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Research Article

On Improving the Students' Combinatorial Thinking Skill in Solving Rainbow Antimagic Colouring Problem on Cryptography for E-Commerce Security Systems under the Implementation of Research-Based Learning with STEM Approach

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Abstract

Combinatorial thinking is the process of obtaining multiple solutions for discrete problem-solving. Combinatorial thinking skills have several indicators: identifying several cases, recognizing patterns from all cases, generalizing all cases, proving mathematically, and considering other combinatorial problems. The learning approach in schools is growing along with the development of science and technology, one of the approaches used in STEM. STEM education focuses students on solving problems in everyday life with the help of science, technology, engineering, and mathematics. This paper aimed to analyze the students' combinatorial thinking skills after using a research-based learning worksheet with a STEM approach to solving the rainbow antimagic colouring problem. This study uses a mixed-method where this research combines qualitative and quantitative methods. The subjects of this research are undergraduate students of Mathematics Education, University of Jember. This study involved two classes, namely the experimental and control classes, where the researcher gave the two classes different treatments. The t-test compares students' combinatorial thinking skills between the experiment and control classes after using research-based learning tools with a STEM approach for solving the rainbow antimagic problem. The homogeneity test results on the pre-test items indicated a significance value of .789 > .05, which indicates that the two classes are homogenous. The independent sample t-test score is .020 < .05, which means the difference is significant. It concludes that implementation of the research-based learning materials with a STEM approach can improve the students' combinatorial thinking skills in solving the rainbow antimagic colouring.

Keywords: combinatorial thinking, rainbow antimagic colouring, research-based learning, STEM approach

Introduction

Learning in the 21st century has entered the era of the industrial revolution 4.0, where the learning process has been made easier by using technology. The learning approach in schools is growing along with the development of science and technology. One of the approaches used is the STEM approach. The STEM acronym stands for Science, Technology, Engineering, and Mathematics. In STEM education, students solve real-life problems using science, technology, engineering, and mathematics. STEM education can help students develop technology skills, sharpen their cognitive and affective abilities, and apply knowledge (Torlakson, 2014, p. 12). Schools can use STEM education to train students in designing solutions for environmental problems through technology by using STEM Education (Ebal et al., 2019, p. 2; Gita et al., 2021, p. 5; Subekti et al., 2018, p. 2). The learning model currently being researched is research-based learning (RBL). RBL focuses on analysis, synthesis, and evaluation as a learning model and how students and lecturers assimilate and apply knowledge (Susiani et al., 2018,

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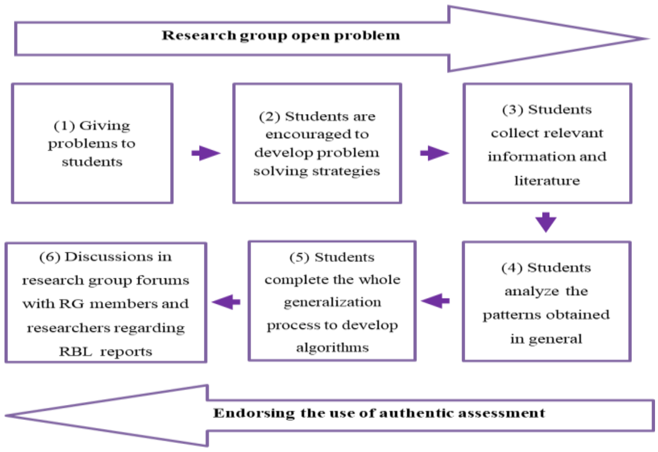
p. 2). Learning that uses RBL models helps students in developing various mathematical thinking skills. Through RBL, students get the opportunity to learn by doing, which makes learning more meaningful. Arifin (2019, p. 83) explains that through RBL, students can learn fundamental concepts, apply robust methodologies, solve problems logically and scientifically, and have an attitude of seeking the truth and being open and honest. Specifically, RBL consists of background problems, research procedures, implementation, results, discussion, and publication of research results. More studies of RBL model (Maylisa et al., 2020, p. 3; Nazula et al., 2019, p. 2; Singh, 2014, p. 21). Figure 1 shows the RBL syntax we used in this study.

Figure 1
The Syntax of Research-Based Learning



One of the mathematical thinking skills that can be developed through the use of RBL is combinatorial thinking skills (CTS). Combinatorial thinking is the process of obtaining multiple solutions to discrete problem- solving (Syahputra, 2016, p. 3). Combinatorial thinking skills are needed to find a solution to a graph problem. Students use combinatorial thinking skills to systematically look for possible solutions to these problems and ensure that the results obtained are correct and can be accounted for. Table 1 demonstrates the use of indicators of CTS based on (Anggraeni et al., 2019, p. 2).

Table 1
The Indicators of CTS and their Development

Indicator	Indicator development
Identify some cases	a. Students are able to identify problems related to the concept of rainbow antimagic colouring b. Students are able to apply the concept of rainbow antimagic colouring on simple graphs
Recognizing the pattern of all cases	a. Students are able to find colouring patterns based on the concept of rainbow antimagic colouring that has been applied b. Students are able to expand the colouring pattern to the n^{th} order on a graph that has applied the concept of rainbow antimagic colouring
Generalizing all cases	a. Students are able to provide notation on graphs b. Students are able to determine the set of vertices and edges of graphs c. Students are able to determine the set of vertices and edges of a graph to the n^{th} order
Prove mathematically	a. Students are able to determine the cardinality of a graph to the n^{th} order b. Students are able to create vertex functions and edge weight functions from a graph that has applied the concept of rainbow antimagic colouring c. Students are able to test the correctness of the vertex function and edge weight function of a graph that has applied the concept of rainbow antimagic colouring that has been found d. Students are able to find the number of rainbow antimagic connections from a graph that has applied the concept of rainbow antimagic colouring e. Students are able to test the correctness of an antimagic rainbow connection number from a graph to the n^{th} order
Considering another combinatorial problem	a. Students are able to explain the definition of rainbow antimagic colouring and the flow/stages of searching for rainbow antimagic connection numbers from graphs b. Students are able to find open problems related to the concept of rainbow antimagic colouring c. Students are able to find new problems related to the concept of rainbow antimagic colouring d. Students are able to find applications related to the concept of rainbow antimagic colouring in everyday life

