

The impact of new varieties on aggregate productivity growth*

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Abstract

Although there is an extensive body of literature on aggregate productivity growth, reallocation, and firm turnover, the contribution to overall productivity growth from new firms that produce new varieties is not well understood. In this paper, we propose a framework for aggregating productivity that identifies the contribution from new firms that produce new varieties. Our framework generalizes the frameworks currently used in the literature. To illustrate the decomposition, we analyse the case of firm turnover in Norway. We find that the net creation of new varieties due to firm turnover contributes about half a percentage point to annual aggregate labour productivity growth in the manufacturing sector.

Keywords: Aggregation; demand elasticity; labour productivity; multifactor productivity; new varieties; panel data; pooled Feenstra–Soderbery estimator; productivity growth

JEL classification: C43; E24; O47

1. Introduction

It is generally agreed among economists that labour productivity growth is the main determinant of long-run economic development (Krugman, 1990). Several determinants of firm-specific productivity growth have been identified in the literature, including R&D, new technologies, and organizational structure. At the aggregate level, productivity growth can also be driven by reallocation effects (i.e., a change in the composition of firms in the economy). If the dispersion of productivity across establishments and industries is high (Bartelsman and Wolf, 2018), then aggregate productivity can increase substantially if resources are reallocated from low- to high-productivity firms. Firm turnover represents one such means of resource

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reallocation. There is extensive theoretical and empirical research showing that firm turnover affects aggregate productivity growth; see Griliches and Regev (1995), Baily et al. (1992), Foster et al. (2001, 2006, 2008), Diewert and Fox (2010), and Acemoglu et al. (2017).

Firm turnover can also increase productivity through another channel: the creation of new varieties. If new firms create new varieties that increase the range of produced goods available to other firms as inputs, then this will expand aggregate production possibilities and fuel economic growth. Feenstra et al. (1992) found that new produced inputs are an important explanation for productivity growth in a sample of Korean industries. Goldberg et al. (2010) found that Indian firms responded to the 1991 trade liberalization by importing new intermediate goods, which in turn enabled the production of new outputs. In a study of private-sector firms in the US non-farm economy, Garcia-Macia et al. (2019) found that most growth seems to occur through the improvement of existing varieties rather than the creation of new varieties. Aghion et al. (2019) found that output growth was about half a percentage point higher per year for non-farm businesses between 1983 and 2013 due to firm turnover and new varieties, which they referred to as “creative destruction”.

Despite the extensive literature on new varieties and firm turnover, up until now there has been no formal framework for aggregating productivity growth from firm level to economy-wide level that specifically identifies the contribution from new firms producing new varieties. The frameworks used in the literature so far (see, e.g., Baily et al., 1992; Griliches and Regev, 1995; Diewert and Fox, 2010) are based on a weighted average of productivity levels, and thus implicitly assume that all products are perfect substitutes (see Appendix A). However, new goods fuel growth precisely because they hold some new characteristics (i.e., they are not perfect substitutes for existing goods).

In this paper, we provide a framework for aggregating productivity growth that identifies the contribution from new firms producing new varieties. To this end, we adopt the procedure developed by Feenstra (1994), who calculated consumer gains from new varieties, to analyse the case of firm turnover and aggregate productivity growth. We show that the net effect of firm turnover on aggregate productivity growth is approximately given by $(s^N - s^X)/(\sigma - 1)$, where s^N and s^X are the output shares of new and exiting firms, respectively, and σ is the constant elasticity of substitution between varieties. The framework we propose generalizes the frameworks used in the literature on firm turnover: if products are perfect substitutes, which is the benchmark case implicitly assumed in the literature, the elasticity of substitution tends to infinity and there is no extra gain from new varieties.

We illustrate the framework for aggregating productivity growth using the case of firm turnover in Norway and firm-level panel data obtained

by linking producer price survey data for the manufacturing sector with administrative registers. To estimate the elasticity of substitution, σ , we use a modified version of the estimator most commonly used in the literature on new goods (see Feenstra, 1994; Broda and Weinstein, 2006; Soderbery, 2015). Estimates of σ range from 2.1 to infinity. Based on these estimates, we find that annual aggregate productivity growth has, on average, been downward-biased by about half a percentage point, which is substantial compared to the average productivity growth of 3.4 percent annually in the period 1996–2018.

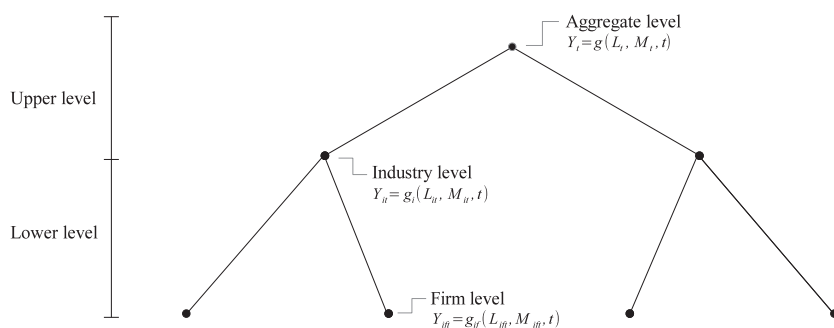
As pointed out by Diewert and Feenstra (2019), the constant elasticity of substitution (CES) framework might overestimate the true impact of firm turnover because this functional form implies a reservation price of infinity. Drawing on Hausman (1999), Diewert and Feenstra (2019) propose a local linear approximation of the demand curve to estimate the gains attributable to new varieties, which they refer to as a lower bound estimator. When applying this lower bound estimator, our results still show that, on average, annual aggregate productivity growth has a significant downward bias.

The rest of this paper is organized as follows. In Section 2, we outline the decomposition of aggregate productivity growth and identify the impact from an expanding set of intermediate goods. In Section 3, we give details of the econometric framework. In Section 4, we describe the data and apply our decomposition empirically. We provide a conclusion in Section 5.

2. Aggregation of productivity growth

Productivity is commonly defined as the ratio of outputs to inputs, both measured as volumes. In the case of one type of aggregate output and input, the standard measure of aggregate productivity growth can be written as Q_Y/Q_L , where Q_Y represents an index of overall output and Q_L represents an index of overall input (see Diewert and Nakamura, 2003; Balk, 2021). Multi-factor productivity generalizes the above definition to include several types of inputs. We derive expressions for both productivity concepts below, although our focus is on labour productivity (i.e., with Q_L as an index of labour input).

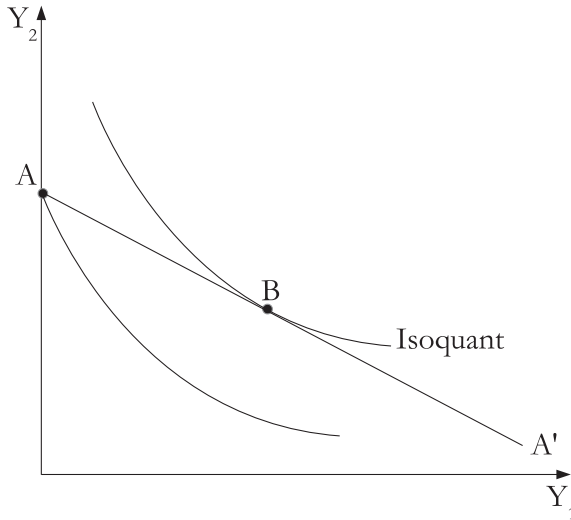
Figure 1 illustrates the two-level aggregation scheme we propose for measuring aggregate productivity growth. At the lower level, productivity is aggregated across firms to the industry level. At the upper level, productivity growth is aggregated across industries to the aggregate economy-wide level. It is at the lower level of aggregation that we identify the extra contribution to productivity growth from new firms producing new varieties. To understand how firm turnover affects overall productivity growth, the output and input indices at the lower level can be decomposed into

Figure 1. Two-level aggregation framework

contributions from continuing, entering, and exiting firms. To that end, we outline in the following sections how both inputs and outputs are aggregated, and highlight the link between the literature on firm turnover and productivity growth, on the one hand, and the literature on new intermediate goods and gains attributable to variety, on the other.

Our point of departure is the economic approach to index numbers. This approach provides an index formula that is consistent with economic theory (International Labour Organization et al., 2004, Chapters 16 and 17). In contrast, the axiomatic approach to index numbers evaluates an index formula based on a set of desirable properties. Within the economic approach to index numbers, the output index can be interpreted as the change in production volume while prices are kept constant (Diewert, 1987). This interpretation provides a clear link between the literature on firm turnover and productivity and the literature showing how an increasing range of intermediate inputs fuels economic growth (see, e.g., Feenstra et al., 1992; Goldberg et al., 2010).

Figure 2 illustrates how product innovation can affect the output index, Q_Y , through the channel of intermediate goods production. Assume a representative profit-maximizing firm that uses labour and two varieties of a composite intermediate good as inputs. The isoquants represent the combinations of the two varieties, Y_1 and Y_2 , at the given level of the composite good, and the isocost line AA' shows the combinations that yield the same intermediate cost level. In the time period $t-1$, only variety Y_2 is available and input use is at point A . In period t , the variety Y_1 becomes available. The introduction of the new intermediate good increases the overall utility for the representative firm: at the given isocost line, the attained indifference curve shifts outwards and the level of intermediate input use is at point B , where profits and output are higher.

