Energy sinks: Vibration absorption by an optimal set of undamped oscillators

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This paper describes a new concept referred to here as "energy sinks" as an alternative to conventional methods of vibration absorption and damping. A prototypical energy sink envisioned here consists of a set of oscillators attached to, or an integral part of, a vibrating structure. The oscillators that make up an energy sink absorb vibratory energy from a structure and retain it in their phase space. In principle, energy sinks do not dissipate vibratory energy as heat in the classical sense. The absorbed energy remains in an energy sink permanently (or for sufficiently long durations) so that the flow of energy from the primary structure appears to it as damping. This paper demonstrates that a set of linear oscillators can collectively absorb and retain vibratory energy with near irreversibility when they have a particular distribution of natural frequencies. The approach to obtain such a frequency distribution is based on an optimization that minimizes the energy retained by the structure as a function of frequency distribution of the oscillators in the set. The paper offers verification of such optimal frequency spectra with numerical simulations and physical demonstrations. © *2005 Acoustical Society of America.* [DOI: 10.1121/1.2074807]

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I. INTRODUCTION

Considerable advances over the years have been made in reducing structural vibrations by means of damping. These range from passive methods to active-control methods. Passive methods include contact damping, fluid-layer damping, and those that use energy absorption materials, such as viscoelastic and granular materials. Selection of damping methods is based on cost and suitability to a given application; contact damping for jet engine blade vibrations, fluid layer damping for rotating thin disks, and viscoelastic layers for stationary large panels are such examples.

Energy sinks function as a substructure attached to, or an integral part of, a primary structure from which they can absorb and trap vibrational energy without adversely affecting its performance. As such, energy sinks provide a suitable alternative to application of conventional damping treatments under conditions when they are not suitable. For example, transient vibrations of large structures that have very low frequencies such as naval vessels and those deployed in space and buildings fall into this category.

The energy sinks described here consist of a set of linear oscillators. When attached to a primary structure, for example an oscillating rigid platform, the set of oscillators absorbs and retains the vibratory energy from the primary structure. Energy sinks described here, in principle, do not require the presence of loss mechanisms, or damping in the classical sense. Energy is conserved and the absorbed energy remains in the collective phase space of the attached oscillators. Depending on design parameters of the set, the absorbed energy may remain in the set permanently (or for sufficiently long duration) so that the flow of energy from the primary structure appears as damping. As such, energy sinks induce "apparent damping" to the primary structure.[1]

Irreversible absorption of energy is usually associated with nonlinear systems, such as a lattice of atoms in a solid excited by friction.[2] Corresponding linear systems normally require the presence of loss mechanisms to irreversibly absorb energy because in a conservative linear system energy exchange between a primary structure and its satellites is known to exhibit periodicity, or recurrence, determined by the system configuration. For example, considering transient cases, when a primary structure responds to an initial excitation, energy transferred to the attached oscillators returns to the primary structure after a delay during which the oscillators undergo their own periodic motions. However, as shown in this paper, under certain conditions conservative linear systems may also absorb and retain energy with near irreversibility.

The concept of energy sink described here differs from the previous similar proposals, such as spatial containment or single nonlinear attachments, or those that consider influence of internal degrees of freedom on a structure, viz. Refs. 3–7. The method used here relies on the use of a *set* of lossless,

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linear oscillators that absorbs the vibratory energy of the primary structure to which it is attached.

Multiple tuned mass dampers with linear stiffness and damping function effectively under steady-state excitation. Zuo and Nayfeh in a series of studies showed optimum distributions of these properties for vibration reduction in single- and multiple-degree-of-freedom systems.[8,9]

Among the many studies that examined energy absorption from a primary structure by attached oscillators, viz., Refs. 10–24, several showed that numerous oscillators attached to a primary structure collectively act like a viscous damper.[10,12–14] Most of these studies also demonstrate a trade off between the number of oscillators and the need for presence of a loss mechanism in the oscillators; a set of oscillators absorbs energy even for vanishing values of loss factor in each oscillator so long as the number of oscillators remain large, approaching infinity.[12]

For practical cases, however, where the primary structure has a finite number of oscillators attached to it, the assertions for vanishing loss factors hold true only during a transient period, described as the return time t^*, during which energy flows into the satellite oscillators before returning to the primary structure. If the attached oscillators do not possess any loss mechanisms, even for very large number of oscillators, energy returns to the master with a return time that depends on the number of oscillators.[1,16] For a very large number of oscillators, the transient part has a very long duration. The return time corresponds to the smallest difference among the natural frequencies of the oscillators.

The present study shows that energy can be trapped with near irreversibly by a finite number of linearly attached oscillators even in the absence of dissipative mechanisms. In this case, energy absorption by the oscillators is governed by their frequency distribution, a subject which has not yet received much attention. The results show the existence of an optimal frequency distribution that minimizes the total energy returned to the primary structure from the set of oscillators.

The underlying physics of energy absorption also relates to the return times associated with the optimum set of frequencies. The optimization method described in the following produces an optimal distribution for the fundamental frequencies of the oscillators such that the combination of the associated return times minimizes the energy retained by the primary structure. Corresponding experiments demonstrate the feasibility of energy sinks.

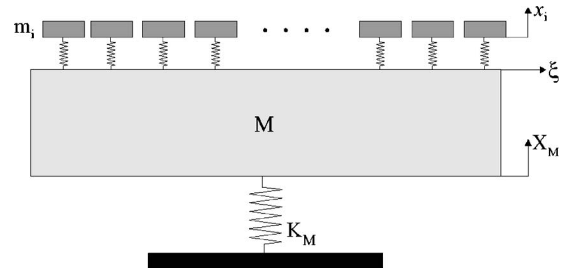

FIG. 1. Schematic description of a primary structure and attached set of oscillators.

II. MODEL

The prototypical system under consideration consists of a rigid primary structure with a substructure comprised of a set of oscillators attached to it as depicted in Fig. 1. The system does not possess any mechanism of dissipation in the classical sense, thus stiffness alone characterizes the connections between the substructure and the primary structure. The total mass, m ($m=\Sigma m_i$), of the attachments is a fraction of the primary mass, M, always $m/M \leqslant 0.1$

Dynamic response of such a system can be described by a set of coupled equations:

$$M\ddot{x}_M + K_M x_M + \sum_{i=1}^{N} k_i[x_M - x_i] = 0, \tag{1}$$

$$m_i\ddot{x}_i + k_i[x_i - x_M] = 0, \tag{2}$$

where k_i and m_i represent the stiffness and mass of individual oscillators, respectively, and $x_M(t)$ is the displacement of the primary structure and $x_i(t)$ is the displacement of the ith oscillator in the set. [For brevity, the time variable t is omitted in $x_M(t)$ and $x_i(t)$.] For a given mass ratio m/M, energy trapped by the attached set is determined largely by the properties of the mass and stiffness m_i, k_i, or the uncoupled natural frequency distribution ω_i, of the attached oscillators.

Figure 2(b) shows a typical impulse response of the primary structure in the prototype problem described by Eqs. (1) and (2) that has a linear frequency distribution as shown in Fig. 2(a). In this simple example, the natural frequencies of the oscillators have a constant frequency difference between the neighboring frequencies. As expected of linear oscillators with a linear frequency distribution, the response shows a recurrence; as shown in Fig. 2(c), energy periodically returns to the primary structure when the number of

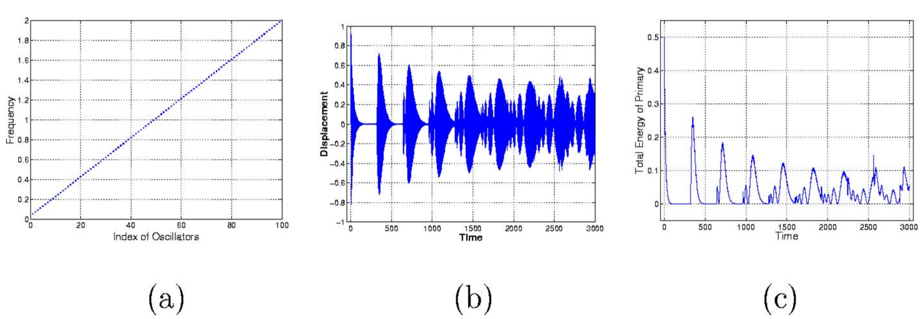

FIG. 2. (Color online) Simulation results using $N=99$ oscillators: (a) linear frequency distribution of the attached oscillators, (b) displacement response, and (c) total energy of the primary structure.

oscillators is finite, in this case $N=100$. The return time corresponds to the constant frequency difference $t^*=2\pi/\Delta\omega$. The optimization method described next seeks to determine a frequency distribution for the attached oscillators that reduces the response amplitude of the primary structure and diminishes the energy it retains.