Graph theory is one of the sciences that help overcome various problems in various disciplines. In mathematics, graphs have many benefits in overcoming social issues and problems of daily life. A graph is a representation of discrete objects and the relationships between them. Visually, a graph represents each object as a vertex, while the relationships between them are shown as an edge. Rainbow antimagic colouring (RAC) combines two notions, namely antimagic labeling and the rainbow concept. A graph G is defined as rainbow antimagic colouring if each vertex is labeled with antimagic labels. Furthermore, the edge weight of antimagic labels is used to assign a rainbow connection, such that there is at least one rainbow path with all different edge weights (Sulistiyono et al., 2020, p. 1). The rainbow antimagic connection number, denoted by $rac(G)$, is defined as the smallest number of colors in which at least one rainbow path with all different side weights. More studies on rainbow antimagic colouring (Al Jabbar et al., 2020, p. 1; Budi et al., 2020, p. 2; Dafik et al., 2021, p. 278; Septory et al., 2021, p. 3). An illustration of RAC is provided in Figure 2.

Figure 2
RAC of a Path with Order 7

Research Problem

Is the implementation of research-based learning with STEM approach able to improve the students' combinatorial thinking skills in solving rainbow antimagic colouring problems on cryptography for e-commerce security systems?

Hypothesis

The hypothesis is written in the form of a pair of the null hypothesis (H_0) and alternative hypothesis (H_1). To consider the acceptance of the hypothesis, if sig > .05, then H_0 is accepted, but if sig < .05, then H_0 is rejected. The following hypotheses will be tested in this study:

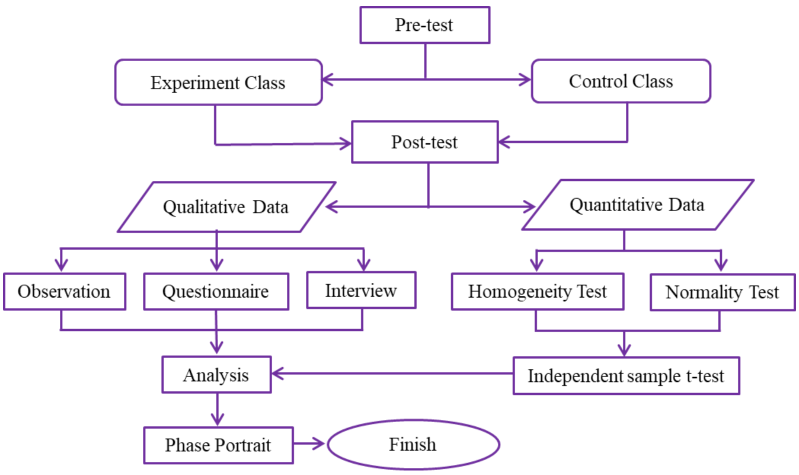
Hypothesis 1: (H_0) There is no significant difference in students' combinatorial thinking skills in solving rainbow antimagic colouring problems on cryptography for e-commerce security systems.

Hypothesis 2: (H_1) There is a significant difference in students' combinatorial thinking skills in solving rainbow antimagic colouring problems on cryptography for e-commerce security systems.

Methodology

As shown in Figure 3, this study used a mixed-method design. A combination of qualitative and quantitative methods was used to collect data. Test results, observations, questionnaires, and student interviews were analyzed qualitatively, while statistics related to the test of combinatorial thinking skills were analyzed quantitatively.

Figure 3
The Mix Method Research Diagram



For a quasi-experimental study, control and experimental groups have almost identical characteristics initially. Furthermore, there is a pre and post-test control group with one treatment for the control and experimental classes. The STEM approaches were used with RBL models and worksheets in the experimental class. While in the control class, RBL models and worksheets were used without STEM approaches. We used data collection to determine whether students' skills in combinatorial thinking were different after using RBL worksheets with STEM approach to solving RAC problems. Following that, we identified several students as subjects to conduct in-depth interviews and analyze their work on worksheets and tests to determine how their thinking processes flowed according to the stages of combinatorial thinking skills. We classified their CTS into three levels, namely low combinatorial thinking skills (LCTS), medium combinatorial thinking skills (MCTS), and high combinatorial thinking skills (HCTS). This study was conducted using a mixed-methods research design presented in Table 2.

Table 2
The Research Design Using Mixed Methods

Class	Pre-test	Treatment	Post-test
Experimental	O_1	X_1	O_2
Control	O_3	X_2	O_4

O_1, O_3 = For the preliminary assessment of the students' work skills, each group was given a pre-test, and we expected that both groups would have similar skills

X_1 = RBL with a STEM approach and utilizing worksheets for students that have been created
X_2 = RBL and utilizing worksheets for students that have been created
O_2, O_4 = Post-test results from both groups

Sample

The research subjects were students in Math. Education at the University of Jember.