Figure 2. New intermediate varieties and firm output

The size of the output increase depends on the curvature of the indifference curve, that is, on how easy it is to substitute one variety for another, as expressed by the elasticity of substitution. When there is some sort of complementarity between input varieties, the indifference curves will show a curvature as illustrated in Figure 2. The lower the elasticity of substitution, the higher the output increase as a result of having a new input variety available. We apply this insight below in the lower level of the aggregation framework. However, if varieties are perfect substitutes, then the elasticity of substitution tends to infinity and the indifference curves become straight lines. In this scenario, new firms could also lead to increased output in other industries if intermediate goods are sold at lower (quality-adjusted) prices. This would happen, for example, if there were new entrants to a Cournot oligopoly market.

2.1. Lower level of aggregation

For the rest of this paper, we adopt the notation that f refers to firm, i to industry, and t to time. For any variable x_{ift} , x_{it} denotes the same variable aggregated to industry level, and x_t denotes the variable aggregated to the level of all industries $i \in I$. A bar (\bar{x}_{ift} or \bar{x}_{it}) denotes an appropriate mean value (of x_{ift} or x_{it}) over $[t-1, t]$. Bold italic characters, such as x_{it} , denote vectors.

Let

$$Y_{ift} = g_{if}(L_{ift}, M_{ift}, t) \quad (1)$$

denote the production function at firm level $f \in \mathcal{F}_{it}$, where Y_{ift} is output, L_{ift} is labour input measured as hours worked, and M_{ift} is an aggregate of all produced inputs used in production, henceforth referred to as material inputs. As in Hulten (1978), we consider capital services as produced by fixed (tangible) assets (machinery, buildings, etc.) and therefore included in M_{ift} . The nominal value of output equals $V_{ift} = p_{ift}Y_{ift}$, where p_{ift} is the producer price.

The set of firms \mathcal{F}_{it} in industry i varies over time. Let C_{it} denote the set of firms that exists in two consecutive time periods, $t - 1$ and t . We refer to these as “continuing” firms. “Entering” firms, denoted \mathcal{N}_{it} , exist in period t but not in $t - 1$. “Exiting” firms, denoted \mathcal{X}_{it} , operate in period $t - 1$ but not in t . It then follows that the number of firms producing a variety of good i in period t is the union of the set of continuing firms and the set of entering firms: $\mathcal{F}_{it} = C_{it} \cup \mathcal{N}_{it}$. Correspondingly, the number of firms producing a variety at $t - 1$ can be written as the union of the set of continuing and exiting firms in t : $\mathcal{F}_{i,t-1} = C_{it} \cup \mathcal{X}_{it}$.

We next assume the existence of industry-level production functions

$$Y_{it} = g_i(L_{it}, M_{it}, t) \text{ for } i \in I. \quad (2)$$

That is, firm-level outputs, Y_{ift} , can be aggregated into an industry-level composite good, Y_{it} , and the same applies to the inputs. As the purpose of industry classifications is to organize firms into industrial groupings based on similar products and activities, each industry can be seen as representing a composite good, and each firm within the industry as producing a variety of the composite good.

We now consider the aggregation of firm-level output Y_{ift} to industry-level output, Y_{it} . Regardless of the type of good produced, it is assumed that Y_{ift} is purchased by firms (k) in other industries (j) as material inputs. This could be firms in services, such as retail trade, or in industries that use the goods as intermediate inputs in their production of physical goods. That is,

$$Y_{ift} = \sum_j \sum_k Y_{ift}^{(jk)}, \quad (3)$$

where $Y_{ift}^{(jk)}$ is the amount of variety f in industry i purchased by firm k in industry j . Thus, the superscript refers to the buyer (firm k in industry j) and the subscript to the variety (firm f in industry i). Importantly, it is implicitly assumed that there are no intra-industry purchases, i.e. $Y_{ift}^{(jk)} = 0$ if $i = j$.

Let $\mathbf{y}_{it}^{(jk)} = \{Y_{ift}^{(jk)}\}_{f \in \mathcal{F}_{it}}$ be the vector of material inputs firm k in industry j (firm jk) purchases from each firm f in industry i (firm if). The prices corresponding to each of the varieties in $\mathbf{y}_{it}^{(jk)}$ are

$$\mathbf{p}_{it} = \{p_{ift}\}_{f \in \mathcal{F}_{it}},$$

where the price vector is assumed to be the same for all buyers (jk). Following Pollak and Wales (1987), we assume that firm (jk)’s composite material input, M_{jkt} (see equation (1)), can be represented as a multi-stage technology. At the first level, the purchased varieties from industry i are aggregated by means of a CES function¹:

$$\phi_i(\mathbf{y}_{it}^{(jk)}) = \left[\sum_{f \in \mathcal{F}_{it}} \gamma_{if}^{1/\sigma_i} (Y_{ift}^{(jk)})^{(\sigma_i-1)/\sigma_i} \right]^{\sigma_i/(\sigma_i-1)}. \tag{4}$$

At the second level, the vector of industry-specific aggregates, $\boldsymbol{\phi}_{jkt} = \{\phi_i(\mathbf{y}_{it}^{(jk)})\}_i$, are combined into composite material inputs:

$$M_{jkt} = m_{jk}(\boldsymbol{\phi}_{jkt}, t), \tag{5}$$

where $m_{jk}(\cdot, t)$ is an arbitrary upper-level aggregator specific to (jk). At the final level, M_{jkt} is combined with labour, L_{jkt} , to produce output, Y_{jkt} (see equation (1)).

The industry-level composite good, Y_{it} , is defined by the relation:

$$V_{it} = c_i(\mathbf{p}_{it})Y_{it}, \tag{6}$$

where

$$c_i(\mathbf{p}_{it}) = \left(\sum_{f \in \mathcal{F}_{it}} \gamma_{if}^{\sigma_i} p_{ift}^{1-\sigma_i} \right)^{1/(1-\sigma_i)}. \tag{7}$$

The derivation of equations (6) and (7) is given in Appendix A (see also equation (B2) in Appendix B).

The volume index $Y_{it}/Y_{i,t-1}$ follows from the price index $c_i(\mathbf{p}_{it})/c_i(\mathbf{p}_{i,t-1})$ and equation (6). Sato (1976) and Vartia (1976) showed how to calculate $c_i(\mathbf{p}_{it})/c_i(\mathbf{p}_{i,t-1})$ when the goods are the same in two different periods. Feenstra (1994) generalized the results of Sato

¹The CES approach to calculating welfare gain from new goods is not uncontroversial; see, for example, the comment by Zvi Griliches to Feenstra and Shiells (1996, pp. 273–276). Diewert and Feenstra (2017) compare the CES function with a flexible functional form where the reservation price is finite. However, the need for estimating large systems of reservation prices for unavailable varieties in each period makes their method infeasible when the number of varieties (firms) is large, as is the case in our application, with several thousand entering and exiting firms (see Section 4).

(1976) and Vartia (1976) to handle situations where the number of goods changes over time, which is the case for the set of firms \mathcal{F}_{it} producing a variety of good i .

The price index refers to a unit bundle of varieties, representing the weighted average (“representative”) buyer of goods from the given industry. The output measure Y_{it} is obtained by aggregating all output from the industry. The aggregation is feasible under the assumption that buyers differ with respect to the amount of bundles they buy, not the composition of their bundles.

Before we can proceed to the decomposition of industry productivity growth, we need to define output and input weights, for continuing, entering, and exiting firms. First, we let s_{it}^N denote the total nominal output share of entering firms at time t within industry i ,

$$s_{it}^N = \frac{\sum_{f \in \mathcal{N}_{it}} V_{ift}}{\sum_{f \in \mathcal{F}_{it}} V_{ift}}, \tag{8}$$

and we let

$$s_{it-1}^X = \frac{\sum_{f \in \mathcal{X}_{it}} V_{if,t-1}}{\sum_{f \in \mathcal{F}_{i,t-1}} V_{if,t-1}} \tag{9}$$

denote the total nominal output share in $t - 1$ of exiting firms (operating in $t - 1$ but not t). Next, let

$$\bar{w}_{ift} = \frac{M(s_{ift}, s_{if,t-1})}{\sum_{f \in C_{it}} M(s_{ift}, s_{if,t-1})}, \tag{10}$$

where

$$s_{ift} = \frac{V_{ift}}{\sum_{f \in C_{it}} V_{ift}}, \tag{11}$$

and $M(y, z)$ is the logarithmic mean of y and z .² Correspondingly, on the input side, let h_{it}^N and $h_{i,t-1}^X$ denote the shares of hours worked in entering and exiting firms in industry i ,

$$h_{it}^N = \frac{\sum_{f \in \mathcal{N}_{it}} L_{ft}}{\sum_{f \in \mathcal{F}_{it}} L_{ift}}, \tag{12}$$

$$h_{i,t-1}^X = \frac{\sum_{f \in \mathcal{X}_{it}} L_{if,t-1}}{\sum_{f \in \mathcal{F}_{i,t-1}} L_{if,t-1}}, \tag{13}$$

