A Paraconsistent Pavelka Technique for Multiple Criteria Decision Analysis

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Decision making involving multiple alternatives or strategies in respect of a number of conflicting criteria is usually a daunting problem for practitioners from across diverse fields. As a result, academics, particularly decision science researchers have been working so hard over the years to develop appropriate multicriteria decision making methods to aid decision makers in their quest to establish the optimum decisions in their respective fields and problems. In this paper, a technique for decision making in environments characterised by the availability of incompletely ignorant facts, partially conflicting and vague information is proposed. The technique is dependent on paraconsistent logic, Pavelka style fuzzy logic and fuzzy similarity relations. To demonstrate the use of the technique, it is applied to the selection of the best or optimal five energy mix from eight electricity generation sources for Ghana.

Key words: Multiple Criteria; Paraconsistency; Multiple-Valued logic; Decision-Analysis; Pavelka logic; Fuzzy Similarities

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1 INTRODUCTION

Decision making based on partially conflicting and vague information is a challenging task that has been addressed by many researchers in ([3], [5], [4], [12] and many others). In this paper, we introduce a new approach, an approach intended to be transparent to the end user. The technique combines paraconsistent logic, Pavelka-style fuzzy logic, especially, as developed in the paraconsistent Pavelka-style fuzzy logic defined in [15], and total fuzzy similarity relation proposed in [10].

Problem statement: the problem of most fuzzy multi-attribute decision making methods in the existing literature has to do with the size of a decision problem [13]. In any case, the size of a decision problem is determined by the number of available options and the number of attributes in such a problem. Although the existing fuzzy multi-attribute decision making methods such as Yager's model; Baas and Kwakernaak's model; Kwakernaak's model[18], [13]; Fuzzy Analytic Hierarchy Process (FAHP); Fuzzy Analytic Network Process (FANP); Fuzzy Simple Additive Weighting (FSAW); Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) and many others have been shown to have a sound theoretical foundation, they are so cumbersome to apply even to decision problems with few options (the number of options less than 10) and few criteria (the number of criteria less 10) let alone those with a large number of alternatives and a large number of criteria and sometimes sub-criteria. Therefore, the mathematical model we introduce herein is part of the global effort to develop novel approaches that have the efficiency to solve large size decision problems with less difficulty. Eventhough this new technique bears some semblance to our earlier method in [9], it is simpler, easier and friendlier to the end user than the former. The current method is most particularly suitable for experts and decision makers with less familiarity with complex mathematical computations. To illustrate the efficiency of the new approach, we have applied it to Ghana's energy decision making problem involving eight (8) electricity generation sources in relation to twenty six (26) criteria.

Pavelka-style fuzzy logic was introduced in the mid sixties following the development of fuzzy set theory by L. A. Zadeh. Pavelka-style logic is a kind of many-valued logic in which the interval [0,1] constitutes the truth value set for all formulas. Pavelka logic, thus, subsumes everything of classical logic by adopting a graded approach to truth so that the truth value of every statement or formula ranges between totally true (1) and totally false (0) values. Moreover, according to Pavelka, a given logic in the realm of

Pavelka-style logics is Pavelka-style complete if and only if the unit interval [0, 1] as the truth value set for that logic is endowed with Łukasiewicz operations [7]. By the way, the fact that Pavelka's axiomatisation of other logics than Łukasiewicz logics yielded no positive outcome does not mean that the attainment of Pavelka-style completeness in Pavelka-style logics is only possible through the Łukasiewicz operations. As a matter of fact, there exists some Pavelka-style complete logics that are based on other operations than the ones associated the standard MV-algbra [2]. Turunen did further studies on this and showed that Pavelka-style completeness holds granted that the truth value set is an injective MV-algebra [16]. Hence, in the field of decision making, Pavelka-style fuzzy logic enables us to incorporate partly ignorant facts, imprecise information, non-numerical data, unattainable data, and vague information into the decision making model.

Paraconsistent logic is another generalised version of classical logic. A logical system in which inconsistencies do not imply anything is referred to as a *paraconsistent logic* [8]. Let us assume that ϕ, $\neg\phi$ and ψ are any three well formed formulas. If we denote any logical consequence relation by $\vdash$ then such a consequence relation is said to be *explosive* granted it upholds the principle that $\phi,\, \neg\phi \vdash \psi$. This principle of explosion holds in classical logic. However, any logical consequence relation that is explosive is not an appropriate medium by which we can draw reasonable conclusions from contradictory information. In other words, meaningful inferences can only be drawn from inconsistencies if the relations do not explode. Therefore, any logical consequence relation that does not explode is *paraconsistent*. Many different paraconsistent logics have developed for different reasons and purposes. The paraconsistent logic we focus on in this paper is the one proposed by Belnap in [1], extended by perny and Tsoukias [11] and further extended by Turunen in [15] into what they called *paraconsistent Pavelka style fuzzy logic.*

Belnap asserted that depending on the proof (evidence) at our disposal any statement β can take one of four possible states, namely: *false, contradictory, unknown* and *true* and not just the usual absolutely true and absolutely false states). This means

i Proposition β is *false* if we have no proof in support of β but we have proof against β. Thus, representing falsity by F, 'there is proof' and 'there is no proof' by 1, 0 respectively, we have $F = (0, 1)$.

ii β is *contradictory* provided we have proof in support of β and at the same time have proof against β. If we denote contradictory by C then, $C = (1, 1)$.

iii β is *unknown* if we neither have proof in support of β nor have proof against β. If we denote the unknown by U then, $U = (0, 0)$ and

iv β is *true* provided we have proof in support of β but have no proof against β. Denoting the quantity true by T, $T = (1, 0)$.

This FOUR valued logic was later widened to cover the entire real unit interval [0,1] in [11] to measure the degree of truth, falsehood, contradiction and unknown in every statement (β). The authors defined these four values over the unit interval [0,1] by:

$$F(\beta) = min(b, 1 - a), \tag{1}$$

$$C(\beta) = max(0, a + b - 1), \tag{2}$$

$$U(\beta) = max(0, 1 - a - b), \tag{3}$$

$$T(\beta) = min(1 - b, a), \tag{4}$$

This logic was further advanced into paraconsistent Pavelka style fuzzy logic by Turunen in [15]. Turunen equipped the truth set: [0,1] here with the Łukasiewicz structure which is an injective MV-algebra; and the accompanied operations are defined for all $a, b \in [0, 1]$, by $a \odot b = max(0, a+b-1)$, $a \wedge b = min(a, b)$, $a^* = 1 - a$ and $a \oplus b = min(1, a + b)$. Further, the various states: $F(\beta)$, $C(\beta)$, $U(\beta)$, and $T(\beta)$ as given in equations (1), (2), (3) and (4) are defined on this structure by the authors as
$F(\beta) = a^* \wedge b$; $C(\beta) = a \odot b$; $U(\beta) = a^* \odot b^*$ and $T(\beta) = a \wedge b^*$. So, any pair $(a, b) \in [0, 1]$ generates the following 2-by-2 matrix:

$$\begin{bmatrix} F(\beta) & C(\beta) \\ U(\beta) & T(\beta) \end{bmatrix} = \begin{bmatrix} a^* \wedge b & a \odot b \\ a^* \odot b^* & a \wedge b^* \end{bmatrix}.$$

These pairs of values (a, b) in [0,1] are referred to as *evidence couple*. Hence, the couples: $F = (0, 1)$; $C = (1, 1)$; $U = (0, 0)$; and $T = (1, 0)$ induce the following corresponding evidence matrices:

$$F = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \; U = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \text{ and } T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The objective of this article is to develop a decision model by associating paraconsistent Pavelka-style logic with fuzzy similarity relations and then illustrate how that model can be applied to solve decision problems involving partly ignorant, conflicting and vague information.

