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# Application of the Prony method for biometric data analysis

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**Abstract.** The paper deals with the processing of numerical series using the spectral estimation methodology. The methodology is based on the Prony transformation. The conversion lets you represent the original number series as a new series. The new series is a linear combination of exponential functions. The article presents the basic concepts and relations of the Prony transformation, and also analyzes the behavior of individual components of the transformation for typical cases. The features of the Prony method are analyzed. Shown are the individual stages of the algorithm, the problems of a specific implementation. The Prony method was used to process the recordings of the electroencephalograms of the operators in their mental representation of movements. This allows us to assume the possibility of using the described methodology in identifying EEG correlates of motor imagination in the space of the roots of polynomials

## 1. Introduction

For a long time, such physical characteristics as voice samples, handwriting samples (dynamic characteristics), hand vein patterns, and corneal prints (static characteristics) were traditionally considered biometric data. However, since the beginning of the 21st century, instrumental biomedical signals such as electroencephalogram (EEG) [1] and electrocardiogram (ECG) [2] have also been actively studied in this regard.

Processing of biomedical signals is largely based on the use of spectral analysis [3]. The methodology of spectral estimation appeared several centuries ago. A detailed overview can be found, for example, in the source[4].

Currently, the discrete Fourier transform (DFT) is widely used due to the fast Fourier transform(FFT) algorithm, which significantly speeds up data processing. However, the DFT has a few disadvantages. This encourages the search for new approaches and methods in time signal processing [5]. Recently, there have been papers in which spectral estimation of signals is based on the Prony method [6].

Therefore, the authors [7] modified the Prony least-squares method for the frequency domain and applied the resulting algorithm to process short time series with recordings of music and speech. This article [8] approximates EEG recordings using the Prony method and shows the possibility of detecting differences in the field of poles of the characteristic polynomial for healthy people and patients with epilepsy.

In this paper, a time series model is constructed using the Prony transform. The model is being tested on the test sequences and used for the analysis of EEG traces.



The purpose of this study is to investigate the Prony method in relation to the analysis of biometric data. One of the goals of the study is to explore the possibility of identifying mental correlates by calculating the roots of Prony.

## 2. Prony method

Briefly, the essence of the method is as follows. Let us have  $n-1$  values of the time series  $x(i)$ , the number  $n$  belongs to a certain range of observed data  $1 \leq n \leq N$  in the time interval  $T$ . Then the value of  $x(n)$  can be predicted by the following sum of  $p$  terms, this sum is also called the  $p$ -term model of complex exponents [4]:

$$x(n) = \sum_{k=1}^p h_k z_k^{(n-1)}, \quad (1)$$

the multipliers  $h_k$  and  $z_k$  are generally complex and are defined by the expressions:

$$h_k = A_k \exp(j\theta_k), \quad z_k = \exp[(\alpha_k + j2\pi f_k)] \quad (2)$$

The notation in formula (2) is generally accepted:  $A$ ,  $\theta$ ,  $\alpha$ ,  $f$  are the amplitude, phase (in radians), attenuation (in  $s^{-1}$ ) and frequency (in Hz) of the  $k$ -th term, respectively.

For series of real numbers, the formula (2) for each complex exponent will contain its complex-conjugate pair and the series (1) will take the following form.

$$x(n) = \sum_{k=1}^{p/2} 2A_k \exp[\theta_k (n-1)T] \cos[2\pi f_k (n-1)T + \theta_k] \quad (3)$$

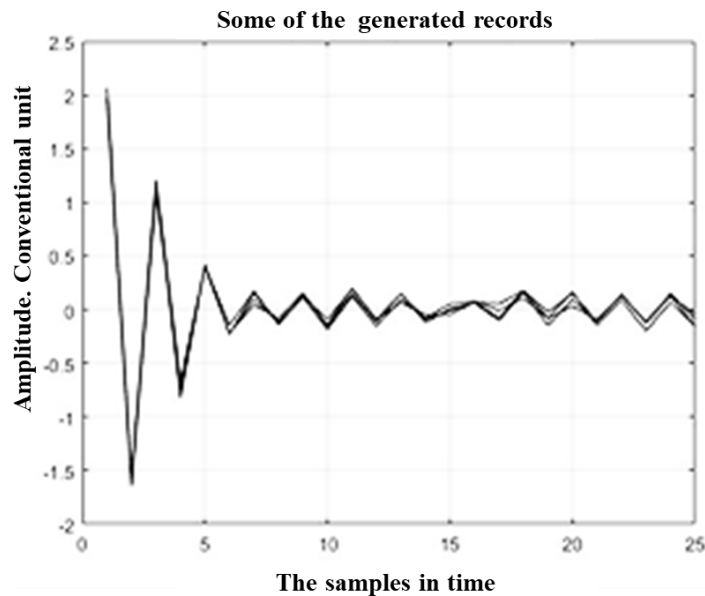
It is important to stress the truncated nature of the series in formula (3) in comparison with a number (2). The number of terms of the series was two times less ( $p/2$  instead of  $p$ ) for the complex conjugate terms.