²The logarithmic mean of (non-negative) numbers y and z is defined as $M(y, z) = 0$ if $y = 0$ or $z = 0$, $M(y, z) = y$ if $y = z$ or $M(y, z) = (y - z)/(\ln y - \ln z)$ otherwise.

where both L_{ift} and L_{it} are sums of hours worked. Moreover, we define the input weights $\bar{\xi}_{ift}$ as follows³:

$$\bar{\xi}_{ift} = \frac{M(L_{ift}, L_{if,t-1})}{M(\sum_{f \in C_{it}} L_{ift}, \sum_{f \in C_{it}} L_{if,t-1})}. \tag{14}$$

Productivity at the industry level is given by the ratio of the growth in outputs to inputs, Q_{iY}/Q_{iL} , where $Q_{iY} = Y_{it}/Y_{i,t-1}$ and $Q_{iL} = L_{it}/L_{i,t-1}$. We now turn to the decomposition that expresses the contribution from product innovation and firm turnover to overall labour productivity growth at the industry level. Our main result is stated in Proposition 1.

Proposition 1 (Decomposition of industry-level productivity growth).

Consider an economy represented by equations (1)–(5). Given cost minimization at the firm level, and input and output weights defined in equations (8)–(14), labour productivity growth at the industry level can be represented as

$$\begin{aligned} \ln \left(\frac{Q_{iY}}{Q_{iL}} \right) &= \underbrace{\sum_{f \in C_{it}} \bar{w}_{ift} \Delta \ln \left(\frac{Y_{ift}}{L_{ift}} \right)}_{(1)} + \underbrace{\sum_{f \in C_{it}} (\bar{w}_{ift} - \bar{\xi}_{ift}) \Delta \ln L_{ift}}_{(2)} \\ &\quad - \underbrace{\ln \left(\frac{1 - s_{it}^N}{1 - h_{it}^N} \right)}_{(3)} + \underbrace{\ln \left(\frac{1 - s_{i,t-1}^X}{1 - h_{i,t-1}^X} \right)}_{(4)} - \underbrace{\left(\frac{1}{\sigma_i - 1} \right) \ln \left(\frac{1 - s_{it}^N}{1 - s_{i,t-1}^X} \right)}_{(5)}. \tag{15} \end{aligned}$$

For the proof, see Appendix A. Proposition 1 expresses the complete decomposition of labour productivity growth at the industry level, where productivity is measured as the ratio of real revenue to hours worked. The term (1) in equation (15) shows the contribution from productivity growth among continuing firms within the industry (i). Term (2) shows the contribution from reallocation within the industry, which depends on the covariance between a firm’s input and the difference between the output weight (\bar{w}_{ift}) and labour input weight ($\bar{\xi}_{ift}$). We can interpret the output–input weight difference, $\bar{w}_{ift} - \bar{\xi}_{ift}$, as a measure of the revenue productivity level of a firm. If inputs are allocated from firms with below-average to above-average productivity levels, reallocation within the industry contributes positively to aggregate labour productivity growth.

³Note that the weights $\bar{\xi}_{ift}$ and \bar{w}_{ift} correspond to the weights in the indices labelled Vartia I and Vartia II, respectively, by Vartia (1976).

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The third and fourth terms represent the contributions from entry (3) and exit (4), respectively, of firms in the absence of product innovation. If entering firms have higher ratio of revenue to hours worked than continuing firms, entering firms contribute positively to overall productivity growth. Productivity growth will also be higher if exiting firms have lower ratio of revenue to hours worked than continuing firms. The last term (5) in equation (15) shows the net effect of creating new varieties. As illustrated diagrammatically in Figure 2, the overall productivity growth attributable to new varieties depends on the elasticity of substitution. The net contribution can be approximated by $(s_{it}^N - s_{i,t-1}^X)/(\sigma_i - 1)$ when the output and input shares are small.⁴ The expression also shows that the impact of new varieties depends on the elasticity of substitution in a highly non-linear manner. The expression also shows that the impact from new varieties depends on the elasticity of substitution in a highly non-linear manner.⁵ To identify the contribution of new varieties to overall productivity growth, it is thus crucial to precisely determine the size of the elasticity of substitution. We return to the issue of identifying the elasticity of substitution in Section 4.

Note that it is not the number of entering and exiting firms that drives the overall impact of firm turnover on aggregate productivity growth. Even when the number of entering and exiting firms is equal, if new varieties from entering firms have a higher quality than varieties produced by exiting firms, the output share of entering firms will exceed the output share of exiting firms: $s_{it}^N > s_{i,t-1}^X$. In this case, we find a positive contribution to overall productivity growth due to net creation of new varieties. Note also that in this framework we only identify the contribution of new varieties from new firms, that is, excluding the impact of product innovations from existing (continuing) firms.

Most of the literature uses a framework based on a weighted average of productivity levels to analyse the contribution of firm turnover to overall productivity growth (see, e.g., Baily et al., 1992; Griliches and Regev, 1995; Foster et al., 2001, 2006, 2008; Diewert and Fox, 2010; Acemoglu et al., 2017). All these studies implicitly assume that products are perfect substitutes. However, it is only within a framework that allows for imperfect substitutes that the extra gain in productivity growth due

⁴The approximation follows from applying $\ln(1+z) \approx z$ when $z \approx 0$. Consider the case where the output share of entering firms is $s_{it}^N = 0.07$ and the output share of exiting firms is $s_{i,t-1}^X = 0.02$. If $\sigma_i = 2$, then the overall contribution to productivity growth attributable to net creation of new varieties is approximately 5 percentage points.

⁵For example, if $\sigma_i = 3$, the contribution to productivity growth drops to approximately 2.5 percentage points, and if $\sigma_i = 4$ it drops to 1.7 percentage points.

to new firms producing new varieties can be identified. In Appendix B, we compare and contrast our decomposition (15) of productivity growth with the frameworks often used in the literature, which we generalize.

The impact of the net creation of new varieties on productivity growth might be overestimated when a CES framework is used, because this functional form implies a reservation price of infinity. Diewert and Feenstra (2019) compares the gain due to new varieties in the CES functional form with a lower bound proposed by Hausman (1999). The lower bound is derived from a linear approximation of the demand curve. Diewert and Feenstra (2019) show that for elasticities of substitution ranging between 2 and 9, the ratio of gains from the linear approximation of the demand curve to the gains from the CES function is about 0.4. More generally, a lower bound to the effect of the net creation of new varieties on productivity growth is given by $(s_{it}^N - s_{i,t-1}^X)/2\sigma_i$. In Section 4, we compare the results from the CES functional form with this lower bound.

2.2. Upper level of aggregation

To obtain an expression of economy-wide productivity growth, we postulate the existence of an aggregate production function that relates aggregate output (Y_t) to aggregate labour input (L_t) and aggregate material input (M_t):

$$Y_t = g(L_t, M_t, t).$$

Following Domar (1961) and Hulten (1978), multi-factor productivity growth from $t - 1$ to t at aggregate level is given by the difference between labour productivity growth and the growth in material intensity:

$$MFP = \sum_{i \in I} \bar{d}_{it} \left[\ln \left(\frac{Q_{iY}}{Q_{iL}} \right) - (1 - \bar{\alpha}_{it}) \ln \left(\frac{Q_{iM}}{Q_{iL}} \right) \right]. \quad (16)$$

Here, α_{it} is the labour cost share in industry i (see equation (A6) in Appendix A) and d_{it} is the so-called Domar weight,

$$d_{it} = \frac{P_{it}Y_{it}}{FD_t},$$

where FD_t denotes the value of final deliveries from I after netting out deliveries within I (see Appendix A for derivations and formal definitions). The Domar weights are based on the information in the input–output matrices (see Baqaee and Farhi, 2020, equation (1)). The sum of the Domar weights, d_{it} , can exceed one, reflecting that the productivity gains in industry i will increase the production possibilities not only of that industry, but also of industries that use the output from i as intermediate inputs. In

the particular case that I represents a closed economy, the value of final deliveries equals value added. However, if there are no deliveries between the industries in I : $FD_t = P_t Y_t$.

From equation (16), we define aggregate labour productivity growth as

$$\ln \left(\frac{Q_Y}{Q_L} \right) = \sum_{i \in I} \bar{d}_{it} \ln \left(\frac{Q_{iY}}{Q_{iL}} \right). \quad (17)$$

Inserting equation (15) into equation (17) yields a decomposition of aggregate labour productivity growth for a set of industries.

The production of new varieties in an industry (i) increases the range of produced goods available to constitute inputs for firms in other industries (j), expanding production possibilities and fuelling economic growth throughout the economy. Although new varieties increase labour productivity in other industries, this increase is exactly offset by an increase in material intensity, leaving their multi-factor productivity unchanged.