From here, the article is organised in the following way: In Section 2, we recollect MV–algebras, paraconsistent Pavelka-style logic, and fuzzy similarity (many–valued similarity) relation as the needed logical and algebraic concepts for the study. In Section 3, the algorithm of the novel technique is presented. In Section 4, we apply the novel technique to the resolution of a real life decision-making problem involving the selection of the best energy mix for an economy and in section 5 we conclude.

2 PRELIMINARIES

The defininitions and concepts in the existing literature relevant to the new model proposed here are as follows:

2.1 MV–algebras

MV-algebras: the algebras of the Łukasiewicz logic was developed by Chang to prove the completeness of this logic [6]. In this study, MV-algebra is the basic algebraic structure of our model. The two most fundamental operations of MV-algebras are the binary operation $\oplus$ and unary operation *.

Definition 2.1. Let 0, 1 be elements of the non-empty set L, and the operations $\oplus$ and * be defined on L such that for any $x, y, z \in L$

$$x \oplus y = y \oplus x, \tag{5}$$

$$x \oplus (y \oplus z) = (x \oplus y) \oplus z, \tag{6}$$

$$x \oplus 0 = x \tag{7}$$

$$x^{**} = x, \tag{8}$$

$$x \oplus 0^* = 0^*, \tag{9}$$

$$0^* = 1, \tag{10}$$

$$1^* = 0, \tag{11}$$

$$(x^* \oplus y)^* \oplus y = (y^* \oplus x)^* \oplus x. \tag{12}$$

Then, the structure $A = \langle L, \oplus, ^*, 0, 1\rangle$ is an MV-algebra.

Moreover, if L is equipped with another binary operation $\odot$ so that for all

$x, y \in L$ $x \odot y = (x^* \oplus y^*)^*$, then the equations

$$x \odot y = y \odot x, \tag{13}$$

$$x \odot (y \odot z) = (x \odot y) \odot z, \tag{14}$$

$$x \odot 1 = x, \tag{15}$$

too hold. This implies $\oplus$ can also be expressed in terms of $\odot$ as $x \oplus y = (x^* \odot y^*)^*$. Also, in every MV-algebra, the binary operation $\rightarrow$ is defined by $x \rightarrow y = x^* \oplus y$; and the operation * known as *complementation* is different from a lattice complementation.

Further, for any $x, y \in L$, the bi-residual operation $\leftrightarrow$ is defined as

$$x \leftrightarrow y = (x \rightarrow y) \wedge (y \rightarrow x). \tag{16}$$

The interval [0,1] endowed with the operations $\oplus$, $\odot$, *, $\vee$, $\wedge$, $\rightarrow$ called *Łukasiewicz structure* (standard MV-algebra) denoted here by $\mathcal{L}$ is a classic instance of an MV-algebra. These operations are defined as follows:

$$x \oplus y = min(x + y, 1), \tag{17}$$

$$x \odot y = max(0, x + y - 1), \tag{18}$$

$$x^* = 1 - x, \tag{19}$$

$$x \vee y = max(x, y), \tag{20}$$

$$x \wedge y = min(x, y), \tag{21}$$

$$x \rightarrow y = min(1, 1 - x + y). \tag{22}$$

An MV-algebra A is *complete* if only it is closed with respect to the infimum and supremum of every subset of L; it is divisible if for every element $a \in L$ there exists $n - divisors$ such that $n \in N$. Hence, an MV–algebra A is said to be *injective* provided it is complete and divisible. The Łukasiewicz structure is an injective MV-algebra [17]. Turunen et al in [15] showed that in an injective MV-algebra A any ordered pair (x, y) in $L \times L$ generates a corresponding 2-by-2 matrix M defined by

$$M = \begin{bmatrix} x^* \wedge y & x \odot y \\ x^* \odot y^* & x \wedge y^* \end{bmatrix}.$$

As previously stated, the couples: (0,1); (1,1); (0,0); and (1,0) induce the following matrices respectively:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

So, the set $\mathbb{M}$ of all 2-by-2 matrices generated by such ordered pairs can be equipped with the binary operation $\oplus$ and the unary operation $^{\perp}$ so that $\mathbb{M} = (M, \oplus, ^{\perp}, 0, 1)$ forms an injective MV– algebra. Now, if the evidence matrices D, M in $\mathbb{M}$ are obtained from the evidence couple (c, d) and (x, y) respectively then, according to the authors in [15], the evidence couple for the evidence matrix $D \oplus M$ is $(c \oplus x, d \odot y)$, and the respective evidence matrix

$$D \oplus M = \begin{bmatrix} (c \oplus x)^* \wedge (d \odot y) & (c \oplus x) \odot (d \odot y) \\ (c \oplus x)^* \odot (d \odot y)^* & (c \oplus x) \wedge (d \odot y)^* \end{bmatrix},$$

The same way, the evidence couple (x^*, y^*) produces the evidence matrix

$$M^{\perp} = \begin{bmatrix} x \wedge y^* & x^* \odot y^* \\ x \odot y & x^* \wedge y \end{bmatrix}.$$

Further, it is important to point out that there is a one to one correspondence between evidence couples and evidence matrices. This means for all $D, M \in \mathbb{M}$, $D = M$ provided the corresponding components of the evidence couples are equal (i.e., $c = x$ and $d = y$).

2.2 Fuzzy similarity relations

The definitions and findings made in this subsection are all in [10], [9]. Assume X is a non empty set and A is an injective MV-algebra. The binary operation S defined on X is a fuzzy similarity relation if for all $x, y, z \in X$ S fulfils these three conditions:

$$S(x, x) = 1, \tag{23}$$

$$S(x, y) = S(y, x), \tag{24}$$

$$S(x, y) \odot S(y, z) \leq S(x, z). \tag{25}$$

Thus, S is reflective, symmetrical and weakly transitive as found in equations (23), (24) and (25) respectively. Moreover, S is a fuzzy equivalence relation and as a result a generalisation of the equivalence relation in classical logic. Also, in any residuated lattice L, the binary operation $\leftrightarrow$ is defined for each

$x, y \in L$ as

$$x \leftrightarrow y = (x \to y) \wedge (y \to x).$$

The bi-residuum operation $\leftrightarrow$ fulfils these four conditions:

$$x \leftrightarrow x = 1, \tag{26}$$
$$x \leftrightarrow y = y \leftrightarrow x, \tag{27}$$
$$(x \leftrightarrow y) \odot (y \leftrightarrow z) \leq (x \leftrightarrow z), \tag{28}$$
$$x \leftrightarrow 1 = x. \tag{29}$$