## 3. Processing of model data

Following the source [4], we consider an analytically defined time series

$$\begin{aligned} s_1 &= -0,1 + 0,52(2\pi i), \quad s_2 = -0,2 + 0,42(2\pi i), \\ x[n] &= \exp(s_1 n) + \exp(s_2 n) + w[n], \quad n = 0, \dots, 24. \end{aligned} \quad (4)$$

Here  $w$  is a complex additive white Gaussian noise. For a signal-to-noise ratio of 20 dB, several generated records are shown in figure 1



**Figure 1.** Synthetic recordings with a 20dB signal-to-noise ratio.

Using the example of series (4), we will consider the procedure of applying the Prony method.

For estimation of the set values of  $x(n)$  in compliance with (1), it is necessary to calculate estimation errors

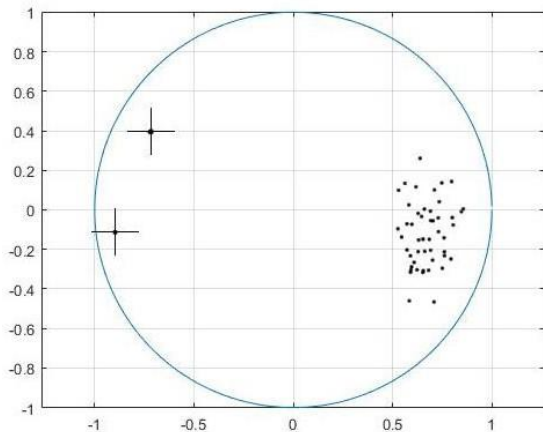
$$\varepsilon(n) = x(n) - \sum_{k=1}^p h_k z_k^{(n-1)}, \quad (5)$$

Then it is necessary minimize the sum of squared errors for parameters  $h_k$ , parameters  $z_k$  and the order of evaluation  $p$ . This problem is essentially nonlinear even for a given  $p$  [4]. The difficulties encountered in minimizing the sum of squared errors resulted in the development of suboptimal optimization procedures called the Prony method, or the Prony least-squares method.

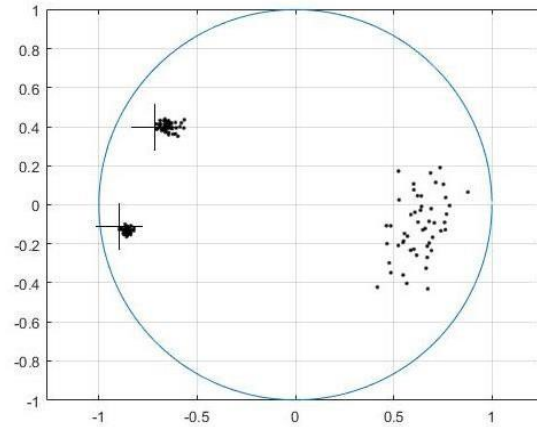
Equation (1), written in matrix form, is itself the solution of a linear difference equation with constant coefficients. A polynomial associated with a linear difference equation is called a characteristic polynomial. Thus, the first step is to find a solution for the coefficients of the polynomial. At the second stage, the roots of the characteristic polynomial are found. These roots let us determine the attenuation coefficient and the frequency of the sinusoid (2), and then use the same roots to find complex parameters.