3. Estimation of demand elasticities

In the literature on new goods, the key idea when estimating demand elasticities has been to overcome the simultaneity problem caused by an upward-sloping supply curve by utilizing the panel structure of the data set and reformulating the model in terms of second-order moments of prices and expenditure shares. This approach was originally proposed by Feenstra (1994). Later, Broda and Weinstein (2006) and Soderbery (2015) extended this framework along several dimensions. In particular, Soderbery (2015) created a hybrid estimator combining the unrestricted (first-stage) Feenstra estimator with a restricted non-linear limited information maximum likelihood (LIML) algorithm to be executed when the unrestricted estimator yields inadmissible values ($\hat{\sigma}$ less than one or complex-valued). Soderbery showed that his method is more robust to outliers and less prone to weak instrument bias, which is a prevalent problem with this methodology when the number of time periods is small (see the discussion in Soderbery, 2015, p. 15). A weakness of the Feenstra–Soderbery estimator is that it is not robust to the (ad hoc) choice of reference unit. To eliminate this problem, we average the unrestricted (first-stage) Feenstra–Soderbery estimator over all possible choices of reference units to create a “pooled” first-stage estimator, which in Monte Carlo simulations have been shown to be much more efficient than the original Feenstra–Soderbery estimator (see von Brasch and Raknerud, 2021).

3.1. Structural econometric framework

In order to identify structural parameters in a system of demand and supply equations using panel data on prices and expenditures, we follow Broda and Weinstein (2006). Dropping the industry subscript i for notational convenience, the demand, x_{ft}^D , for the variety produced by firm f at t is assumed to be given by

$$\ln x_{ft}^D = -\sigma \ln p_{ft} + \lambda_t^D + u_f^D + e_{ft}^D, \tag{18}$$

where p_{ft} is the price, λ_t^D and u_f^D represent fixed time and firm effects, e_{ft}^D is an error term with mean zero and $\sigma > 1$. The inverse supply equation is assumed to be given by

$$\ln p_{ft} = \omega \ln x_{ft}^S + \frac{1}{\omega + 1} (\lambda_t^S + u_f^S + e_{ft}^S), \tag{19}$$

where $\omega \geq 0$ is the inverse elasticity of supply – the scaling factor $(1 + \omega)^{-1}$ is for notational convenience. In equilibrium, supply equals demand ($x_{ft}^S = x_{ft}^D = x_{ft}$) and expenditure equals $s_{ft} = p_{ft}x_{ft}$. It follows from equations (18) and (19) that

$$\begin{aligned} \ln s_{ft} &= \beta \ln p_{ft} + \lambda_t^D + u_f^D + e_{ft}^D, \\ \ln p_{ft} &= \alpha \ln s_{ft} + \lambda_t^S + u_f^S + e_{ft}^S, \end{aligned} \tag{20}$$

where $\beta = 1 - \sigma < 0$ and $\alpha = \omega/(1 + \omega)$.

For any variable z_{ft} , the double difference operator with firm k as the reference firm is defined as

$$\Delta^{(k)} z_{ft} = \Delta z_{ft} - \Delta z_{kt}.$$

From equations (20) and after some manipulation, we have

$$(\Delta^{(k)} \ln p_{ft})^2 = \theta_1 (\Delta^{(k)} \ln s_{ft})^2 + \theta_2 (\Delta^{(k)} \ln p_{ft} \Delta^{(k)} \ln s_{ft}) + U_{ft}^{(k)}, \tag{21}$$

where

$$\theta_1 = -\frac{\alpha}{\beta}, \quad \theta_2 = \frac{1}{\beta} + \alpha, \quad \text{and} \quad U_{ft}^{(k)} = \Delta^{(k)} e_{ft}^D \Delta^{(k)} e_{ft}^S.$$

The restrictions on β and α are equivalent to $\theta_1 \geq 0$ and $\theta_1 + \theta_2 \leq 1$, which define the set of admissible θ values, say, Θ .⁶ Moreover, it is well

⁶The first inequality is obvious (the boundary $\theta_1 = 0$ corresponds to $\alpha = 0$ or $\beta = -\infty$), while the second inequality follows from $\alpha^{-1} = (-\theta_2 + \sqrt{\theta_2^2 + 4\theta_1})/2\theta_1$ (see Feenstra, 1994) and $\alpha^{-1} \geq 1 \Leftrightarrow \sqrt{\theta_2^2 + 4\theta_1} \geq 2\theta_1 + \theta_2 \Leftrightarrow \theta_2^2 + 4\theta_1 \geq 4\theta_1^2 + \theta_2^2 + 4\theta_1\theta_2 \Leftrightarrow \theta_1 - \theta_1^2 - \theta_1\theta_2 \geq 0 \Leftrightarrow 1 - \theta_1 - \theta_2 \geq 0 \Leftrightarrow \theta_1 + \theta_2 \leq 1$.

known that β is the unique negative solution to $\theta_1 s^2 + \theta_2 s - 1 = 0$ (see, e.g., Feenstra, 1994); that is,

$$\sigma = 1 + \frac{\theta_2 + \sqrt{\theta_2^2 + 4\theta_1}}{2\theta_1} > 1 \quad (22)$$

for all $\theta \in \Theta$.⁷ Under the identifying assumptions of Feenstra (1994), the idiosyncratic error terms e_{ft}^D and e_{ks}^S are assumed to be independent for any ft and ks (i.e., supply affects demand only through the price, p_{ft}), implying

$$E(U_{ft}^{(k)}) = 0.$$

Note that equation (21) is not a valid regression equation for estimating θ , because the regressors $\Delta^{(k)} \ln s_{ft}^2$ and $\Delta^{(k)} \ln p_{ft} \Delta^{(k)} \ln s_{ft}$ are correlated with $U_{ft}^{(k)}$, and must therefore be estimated using a generalized method of moments (GMM) estimator, such as Feenstra's two-stage least-squares estimator or the Feenstra–Soderbery LIML estimator. Both these estimators can be seen as instrumental variable estimators, with variety indicators as instruments (see Feenstra, 1994, p. 164), and they suffer from weak instrument bias when the number of observation periods is small or moderate (see the Monte Carlo results and discussions in Soderbery, 2015).

3.2. Pooling of estimates across reference firms

A drawback of the Feenstra–Soderbery estimator is that a fixed firm (k) must be chosen as an (ad hoc) reference firm. This renders all the error terms $U_{ft}^{(k)}$ highly correlated across firms, and therefore makes the estimator non-robust to the choice of reference firm. A simple remedy is to generate a sequence of unrestricted Feenstra–Soderbery estimators for each possible reference firm, k (i.e., $\{\hat{\theta}_{-k}^{(u)}\}_{k=1}^N$), and then choose as an unrestricted “pooled” estimator a weighted average:

$$\hat{\theta}^{(P)} = \sum_{k=1}^N W_k \hat{\theta}_{-k}^{(u)}.$$

If $W_k = (\sum_{k=1}^N H_k)^{-1} H_k$, where H_k is the Hessian of the k th GMM criterion function, then $\hat{\theta}^{(P)}$ is the unconstrained minimizer of the sum of N quadratic GMM criterion functions:

⁷The inequality (> 1) follows because $\sqrt{\theta_2^2 + 4\theta_1} > |\theta_2|$ for $\theta_1 > 0$. The boundary case $\theta_1 = 0$ is the (continuous) limit of the above formula as θ_1 approaches zero, in which case $\sigma = 1 - (1/\theta_2)$ if $\theta_2 < 0$ and $\sigma = \infty$ if $\theta_2 \geq 0$.

$$\sum_{k=1}^N (\tilde{\theta}_{-k}^{(u)} - \theta)' H_k (\tilde{\theta}_{-k}^{(u)} - \theta). \quad (23)$$

Our second-stage GMM estimator (i.e., $\hat{\theta}^{(r)}$) is the admissible θ value that minimizes the pooled GMM criterion (23). Thus, $\hat{\theta}^{(r)}$ equals $\hat{\theta}^{(P)}$ if the latter is admissible. If not, $\hat{\theta}^{(r)}$ is the trivial minimizer of the quadratic criterion (23) at the boundary of Θ .

Monte Carlo simulations show that pooling substantially reduces the variance of the first-stage estimator over the whole parameter space (see von Brasch and Raknerud, 2021). In particular, if the true θ is an interior point of Θ , the probability that $\hat{\theta}^{(P)}$ will be outside Θ will be lower than for the first-stage Feenstra–Soderbery estimator. This finding is also supported by our results in Section 4.

4. Empirical application

4.1. Data and operationalizations

Our population for productivity decomposition consists of all limited liability firms in the manufacturing industries observed during the period 1995–2018.⁸ However, for the estimation of industry-specific elasticities of substitution (σ_i), only the subset of firms covered by the Producer Price Survey (PPS) is included in the estimation sample. This is the sample of firms for which we observe actual, firm-specific commodity prices.