From these four conditions, it is obvious that the operation $\leftrightarrow$ as well is reflexive, symmetric and weakly transitive. A fuzzy subset B is an ordered pair (X, μ_B), where X is a set of elements and $\mu_B : X \to L$ a function called a *membership function*. A fuzzy subset B with the membership function μ_B of any given set X measures the degree to which an element $x \in X$ is a member of B. So, every fuzzy subset B with the membership function μ_B that is defined on the non empty set X induces a fuzzy similarity relation S on the set X via

$$S_B(x, y) = \mu_B(x) \leftrightarrow \mu_B(y) \;\; for \;\; any \;\; x, y \in X. \tag{30}$$

Particularly, if A is the Łukasiewicz structure $\mathcal{L}$ on the interval [0,1] and B is provided then for any $x, y \in X$,

$$S_B(x, y) = 1 - |\mu_B(x) - \mu_B(y)|. \tag{31}$$

In the field of Multi-Criteria Decision Making, every fuzzy subset represents a unique criterion. So, the fuzzy subset B is the same as criterion B. The elements of X, on the other hand, are the available alternative courses of action or options in the decision problem. Hence, the value $S_B(x, y)$ determines the extend to which any two elements x, y in X are identical with respect to B. Assume the element $y \in X$ has a full membership degree in B. The focus then will be to calculate the degree to which every other element in X is alike to the full member y in relation to B. Thus, for each member x in X the formula:

$$S_B(x,y) = \mu_B(x) \leftrightarrow \mu_B(y) = \mu_B(x) \leftrightarrow 1 = \mu_B(x). \tag{32}$$

is true.
All in all, if we have k criteria against m options in a decision making problem then that means that we have k fuzzy subsets (X, μ_j) and k many-valued similarity relations $S_j(x,y)$ with respect to the set X containing m elements or options, where $j = 1, \cdots, k$. Now, on an injective MV-algebra, what is referred to as the *total fuzzy similarity relation* between every pair of elements x, y in X is determined using the formula

$$S(x,y) = \frac{S_1(x,y)}{k} \oplus \cdots \oplus \frac{S_k(x,y)}{k} = \frac{1}{k}\sum_{j=1}^{k} S_j(x,y), \tag{33}$$

such that the operation $\oplus$ is the MV-addition operation and $\frac{S_1(x,y)}{k}$ is the k-divisor of every similarity relation value $S_j(x,y)$, for $j = 1, \cdots, k$. Especially, if A is the Łukasiewicz structure then,

$$S(x,y) = \frac{1}{k}\sum_{j=1}^{k} S_j(x,y). \tag{34}$$

Moreover, if varied values of weight are assigned to the fuzzy subsets, then the total fuzzy similarity relation is determined via the weighted mean by

$$S(x,y) = \frac{w_1 S_1(x,y)}{W} \oplus \cdots \oplus \frac{w_k S_k(x,y)}{W}, \tag{35}$$

where $W = \sum_{j=1}^{k} w_j$, and $w_j \in \mathbb{N}$. Just like equation (34), if A is the Łukasiewicz structure then,

$$S(x,y) = \frac{1}{W}\sum_{j=1}^{K} w_j S_j(x,y). \tag{36}$$

At this point, let us assume that the set of options or alternative courses of action at the disposal of the decision maker is denoted by X. Assume also that there are k attributes which are expressed as fuzzy subsets by $B_1, \cdots, B_k$.

We suppose that every fuzzy subset B_j, $j = 1, \cdots, k$ contains an ideal solution y which takes a membership value of 1. So, to measure the proximity of any option x in X to the perfect solution y is to establish the magnitude of the total similarity between the option x and that of y. We achieve this through the formula of the weighted mean:

$$S(x, y) = \frac{1}{W} \sum_{j=1}^{k} w_j \mu_{B_j}(x) \tag{37}$$

such that $\mu_{B_j}(x)$ is the membership degree of x in the fuzzy set B_j, where $j = 1, \cdots, k$.

3 AN ALGORITHM ON DECISION-MAKING UNDER CONFLICTING AND VAGUE INFORMATION

Let us assume that there are a dozen of choices (options, alternatives) β_i, $i = 1, \cdots, n$, from which we should choose the best option. Each option has p prospective criteria (desired properties) P_j, $j = 1, \cdots, p$ and q consequence criteria (undesired features) C_k, $k = 1, \cdots, q$; P_j, C_k may be conflicting. Some information may also be lacking. Our method comprises such decision tasks via the following steps.

1. A decision maker, a human expert, expresses by means of fuzzy sets and membership functions to what grade an option β_i, $i = 1, \cdots, n$ satisfies the given criteria, i.e. to what degree $\mu_{P_j}(\beta_i) \in [0, 1]$ and $\mu_{C_j}(\beta_i) \in [0, 1]$ the option β_i belongs to the corresponding fuzzy sets. Notice that pros and cons may be completely independent of each other. Our approach allows also a situation where there is an option β_i possessing all pros and all cons; in this case $\mu_{P_j}(\beta_i) = \mu_{C_j}(\beta_i) = 1$ for all $j = 1, \cdots, p, k = 1, \cdots, q$; this means maximally contradiction in evidence. Correspondingly, there may be an option β_i that does not have any pros nor cons, i.e. $\mu_{P_j}(\beta_i) = \mu_{C_j}(\beta_i) = 0$ for all $j = 1, \cdots, p$, $k = 1, \cdots, q$; in this case evidence is a complete unknown.

2. Now, each individual membership degree corresponds to a particular *partial fuzzy similarity degree*. These partial fuzzy similarity degrees are then combined to two *total fuzzy similarity degrees* $x_i, y_i \in [0, 1]$,

$i = 1, \cdots, n$, where x_i corresponding to all the pros and y_i corresponding to all the cons related to the option β_i. The significance of various criteria P_j and C_k may be expressed by criteria weights $m_j, n_k \in \mathcal{M}$, where $\mathcal{M}$ is the set of weights of the criteria under consideration.

3. Any evidence couple (x_i, y_i) is related to the option β_i, and thereby produces the evidence matrix

$$M_i(\beta_i) = \begin{bmatrix} F(\beta_i) & C(\beta_i) \\ U(\beta_i) & T(\beta_i) \end{bmatrix} for \;\; i = 1, \cdots, n.$$

As we have already mentioned, the F denotes falsehood, C denotes contradiction, U represents unknown, and T represents truth of any given option β_i.

4. The set of evidence matrices denoted by $\mathbb{M}$ is an injective MV-algebra. Thus, any pair of options β_l, β_m are comparable through the corresponding evidence matrices $M_l(\beta_l), M_m(\beta_m)$ and if $M_l(\beta_l) \leq M_m(\beta_m)$ we say that the criterion P_j or C_j is more satisfied by alternative β_m than β_l. Algebraically, $M_l(\beta_l) \leq M_m(\beta_m)$ provided $M_l(\beta_l)^{\perp} \oplus M_m(\beta_m) = 1$. In the set $\mathbb{M}$, the matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

corresponding to the evidence couples $(0, 1)$ and $(1, 0)$ are respectively the bottom and the top elements of $\mathbb{M}$. However, $\mathbb{M}$ is not a chain since there are matrices in $\mathbb{M}$ that are not comparable. In this article, we shall show how the incomparabilities are resolved to have a chain of elements.