Therefore, the complex roots of the characteristic polynomial completely determine the character of the analyzed data. Finding complex roots is part of the common Prony algorithm

Figures 2 and 3 show the results of applying the Prony procedure to data (4), with the estimation parameters  $p=3$ . Some of the roots of polynomials to short segments of data can describe a undamped sinusoid, so for convenience, the graphs show unit circles. Quasiharmonic components are defined by roots that get into any of the quadrants inside the circle; the abscissa axis corresponds to the exponential terms.



**Figure 2.** The space of zeros of the characteristic polynomial with a signal-to-noise ratio of 20dB, the accurate values of the roots are shown by crosshairs.



**Figure 3.** The same as in figure 2, but for a signal-to-noise ratio of 40dB.

A comparison of figures 2 and 3 shows the following.

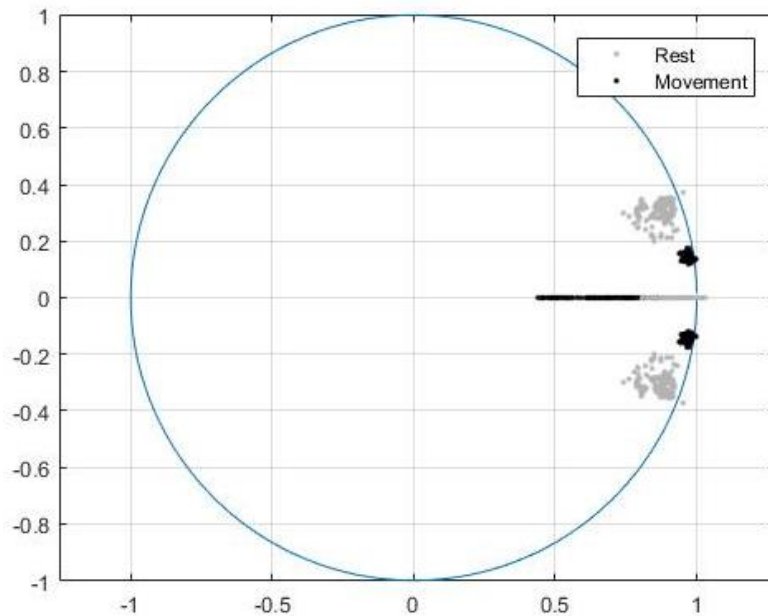
The root  $s_2$ , which corresponds to the second term in (4) (in figures 2 and 3, the crosshair in the second quadrant), has a greater attenuation coefficient than the root  $s_1$ . This leads to a greater bias in the estimates of terms that characterize attenuation and frequency. This corresponds to the results [4]. It is also necessary to note the obvious increase in the accuracy of determining the main tone of the signal (4) with a decrease in the part of noise.

One of the characteristic features of the Prony method is the absence of a strict set of frequencies for this set of samples. You can talk about bandwidth. This is the frequency range in Hz that corresponds to the given harmonic number. The estimation procedure provides for calculating the accurate values of the spectra in frequency increments that are independent of the number of observations. It makes Prony method is convenient in the processing of even small samples of data.

#### 4. Processing of biometrics data using the Prony method

For work with biomedical data, the open access database of ECG and EEG of the Department of higher nervous activity of Lomonosov Moscow state University was used. EEG recordings of two subjects were selected. The recordings were made during the subject's mental representation of the movements of the fingers of the left and right hands. Control records were also taken, which were carried out in a calm state. The scheme of arrangement of electrodes - 10-20% with additional electrodes. Entries were selected for the F3 (left frontal area) and F4 (right frontal area) electrodes.

The record length is about five in increments of 0.002 s. The section from the 2nd second to the 3rd second (500 points) was taken from the recording. The sliding processing window is 50 samples, the order of estimation is  $p=3$ . The choice of the sliding window size (100 ms) was determined by a preliminary visual evaluation of the records and followed the recommendations for using the EEGLAB processing environment, which was developed by the Schwartz Center for Computational Neuroscience. A description of this environment can be found on the Center's website. The overlap area when sliding the window is 96 ms. Figure 4 shows the result of processing the operator's EEG recording in a calm state (light points) and with a mental representation of the movement of the right hand (black points) for channel F3, and figure 5 shows the same data for channel F4.