Students' Task

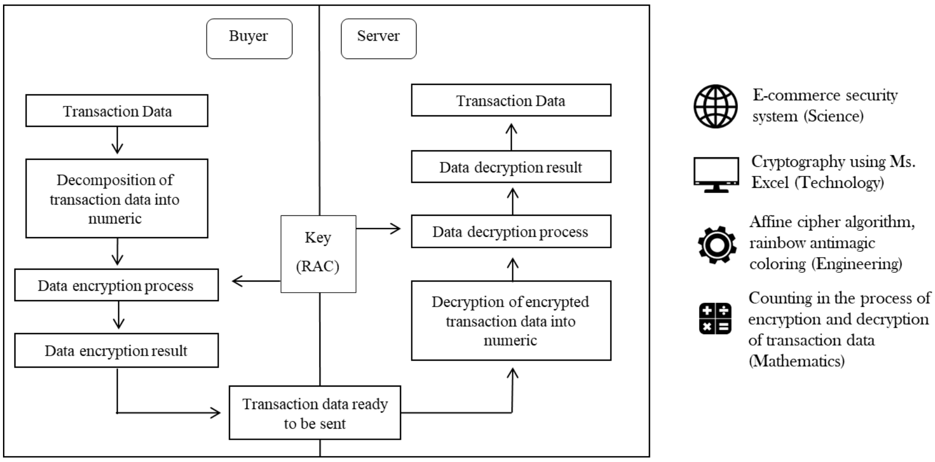
We will give two types of assignments to students: tasks related to STEM problems and application of the RAC concept to the vertex amalgamations of star graphs with order four denoted by $Amal(S_4, v, m)$. The explanation of the two tasks is as follows:

STEM problem. The shop opened by Nina sells all kinds of accessories for women, such as handbags, purses, wallets, bracelets, etc. Due to the COVID-19 pandemic, Nina's shop has become increasingly quiet, and the buyers are getting rare. Thus, she decided to start an online shop to make transactions easier. Nina needs an e-commerce security system to maintain both security and convenience in the process.
The e-commerce security system uses cryptography to maintain the confidentiality of the buyer's transaction data. The cryptographic algorithm used in the e-commerce security system on Nina's online shop website is the affine cipher algorithm, where the key used is a vertex amalgamation of star graphs with order four denoted by $Amal(S_4, v, m)$ that applies the concept of rainbow antimagic colouring and for the encryption process using the y_i=(x_i+k_i)(mod 26) functions, while the decryption process uses the x_i=(y_i-k_i)(mod 26) functions.
Explanation:
x_i = The numeric form of plaintext
k_i = Vertex labels and edge labels of a graph that applies the concept of rainbow antimagic colouring starting from the smallest that is used as the key
y_i = The numeric form of ciphertext

After Nina's online shop website was equipped with an e-commerce security system, Nina found a buyer who ordered a blue forever young women's wallet. The transaction is converted into a short message "DOMPET FY BLUE" without regard to spaces. Convert the transaction data into a secret text using the affine cipher algorithm thus that the buyer's type and the number of products are accurate by answering the following questions:
1. Determine the plaintext and key to be used!
2. Apply the concept of rainbow antimagic colouring to the keys used!
3. Describe the encryption process that occurs on the website!
4. Describe the decryption process that occurs on the website!
5. Write down the results of the encryption and decryption process on the website!

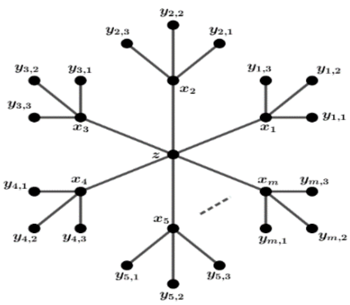
In Figure 4, we can see a depiction of how an e-commerce security system works on Nina's website when viewed from the buyer's perspective.

Figure 4
The E-Commerce Security System Process from the Buyer's Perspective



Rainbow antimagic colouring problem. The students are given $Amal(S_4, v, m)$ graph. The illustration of the graph can be seen in Figure 5. After students have observed the illustration image of the given graph, the students must define the notation and name of the graph, find the vertex set, edge set, the cardinality of the graph, calculate the vertex labeling and edge weight functions, calculate the rainbow antimagic connection number, and the generalization of each function. This worksheet contains a research report that explains the concept of rainbow antimagic colouring for graphs that have not yet been studied, followed by a forum discussion of the results with the expert in FGD.

Figure 5
$Amal(S_4, v, m)$ Graph



Results

First, we gave these two classes a pre-test to see the initial level of combinatorial thinking skills possessed by students regarding the concept of rainbow antimagic colouring we gave the treatment. Afterward, we gave the two classes different treatments. The treatment given to the experimental class was in the form of learning using the RBL models with the STEM approach and the developed worksheets, while treatment in the control class was in the form of learning using the RBL models and the designed worksheets. After giving the treatment, we gave a post-test with the same concept to see the final level of students' CTS.

After all, the data was collected, we carried out the analysis using the qualitative method. The statistical tests were carried out to analyze the data obtained from observing student activities during the learning activities. There are two variables tested in this study: the independent variable, namely research-based learning tools with a STEM approach, and the dependent variable, namely students' combinatorial thinking skills. The statistical test used in this study is the independent sample t-test. The condition to do the independent sample t-test is the data must have the similarity of variance using the homogeneity test, and the data must be normally distributed using the normality test. The data analysis uses the SPSS software.

Table 3
Homogeneity test results on pre-test

		Levene statistic	*df1*	*df2*	Sig.
Value	Based on mean	.073	1	48	.789
	Based on median	.031	1	48	.861
	Based on median and with adjusted *df*	.031	1	47.9	.861
	Based on trimmed mean	.078	1	48	.781

Table 4
Normality Test Results on Pre-test

Class		Kolmogorov-smirnov[a]			Shapiro-wilk		
		Statistic	*df*	Sig.	Statistic	*df*	Sig.
Value	Control class	.161	25	.096	.910	25	.031
	Experiment class	.168	25	.068	.912	25	.034

Note. [a] Lilliefors significance correction.