The PPS measures, for more than 600 commodities, price developments in first-hand sales of products from Norwegian manufacturing to the Norwegian market and for export. The PPS is used by Statistics Norway to calculate the Producer Price Index (PPI) at different aggregation levels for selected industries, including all manufacturing industries at the two-digit 2007 Standard Industrial Classification (SIC) level (i.e., NACE 2, 10–33). Because some firms produce more than one product, we have aggregated price data for each firm covered by the PPS to create a firm-specific price index, $\Delta \ln p_{ift}$. For this purpose, we use a standard Törnqvist index, where the weights are the moving average of production shares between two consecutive time periods.

Table 1 shows the total number of firm–years with price observations from the PPS by industry (Column 2) and, for each industry, the share

⁸The registry data are managed by Statistics Norway under the Norwegian Statistics Act. This act prohibits us from making the data available to other users, but Statistics Norway can provide access subject to the approval of an application. See the README document included with the replication files for information about data usage.

Table 1. Number of firm–year observations, and gross value of output in the PPI sample versus total manufacturing

Industry	NACE 2	PPI sample		Total manufacturing	
		Firm–years	Sample share	Firm–years	Industry share
Food products	10	895	0.25	18,996	0.19
Beverages	11	89	0.47	1,104	0.02
Textiles	13	174	0.21	3,388	0.01
Wearing apparel	14	106	0.32	2,099	0.00
Leather products	15	71	0.99	408	0.00
Wood products	16	504	0.24	12,538	0.05
Paper products	17	253	0.99	1,156	0.02
Chemical products	20	382	0.35	2,684	0.09
Pharmaceuticals	21	71	0.48	574	0.02
Rubber and plastic	22	312	0.30	4,683	0.02
Mineral products	23	477	0.21	6,527	0.05
Basic metals	24	290	0.67	1,699	0.14
Metal products	25	421	0.11	18,589	0.07
Computer products	26	240	0.29	3,529	0.04
Electronic equipment	27	205	0.24	4,592	0.03
Machinery	28	426	0.17	11,791	0.12
Motor vehicles	29	162	0.51	1,700	0.01
Furniture	31	217	0.30	5,622	0.02
Other manufacturing	32	213	0.25	6,555	0.01
Repair and installation	33	80	0.09	11,573	0.08
Total		5,588	0.32	119,807	1.00

Notes: The PPI sample is the sample used for estimating elasticities of substitution for each industry. “Total manufacturing” includes all limited liability firms. “Sample share” denotes the sample share of industry output, which is the gross value of output for the firms in the PPI sample (in the given industry) as a share of gross value of output for the whole industry. “Industry share” denotes the industry share of total manufacturing output, which is the gross value of output for the whole industry as a share of total manufacturing gross output.

of gross output covered relative to the whole industry (Column 3). The table also shows the number of firm–years for the whole industry (Column 4), and each industry’s share of total manufacturing gross output (Column 5). The PPI sample covers 32 percent of total manufacturing output.

In contrast to the estimation of industry-specific elasticities of substitution, the productivity decomposition population comprises all (limited liability) firms in manufacturing. For these firms, we calculate labour productivity as gross value of production per our worked in real prices, using, as explained in Section 2, the PPI⁹ at the lowest available aggregation level as a deflator. Our data source for labour inputs is Statistics

⁹See <https://www.ssb.no/en/ppi/>.

Norway's Employer–Employee Register, which is a matched employer–employee dataset containing contracted hours of work for each employer–employee combination, which we aggregate to firm–year level.

4.2. Empirical results

4.2.1. Estimates of σ . The first two columns of Table 2 show the pooled estimates, $\hat{\theta}^{(P)}$, obtained by pooling the unrestricted Feenstra–Soderbery estimator, $\hat{\theta}^{(u)}$, across all possible reference firms. The unrestricted (i.e., first-stage) estimates of θ obtained by using the Feenstra–Soderbery estimator are also shown in Table 2. Any differences between these

Table 2. Estimates of parameters

NACE 2	Two-stage estimator					Feenstra–Soderbery estimator		
	Pooled estimator		Second stage			Unrestricted	Second stage	
	$\hat{\theta}_1^P$	$\hat{\theta}_2^P$	$\hat{\sigma}$	SE($\hat{\sigma}$)	95% CI	$\hat{\theta}_1^{(u)}$	$\hat{\theta}_2^{(u)}$	$\hat{\sigma}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10	−0.01	0.37	$\infty^{(r)}$	NA	NA	−0.04	1.03	2.49 ^(nl)
11	0.14	0.03	3.78	1.40	[2.0, 8.6]	0.00	0.86	184.00
13	0.00	0.27	62.83	∞	[38.7, ∞]	0.00	−0.68	2.47
14	0.05	−0.51	2.70	0.64	[1.8, 4.6]	−0.08	−0.04	1.15 ^(nl)
15	0.00	−0.32	4.12	1.86	[2.0, 11.2]	−0.02	−0.46	2.13 ^(nl)
16	−0.02	0.21	$\infty^{(r)}$	NA	NA	NA	NA	NA
17	0.05	0.43	12.09	2.55	[8.0, 18.6]	0.08	0.59	9.68
20	0.00	0.07	23.60	∞	[8.3, ∞]	0.00	0.04	36.33
21	0.02	0.19	12.12	4.1	[3.9, 21.2]	−0.03	−0.10	1.27 ^(nl)
22	0.00	−0.35	3.73	0.37	[3.1, 4.6]	0.06	−2.84	1.35
23	0.04	0.58	18.95	1.99	[15.4, 23.4]	0.00	0.50	145.41
24	0.26	0.95	17.78 ^(r)	5.9	[5.5, 31.5]	−0.35	1.81	1.70 ^(nl)
25	0.39	−0.46	2.12	0.51	[1.2, 3.1]	0.07	0.40	8.29
26	0.00	0.02	$\infty^{(r)}$	NA	NA	NA	NA	NA
27	0.10	0.44	7.18	2.06	[4.2, 13.0]	0.07	0.78	13.25
28	−0.01	−0.47	3.14 ^(r)	1.74	[1.4, 11.8]	NA	NA	NA
29	0.02	0.28	17.92	3.16	[12.6, 25.6]	0.06	−0.21	3.66
31	−0.01	1.17	13.56 ^(r)	2.98	[8.8, 21.2]	−0.01	0.68	1.77 ^(nl)
32	0.08	0.53	9.40	2.27	[5.9, 15.4]	0.03	0.43	15.89
33	0.01	−0.33	3.90	0.44	[3.1, 4.9]	0.01	−0.18	5.78

Notes: For the two-stage estimator, the pooled estimator is an average of the unrestricted (first-stage) Feenstra–Soderbery estimator across all possible reference firms. If the pooled estimate is inadmissible, parameter restrictions are imposed and a solution at the boundary of the parameter space is found in the second stage (marked ^(r)). For the Feenstra–Soderbery estimator, in the second stage, a restricted non-linear LIML routine is executed if the unrestricted estimate is inadmissible (marked ^(nl)). SE($\hat{\sigma}$) and 95% CI are obtained by bootstrapping. ^(r) denotes a binding parameter restriction (an estimate at the boundary of the parameter space). ^(nl) signifies that a restricted non-linear LIML routine is executed (inadmissible unrestricted estimate).

unrestricted estimates and our pooled estimates are due to pooling across reference firms.

Our final (second-stage) estimates of σ , denoted $\hat{\sigma}$, are shown in Columns 4 and 5. We see that $\hat{\theta}^{(P)}$ is inadmissible in six of the 20 industries. In three of these cases, the second-stage estimator yields an estimate $\hat{\sigma} = \infty$ (NACE 2 industry 10, 16, and 26). The 17 finite estimates of σ shown in Column 5 of Table 2 lie in the range 2.1–62.8.

It is interesting to compare our estimates with the Feenstra–Soderbery estimator shown in the last two columns of Table 2. This estimator does not provide standard errors in the six cases when $\hat{\theta}^{(u)}$ is inadmissible. Moreover, there are three additional cases where the Feenstra–Soderbery methodology does not produce any estimates. For all the six industries where our pooled estimates are inadmissible, the Feenstra–Soderbery method also yields inadmissible unrestricted estimates. Conversely, in all industries where the Feenstra–Soderbery method yields admissible unrestricted estimates (11 cases), the pooled estimator does so too. We conclude that our method of pooling reduces the need for executing the second stage of the estimation.

In general, the two sets of σ -estimates differ significantly: of the 20 industries, in only three cases does the Feenstra–Soderbery estimator $\hat{\sigma}$ lie within the 95 percent confidence interval of the pooled method. The most striking difference is that the Feenstra–Soderbery estimate is less than 2 in five of the industries, the lowest estimate being 1.15, implying that new varieties have a large impact on the output growth index. In contrast, only one σ estimate is less than 3 with the pooled method, the lowest estimate being 2.12. The standard errors and confidence intervals in Table 2 are estimated using bootstrap methodology (re-sampling of firms in the sample with replacement). The estimates show that σ is generally not precisely estimated, especially for high values of $\hat{\sigma}$.