III. OPTIMIZATION

The optimum distribution of the natural frequencies of attached oscillators is sought by finding the frequencies that minimize the objective function, which is based on the integral of the energy of the primary structure:

$$\mathcal{L}_M \sim \int_0^{\infty} x_M^2(t)dt. \tag{3}$$

Because the approach used here requires that the integral is finite, a small amount of "damping" will be introduced for optimization to find the desired distribution. However, the frequency distribution obtained from this approach will then be used in the absence of any damping in the system. With Parseval's theorem, an equivalent expression to Eq. (3) in the frequency domain becomes

$$\mathcal{L}_M \sim \int_{-\infty}^{+\infty} |X_M(\omega)^2| d\omega. \tag{4}$$

Using nondimensional parameters, substituting in Eq. (4) the equations of motion of the system (1) and (2) produces

$$\mathcal{L}_M \sim \int_{-\infty}^{+\infty} \frac{d\Omega}{\left|1 + j\eta_M - \Omega^2 - (m/MN)\Sigma_{i=1}^{N} \frac{\Omega^2(1+j\eta_i)}{1+j\eta_i-(\Omega/\Omega_i)^2}\right|^2}, \tag{5}$$

where $\Omega=\omega/\omega_M$, $\Omega_i=\omega_i/\omega_M$, with $\omega_M=\sqrt{K_M/M}$ and $\omega_i=\sqrt{k_i/m_i}$, and η_i, η_M represent the loss factors associated with the oscillators and primary structure, respectively.

Optimization searches for the minima of the multivariable function $\mathcal{L}_M(\mathbf{\Omega})$ by solving the set of coupled nonlinear equations:

$$\frac{\partial \mathcal{L}_M}{\partial \Omega_i} = 0, \quad i = 1, \ldots, N. \tag{6}$$

For the solution $\mathbf{\Omega}=[\Omega_1, \Omega_2, \ldots, \Omega_N]$ of Eq. (6) to be a local minimum solution of $\mathcal{L}_M(\mathbf{\Omega})$ requires that the Hessian Matrix H_{kj}, evaluated at $\mathbf{\Omega}$, is positive definite with all eigenvalues that satisfy:

$$H_{kj} = \frac{\partial^2 \mathcal{L}_M(\mathbf{\Omega})}{\partial \Omega_k \, \partial \Omega_j} > 0, \quad k = 1, \ldots, N, \quad j = 1, \ldots, N. \tag{7}$$

The numerical results reported in Sec. IV are obtained using the Quasi-Newton optimization method and ODE23t solver, which avoid introduction of numerical damping.

IV. NUMERICAL RESULTS

The optimization process starts with an estimated frequency distribution, seeks to minimize the integral of the energy, $\mathcal{L}_M$, by continuously adjusting the frequency distribution, and stops when $\mathcal{L}_M$ reaches its minimum value. The frequency distribution that produces the lowest value of $\mathcal{L}_M$ is accepted as the optimum distribution. The time it takes for optimization of a particular case depends on the number of oscillators and the value of the loss factor as shown in the following. The effectiveness of the resulting optimum frequency distribution is judged by the vibration amplitude of the primary mass and the total energy it retains following, in this case, an impulsive excitation as compared with the case shown in Fig. 2 for which the attached oscillators have a linear frequency distribution. In the optimization and simulations results presented in this paper, frequencies are normalized with respect to that of the primary oscillator and the frequencies for the energy sinks range between 0 and twice the primary mass resonant frequency.

A. Influence of initial frequency estimates

Optimization starting with different sets of initial estimates for the frequency distribution of the attached oscillators do not show a discernible difference in the case of $N=29$ oscillators. Some differences in the optimum results appear for $N=99$ oscillators, particularly when using very low loss factors, but not enough to affect the response of the primary and the energy it retains, as shown later with simulations.

As shown in Fig. 3, the three different initial estimates of frequency distributions, constant, linearly varying and a nonlinearly varying power-law distribution, which we refer to here as a polynomial distribution, produce nearly identical optimized frequency distributions. Also shown in Fig. 3 is the change in the value of the integral energy, represented by $\mathcal{L}_M$, throughout the optimization process, starting with the result of the first step of optimization. Its initial value and rate of decline depend on the selection of the initial frequency distribution shown in the first column. Each of the initial frequency distributions produce nearly the same minimum integral of energy, within 0.3% of each other.

Similar optimum distributions result in the case of a larger number of attached oscillators ($N=99$), however, with some variation at either end of each frequency distribution as shown in Fig. 4. The largest difference in the minimum value of integral among the three cases changes with the loss factors used in the integral. The values of $\mathcal{L}_M$ corresponding to the optimum frequency distributions shown in Fig. 4 vary 0.5% for $\eta_i=0.01$ and the variation increases to about 16% when $\eta_i=0.001$ is used. The differences that also appear in the corresponding frequency distributions indeed become less if optimization process is continued.

B. Role of loss factor used in optimization of $\mathcal{L}_M$

The loss factors used to ensure a finite value for $\mathcal{L}_M$ in Eq. (4) also determine the duration of the integral considered for optimization. Ideally, the lower the loss factors used, the closer the integral energy represents the energy sink, which does not embody dissipation sources. A lower loss factor leads to minimization of the energy of the primary structure over a longer period of time, which also requires longer computation times. A higher loss factor, on the other hand, short-

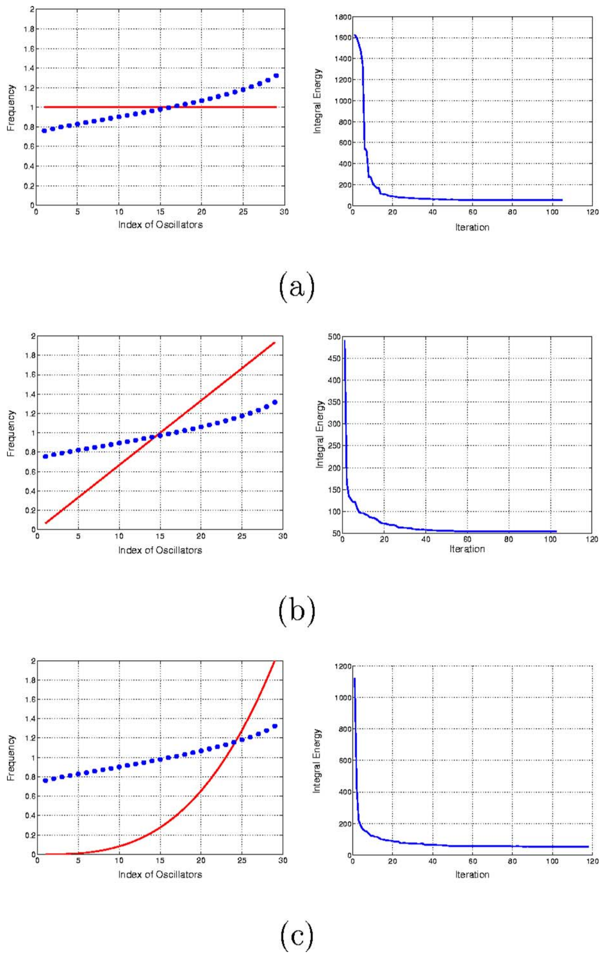

FIG. 3. (Color online) Optimized frequency distributions (dotted lines) for a system of $N=29$ oscillators each with a loss factor $\eta_i=0.001$. The change of the integral of the energy of the primary structure, Eq. (5), (right-hand column) at each iteration illustrates the optimization process for different initial estimates for the frequency distribution from which optimization starts: (a) all oscillators have the same frequency as the primary structure $\omega_i=1$, (b) linear distribution, and (c) polynomial distribution.