In practise, it is normal to compare two alternatives A and B via their evidence matrices. Assume the evidence matrices M and N generated respectively by the evidence couples (a, b) and (x, y) are the respective evidence matrices for the options A and B. Then, we say that $M \leq N$ if and only if $a \leq x$, and $y \leq b$. Thus, $x^* \leq a^*$, $b^* \leq y^*$. This indicates that more evidence abound backing B than the available evidence backing A and there exists lesser evidence against B than we have against A. From these inequalities, it can be observed that $a \wedge b^* \leq x \wedge y^*$ and $x^* \wedge y \leq a^* \wedge b$. So, it is easy to see that $T(N) - F(N) = (x \wedge y^*) - (x^* \wedge y) \geq (a \wedge b^*) - (a^* \wedge b) = T(M) - F(M)$, where $T(M)$ is the acronym for $T(A)$ and $F(M)$ is the acronym for $F(A)$.

Similarly, $T(N)$, $F(N)$ are the respective acronyms for $T(B)$, $F(B)$. If M, N cannot be compared (i.e. $M \nleq\ngeq N$) then, the situation is more complicated. These together gives rise to definition 3.1:

Definition 3.1. Let the evidence matrices M, N generated respectively by the evidence couples (a, b) and (x, y) be the respective matrices for the pair of options A, B. It is said that option B is more desirable than option A written as $A \preceq B$ if (i) $M \leq N$ or (ii) $M \nleq\ngeq N$ but $T(N) - F(N) > T(M) - F(M)$. Particularly, if $M = N$, then A, B are equally desirable and it is written as $A \equiv B$. If $M \nleq\ngeq N$, but $T(N) - F(N) = T(M) - F(M)$, then A, B are weakly equally desirable and it is represented by $A \equiv_w B$.

For example, for every evidence matrix $M \in \mathbb{M}$:

$$M = \begin{bmatrix} a^* \wedge b & a \odot b \\ a^* \odot b^* & a \wedge b^* \end{bmatrix},$$

we have $F(M) = a^* \wedge b$, and $T(M) = a \wedge b^*$, while $C(M) = a \odot b$ and $U(M) = a^* \odot b^*$.

Theorem 1. The relation $\equiv$ is an equivalence relation on the set of options whereas the relation $\equiv_w$ is not. The relation $\preceq$ defines a quasi–order over the set of options.

Proof. The relation $\equiv$ is obviously an equivalence relation. Now, for the fact that $A \equiv_w B$ and $B \equiv_w A$ does not mean $A \equiv_w A$, $\equiv_w$ is not reflexive and it is not symmetric either, and so it is not an equivalence relation. Further, the relation $\preceq$ is clearly reflexive. Now, suppose $A \preceq B$ and $B \preceq C$, where C is denoted by the evidence matrix P which is produced by the evidence couple (p, q). In proving that $A \preceq C$, we have a number of cases and subcases to look at. These cases and subcases are as follows:

Case 1. The case that $M \leq N$, $N \leq P$ trivially implies $M \leq P$, therefore $A \preceq C$.

Case 2. If $M \nleq\ngeq N$ but $T(N) - F(N) > T(M) - F(M)$, and $N \nleq\ngeq P$ but $T(P) - F(P) > T(N) - F(N)$ then there are two sub cases here.

Sub case I. (i) $a < x$, (ii) $b < y$, (iii) $x^* < a^*$, (iv) $y^* < b^*$, (v) $(x \wedge y^*) - (x^* \wedge y) > (a \wedge b^*) - (a^* \wedge b)$ (vi) $x < p$, (vii) $y < q$, (viii) $p^* < x^*$, (ix) $q^* < y^*$, (x) $(p \wedge q^*) - (p^* \wedge q) > (x \wedge y^*) - (x^* \wedge y)$.
So, by (i), (vi); $a < p$. By (ii), (vii); $b < q$. Thus $M \nleq\ngeq P$.
However, by (x) and (v), $T(P) - F(P) > T(M) - F(M)$
Hence, $A \preceq C$.

Sub case II. (i) $x < a$, (ii) $y < b$, (iii) $a^* < x^*$, (iv) $b^* < y^*$, (v) $(x \wedge y^*) - (x^* \wedge y) > (a \wedge b^*) - (a^* \wedge b)$ (vi) $p < x$, (vii) $q < y$, (viii) $x^* < p^*$, (ix) $y^* < q^*$, (x) $(p \wedge q^*) - (p^* \wedge q) > (x \wedge y^*) - (x^* \wedge y)$.
By (vi) and (i), $p < a$. And by (vii), (ii); $q < b$, hence $M \nleq\ngeq P$. Now, through (x) and (v), $T(P) - F(P) > T(M) - F(M)$. Therefore, $A \preceq C$.
Alternatively, if $T(N) - F(N) > T(M) - F(M)$, and $T(P) - F(P) > T(N) - F(N)$, then $T(P) - F(P) > T(M) - F(M)$ and so $A \preceq C$. Assume $P \leq M$. This implies $p \leq a$ and $b \leq q$. This further implies $a^* \leq p^*$ and $q^* \leq b^*$. Thus, $p \wedge q^* \leq a \wedge b^*$ and $a^* \wedge b \leq p^* \wedge q$. This implies $(a \wedge b^*) - (a^* \wedge b) \geq (p \wedge q^*) - (p^* \wedge q)$. Therefore, $(a \wedge b^*) - (a^* \wedge b) > (x \wedge y^*) - (x^* \wedge y)$, which is a contradiction to the initial assumption. Hence, either $M \leq P$ or $M \nleq\ngeq P$. In both scenarios, $A \preceq C$.

Case 3. Let $M \leq N$, $N \nleq\ngeq P$, but $T(P) - F(P) > T(N) - F(N)$.
Again there are two sub cases.

Sub case I. (i) $a \leq x$, (ii) $y \leq b$, (iii) $x^* \leq a^*$, (iv) $b^* \leq y^*$, (v) $x < p$, (vi) $p^* < x^*$, (vii) $y < q$, (viii) $q^* < y^*$, (ix) $T(P) - F(P) > T(N) - F(N)$ [i.e., $(p \wedge q^*) - (p^* \wedge q) > (x \wedge y^*) - (x^* \wedge y)$].
By (i), (v); $a < p$. So, if $q \leq b$ then $M \leq P$. Hence, $A \preceq C$. But, if $b < q$ then $M \nleq\ngeq P$. In this situation, by (i), (iv), $x \wedge y^* \geq a \wedge b^*$. And by (ii),(iii); $x^* \wedge y \leq a^* \wedge b$. Therefore, $(x \wedge y^*) - (x^* \wedge y) \geq (a \wedge b^*) - (a^* \wedge b)$. Recalling (ix), $(p \wedge q^*) - (p^* \wedge q) > (a \wedge b^*) - (a^* \wedge b)$ [i.e., $T(P) - F(P) > T(M) - F(M)$]. So, $A \preceq C$.