Tables 3 and 4 provide the pre-test's homogeneity and normality test results. Based on the result shown in Table 3, the significance value is .789, which is more than .05. Thus, we can say that the variance of both groups is the same. It indicates that both groups are homogeneous. Table 4 shows that in the Kolmogorov-Smirnov column, the significance value for experimental data and control data was .068 experimental data and .096 for the control data, which were both greater than .05, which indicates that the data was normally distributed in both classes.

Table 5
Independent Sample t-test on Pre-test

		Levene's Test for equality of variances		t-test for Equality of Means						
		F	Sig.	*t*	*df*	Sig. (2-tailed)	Mean difference	*SE* difference	95% Confidence interval of the difference	
									Lower	Upper
Value	Equal variances assumec	.073	.789	-.760	48	.451	-1.400	1.843	-5.106	2.306
	Equal variances not assumed			-.760	47.999	.451	-1.400	1.843	-5.106	2.306

After obtaining the results of the homogeneity and normality tests, we performed an independent sample t-test to compare the mean of both classes, as seen in Table 5. As illustrated in Table 5, neither the experimental nor the control classes have different average student learning outcomes since the paired (2-tailed) significance value is .451 greater than .05.

Figure 6
Distribution of CTS on Pre-test

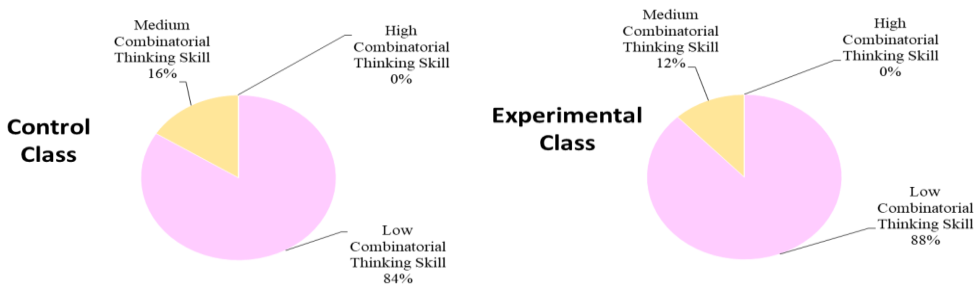


Figure 6 shows the pre-test results in the control class. These are 84% students with LCTS, 16% students with MCTS, and 0% students with HCTS, while in the experimental class, 88% students with LCTS, 12% students with MCTS, and 0% students with HCTS.

Table 6
Homogeneity Test Results on Post-test

		Levene statistic	*df1*	*df2*	Sig.
Value	Based on mean	2.518	1	48	.119
	Based on median	1.871	1	48	.178
	Based on median and with adjusted df	1.871	1	47.2	.178
	Based on trimmed mean	2.401	1	48	.128

Table 7
Normality Test Results on Post-test

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Value	Control Class	.143	25	.200*	.952	25	.282
	Experiment Class	.154	25	.128	.939	25	.138

Note. [a] Lilliefors significance correction. * This is a lower bound of the true significance.

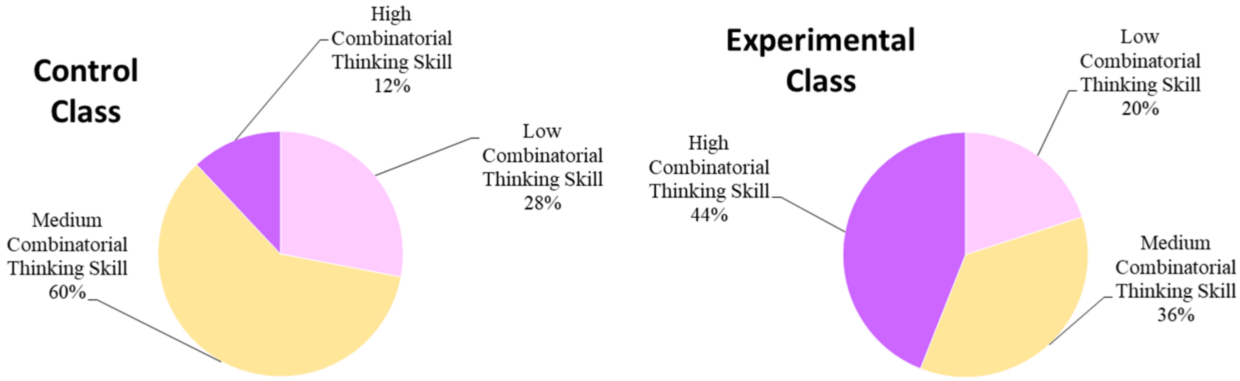
After we had done the pre-test, we gave the two classes different treatments. At the end of the lesson, the two classes took a post-test. Tables 6 and 7 provide the post-tests homogeneity and normality test results. Based on the result shown in Table 6, the significance value is .119, which is more than .05. Thus, we can say that the variance of both groups is the same. It indicates that both groups are homogeneous. Table 7 shows that in the Kolmogorov-Smirnov column, the significance value for experimental data and control data was .128 for experimental data and .200 for the control data, which were both greater than .05, which indicates that there was a normal distribution of data across both classes.