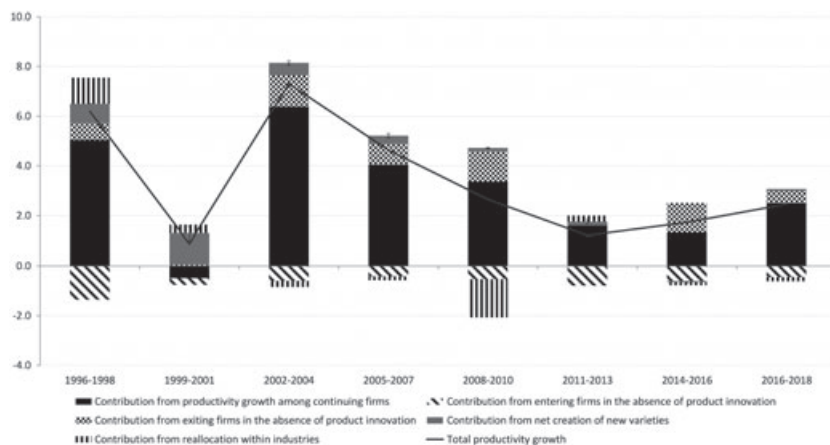
4.2.2. Productivity growth decomposition. The results of the decomposition of aggregate productivity growth for manufacturing over the observation period 1995–2018 are shown in Table 3 and Figure 3. We have split the whole observation period into eight intervals, each covering three years, except the last (two-year) interval 2017–2018. For each interval, we present average annual growth rates in percent corresponding to $\sigma = \infty$ (new varieties have no impact) and $\sigma_i = \hat{\sigma}_i$ (the estimated demand elasticity for each industry). The results for all industries involve a weighted average of contributions from the individual industries.

The decomposition in Table 3 shows the contributions from five sources: (1) intra-firm productivity growth among continuing firms; (2) reallocation

Table 3. Sources of aggregate productivity growth, by year

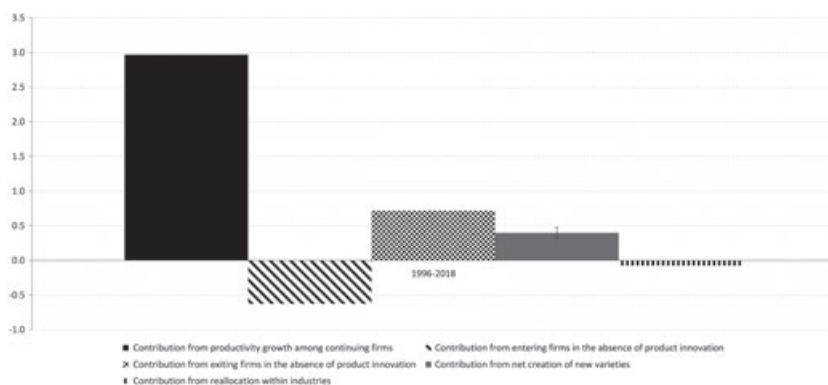
Source	96–98	99–01	02–04	05–07	08–10	11–13	14–16	17–18	Mean
(1) Continuing firms	5.04	-0.49	6.38	4.04	3.37	1.60	1.32	2.50	2.97
(2) Within industry realloc.	1.05	0.34	-0.25	-0.16	-1.54	0.25	-0.13	-0.17	-0.08
(3) Entering firms	-1.36	-0.26	-0.60	-0.42	-0.53	-0.79	-0.61	-0.45	-0.63
(4) Exiting firms	0.68	0.06	1.26	0.85	1.22	0.00	1.18	0.49	0.72
(5) New varieties ($\sigma = \hat{\sigma}$)	0.78	1.23	0.50	0.33	0.13	0.16	-0.04	0.09	0.40
SE	0.06	0.28	0.08	0.09	0.01	0.08	0.02	0.03	0.08
Lower bound	0.24	0.44	0.18	0.11	0.04	0.05	0.02	0.02	0.13
	(0.03)	(0.14)	(0.04)	(0.04)	(0.01)	(0.04)	(0.01)	(0.01)	(0.04)
Total productivity growth	6.19	0.88	7.30	4.64	2.65	1.21	1.73	2.46	3.38

Notes: Growth rates in percent. The terms (1)–(5) refer to the numbered terms of equation (15) (Proposition 1). “SE” is the standard error (attributable to the estimator $\hat{\sigma}$) of the estimated net contribution from new and disappearing varieties. “Lower bound” is the lower bound estimate of the net contribution from new and disappearing varieties, with the standard error of the lower bound in parentheses; it is based on a linear approximation of the demand curve (see Diewert and Feenstra, 2019).

Figure 3. Productivity growth in manufacturing 1996–2018

Notes: Average annual growth rates are in percent.

between continuing firms in the same industries; (3) entering firms when all products are assumed to be perfect substitutes ($\sigma = \infty$); (4) exiting firms when $\sigma = \infty$; and (5) net creation of new varieties with $\sigma_i = \hat{\sigma}_i$. The results for all industries then involve a weighted average of all the industry-specific estimates, $\hat{\sigma}_i$. The terms (1)–(5) correspond to the terms (1)–(5) in equation (15) (Proposition 1). The decomposition of labour productivity growth into its various sources – and over time – is illustrated in Figure 3,

Figure 4. Contributions to aggregate productivity, 1996–2018

Notes: Average annual growth rate in percent.

with 95 percent confidence intervals for contributions from the net creation of new varieties indicated by markers.

Average labour productivity growth in manufacturing was 3.4 percent annually in 1996–2018. The lowest growth rate was 0.9 percent in 1999–2001, immediately followed by the exceptionally high growth rate of 7.3 percent in 2002–2004. This period overlaps with the height of the oil-fuelled boom period (when oil prices surged from \$20 to more than \$100 per barrel), which lasted from 2001 until the financial crisis of 2008. The period 2005–2013 displayed a monotonic downward trend in productivity growth, with only 1.2 percent annual growth in the period 2011–2013. Since then, the growth rate has increased again, reaching 2.5 percent in 2017–2018, which is still below the overall mean of 3.4 percent in 1996–2018.

To highlight the contribution from each source, Figure 4 shows the average, for all years, of the decomposition shown in Figure 3. New varieties contribute significantly to overall productivity growth: average annual productivity growth in the period 1996–2018 is 3.0 percent with no allowance for new varieties and 3.4 percent when the (net) contribution of new (and disappearing) varieties is taken into account. This is shown in the last column of Table 3. Thus, the estimated contribution of new varieties to total productivity growth is 0.4 percentage points annually. This contribution is statistically significant, as seen from the estimated standard error of 0.08 (see Table 3) and the indicated 95 percent confidence interval in Figure 4. Table 3 also shows the annual estimated lower bound estimates for the contributions of new varieties, with standard errors in parentheses. In contrast to the CES functional form, the lower bound is based on a linear

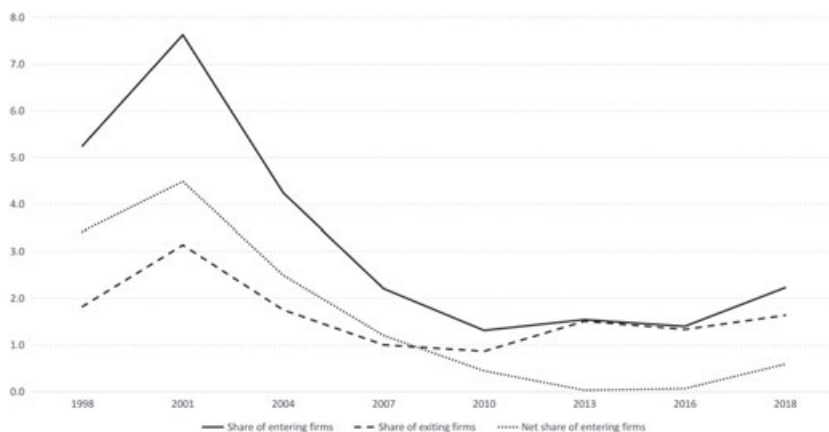
approximation of the demand curve (see Diewert and Feenstra, 2019). Each of the annual lower bound estimates is significantly different from zero (p -value < 0.05). The average for all years is 0.13, with a standard error of 0.04. The lower bound estimate for each year is almost exactly one-third of the estimate based on the assumption of a constant elasticity of substitution σ .

Of the other sources of aggregate productivity growth, intra-firm productivity growth is the most important, contributing 3.0 percentage points annually to the overall annual average productivity growth rate of 3.4 percent. Reallocation of labour within industries is of minor importance to long-run aggregate productivity growth (−0.1 percentage point annually), as positive reallocation effects in some years are offset by negative effects in others.

Disregarding the impact of new varieties, on the one hand, exiting firms contribute positively to annual productivity growth (0.7 percentage points, on average), reflecting the fact that exiting firms have lower productivity levels than survivors. On the other hand, entering firms contribute negatively to productivity growth (disregarding the impact of new varieties). This might seem to contradict conventional wisdom that entry and exit dynamics contribute to “creative destruction”, whereby inefficient old firms are replaced by new and more efficient firms. However, our finding is not surprising in view of the high exit rates among young firms, and is consistent with the results of Golombek and Raknerud (2018), who document strong selection based on productivity among start-up firms. Moreover, our results are in line with conventional decompositions of aggregate productivity growth for the whole mainland Norwegian economy (Iancu and Raknerud, 2017).