Sub case II. (i) $a \leq x$, (ii) $y \leq b$, (iii) $x^* \leq a^*$, (iv) $b^* \leq y^*$, (v) $p < x$, (vi) $x^* < p^*$, (vii) $q < y$, (viii) $y^* < q^*$, (ix) $(p \wedge q^*) - (p^* \wedge q) > (x \wedge y^*) - (x^* \wedge y)$.
By (ii), (vii), $q < b$. If $a \leq p$, then $M \leq P$. Therefore, $A \preceq C$. However, if $p < a$, then $M \nleq\ngeq P$. So, it is reasoned that by (i), (iv); $x \wedge y^* \geq a \wedge b^*$ and by (ii), (iii);

$x^*\wedge y \le a^*\wedge b$. Hence, $(x\wedge y^*)-(x^*\wedge y) \ge (a\wedge b^*)-(a^*\wedge b)$. And by recalling (ix), $T(P) - F(P) > T(M) - F(M)$. Thus, $A \preceq C$.

Case 4 Let $M \nleq \ngeq N$ but $T(N) - F(N) > T(M) - F(M)$, and $N \le P$. This combination too has two sub cases.

Sub case I. (i) $a < x$, (ii) $x^* < a^*$, (iii) $b < y$, (iv) $y^* < b^*$, (v) $(x\wedge y^*)-(x^*\wedge y) > (a\wedge b^*)-(a^*\wedge b)$, (vi) $x \le p$, (vii) $p^* \le x^*$, (viii) $q \le y$, (ix) $y^* \le q^*$,
By (i) and (vi), $a < p$. If $q \le b$ then $M \le P$ and therefore $A \preceq C$. If $b < q$, then $M \nleq \ngeq P$. By (vi) and (ix), $p \wedge q^* \ge x \wedge y^*$. And by (vii) and (viii), $p^* \wedge q \le x^* \wedge y$. Hence, $(p \wedge q^*) - (p^* \wedge q) \ge (x \wedge y^*) - (x^* \wedge y)$. So, by (v), $T(P) - F(P) > T(M) - F(M)$. Therefore, $A \preceq C$.

Sub case II. (i) $x < a$, (ii) $a^* < x^*$, (iii) $y < b$, (iv) $b^* < y^*$, (v) $(x\wedge y^*)-(x^*\wedge y) > (a\wedge b^*)-(a^*\wedge b)$, (vi) $x \le p$, (vii) $p^* \le x^*$, (viii) $q \le y$, (ix) $y^* \le q^*$.
Now, from (viii), (iii); $q < b$, and if $a \le p$ then $M \le P$, hence $A \preceq C$. However, if $p < a$ then $M \nleq \ngeq P$. Recalling (vi), (ix); $x \wedge y^* \le p \wedge q^*$ and via (vii) and (viii), $p^* \wedge q \le x^*\wedge y$. This implies $(p\wedge q^*)-(p^*\wedge q) \ge (x\wedge y^*)-(x^*\wedge y)$. And by (v), $T(P) - F(P) > T(M) - F(M)$ so, $A \preceq C$.
Hence, from cases 1, 2, 3 and 4; the relation $\preceq$ is transitive, and the proof is complete.

□

Examples (i), (ii) below illustrate how the technique proposed here operates.
Example (i): Assume matrices C, D are generated by the couples (0.4, 0.6), (0.8, 0.3) sequentially. If C is the evidence matrix of option β_1 and D is the evidence matrix of option β_2 which alternative is more desirable?
If the matrices C, D are obtained from (0.4, 0.6), (0.8, 0.3) correspondingly, then

$$C = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.4 \end{bmatrix}, D = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.7 \end{bmatrix}.$$

Also, the evidence couple of $C^\perp$ is $(0.4^*, 0.6^*) = (0.6, 0.4)$ and that of $C^\perp \oplus D$ is $(0.6 \oplus 0.8, 0.4 \odot 0.3) = (1, 0)$. So, in terms of matrices, we have

$$C^\perp \oplus D = \begin{bmatrix} 1^* \wedge 0 & 1 \odot 0 \\ 1^* \odot 0^* & 1 \wedge 0^* \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

hence, $C \leq D$. Similarly, the evidence couple of $D^{\perp}$ is $(0.8^*, 0.3^*) = (0.2, 0.7)$. The evidence couple for $D^{\perp} \oplus C = (0.2 \oplus 0.4, 0.7 \odot 0.6) = (0.6, 0.3)$. the corresponding matrix becomes,

$$D^{\perp} \oplus C = \begin{bmatrix} 0.6^* \wedge 0.3 & 0.6 \odot 0.3 \\ 0.6^* \odot 0.3^* & 0.6 \wedge 0.3^* \end{bmatrix} = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0.6 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $D \nleq C$. However, from $C^{\perp} \oplus D$ we have realised that $C \leq D$, hence β_2 is more desirable than β_1. The same final conclusion: β_2 is more desirable than β_1 will be drawn if one considers the difference between the truth value and the falsehood value of each of the two matrices. That is, $T(D) - F(D) = 0.7 - 0.2 = 0.5 > -0.2 = 0.4 - 0.6 = T(C) - F(C)$.
Example (ii): Let us suppose that the two evidence coupls (0.7, 0.5) and (0.4, 0.3) induce the matrices E, G respectively for the two alternatives, α, β correspondingly. What alternative between these two is the more desirable one?

Now, given that the evidence couple for E is $(0.7, 0.5)$, and that of G is $(0.4, 0.3)$, their corresponding matrices are

$$E = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.5 \end{bmatrix}, G = \begin{bmatrix} 0.3 & 0 \\ 0.3 & 0.4 \end{bmatrix}.$$

Further, the matrix $E^{\perp}$ is produced by the couple $(0.7^*, 0.5^*) = (0.3, 0.5)$ and so $E^{\perp} \oplus G = (0.3 \oplus 0.4, 0.5 \odot 0.3) = (0.7, 0)$. Thus, the respective evidence matrix :

$$E^{\perp} \oplus G = \begin{bmatrix} 0.7^* \wedge 0 & 0.7 \odot 0 \\ 0.7^* \odot 0^* & 0.7 \wedge 0^* \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0.3 & 0.7 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

hence, $E \nleq G$.
Reciprocally, $G^{\perp}$ is induced by $(0.4^*, 0.3^*) = (0.6, 0.7)$. Hence, $G^{\perp} \oplus E = (0.6 \oplus 0.7, 0.7 \odot 0.5) = (1, 0.2)$. Therefore, the matrix :

$$G^{\perp} \oplus E = \begin{bmatrix} 1^* \wedge 0.2 & 1 \odot 0.2 \\ 1^* \odot 0.2^* & 1 \wedge 0.2^* \end{bmatrix} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.8 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

so, $G \nleq E$. Thus, the two matrices E, G cannot be compared. However, $T(E) - F(E) = 0.5 - 0.3 = 0.2 > 0.1 = 0.4 - 0.3 = T(G) - F(G)$. Therefore, we conclude that alternative α is more desirable than alternative β.