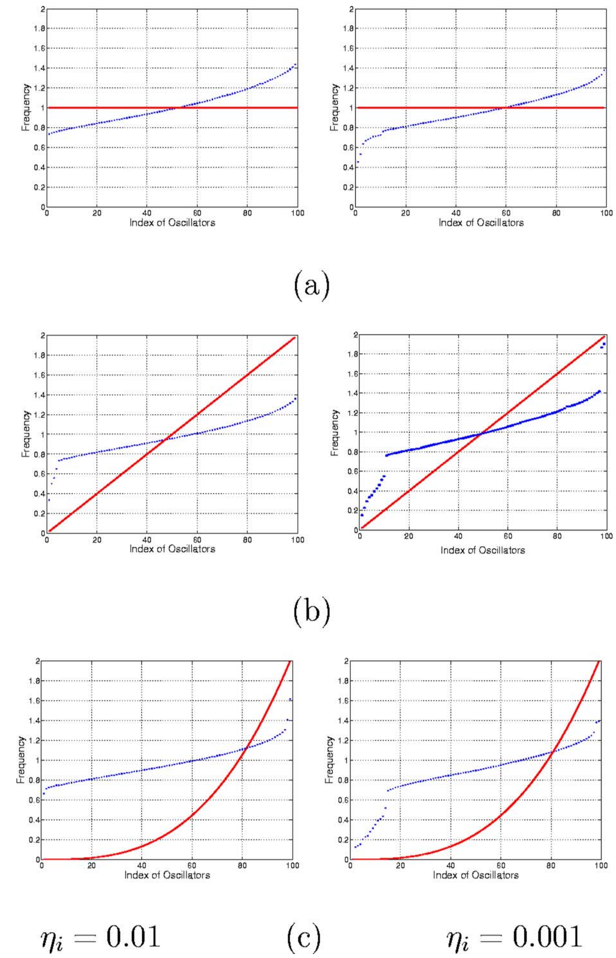

FIG. 4. (Color online) Optimized frequency distributions for $N=99$ oscillators that result from different initial frequency distributions for two different loss factors η_i: (a) all oscillators have the same frequency as the primary structure $\omega_i=1$, (b) linear distribution, and (c) polynomial distribution.

ens the period of the integral, thus reducing the time over which energy should be minimized. However, loss factors η_i also reduce the vibration amplitude of individual oscillators in the energy sink and thus reducing their effectiveness in transferring energy from the primary structure as simulations demonstrate.

Figures 5 and 6 show examples of optimization results for low and moderate values of loss factors for $N=29$ and 99 oscillators, respectively. In each case, the initial frequency distribution of the set has a constant value the same as that of the primary mass, $\omega_M=1$, as indicated by the solid line.

Optimization spreads the frequencies over a band with a higher density around the frequency of the primary mass. The optimum frequency sets in Fig. 5, obtained using loss factors $\eta_i=0.1$, 0.01, and 0.001, display very similar distributions except at each end of the frequency bands. The corresponding optimum frequency distributions for $N=99$ oscillators, shown in Fig. 6, also exhibit similar trends.

The simulations presented next show the relative effectiveness of optimum frequency distributions in reducing the vibration amplitude of the primary mass and the energy it retains. Results also show that the use of large loss factors speeds up the optimization process but produces less than optimal distributions as demonstrated later with simulations. Very small values of loss factors prolong the optimization process and introduce computational errors.

C. Simulations

The simulation results presented in the following use the optimum frequency distributions, obtained as described earlier, in solving Eqs. (1) and (2) without any damping in the system.

As an example, Fig. 7 demonstrates simulation results that correspond to the optimum frequency set shown in Fig. 5(c) obtained for $N=29$ oscillators with loss factors $\eta_i=0.001$ and compares them with the corresponding responses for the initial frequency distribution with which optimization started. The amplitude response and the total energy of the primary mass presented over a long time period show a distinct reduction from the initial conditions from which optimization started when all oscillators had the same frequency as the primary, making the initial configuration essentially a two-degree-of-freedom system. The response spectrum 7(b) of the primary mass reflects the presence of a distributed set

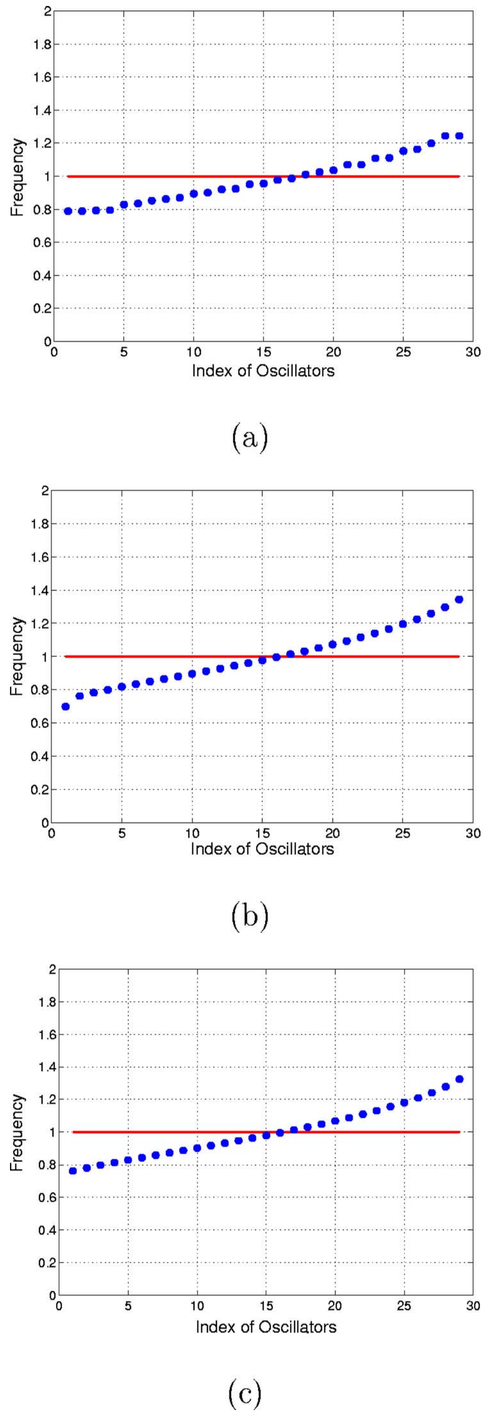

FIG. 5. (Color online) Influence of loss factors η_i on the resulting optimized frequency distributions for a system with $N=29$ oscillators, which initially have the same frequencies as that of the primary $\omega_i=\omega_M=1$. (a) $\eta_i=0.1$, (b) $\eta_i=0.01$, and (c) $\eta_i=0.001$.