Figure 5 shows that the (gross) output share of entering firms and the net share of entering firms (share of entering firms less that of exiting firms) have both been decreasing monotonically since 1998. The net share even became slightly negative in 2014–2016, leading to slightly negative (value-weighted) net creation of new varieties in 2014–2016.

Finally, we investigate the sensitivity of our results with regard to the choice of a three-year time window for defining and measuring the output shares of exiting and entering firms. To do the sensitivity analysis, we have recalculated the productivity decomposition using a five-year window (except for the three-year window 2015–2018). One might expect that the use of a longer time window would increase the contribution of entering firms, because it takes time to build up firms after entry. However, our results are remarkably robust to the change in window size. The only non-negligible change is a slight decrease in the impact of new varieties – from 0.40 to 0.35. We find a slight increase in the output share of entering firms, but this is more than offset by an increase in the output share of exiting

Figure 5. Output shares of entering and exiting firm (in percent)

Notes: Average annual growth rate in percent.

firms. There is no impact on the contribution from continuing firms, which is dominated by a relatively small group of large incumbent firms.

5. Conclusion

Applying the economic approach to index numbers, we have provided a decomposition of aggregate productivity growth, which is rooted in economic theory and which identifies the contribution from new firms producing new varieties. The novelty of this decomposition lies in the way we have reconciled the literature on how new goods affect prices and the literature on aggregate productivity growth and firm turnover. The decomposition provided in this paper encompasses many of the frameworks currently adopted in the literature.

Our results indicate that the effect of new varieties on aggregate productivity growth is both statistically and economically significant and amounted to almost half a percentage point annually for Norwegian manufacturing industries in the period 1996–2018. This result is based on estimates of demand elasticity ranging from 2.1 to infinity.

Appendix A. Decomposition of industry-level productivity growth

We split this appendix into three parts. First, we define the concept multi-factor productivity at industry level for a set of continuing firms, C_{it} .

Second, we prove equation (6), and we outline the output and input indices at the industry level and the economy-wide level for entering and exiting firms. Lastly, we formulate the input index. The proof of Proposition 1 follows by considering the ratio of the output and input indices.

A.1. Multi-factor productivity growth with continuing firms

Multi-factor productivity growth (MFP) is defined as the relative (logarithmic) shift in the production function for a given set of inputs. Assuming constant returns to scale and differentiability with respect to time, which is realistic for the set of continuing firms C_{it} , the growth rate of MFP at firm level is

$$\frac{\partial \ln g_{if}(\cdot, t)}{\partial t} = \frac{d \ln(Y_{ift})}{dt} - \alpha_{ift} \frac{d \ln(L_{ift})}{dt} - (1 - \alpha_{ift}) \frac{d \ln(M_{ift})}{dt},$$

with

$$\alpha_{ift} = \frac{q_{ift}^L L_{ift}}{C_{ift}}, \tag{A1}$$

where q_{ift}^L is the wage rate, L_{ift} is man-hours, and C_{ift} is total factor costs (see, e.g., Balk, 2009).

According to Domar’s theorem of aggregation,

$$\frac{\partial \ln g_i(L_{it}, M_{it}, t)}{\partial t} = \sum_{f \in C_{it}} d_{ift} \frac{\partial \ln g_{if}(L_{ift}, M_{ift}, t)}{\partial t}, \tag{A2}$$

where the (Domar) weights at the lower level of aggregation are

$$d_{ift} = \frac{P_{ift} Y_{ift}}{FD_{it}} = \frac{V_{ift}}{\sum_{f \in C_{it}} V_{ift}} = \frac{C_{ift}}{\sum_{f \in C_{it}} C_{ift}},$$

and FD_{it} denotes the value of final deliveries from C_{it} . The first equality is the standard expression for the Domar weight, the second follows from the assumption that there are no intra-industry deliveries, and the third from the assumption that the price is an industry-specific markup on marginal costs (possibly equal to one).¹⁰ After some manipulation, it follows that

$$\frac{\partial \ln g_i(\cdot, t)}{\partial t} = \sum_{f \in \mathcal{F}_{it}} \left[\omega_{ift} \frac{d \ln(Y_{ift})}{dt} - \alpha_{it} \xi_{ift} \frac{d \ln L_{ift}}{dt} - (1 - \alpha_{it}) \nu_{ift} \frac{d \ln(M_{ift})}{dt} \right], \tag{A3}$$

¹⁰In the case of the CES production function with elasticity of scale ϵ_i and demand elasticity equal to e_i , $C_{ift} = [e_i(e_i - 1)/e_i]V_{ift}$. This example illustrates why it is not possible to separately identify elasticity of scale and demand elasticity from expenditure data. This is discussed in more detail in Golombek and Raknerud (2018).

where

$$\begin{aligned} \frac{d \ln(Y_{it})}{dt} &= \sum_{f \in C_{it}} \omega_{ift} \frac{\partial \ln(Y_{ift})}{dt}, \\ \frac{d \ln(L_{it})}{dt} &= \sum_{f \in C_{it}} \xi_{ift} \frac{\partial \ln(L_{ift})}{dt}, \\ \frac{d \ln(M_{it})}{dt} &= \sum_{f \in C_{it}} \nu_{ift} \frac{\partial \ln(M_{ift})}{dt}, \end{aligned} \tag{A4}$$

with

$$\begin{aligned} \omega_{ift} &= \frac{V_{ift}}{\sum_{f \in C_{it}} V_{ift}}, \\ \xi_{ift} &= \frac{q_{ift}^L L_{ift}}{\sum_{f \in C_{it}} q_{ift}^L L_{ift}}, \\ \nu_{ift} &= \frac{q_{ift}^M M_{ift}}{\sum_{f \in C_{it}} q_{ift}^M M_{ift}}, \end{aligned} \tag{A5}$$

and

$$\alpha_{it} = \frac{\sum_{f \in C_{it}} q_{ift}^L L_{ift}}{\sum_{f \in C_{it}} C_{ift}}. \tag{A6}$$

It follows that

$$\frac{\partial \ln g_i(\cdot, t)}{\partial t} = \frac{d \ln(Y_{it}/L_{it})}{dt} - (1 - \alpha_{it}) \frac{d \ln(M_{it}/L_{it})}{dt}. \tag{A7}$$

Multi-factor productivity growth in discrete time, from $t - 1$ to t , is the integrated growth rate (see Balk, 2009),

$$MFP_i = \int_{t-1}^t \frac{\partial \ln g_i(\cdot, u)}{\partial u} du \tag{A8}$$

$$= \ln \left(\frac{Q_{iY}}{Q_{iL}} \right) - (1 - \bar{\alpha}_{it}) \ln \left(\frac{Q_{iM}}{Q_{iL}} \right), \tag{A9}$$

where

$$Q_{iY} = \frac{Y_{it}}{Y_{i,t-1}}, \quad Q_{iL} = \frac{L_{it}}{L_{i,t-1}}, \quad Q_{iM} = \frac{M_{it}}{M_{i,t-1}},$$

and the bar symbol denotes an appropriate mean value over $[t - 1, t]$. Equation (A9) expresses the familiar relationship that multi-factor productivity growth at industry level equals labour productivity growth minus weighted material intensity growth.

A.2. The output index with entering and exiting firms

So far we have ignored entry and exit of firms, which cause discontinuities and non-differentiability at the lower level of aggregation. While equations (A7)–(A9) are valid for $f \in C_{it}$, the MFP decomposition expressed in equation (A9) must be augmented to account for $f \in \mathcal{N}_{it} \cup \mathcal{X}_{it}$. To do so, we start by noting that, according to Hotelling’s lemma and the well-known properties of the CES function, the cost-minimizing demand, $Y_{ift}^{(jk)}$, given $\phi_i(\mathbf{y}_{it}^{(jk)}) = Y_{it}^{(jk)}$ is

$$Y_{ift}^{(jk)} = Y_{it}^{(jk)} \partial c_i(\mathbf{p}_{it}) / \partial p_{ift} \quad \text{for all } f \in \mathcal{F}_{it} \tag{A10}$$

(see equations (4) and (7) for definitions). It follows from equations (3) and (A10) that

$$Y_{ift} = \frac{Y_{it} \partial c_i(\mathbf{p}_{it})}{\partial p_{ift}},$$

for any $Y_{it} = \sum_j \sum_k Y_{it}^{(jk)} \geq 0$ and $f \in \mathcal{F}_{it}$. Because V_{it} is total (cost minimizing) expenditures, equations (6) and (7) follow.