4 A CASE STUDY: RANKING ENERGY PRODUCTION METHODS IN GHANA

The above paraconsistency approach was applied to the energy data from Ghana to identify the best (optimal) energy mix for the country. At present, there are eight potential sources from which the country can generate electricity. The energy sources; the options from which to select the best one, constitutes the set X:

$$X = \{\text{hydro, wind, solar, natural gas, nuclear, biomass, oil, coal}\}.$$

These options are also denoted for short by HYD, WIN, SOL, GAS, NUC, BIO, OIL and COA, respectively. Elements of the set X are denoted by α, β, γ or α_i. The authors of this paper consulted energy experts at Ghana Ministry of Energy. After given the two sets of selection criteria, namely the pros set and the cons set, energy experts evaluated each of the eight options. The positive criteria set, the pros, consists of eleven criteria whereas the negative criteria set, the cons, contains fifteen. To reflect the relative importance of various criteria in the search for more reliable and sustainable power, all the criteria were assigned weight*, given by the energy experts at the said ministry. The two sets of criteria together with their weights are displayed in Table 1 and Table 2, respectively. $\star$

TABLE 1
Prospect criteria

* The given scale was from 1 to 10, however, the experts did not use the low end of the scale at all.

Criteria	weight
1. Availability	7
2. Energy storage versatility	8
3. Energy storage capability	8
4. Self-sufficiency and reliability	8
5. Energy yield	8
6. Renewable	6
7. Job creation	8
8. Other benefits(including those from by-products)	8
9. Plant's versatility/flexibility	9
10. Life span	9
11. Technological impact	9

TABLE 2
consequence criteria

Criteria	weight
1. Energy outsourcing	4
2. Green house gases emission	3
3. Rainfall fluctuation	6
4. Ecosystem and livelihood	6
5. Pollution	4
6. Waste management	8
7. Capital cost	7
8. Operational cost	8
9. Price volatility	8
10. Human consequence	6
11. Inter and/or intra boundary disputes	3
12. Civil unrest or social disorder	6
13. Location	4
14. Land mass or space consumption	3
15. Political interference	8

Every criterion generates a fuzzy subset of X, and so there are in all 26 fuzzy subsets, 11 of which are the pros while the rest are the cons. In Table 3 and Table 4, it is displayed to which degree each options $\alpha \in X$ satisfy the criteria.

TABLE 3
Pros: Membership functions

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{P_1}	1	0.5	0.75	0.75	0.25	0.75	0.75	0.25
μ_{P_2}	1	0.25	0.5	0.75	0.25	0.5	0.5	0.25
μ_{P_3}	1	0.25	0.5	0.75	0.25	0.75	0.75	0.5
μ_{P_4}	0.5	0.75	0.5	0.75	0.25	0.75	0.75	0.25
μ_{P_5}	1	0.5	0.25	0.5	0.75	0.5	0.5	1
μ_{P_6}	1	1	1	0	0.25	1	0	0
μ_{P_7}	1	0.75	0.5	0.75	0.75	0.75	1	1
μ_{P_8}	1	0.5	0.5	0.75	0.25	0.75	1	1
μ_{P_9}	0.75	0.75	0.75	0.75	0.25	0.75	0.5	0.5
$\mu_{P_{10}}$	1	0.75	0.5	0.75	0.75	0.75	0.5	1
$\mu_{P_{11}}$	0.75	0.75	0.75	0.75	0.5	0.75	0.75	0.75

TABLE 4
Cons: Membership functions

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{C_1}	0.75	0.5	0.5	0.75	0.5	0.5	0.5	0.5
μ_{C_2}	0	0	0	0.25	0.75	0.25	0.75	0.75
μ_{C_3}	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
μ_{C_4}	0.75	0.5	0.75	0.75	1	0.5	0.75	0.75
μ_{C_5}	0	0.25	0.25	0.5	0.75	0.5	0.75	0.75
μ_{C_6}	0	0.25	0.25	0.25	0.75	0.5	0.25	0.75
μ_{C_7}	0.75	0.75	0.75	0.75	0.75	0.25	0.75	0.75
μ_{C_8}	0	0	0	0.5	0.25	0.5	0.5	0.5
μ_{C_9}	0	0	0	0.75	0.25	0.5	0.75	0.5
$\mu_{C_{10}}$	0.75	0.5	0.5	0.75	1	0.5	0.75	0.5
$\mu_{C_{11}}$	0.75	0.5	0.5	0.75	0.5	0.5	0.75	0.5
$\mu_{C_{12}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\mu_{C_{13}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\mu_{C_{14}}$	0.75	0.5	0.75	0.5	0.25	0.5	0.5	0.5
$\mu_{C_{15}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

By means of the weighted mean approach, the evidence couples of the eight power sources $\alpha \in X$ were calculated. They are displayed in Table 5:

TABLE 5
Evidence couples

Option α	Evidence for α	Evidence against α
HYD	0.9034	0.4196
WIN	0.6108	0.3899
SOL	0.5824	0.4167
GAS	0.6761	0.5686
NUC	0.4176	0.5804
BIO	0.7216	0.5119
OIL	0.6705	0.5863
COA	0.6136	0.5833

These evidence couples induced the following evidence matrices. However, for the sake of simplicity, we denote the evidence couples and their corresponding evidence matrices for the alternatives: HYD, WIN, SOL, GAS, NUC, BIO, OIL, and COA by H, W, S, G, N, B, O and C respectively. Hence, the evidence matrices are :

$$H = \begin{bmatrix} 0.0966 & 0.3231 \\ 0 & 0.5804 \end{bmatrix},\ W = \begin{bmatrix} 0.3892 & 0.0007 \\ 0 & 0.6101 \end{bmatrix},\ S = \begin{bmatrix} 0.4167 & 0 \\ 0.0009 & 0.5824 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.3229 & 0.2446 \\ 0 & 0.4316 \end{bmatrix},\ N = \begin{bmatrix} 0.5804 & 0 \\ 0.0020 & 0.4176 \end{bmatrix},\ B = \begin{bmatrix} 0.2784 & 0.2335 \\ 0 & 0.4881 \end{bmatrix}$$

$$O = \begin{bmatrix} 0.3295 & 0.2568 \\ 0 & 0.4137 \end{bmatrix},\ C = \begin{bmatrix} 0.3864 & 0.1970 \\ 0 & 0.4167 \end{bmatrix}.$$

Furthermore, through the evidence couples in table 5, we induced the following evidence couples and their corresponding evidence matrices:

$H^{\perp} \oplus W = (0.7074, 0)$, $W^{\perp} \oplus H = (1, 0.0297)$.

$$H^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0.2926 & 0.7074 \end{bmatrix},\ W^{\perp} \oplus H = \begin{bmatrix} 0 & 0.0297 \\ 0 & 0.9703 \end{bmatrix}.$$

$H^{\perp} \oplus S = (0.5142, 0.1637)$, $S^{\perp} \oplus H = (1, 0.009)$.

$$H^{\perp} \oplus S = \begin{bmatrix} 0.1637 & 0 \\ 0.3221 & 0.5142 \end{bmatrix},\ S^{\perp} \oplus H = \begin{bmatrix} 0 & 0.0029 \\ 0 & 0.9971 \end{bmatrix}.$$

$H^{\perp} \oplus G = (0.7727, 0.149)$, $G^{\perp} \oplus H = (1, 0)$.

$$H^{\perp} \oplus G = \begin{bmatrix} 0.149 & 0 \\ 0.0783 & 0.7727 \end{bmatrix},\ G^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

$H^{\perp} \oplus N = (0.5142, 0.1608)$, $N^{\perp} \oplus H = (1, 0)$.