Table 8
Independent Sample t-test on Post-test

		Levene's test for equality of variances		t-test for equality of means						
		F	Sig.	*t*	*df*	Sig. (2-tailed)	Mean difference	*SE* difference	95% Confidence interval of the difference	
									Lower	Upper
Value	Equal variances assumed	2.518	.119	-2.403	48	.020	-6.400	2.663	-11.755	-1.045
	Equal variances not assumed			-2.403	44.766	.020	-6.400	2.663	-11.765	-1.035

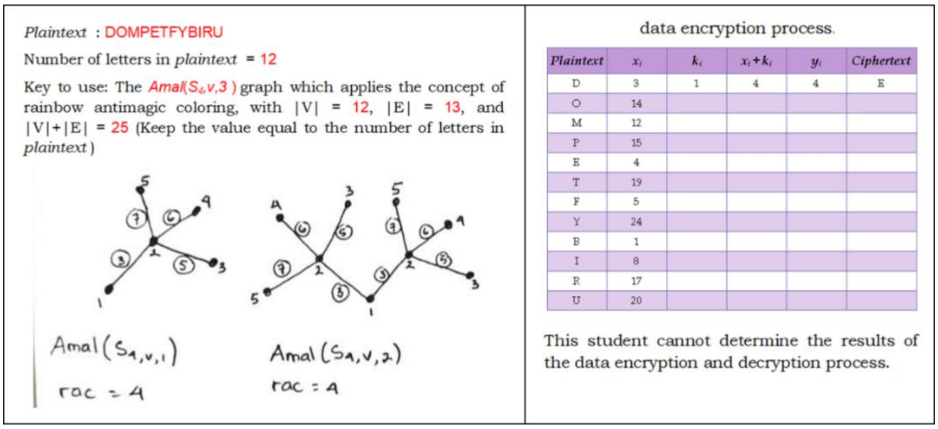
Following the results of the homogeneity and normality tests, we conducted an independent sample t-test to compare the means of the two classes, as illustrated in Table 8. Table 8 shows that the average student learning outcomes in an experimental group are different from those in a control group since the paired (2-tailed) significance value is .020, which is less than .05. It means $t_{count} > t_{table}$, and it implies that H_0 is rejected and H_1 is accepted. Thus, there is a significantly different in the students' combinatorial thinking skills in solving rainbow antimagic colouring problems on cryptography for e-commerce security systems between the control class and experiment class. The conclusion is the implementation of research-based learning with STEM approach can improve the students' combinatorial thinking skills in solving rainbow antimagic colouring problems on cryptography for e-commerce security systems.

Figure 7
Distribution of CTS on Post-test



Based on Figure 7, the post-test results from the control class showed that there were 28% students with LCTS, 60% students with MCTS, and 12% students with HCTS. While the post-test results from the experimental class showed that there were 20% students with LCTS, 36% students with MCTS, and 44% students with HCTS. Furthermore, the results of interviews on one sample from each category of combinatorial thinking skills related to the problem-solving process will be displayed as follows.

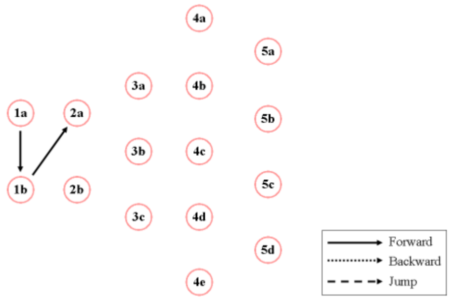
Figure 8
The Work of Students with LCTS



Plaintext	x_i	k_i	x_i+k_i	y_i	Ciphertext
D	3	1	4	4	E
O	14				
M	12				
P	15				
E	4				
T	19				
F	5				
Y	24				
B	1				
I	8				
R	17				
U	20				

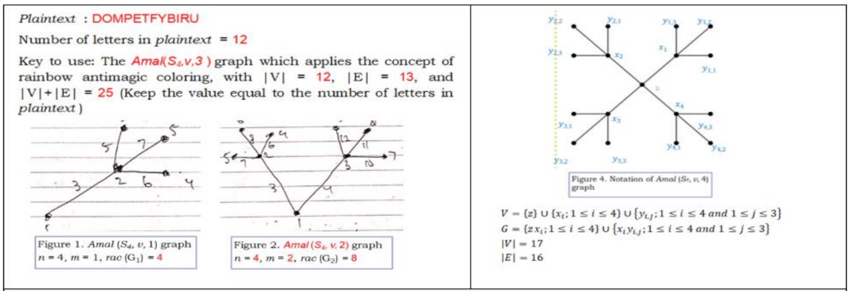
The first work is illustrated in Figure 8. The student could not solve the STEM problem and the rainbow antimagic colouring problem in the first work. In STEM problems, LCTS students can determine the plaintext and graph used as keys. Next, this student gave vertex labels and edge weights according to the concept of rainbow antimagic colouring. However, this student's vertex labeling provided is still repeated, even though there should be no repeated labels in antimagic labeling. This student also cannot determine the data encryption and decryption process results. While in the rainbow antimagic colouring problem, students cannot determine the vertex labels and edge weights according to the concept of RAC. The labeling of vertex given by students is still repeated, even though there should be no repeated labels in antimagic labeling.

Figure 9
The Phase Portrait Results of Students with LCTS



After conducting in-depth interviews, we can find out the students' combinatorial thinking processes in detail. To describe the combinatorial thinking process of students, we can use the help of a phase portrait to find out the students' thinking steps from beginning to end in solving the STEM problem and the rainbow antimagic colouring problem. Next, we connect each step using direct lines to form a phase portrait diagram. Figure 9 illustrates the phase portrait of a student with LCTS. As a result of the work of LCTS students, Figure 8 is shown, wherein steps 1a to 1b, LCTS students can only determine the plaintext, the graph used as the key, and can only determine $rac(G)$ in a certain order. Furthermore, the LCTS student proceeded to step 2a, but the LCTS student could not determine the generalization of the colouring pattern thus, this student stopped at this step.