The volume index, Q_{iY} , of the CES aggregate Y_{it} in equation (6) follows from Feenstra (1994), who generalized the results of Sato (1976) and Vartia (1976) to handle situations where the number of goods changes over time, which is the case for the set of firms \mathcal{F}_{it} producing a variety of good i . The Sato–Vartia–Feenstra index theory yields the following expression for the aggregate output index, Q_{iY} :

$$\ln Q_{iY} = \sum_{f \in C_{it}} \bar{w}_{ift} \Delta \ln Y_{ift} - \ln(1 - s_{it}^N) + \ln(1 - s_{i,t-1}^X) - \left(\frac{1}{\sigma_i - 1} \right) \ln \left(\frac{1 - s_{it}^N}{1 - s_{i,t-1}^X} \right). \tag{A11}$$

The first term is the standard Sato–Vartia index across continuous firms that produce the same composite good. The second and third terms represent the contributions from firm turnover in the absence of product innovation. The last term shows the net effect on output of the creation of new varieties.

A.3. Aggregate labour input

It is common, but not uncontroversial, to aggregate labour input as a simple sum of hours worked across firms. Although there is an extensive body of literature on quality adjustment of labour services dating back to at least Jorgenson and Griliches (1967), we proceed at industry level with the standard approach of using the sum of hours worked to derive the index

for labour input Q_{iL} .¹¹ In conformity with our theoretical framework of aggregation, the implicit assumption is that labour is a homogeneous input within the same industry, but not necessarily across industries. The weight ξ_{ift} in equation (A3) then coincides with the real weight L_{ift}/L_{it} , where both L_{ift} and L_{it} are sums of hours worked.

For a particular industry, i , the index of hours worked can be decomposed according to whether firms are continuing, entering, or exiting, as follows (von Brasch et al., 2018):

$$\ln Q_{iL} = \sum_{f \in C_{it}} \bar{\xi}_{ift} \Delta \ln L_{ift} - \ln(1 - h_{it}^N) + \ln(1 - h_{i,t-1}^X), \quad (\text{A12})$$

where h_{it}^N , $h_{i,t-1}^X$, and $\bar{\xi}_{ift}$ are defined in equations (12)–(14).

Appendix B. Relation to existing literature

Much of the literature analyses the contribution of firm turnover to overall productivity growth by applying a framework based on a weighted average of productivity levels (see, e.g., Griliches and Regev, 1995; Baily et al., 1992; Foster et al., 2001, 2006, 2008). In the following, we point out similarities and differences between the standard frameworks used in the literature and the novel framework outlined in equation (15). In particular, we show how the framework outlined in equation (15) generalizes the frameworks typically used in the literature.

Following the notation used in the main text, a firm's level of productivity is defined as the ratio of outputs to inputs in real terms Y_{ift}/L_{ift} . A weighted arithmetic average productivity level across all firms can then be written as

$$\Pi_{it} = \sum_{f \in F_{it}} \pi_{ift} \left(\frac{Y_{ift}}{L_{ift}} \right), \quad (\text{B1})$$

where the weights π_{ift} sum to unity and F_{it} denotes the set of all firms producing a variety of good b . For this average to have a meaningful interpretation, all firms must be producing identical or homogeneous products. For example, if one firm is producing 1,000 cellular phones per hour worked and another firm is producing 50 tablets per hour worked, it is not meaningful to compare productivity levels across firms. In general, if firms are not producing homogeneous products, taking the average in equation (B1) is like comparing apples and oranges. This insight relates to

¹¹The input index we derive in this paper can alternatively be defined within the theory of quality adjustment (see, e.g., von Brasch et al., 2018).

the basic index number problem and illustrates the restrictiveness of using equation (B1) as a starting point for decomposing aggregate productivity growth.

The assumption of homogeneous products implicitly underlying equation (B1) can be made explicit in terms of the framework outlined in Section 2. Consider aggregation of varieties in equation (6), which can be written explicitly as

$$Y_{it} = \left(\sum_{f \in \mathcal{F}_{it}} \gamma_{if} Y_{ift}^{(\sigma_i-1)/\sigma_i} \right)^{\sigma_i/(\sigma_i-1)}. \tag{B2}$$

All the varieties are homogeneous if the following assumptions hold:

$$\gamma_{if} = 1 \quad \text{and} \quad \sigma_i \rightarrow \infty \quad \text{for all } f \in \mathcal{F}_{it}, i \in I.$$

Given these assumptions, aggregation of output is reduced to a summation across homogeneous products (i.e., $Y_{it} = \sum_{f \in \mathcal{F}_{it}} Y_{ift}$). One way to measure the average productivity level in equation (B1) is by the ratio of outputs to inputs

$$\Pi_{it} = \frac{Y_{it}}{L_{it}} = \frac{\sum_{f \in \mathcal{F}_{it}} Y_{ift}}{\sum_{f \in \mathcal{F}_{it}} L_{ift}} = \sum_{f \in \mathcal{F}_{it}} \pi_{ift} \left(\frac{Y_{ift}}{L_{ift}} \right),$$

where the weights are now defined as input shares: $\pi_{ift} = L_{ift}/(\sum_{f \in \mathcal{F}_{it}} L_{ift})$. For example, Iancu and Raknerud (2017) employ this weighting scheme. It is more common, however, to base the weights π_{ift} on output shares.¹² After comparing productivity levels by industry (or product), the results can be averaged across industries, i.e.

$$\Pi_t = \sum_{i \in I} \pi_{it} \Pi_{it}. \tag{B3}$$

Aggregate output shares are typically used as weights. The change in average productivity, as defined by equations (B1) and (B3), can be decomposed as

$$\begin{aligned} \Delta \Pi_t = & \sum_{i \in I} \pi_{it} \left[\sum_{f \in \mathcal{C}_{it}} \bar{\pi}_{ift} \Delta \left(\frac{Y_{ift}}{L_{ift}} \right) + \sum_{f \in \mathcal{N}_{it}} \pi_{ift} \left(\frac{\bar{Y}_{ift}}{L_{ift}} - \bar{\Pi}_i \right) \right. \\ & \left. - \sum_{f \in \mathcal{X}_{it}} \pi_{if,t-1} \left(\frac{\bar{Y}_{if,t-1}}{L_{if,t-1}} - \bar{\Pi}_i \right) \right] + \widetilde{RWI}_t + \widetilde{RBI}_t. \end{aligned} \tag{B4}$$

The first term in the square brackets represents a within-industry component showing the weighted average of productivity growth across continuing

¹²One way of doing this is to take the reciprocal of the aggregate inverse productivity measure (see Diewert and Fox, 2010).

firms. The last two terms inside the square brackets represent the contributions of entering and exiting firms. Note that the impact of firm turnover on productivity growth depends on the productivity levels of entering and exiting firms relative to the average productivity level: aggregate productivity increases either if entering firms are more productive than the average or if exiting firms are less productive than the average.

The last two terms represent reallocation effects within and between industries, and are given by

$$\overline{RWI}_t = \sum_{i \in I} \pi_{it} \sum_{f \in C_{it}} \left(\frac{\overline{Y}_{ift}}{L_{ift}} - \bar{\Pi}_i \right) \Delta \pi_{ift} \quad (\text{B5})$$

$$\overline{RBI}_t = \sum_{i \in I} \bar{\Pi}_i \Delta \pi_{it}. \quad (\text{B6})$$

Reallocation within industries (\overline{RWI}) contributes positively to aggregate productivity if the weight of high-productivity firms increases. Reallocation between industries (\overline{RBI}) contributes positively to aggregate productivity if the weight of high-productivity industries increases.

The framework outlined in equation (B4) is conceptually very similar to that typically found in the literature. For example, Foster et al. (2008) also starts out with a weighted average of productivity levels across firms in the first stage of the aggregation equation (B1). However, instead of applying a weighted average of productivity levels at the second stage of aggregation equation (B3), Foster et al. (2008) calculate a weighted average of changes in productivity levels at the industry/product level (i.e., $\Delta \Pi_t = \sum_{i \in I} \pi_{it} \Delta \pi_{it}$). The only difference between that approach and equation (B4) is that equation (B4) also contains the impact of reallocation between industries. Importantly, the underlying assumption that products are homogeneous within industries is common to the decompositions typically used in the literature and also equation (B4).

There are both similarities and differences between the decompositions in equations (B4) and (15). First, while equation (15) decomposes productivity growth, measured as the difference between the log change of the output and the input indices, equation (B4) shows the absolute change in the weighted average of productivity levels. Second, the weighting scheme in the two decompositions might differ, depending on how the weights π_{it} and π_{ift} are defined.

The most important difference between the two decompositions is that equation (15) generalizes the framework underlying the decomposition in equation (B4); that is, it allows for products being imperfect substitutes ($\sigma_i < \infty$). When products are imperfect substitutes the entry of a new firm increases the number of varieties and the overall level of output. In

equation (15), the net impact of new varieties on aggregate productivity growth is given by the term

$$\left(\frac{1}{1 - \sigma_i} \right) \ln \left(\frac{1 - s_{it}^N}{1 - s_{i,t-1}^X} \right).$$

This effect on aggregate productivity growth is absent in equation (B4).

Supporting information

Additional supporting information may be found online in the supporting information section at the end of the article.

Replication files

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