$$H^{\perp} \oplus N = \begin{bmatrix} 0.1608 & 0 \\ 0.325 & 0.5142 \end{bmatrix},\ N^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$H^{\perp} \oplus B = (0.8182, 0.0923)$, $B^{\perp} \oplus H = (1, 0)$.

$$H^{\perp} \oplus B = \begin{bmatrix} 0.0923 & 0 \\ 0.0895 & 0.8182 \end{bmatrix}, \; B^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$H^{\perp} \oplus O = (0.7671, 0.1667)$, $O^{\perp} \oplus H = (1, 0)$.

$$H^{\perp} \oplus O = \begin{bmatrix} 0.1667 & 0 \\ 0.0662 & 0.7671 \end{bmatrix}, \; O^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$H^{\perp} \oplus C = (0.7102, 0.1637)$, $C^{\perp} \oplus H = (1, 0)$.

$$H^{\perp} \oplus C = \begin{bmatrix} 0.1637 & 0 \\ 0.1261 & 0.7102 \end{bmatrix}, \; C^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$W^{\perp} \oplus S = (0.9716, 0.0268)$, $S^{\perp} \oplus W = (1, 0)$.

$$W^{\perp} \oplus S = \begin{bmatrix} 0.0268 & 0 \\ 0.0016 & 0.9716 \end{bmatrix}, \; S^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$W^{\perp} \oplus G = (1, 0.1787)$, $G^{\perp} \oplus W = (0.9347, 0)$.

$$W^{\perp} \oplus G = \begin{bmatrix} 0 & 0.1787 \\ 0 & 0.8213 \end{bmatrix}, \; G^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0.0653 & 0.9347 \end{bmatrix}$$

$W^{\perp} \oplus N = (0.8068, 0.1905)$, $S^{\perp} \oplus W = (1, 0)$.

$$W^{\perp} \oplus N = \begin{bmatrix} 0.1905 & 0 \\ 0.0027 & 0.8068 \end{bmatrix}, \; N^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$W^{\perp} \oplus B = (1, 0.122)$, $B^{\perp} \oplus W = (0.8892, 0)$.

$$W^{\perp} \oplus B = \begin{bmatrix} 0 & 0.122 \\ 0 & 0.878 \end{bmatrix}, \; B^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0.1108 & 0.8892 \end{bmatrix}$$

$W^{\perp} \oplus O = (1, 0.1964)$, $O^{\perp} \oplus W = (0.9403, 0)$.

$$W^{\perp} \oplus O = \begin{bmatrix} 0 & 0.1964 \\ 0 & 0.8036 \end{bmatrix}, \; O^{\perp} \oplus W = \begin{bmatrix} 0.0597 & 0 \\ 0 & 0.9403 \end{bmatrix}$$

$W^{\perp} \oplus C = (1, 0.1934)$, $C^{\perp} \oplus W = (0.9972, 0)$.

$$W^{\perp} \oplus C = \begin{bmatrix} 0 & 0.1934 \\ 0 & 0.8066 \end{bmatrix}, \; C^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0.0028 & 0.9972 \end{bmatrix}$$

$S^{\perp} \oplus G = (1, 0.1519)$, $G^{\perp} \oplus S = (0.9063, 0)$.

$$S^{\perp} \oplus G = \begin{bmatrix} 0 & 0.1519 \\ 0 & 0.8481 \end{bmatrix}, \; G^{\perp} \oplus S = \begin{bmatrix} 0. & 0 \\ 0.0937 & 0.9063 \end{bmatrix}$$

$S^{\perp} \oplus N = (0.8352, 0.1637)$, $N^{\perp} \oplus S = (1, 0)$.

$$S^{\perp} \oplus N = \begin{bmatrix} 0.1637 & 0 \\ 0.0011 & 0.8352 \end{bmatrix}, \; N^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$S^{\perp} \oplus B = (1, 0.0952)$, $B^{\perp} \oplus S = (0.8608, 0)$.

$$S^{\perp} \oplus B = \begin{bmatrix} 0 & 0.0952 \\ 0 & 0.9048 \end{bmatrix}, \; B^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0.1392 & 0.8608 \end{bmatrix}$$

$S^{\perp} \oplus O = (1, 0.1696)$, $O^{\perp} \oplus S = (0.9119, 0)$.

$$S^{\perp} \oplus O = \begin{bmatrix} 0 & 0.1696 \\ 0 & 0.8304 \end{bmatrix}, \; O^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0.0881 & 0.9119 \end{bmatrix}$$

$S^{\perp} \oplus C = (1, 0.1666)$, $C^{\perp} \oplus S = (0.9688, 0)$.

$$S^{\perp} \oplus C = \begin{bmatrix} 0 & 0.1666 \\ 0 & 0.8334 \end{bmatrix}, \; C^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0.0312 & 0.9688 \end{bmatrix}$$

$G^{\perp} \oplus N = (0.7415, 0.0118)$, $N^{\perp} \oplus G = (1, 0)$.

$$G^{\perp} \oplus N = \begin{bmatrix} 0.0118 & 0 \\ 0.2467 & 0.7415 \end{bmatrix},\ N^{\perp} \oplus G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$G^{\perp} \oplus B = (1, 0)$, $B^{\perp} \oplus G = (0.9545, 0.0567)$.

$$G^{\perp} \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ B^{\perp} \oplus G = \begin{bmatrix} 0.0455 & 0.0112 \\ 0 & 0.9433 \end{bmatrix}$$

$G^{\perp} \oplus O = (0.9944, 0.0177)$, $O^{\perp} \oplus G = (1, 0)$.

$$G^{\perp} \oplus O = \begin{bmatrix} 0.0056 & 0.0121 \\ 0 & 0.9823 \end{bmatrix},\ O^{\perp} \oplus G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$G^{\perp} \oplus C = (0.9375, 0.0147)$, $C^{\perp} \oplus G = (1, 0)$.

$$G^{\perp} \oplus C = \begin{bmatrix} 0.0147 & 0 \\ 0.0478 & 0.9375 \end{bmatrix},\ C^{\perp} \oplus G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$N^{\perp} \oplus B = (1, 0)$, $B^{\perp} \oplus N = (0.696, 0.0685)$.

$$N^{\perp} \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ B^{\perp} \oplus N = \begin{bmatrix} 0.0685 & 0 \\ 0.2355 & 0.696 \end{bmatrix}$$

$N^{\perp} \oplus O = (1, 0.0059)$, $O^{\perp} \oplus N = (0.7471, 0)$.

$$N^{\perp} \oplus O = \begin{bmatrix} 0 & 0.0059 \\ 0 & 0.9941 \end{bmatrix},\ O^{\perp} \oplus N = \begin{bmatrix} 0. & 0 \\ 0.2529 & 0.7471 \end{bmatrix}$$

$N^{\perp} \oplus C = (1, 0.0029)$, $C^{\perp} \oplus N = (0.804, 0)$.

$$N^{\perp} \oplus C = \begin{bmatrix} 0 & 0.0029 \\ 0 & 0.9971 \end{bmatrix},\ C^{\perp} \oplus N = \begin{bmatrix} 0 & 0 \\ 0.196 & 0.804 \end{bmatrix}$$

$B^\perp \oplus O = (0.9489, 0.0744)$, $O^\perp \oplus B = (1, 0)$.