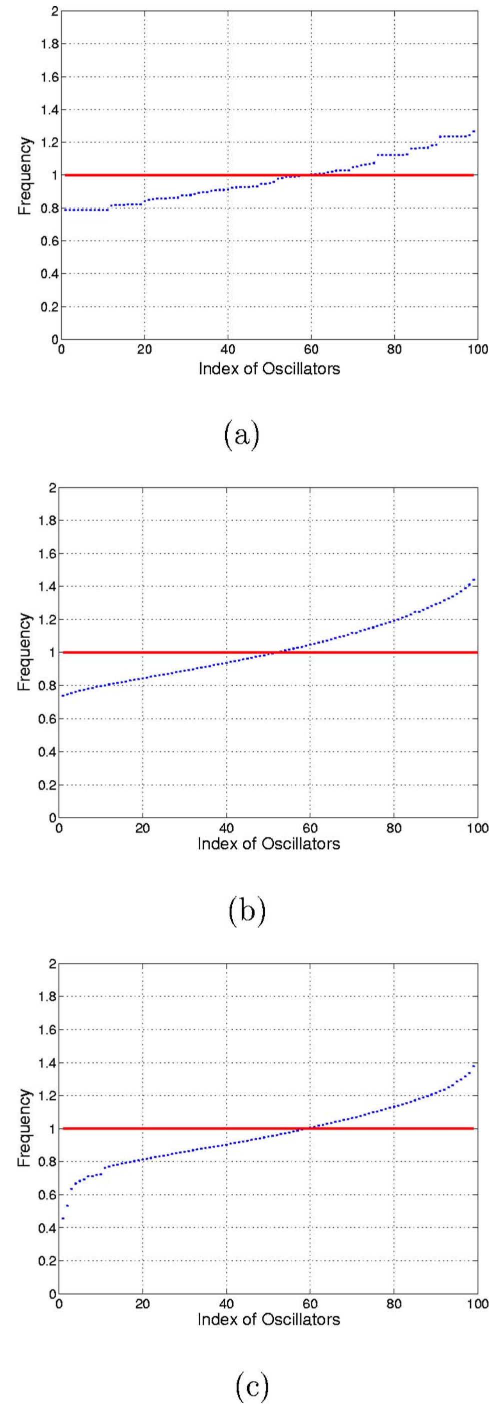

FIG. 6. (Color online) Effect of loss factor on the optimized frequency distribution for $N=99$. Optimization begins with the same initial frequencies for all oscillators and the primary structure. (a) $\eta_i=0.1$, (b) $\eta_i=0.01$, and (c) $\eta_i=0.001$.

of frequencies that contains the effects of each of the 29 oscillators, distinct from the two frequencies with which the initial estimate started out. While the initial amplitude and the energy of the primary structure exhibits a periodic behavior with a constant amplitude, the optimized frequency distribution reduces both the response amplitude and the retained energy each to a fraction of the corresponding initial values.

The impulse response of the primary structure shows that the envelop of its displacement amplitude remains within 40% of its initial response for $N=29$ and the retained energy within 20% of its initial value. The energy initially contained in the two frequencies spreads over to 29 frequencies, each with an amplitude less than 10% of the initial spectral amplitudes. As shown later, increasing oscillator number in the set further reduces the amplitude and retained energy.

Simulations that correspond to the optimized frequency sets obtained from different initial distributions presented in Fig. 4 produce responses shown in Figs. 8 and 9. The slight

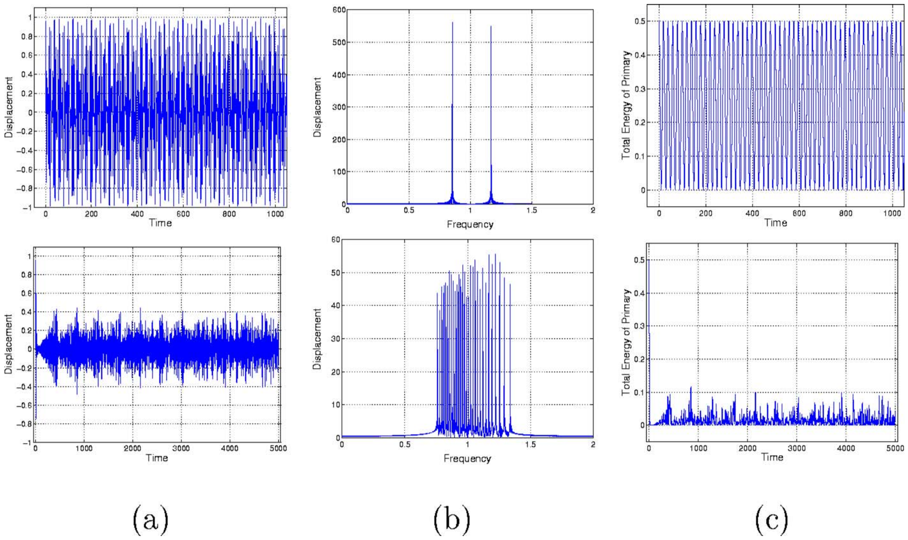

FIG. 7. (Color online) Simulation results for a set of $N=29$ attached oscillators each with a loss factor of $\eta_i=0.001$. The top row shows the response of the primary when all attached oscillators and the primary have the same frequency $\omega_i=\omega_M=1$. The bottom row shows the results obtained using the optimum frequency distribution: (a) displacement time, (b) displacement-frequency response, and (c) total energy of the primary structure.

variations among the optimal frequency distributions lead to barely discernible differences in the response amplitude of the primary and the energy it retains over time. While the differences are small, in this case the optimum frequencies resulting from an initial estimate of constant frequencies and loss factors $\eta_i=0.001$ show a better performance, Fig. 9(a).

An example of how nonoptimal frequency distributions that result from use of large loss factors in Eq. (5) affect the performance of an energy sink is shown in Fig. 10. The frequency distribution presented in Fig. 6(a), obtained using $\eta_i=0.1$, yields larger amplitudes for the displacement of and the energy retained by the primary than the corresponding cases with distributions obtained using lower loss factors. These results are consistent with the observation that the lower the loss factors used in equations of motion (5) the closer they represent the energy sink, which does not have any dissipation at all.

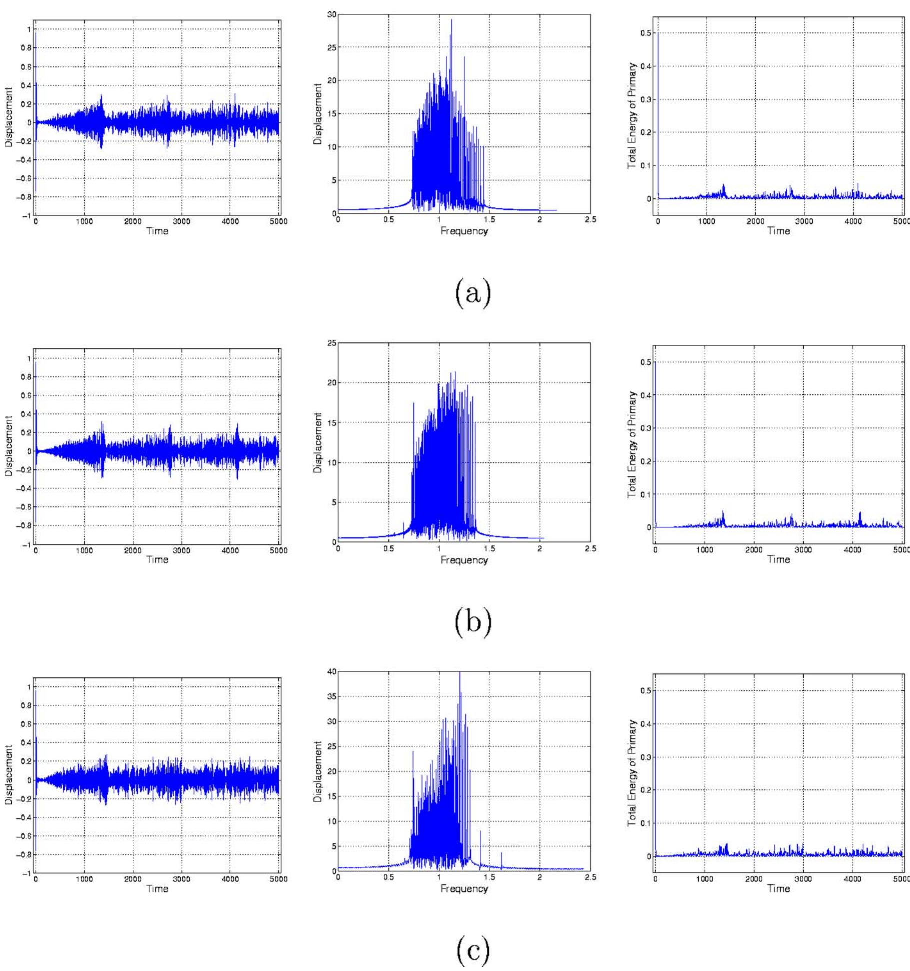

FIG. 8. (Color online) Simulation results for the optimum frequency distributions obtained starting with initial frequencies that correspond to those in Fig. 4 ($\eta_i=0.01$) show negligible difference in the displacement response of the primary in time and frequency domains as well as the total energy it retains. $N=99$. Optimum frequency used in each row corresponds to a different set of initial frequency estimates used in optimization: (a) all oscillators have the same frequency as the primary structure $\omega_i=1$, (b) linear distribution, and (c) polynomial distribution.