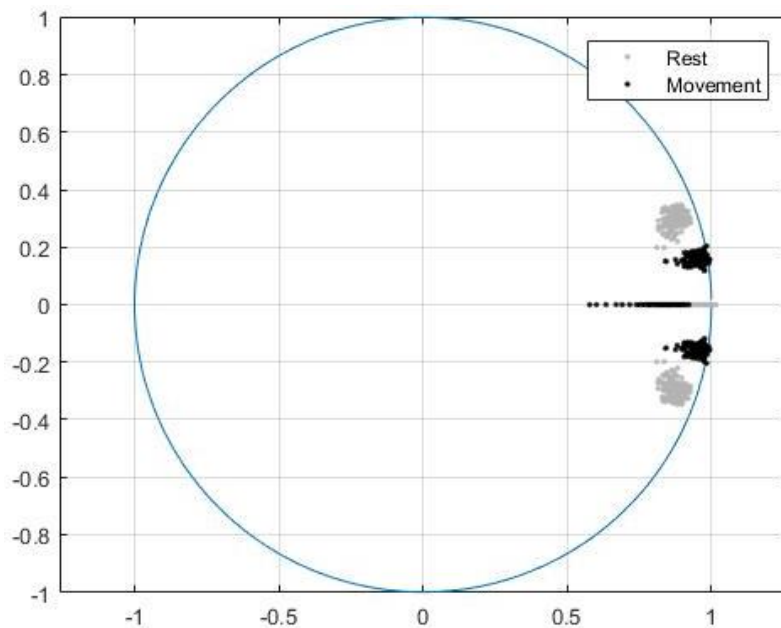
**Figure 4.** Field of zeros of the characteristic polynomial for EEG at rest (light points), and during mental movement (black points), channel F3, epoch 2, evaluation order 3, sliding window of 50 points.

Figures 4 and 5 show the complex roots of characteristic polynomials. for clarity, they are shown together with the unit circle. The roots that fall inside the unit circle provide attenuation of the members of the Prony series [4]. The possibility of analyzing biomedical records by the location of zeros of polynomials is also discussed in [3].

The order of estimation of  $p$  plays an important part in the Prony method. If the order is chosen incorrectly, a significant part of the roots may be outside the unit circle. This worsens the approximation of the series. As already noted, the values of complex-conjugate roots completely determine all the terms of the series (1). for each position of the sliding window, these values will differ from the other values of the roots. The set of harmonics at a fixed window size, as in the case of the DFT, will be unchanged, but the frequency interval will correspond to each harmonic. This feature of the Prony method makes it possible to accurately determine the spectral components of signals, in contrast to the Fourier method.

The order of evaluation here  $p=3$  caused the presence of roots on the abscissa axis for those terms of the series (3) that will contain a pure exponent. This exponent attenuates for the root inside the circle and increases for the root outside the circle. The figures show that an EEG with virtual operator movement (black dots in figures 4 and 5) is accompanied by a narrower band recording than a much wider band signal taken at rest (light dots in the same place).

A comparison of the processing results suggests that the Prony method can be used to identify signs of mental correlates in the EEG. This can be essential both for diagnostic purposes and for the development of biometric authentication methods.



**Figure 5.** Field of zeros of the characteristic polynomial for EEG at rest (light points), and during mental movement (black points), channel F4, epoch 2, evaluation order 3, sliding window of 50 points.

All results of this work are calculated using the author's programs implemented in the GNU Fortran language in accordance with algorithms from sources [4, 9].

## 5. Conclusion

The paper presents the results of calculating the Prony transformation for model and biometric data. Generated attenuated sinusoids with additive noise were taken as model synthetic data and compared with previously published results. Biometric data (EEG records) were taken from the website of the Department of higher nervous activity of Moscow state University. Recordings for correlates of mental movements were selected from the entire database of EEG signals. The features of window processing of EEG records with the formation of the field of zeros of the characteristic polynomial of the Prony least-squares method are considered. Comparison of the field of zeros for recording the EEG of an operator at rest and an operator in virtual motion revealed the possibilities of the Prony methodology for extracting features from EEG recordings. These features can be considered as patterns (correlates) of a person's mental movements.

## Author Contributions

M. M. Nemirovich-Danchenko is the author of the idea of the article. He also implemented scripts for MATLAB and the necessary calculations. M. V. Elenets, under the guidance of M. M. Nemirovich-Danchenko, conducted a study of the subject area and compiled an overview of the theoretical part of the article, as well as translated and edited the article.

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