$$B^\perp \oplus O = \begin{bmatrix} 0.0511 & 0.0233 \\ 0 & 0.9256 \end{bmatrix}, \; O^\perp \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$B^\perp \oplus C = (0.892, 0.0714)$, $C^\perp \oplus B = (1, 0)$.

$$B^\perp \oplus C = \begin{bmatrix} 0.0714 & 0 \\ 0.0366 & 0.892 \end{bmatrix}, \; C^\perp \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$O^\perp \oplus C = (0.9431, 0)$, $C^\perp \oplus O = (1, 0.003)$.

$$O^\perp \oplus C = \begin{bmatrix} 0 & 0 \\ 0.0569 & 0.9431 \end{bmatrix}, \; C^\perp \oplus O = \begin{bmatrix} 0 & 0.003 \\ 0 & 0.997 \end{bmatrix}$$

Thus, from a quick glance at these pairs of matrices, it is obvious that there are several comparabilities and incomparabilities among the alternatives. For instance, apart from wind and solar which are incomparable to hydro power, any other alternative is comparable to hydro power, and indeed hydro power dominates all of them. The remaining options too show numerous incomparabilities among themselves. As a result, Definition 1 and Theorem 1 is applied to rank the alternatives $\alpha \in X$ completely.
So, at a close look at the pairs of evidence matrices, it is clear that there are neither equally desirable options nor weakly equally desirable options, i.e. for no α, β holds $\alpha \equiv \beta$ or $\alpha \equiv_w \beta$. Hence, the relation β is more desirable than α, denoted by $\alpha \preceq \beta$ is studied. As illustrated in examples 1 and 2, these pairs of evidence matrices that represent the comparisons of the eight alternatives are analysed as follows:

$$H^\perp \oplus W = \begin{bmatrix} 0 & 0 \\ 0.2926 & 0.7074 \end{bmatrix}, \; W^\perp \oplus H = \begin{bmatrix} 0 & 0.0297 \\ 0 & 0.9703 \end{bmatrix}.$$

Thus, $H \npreceq, \nsucceq W$ since $W^\perp \oplus H \neq \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \neq H^\perp \oplus W$. However, recall that the various energy options in X have the following individual matrices:

$$H = \begin{bmatrix} 0.0966 & 0.3231 \\ 0 & 0.5804 \end{bmatrix},\ W = \begin{bmatrix} 0.3892 & 0.0007 \\ 0 & 0.6101 \end{bmatrix},\ S = \begin{bmatrix} 0.4167 & 0 \\ 0.0009 & 0.5824 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.3229 & 0.2446 \\ 0 & 0.4316 \end{bmatrix},\ N = \begin{bmatrix} 0.5804 & 0 \\ 0.0020 & 0.4176 \end{bmatrix},\ B = \begin{bmatrix} 0.2784 & 0.2335 \\ 0 & 0.4881 \end{bmatrix}$$

$$O = \begin{bmatrix} 0.3295 & 0.2568 \\ 0 & 0.4137 \end{bmatrix},\ C = \begin{bmatrix} 0.3864 & 0.1970 \\ 0 & 0.4167 \end{bmatrix}.$$

Thus, from matrices H and W, it is observed that $T(H) - F(H) = 0.5804 - 0.0966 = 0.4838 > 0.2209 = 0.6101 - 0.3892 = T(W) - F(W)$. Hence, $WIND \preceq HYDRO$. Similarly,

$$H^{\perp} \oplus S = \begin{bmatrix} 0.1637 & 0 \\ 0.3221 & 0.5142 \end{bmatrix},\ S^{\perp} \oplus H = \begin{bmatrix} 0 & 0.0029 \\ 0 & 0.9971 \end{bmatrix}.$$

So, $H \nleq, \ngeq S$. But, $T(H)-F(H) = 0.5804-0.0966 = 0.4838 > 0.1657 = 0.5824 - 0.4167 = T(S) - F(S)$. Therefore, $SOLAR \preceq HYDRO$. And for hydro and natural gas,

$$H^{\perp} \oplus G = \begin{bmatrix} 0.149 & 0 \\ 0.0783 & 0.7727 \end{bmatrix},\ G^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thus, $H \nleq G$, and $G \leq H$ therefore, NATURAL GAS $\preceq$ HYDRO. In other words, $T(H) - F(H) = 0.5804 - 0.0966 = 0.4838 > 0.1087 = 0.4316 - 0.3229 = T(G) - F(G)$. Therefore, NATURAL GAS $\preceq$ HYDRO. So, extending this analysis to the rest of the pairs, the following results have been obtained:
$T(H) - F(H) = 0.5804 - 0.0966 = 0.4838$,
$T(W) - F(W) = 0.6101 - 0.3892 = 0.2209$,

$T(S) - F(S) = 0.5824 - 0.4167 = 0.1657,$
$T(G) - F(G) = 0.4316 - 0.3229 = 0.1087,$
$T(N) - F(N) = 0.4176 - 0.5804 = -0.1628,$
$T(B) - F(B) = 0.4881 - 0.2784 = 0.2097,$
$T(O) - F(O) = 0.4137 - 0.3295 = 0.0842,$
$T(C) - F(C) = 0.4167 - 0.3864 = 0.0303.$
Hence, the final ranking is as follows:
Nuclear $\preceq$ coal $\preceq$ oil $\preceq$ natural gas $\preceq$ solar $\preceq$ biomass $\preceq$ wind $\preceq$ hydro.
These results show that Ghana's energy production sources superiority depends substantially on the emphasise of the positive aspects or the drawbacks of the production source. Thus, the optimal energy source is hydro power and the worst source is nuclear. Moreover, the best five sources for the energy mix for Ghana are hydro power, wind energy, biomass, solar energy and natural gas.

5 CONCLUSION

In this article, a technique for decision making under conflicting and vague information is introduced. The idea is that every decision alternative say A has its pros as well as cons. Through what is called total fuzzy similarity, these pros are put together to give a single value say x. Similarly, by the total fuzzy similarity approach, the cons are put together to yield a unique value say y. So, the pair of values (x, y) known as the evidence couple defines a 2-by-2 evidence matrix say Q for the option A. So, under normal circumstance, an option B with the 2-by-2 evidence matrix R is more desirable than option A granted that more evidence abound in support of B than what is available in support of A, and lesser amount of evidence against B is available than there is against A. In such a situation, the relationship between Q, R is of the form Q is less than or equal to R ($Q \leq R$). This indicates that in terms of truth and falsity option B is better off than option A. However, the evidence couples and for that matter the corresponding evidence matrices Q, R are not comparable if either of the options A, B is armed with stronger evidence in support of itself and stronger evidence against itself. To solve this incomparability, the difference between the truth value and the falsehood value of the matrix Q [i.e. $T(Q) - F(Q)$] and that of R [i.e. $T(R) - F(R)$] are compared. If $T(R) - F(R) > T(Q) - F(Q)$ then B is more desirable than A and vice versa. Through this process, a complete order is established for all the options.

To demonstrate the use of the method, energy experts in the energy sector in Ghana were consulted to find the best energy mix under vague and contradictory circumstances for the country.
To make the calculation much easier, a programme of the model is in the making for the end user.

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