Figure 10
The Work of Students with MCTS



Plaintext	x_i	k_i	x_i+k_i	y_i	Ciphertext
D	3	1	4	4	E
O	14	1	15	15	P
M	12	1	13	13	N
P	15	1	16	16	Q
E	4	1	5	5	F
T	19	1	20	20	U
F	5	1	6	6	G
Y	24	1	25	25	Z
B	1	1	2	2	C
I	8	1	9	9	J
R	17	1	18	18	S
U	20	1	21	21	V

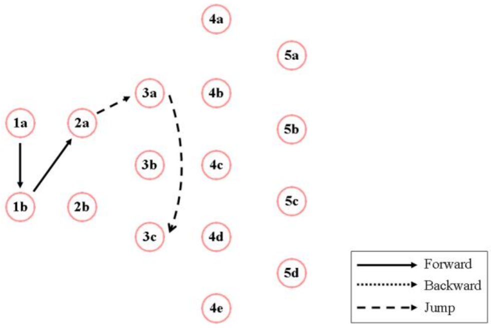
Ciphertext	y_i	k_i	y_i-k_i	x_i	Plaintext
E	4	1	3	3	D
P	15	1	14	14	O
N	13	1	12	12	M
Q	16	1	15	15	P
F	5	1	4	4	E
U	20	1	19	19	T
G	6	1	5	5	F
Z	25	1	24	24	Y
C	2	1	1	1	B
J	9	1	8	8	I
S	18	1	17	17	R
V	21	1	20	20	U

(i) The result of the encryption process : EPNQFUGZCJSV
(ii) The result of the decryption process : DOMPETFYBIRU

The second work is illustrated in Figure 10. In the second work, the student can solve the STEM problem but cannot solve the problem of rainbow antimagic colouring. In STEM problems, MCTS students can determine the plaintext and graph used as keys. Next, this student gave vertex labels and edge weights according to the concept of RAC. The number of colors provided by this student is minimum and correct. After that, the MCTS student calculates the encryption and decryption process results, but the result is incorrect. While in the problem of rainbow antimagic colouring, MCTS students can determine the vertex labels and edge weights according to the concept of RAC. The results obtained are minimal and correct, and students can also find the generalizations of the rainbow antimagic colouring pattern to the n^th order. MCTS students also can create the vertex functions and edge weight functions from a graph that has applied the concept of RAC. Still, this student cannot determine the generalization of the formula for vertex function and edge weight function of a graph that has applied the concept of RAC.

Figure 11

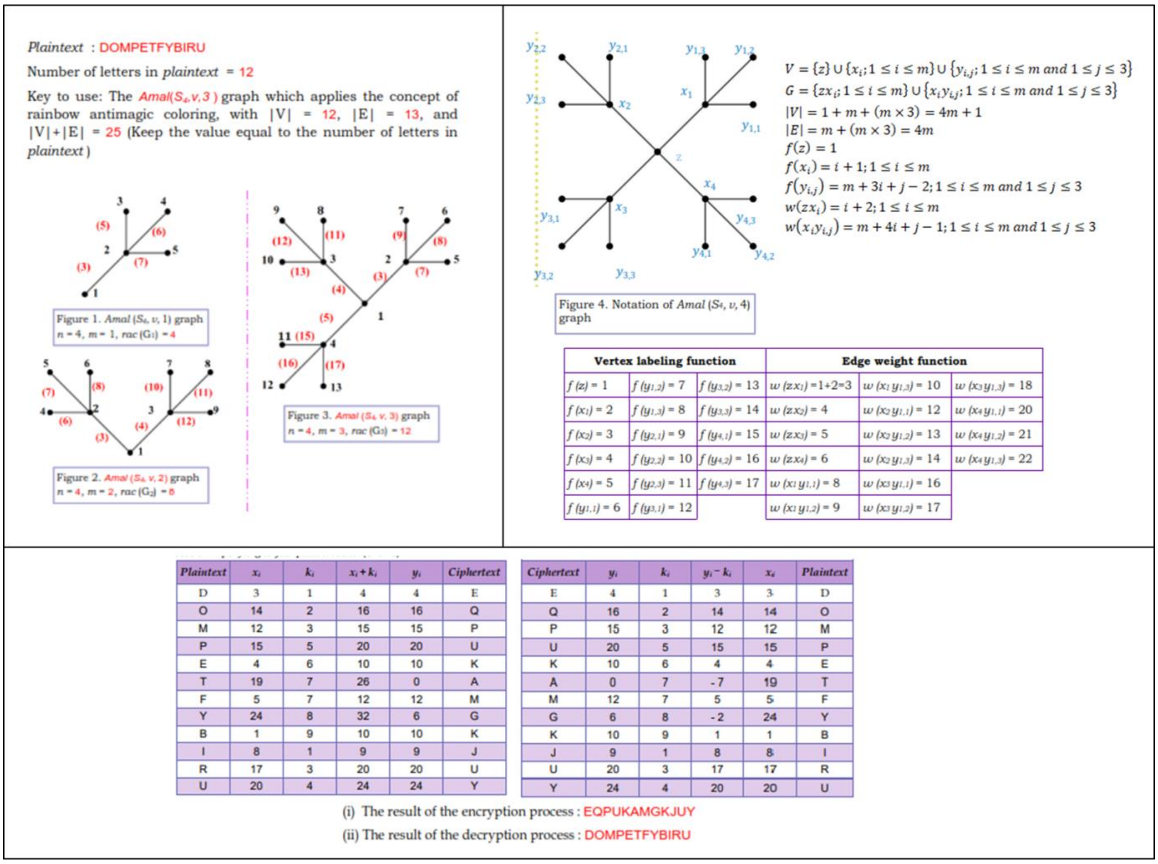
The Phase Portrait Results of Students with MCTS