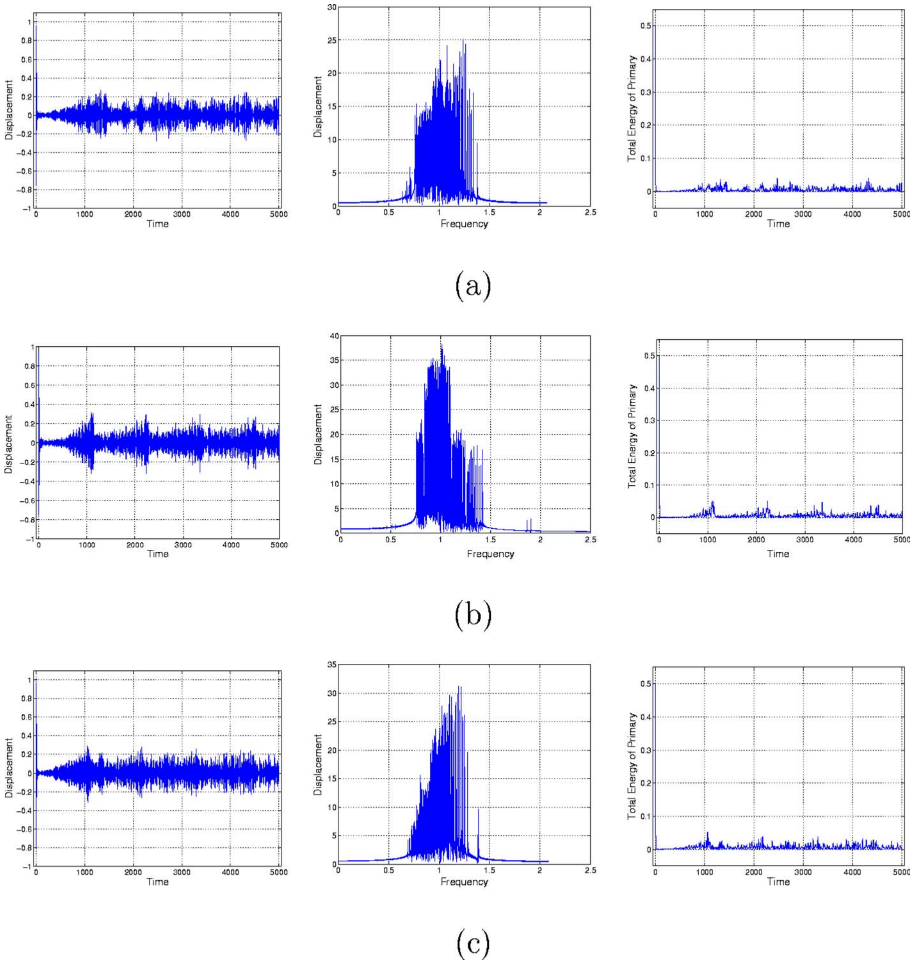

FIG. 9. (Color online) Simulation results for the optimum frequency distributions obtained starting with initial frequencies that correspond to those in Fig. 4 (η_i=0.001): (a) all oscillators have the same frequency as the primary structure ω_i=1, (b) linear distribution, and (c) polynomial distribution. Results show negligible difference in the displacement response of the primary in time and frequency domains as well as the total energy it retains. N=99.

D. Role of damping in an energy sink

Since physical systems have inherent dissipation mechanisms, it is worth examining the influence of losses present in an energy sink on its performance. For low values of loss factors, $\eta_i \simeq 0.01$, the main role of dissipation in the oscillators is to further reduce the oscillation amplitude of the primary structure as seen in Fig. 11 compared with Fig. 9(a). However, the presence of large damping in the system, for example $\eta_i \geqslant 0.1$, reduces the effectiveness, and perhaps the need, for an energy sink.

E. Multiple-degree-of-freedom systems

When optimization is applied to a primary structure with two degrees of freedom depicted in Fig. 12, oscillator frequencies distribute themselves around each of the primary frequencies and absorb energy from both masses. In the example shown, for comparison, the mass ratio between the oscillators and the two-component primary mass has the same value as in the previous examples. As a result of optimization, oscillators assume frequency distributions about the natural frequencies of the primary structure, $\omega_M^{\{1\}}$=0.618 and $\omega_M^{\{2\}}$=1.618, similar to those that resulted for a single-degree-of-freedom platform. As before, simulations show that the optimized frequency distributions reduce the vibration amplitude and the retained energy by the primary structures. The responses of the primary structures have a higher amplitude than the corresponding case with a single primary, in part, as a result of the effectively reduced number of oscillators for each mass.

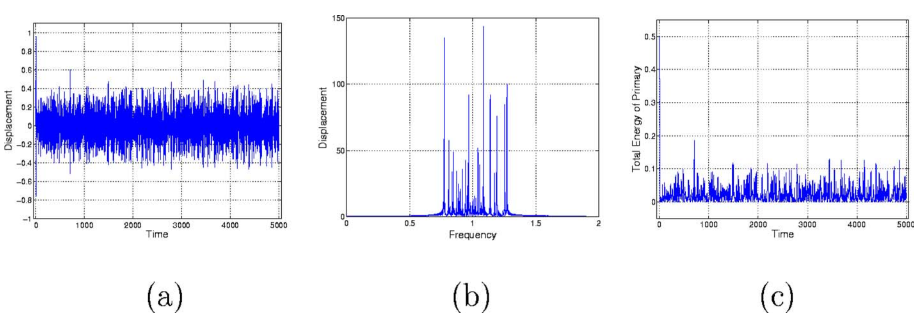

FIG. 10. (Color online) Simulation results for an energy sink with N=99 oscillators. The optimum frequency distribution, shown in Fig. 6(a), is obtained using a rather large loss factor η=0.1 for each of the oscillators: (a) linear frequency distribution of the attached oscillators, (b) displacement response, and (c) energy of the primary structure.

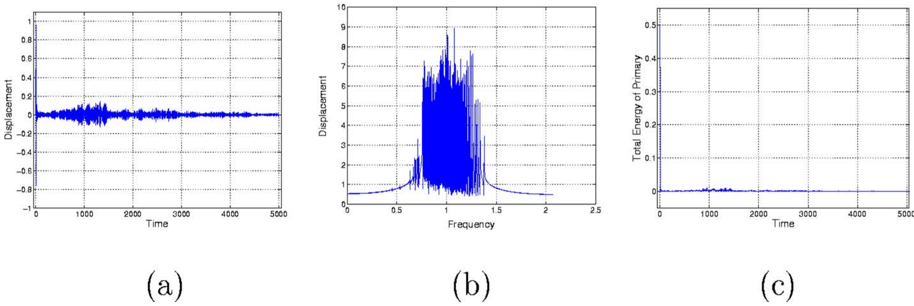

FIG. 11. (Color online) Simulated response of the primary with $N=99$ oscillators corresponding to Fig. 9(a) but each oscillator has a loss factor $\eta_i = 0.001$.

F. An analytical expression for optimal frequency distribution

The frequency distributions obtained by the above-described optimization can be approximated by an analytical expression that depends on a single parameter α:

$$\omega(\xi) = A\left[\frac{2\xi - 1}{|2\xi - 1|}\frac{e^{\alpha|2\xi-1|} - 1}{e^{\alpha} - 1} + 1\right], \tag{8}$$

where $0 < \xi \leq 1$. Discrete values $\omega_i(\xi)$ of frequency distribution are obtained with $\xi_i = i/N$.

