an increased level of errors and at long distances between hosts.

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