After conducting in-depth interviews, we can find out the students' combinatorial thinking processes in detail. To describe the combinatorial thinking process of student, we can use the help of phase portrait to find out the students' thinking steps from beginning to end in solving the STEM problem and the rainbow antimagic colouring problem. Next, we connect each step using direct lines to form a phase portrait diagram. Figure 11 illustrates the phase portrait of a student with MCTS. As a result of the work of MCTS students, Figure 10 is shown, where in steps 1a to 1b, MCTS student can determine the plaintex, the graph used as the key, and apply the concept of RAC to the graph used as the key. However, when MCTS student work on the data encryption and decryption process, this student incorrectly enters the k_i value thus that the data encryption and decryption process results are wrong. Furthermore, the MCTS student goes to step 2a, where the MCTS student can find the colouring pattern on the graph with a certain order and jumps to step 3a to notate the graph, but the MCTS student doesn't know what to do next, thus this student stops at this step.

Figure 12

The Work of Students with HCTS



Plaintext : DOMPETFYBIRU

Number of letters in *plaintext* = 12

Key to use: The *Amal*(S_4,v,3) graph which applies the concept of rainbow antimagic coloring, with |V| = 12, |E| = 13, and |V|+|E| = 25 (Keep the value equal to the number of letters in *plaintext*)

$V = \{z\} \cup \{x_i; 1 \le i \le m\} \cup \{y_{i,j}; 1 \le i \le m \text{ and } 1 \le j \le 3\}$

$G = \{zx_i; 1 \le i \le m\} \cup \{x_i y_{i,j}; 1 \le i \le m \text{ and } 1 \le j \le 3\}$

$|V| = 1 + m + (m \times 3) = 4m + 1$

$|E| = m + (m \times 3) = 4m$

$f(z) = 1$

$f(x_i) = i + 1; 1 \le i \le m$

$f(y_{i,j}) = m + 3i + j - 2; 1 \le i \le m \text{ and } 1 \le j \le 3$

$w(zx_i) = i + 2; 1 \le i \le m$

$w(x_i y_{i,j}) = m + 4i + j - 1; 1 \le i \le m \text{ and } 1 \le j \le 3$

Vertex labeling function			Edge weight function		
$f(z) = 1$	$f(y_{1,2}) = 7$	$f(y_{3,2}) = 13$	$w(zx_1) = 1+2=3$	$w(x_1 y_{1,3}) = 10$	$w(x_3 y_{1,3}) = 18$
$f(x_1) = 2$	$f(y_{1,3}) = 8$	$f(y_{3,3}) = 14$	$w(zx_2) = 4$	$w(x_2 y_{1,1}) = 12$	$w(x_4 y_{1,1}) = 20$
$f(x_2) = 3$	$f(y_{2,1}) = 9$	$f(y_{4,1}) = 15$	$w(zx_3) = 5$	$w(x_2 y_{1,2}) = 13$	$w(x_4 y_{1,2}) = 21$
$f(x_3) = 4$	$f(y_{2,2}) = 10$	$f(y_{4,2}) = 16$	$w(zx_4) = 6$	$w(x_2 y_{1,3}) = 14$	$w(x_4 y_{1,3}) = 22$
$f(x_4) = 5$	$f(y_{2,3}) = 11$	$f(y_{4,3}) = 17$	$w(x_1 y_{1,1}) = 8$	$w(x_3 y_{1,1}) = 16$	
$f(y_{1,1}) = 6$	$f(y_{3,1}) = 12$		$w(x_1 y_{1,2}) = 9$	$w(x_3 y_{1,2}) = 17$	

Plaintext	x_i	k_i	$x_i + k_i$	y_i	*Ciphertext*
D	3	1	4	4	E
O	14	2	16	16	Q
M	12	3	15	15	P
P	15	5	20	20	U
E	4	6	10	10	K
T	19	7	26	0	A
F	5	7	12	12	M
Y	24	8	32	6	G
B	1	9	10	10	K
I	8	1	9	9	J
R	17	3	20	20	U
U	20	4	24	24	Y

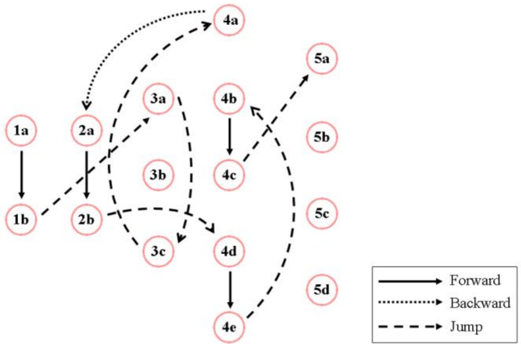
Ciphertext	y_i	k_i	$y_i - k_i$	x_i	*Plaintext*
E	4	1	3	3	D
Q	16	2	14	14	O
P	15	3	12	12	M
U	20	5	15	15	P
K	10	6	4	4	E
A	0	7	-7	19	T
M	12	7	5	5	F
G	6	8	-2	24	Y
K	10	9	1	1	B
J	9	1	8	8	I
U	20	3	17	17	R
Y	24	4	20	20	U