Figure 13 shows a representative set of frequency distributions for different values of α with corresponding simulations in Fig. 14. Very small values of α represent a nearly linear frequency distribution with the same periodic energy return to the primary structure. Values of $\alpha > 5$ assign most of the oscillators the same frequency, producing essentially a two-degree-of-freedom system, with the corresponding results in rows (c) and (d) of Fig. 14 and as discussed earlier. Among these, the distribution that corresponds to $\alpha \approx 2.5$ yields the most optimum result yielding minimum average energy retention by the primary.

An alternative method to reduce the computation time in cases involving a large number of attached oscillators uses the distribution that results from optimization for a small number of oscillators. Figure 15 displays and compares the distributions obtained for $N=29$ and $N=99$ oscillators by direct optimization. Repopulating the distribution for $N=29$ with 99 oscillators and simulating the response of the primary structure as before produces results very close to those obtained by a distribution obtained through direct optimization. A comparison of the results based on interpolated distribution given in Fig. 16 with those in Fig. 9 shows very little difference.

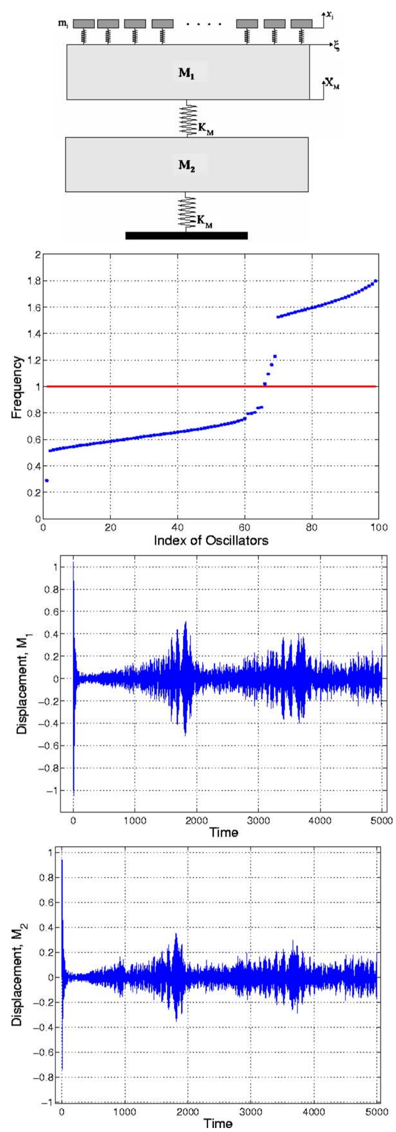

FIG. 12. (Color online) Optimized frequency distribution of $N=99$ oscillators attached to a two-degree-of-freedom primary structure and the responses of the platform with the attached oscillators and platform attached to ground (bottom). The natural frequencies of the primary structure are $\omega_M^{\{1\}} = 0.618$ and $\omega_M^{\{2\}} = 1.618$.

V. EXPERIMENTS

The two energy sinks with different physical configurations demonstrate the efficacy of the proposed energy sinks.

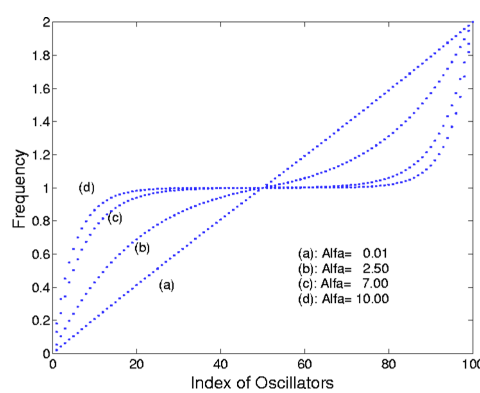

FIG. 13. (Color online) Examples of frequency distribution for different values of α in Eq. (8).

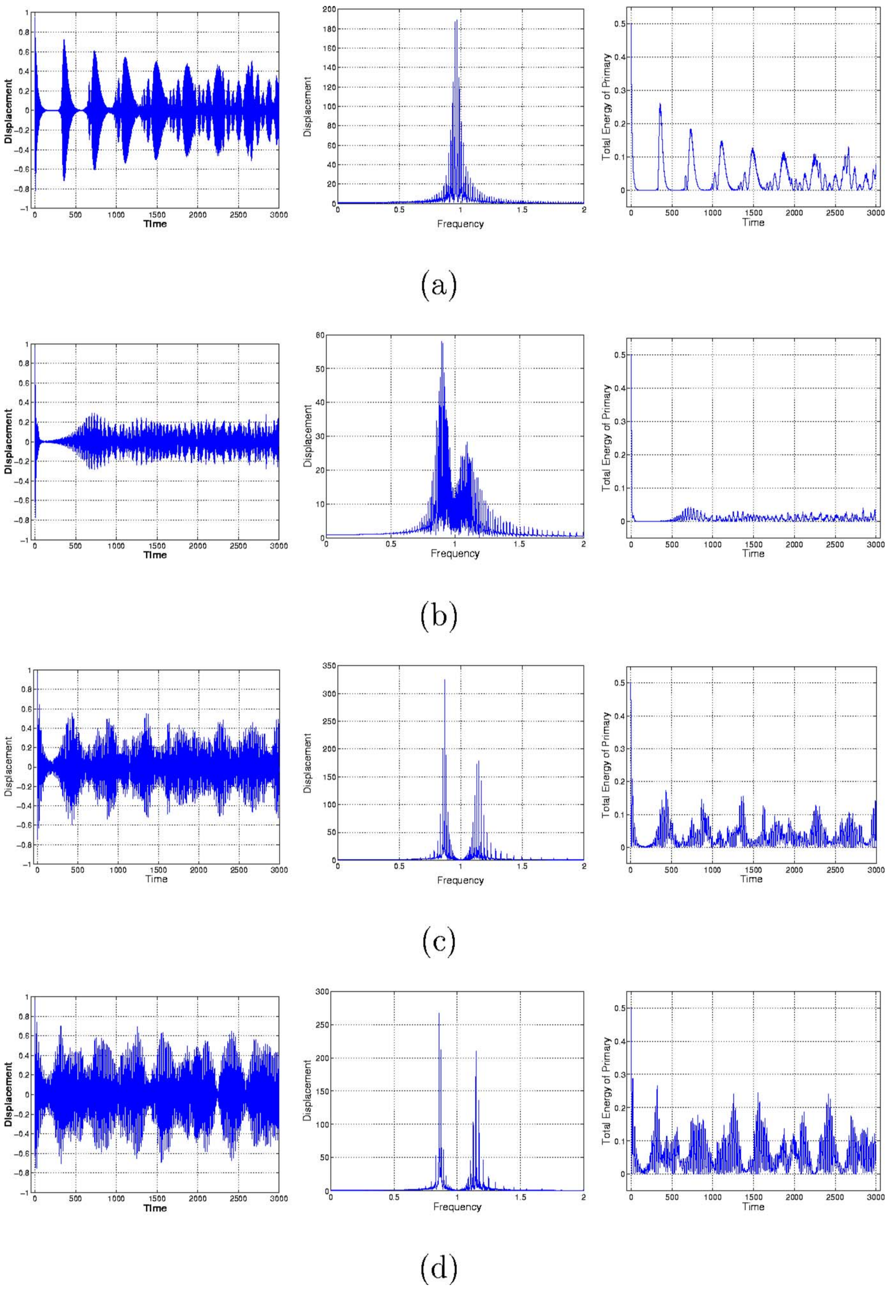

FIG. 14. (Color online) Simulation results for displacement response in time and frequency domains and the energy retained by the primary mass using values of α in Eq. (8): (a) $\alpha=0.01$, (b) $\alpha=2.5$, (c) $\alpha=7.0$, and (d) $\alpha=10.0$.

The ubiquitous presence of inherent losses in physical systems precludes construction of either a primary structure or an energy sink that can oscillate indefinitely. However, use of different frequency distributions delineates the effects of losses in the system from energy absorbed by the attached oscillators.

A. A set of thin beams attached to a *T*-configuration

Figure 17 shows the impulse response of a *T*-configuration of joined beams with and without a set of thin beams attached to it. The first natural frequencies of the thin beams follow the optimum distribution described by the analytical expression in Eq. (8). Without the oscillators, the response of the structure decays as a result of dissipation due to connections and material losses. However, the corresponding response with the attached oscillators exhibit the same behavior as those obtained through simulations in both time

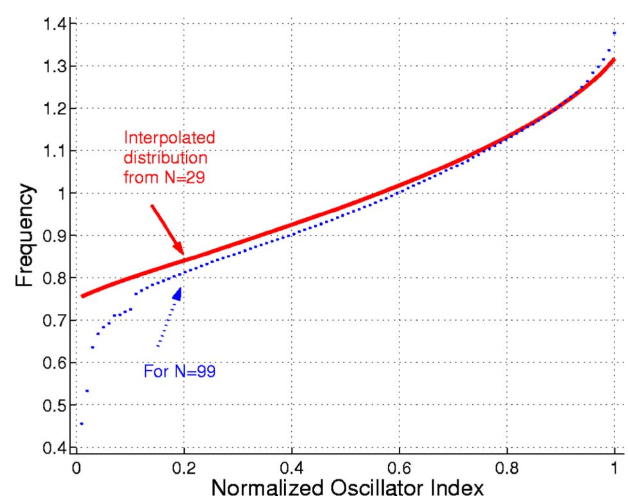

FIG. 15. (Color online) A comparison of the frequency distribution for $N=99$ oscillators obtained by two approaches. Dotted line represents result by direct optimization and the solid line represents interpolation for $N=99$ using the optimum distribution obtained for $N=29$ oscillators.

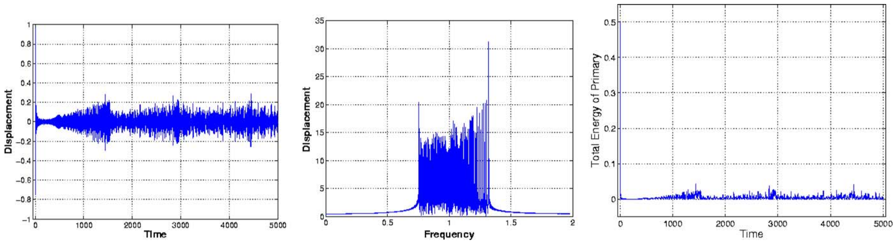

FIG. 16. (Color online) Response of the primary structure with an interpolated frequency distribution. Optimized frequency distribution obtained using $N=29$ oscillators interpolated for $N=99$ oscillators. $\eta_i=0.001$.

and in frequency domains. The response of the primary structure in the frequency domain, shown with both linear and logarithmic scales, clearly shows the effects of the oscillators.

B. A set of flexible beams attached to a rigid oscillator

This demonstration uses as the primary structure a single degree of freedom oscillator that consists of a rigid block which can freely slide in an air bearing. A pair of springs at one end anchors it to an optical table. As displayed in Fig. 18, a lightweight structure built on the block carries a set flexible cantilever wire beams, each with a mass along its axis that acts as an oscillator. Natural frequency of each oscillator is determined by adjusting the position of the mass along the beam. Figure 19 presents velocity response of the block to an impulse for two cases: the oscillators have either a linear or an optimum frequency distribution. When the distribution follows the optimum values obtained using the method described earlier, the energy returned to the block is distributed both in time and frequency compared with the case when oscillators have a linear distribution. The recurrence observed with the linear distribution has a lower amplitude than expected, in part due to the inexact values of the oscillator frequencies. The inherent damping in the physical system also reduces the response amplitudes. To better reflect the effects of losses in the system, the simulation results presented in Fig. 20 include their values as measured from the experimental setup. The simulated responses, both in time and frequency domain, represent the same characteristics as those produced by measurements.

VI. CONCLUDING REMARKS

An energy sink that consists of a set of oscillators can absorb vibration energy from a structure to which it is attached. Following transient excitation of a structure, energy that flows into the oscillators remains in their phase space.

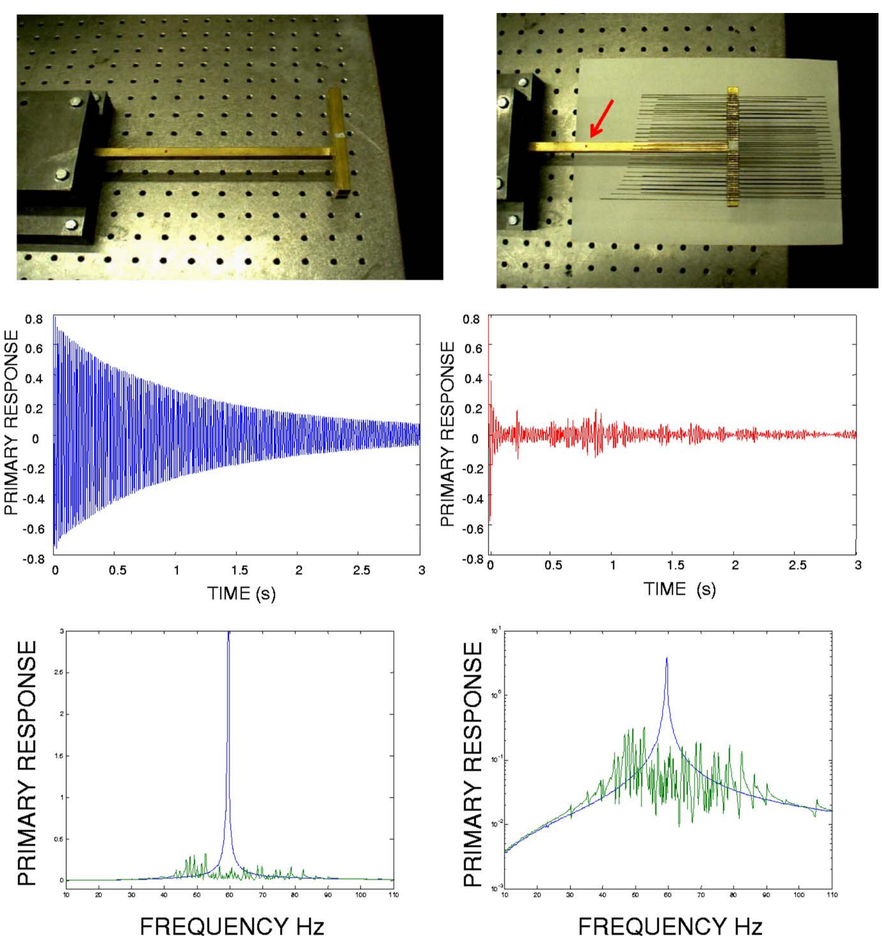

FIG. 17. (Color online) Response of a structure (top) shown with and without the oscillators that make up an energy sink. The first bending frequency of the thin beams that act as oscillators follow the analytical optimum frequency distribution given in Eq. (8). Response of the structure with and without the oscillators is shown (bottom row) in frequency domain using both linear and logarithmic scales.

The net force they collectively exert on the primary structure, and thus the total energy that returns to the primary, stays below a fraction of its initial value.

Because energy sinks, in principle, do not require conventional dissipation sources, they are particularly useful in high temperature or chemically hazardous environments where materials and mechanisms that provide dissipation may not be as effective. The unavoidable loss mechanisms in physical systems, however, induce dissipation in the primary and the attached oscillators. As long as these dissipation rates remain moderate, they do not adversely affect the performance of energy sinks, but assist in reducing the vibration amplitude of the oscillators as well as the primary structure.

Energy absorption by a set of linear oscillators relates to their frequency distribution. The optimization method presented in this paper finds such a distribution. The optimization used here minimizes the energy of the primary mass over a selected time period. Optimization results show a degree of robustness of the process with respect to the initial frequency distributions and the values of loss factors. Optimum distributions increase the density of oscillators near the frequencies of interest, as demonstrated for a two-degree-of-freedom primary structure. This result is consistent with the observation that with linear frequency distributions (and thus constant frequency difference); oscillators near the primary frequency respond with a higher level of energy absorption than those with frequencies away from it. Such a distribution also de-emphasizes the need to finely tune the frequencies as in conventional vibration absorbers.

The simulations and the physical demonstrations support the viability of reducing vibrations of structures with linear energy sinks and that the concept can be extended to primary structures with multiple degrees of freedom. Ability of linear energy sinks to absorb energy independent of dissipation sources sets it apart from many other similar approaches. In particular, energy sinks have an advantage in transient and low frequency applications.

Although this manuscript primarily addresses the role of optimum frequency distribution of energy sinks, their ability

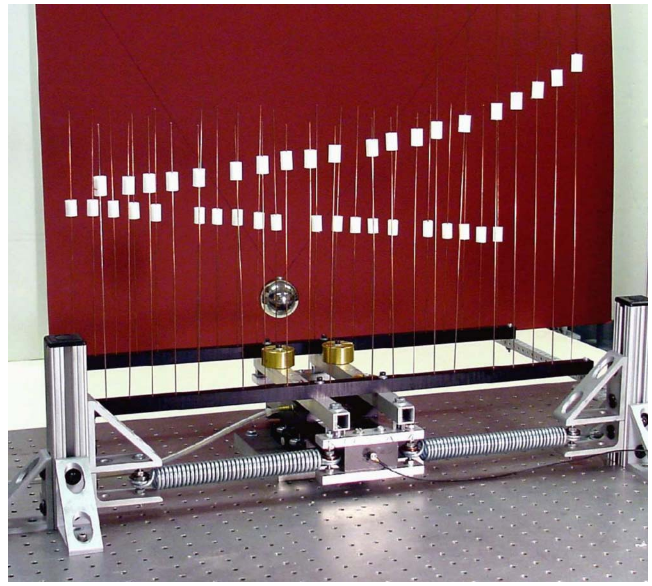

FIG. 18. (Color online) A set of 40 oscillators attached to a primary structure.

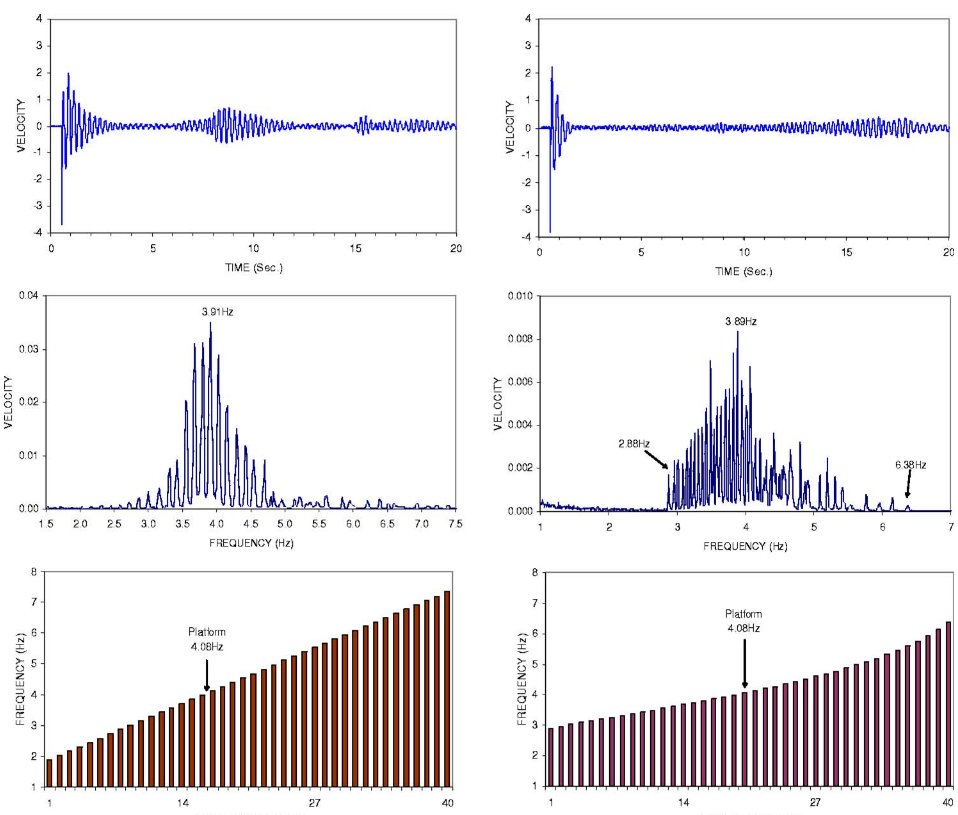

FIG. 19. (Color online) Response of the block with 40 oscillators to an impulse excitation shows that oscillators with an optimum frequency distribution (right) spread the return energy over both time and frequency and reduce its amplitude. Inherent damping in the system also reduces the amplitude of recurrence for the linear distribution, at $t \simeq 8$ s.

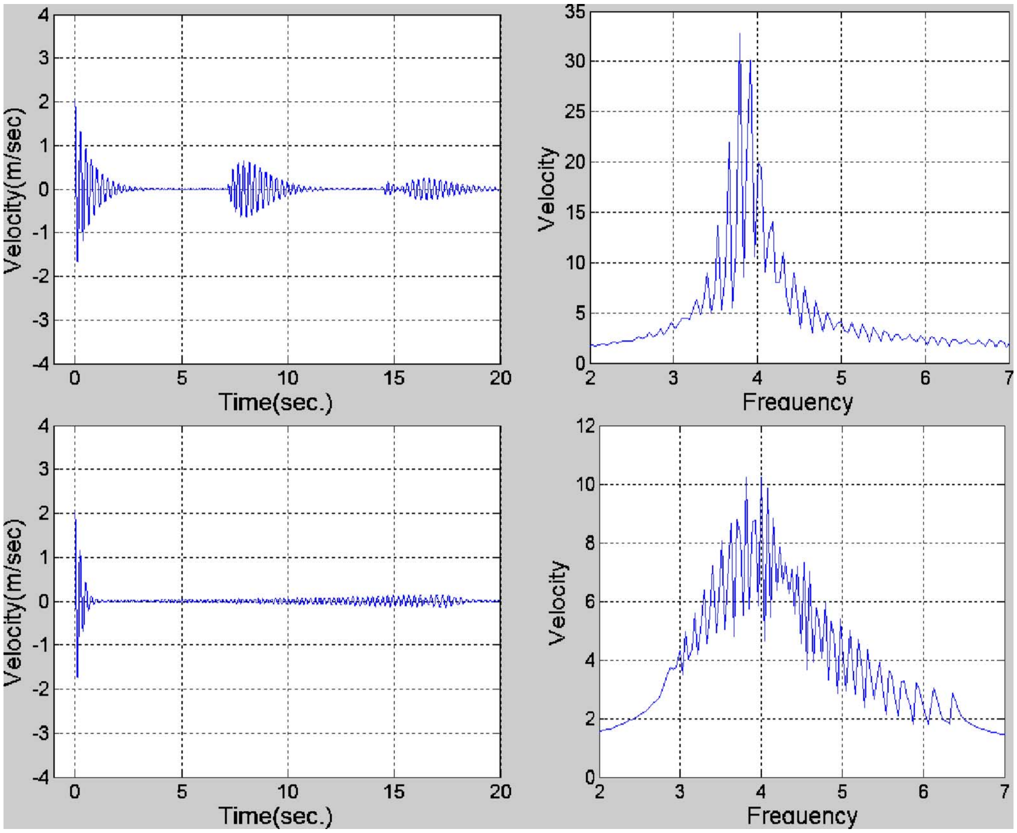

FIG. 20. (Color online) Simulated response of the block corresponding to conditions in Fig. 19 using loss factors measured from the experimental setup as $\eta_M = 0.008$ and $\eta_i = 0.008$.

to absorb energy also depends on other parameters such as the number of oscillators and their mass ratio as discussed in earlier studies, viz. Ref. 1.

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