(i) The result of the encryption process : EQPUKAMGKJUY

(ii) The result of the decryption process : DOMPETFYBIRU

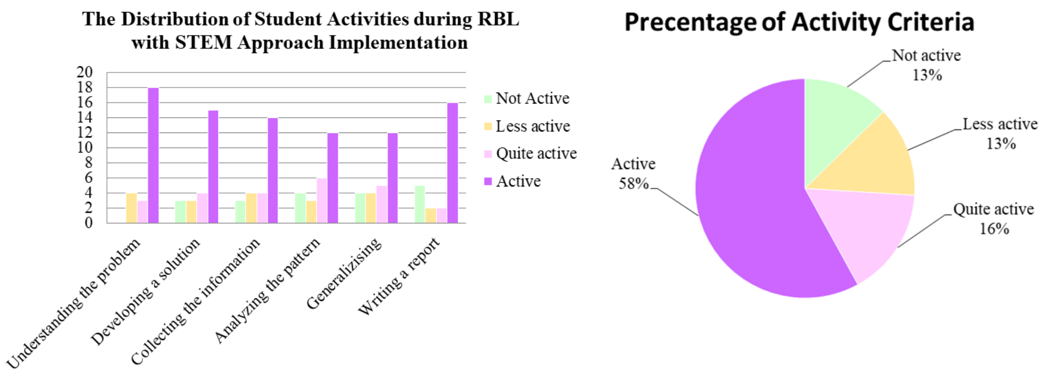
The third work is illustrated in Figure 12. The student can solve STEM problems and rainbow antimagic colouring problems in the third work. In STEM problems, HCTS students can determine the plaintext and graph used as keys. Next, this student gave vertex labels and edge weights according to the concept of RAC. The number of colors provided by this student is minimum and correct. After that, the HCTS student determines the data encryption and decryption process results and writes conclusions. While in the problem of rainbow antimagic colouring, HCTS students can determine the vertex labels and edge weights according to the concept of RAC. The results obtained are minimal and correct, and this student can also find the generalizations of the rainbow antimagic colouring pattern to the nth order. HCTS students can create the vertex labeling functions and edge weight functions and determine the generalizations of the formula for vertex functions and edge weight functions from a graph that has applied the concept of RAC. This student can also find the $rac(G)$ of a graph and determine the generalizations of the $rac(G)$ Formula. Finally, HCTS students can explain the definition of RAC and the flow/steps of searching for $rac(G)$ from a graph.

Figure 13
The Phase Portrait Results of Students with HCTS



After conducting in-depth interviews, we can find out the students' combinatorial thinking processes in detail. To describe the combinatorial thinking process of students, we can use the help of a phase portrait to find out the students' thinking steps from beginning to end in solving the STEM problem and the rainbow antimagic colouring problem. Next, we connect each step using direct lines to form a phase portrait diagram. Figure 13 illustrates the phase portrait of a student with HCTS. As a result of the work of HCTS students, Figure 12 is shown, wherein steps 1a to 1b, HCTS students can determine the plaintext, the graph used as the key, and apply the concept of rac to the graph used as the key. When this student works on the data encryption and decryption process, the HCTS student has correctly entered the k_i The value thus that the data encryption and decryption process results are correct. Furthermore, HCTS student jumps to step 3a, where the student provides notation on the graph and skips to steps 3c & 4a to determine the generalization of vertex set, edge set, and cardinality of the graph. Next, in steps 2a to 2b, this student looks for colouring patterns to nth order. After finding the generalization of the colouring pattern, in steps 4d to 4e, the HCTS student looked for $rac(G)$ until nth order, next, this student looks for vertex functions and edge weight functions and generalizations from these functions according to steps 4b to 4c. Finally, in step 5a, the HCTS student can explain the definition of RAC and how to find the $rac(G)$ of a graph.

Figure 14
The Observation Result of Student Activity in the Experimental Class



The observation of the student activity in the experimental class shows that there are 13% not actively engaged, 13% less actively engaged, 16% quite actively engaged, and 58% actively engaged during the learning process.

Discussion

The focus of this study is to examine how RBL can be applied with a STEM approach to improving students' skills in combinatorial thinking. We used RBL instruments with STEM approaches in the experimental class, while the control class used just RBL instruments. The STEM problem discussed in this study is the use of an e-commerce security system with the help of cryptography with algorithm affine cipher and the concept of RAC as the key to the encryption and decryption process thus that there are no errors in the number and type of goods ordered by the buyer. After solving the problem, students are expected to apply the concept of RAC to other graphs that have not been studied.

According to the independent sample t-test result for the pre-test, there was no significant difference between the two classes' results. There is a significant difference between the post-test results of the two classes where the (2-tailed) significance value is .020 (.020 ≤ .05). In analyzing the post-test results of 50 students from two classes, it appears that both the control class and the experimental class have improved in terms of their combinatorial thinking skills. As compared with the control class of 25 students, students with LCTS decreased to 28% (7 students), students with MCTS increased to 60% (15 students), and students with HCTS increased to 12% (3 students). Meanwhile, from the experimental class of 25 students, students with LCTS decreased to 20% (5 students), students with MCTS increased to 36% (9 students), and students with HCTS increased to 44% (11 students).

Compared to traditional learning, combinatorial thinking skills can be improved significantly under the implementation of RBL with the STEM approach. This learning encourages students to be more active and to try new things; thus, learning occurs in both directions and positively impacts them. It is in line with the research done by (Gita et al., 2021, p.5). According to the test results and students' activities, the RBL model with the STEM approach can improve students' combinatorial thinking skills. The results of this study have similar results to research conducted by Subekti et al. (2018, p. 6), where student learning outcomes in the classes that apply RBL-STEM approach are better than student learning outcomes in the classes which was not applied RBL-STEM approach. It implies that the implementation of RBL-STEM gives a positive contribution to the improvement of learning outcomes.

Conclusion

We can conclude that there are significant differences in students' combinatorial thinking skills between the experimental class and the control class. The students' combinational thinking skills can be improved by applying the RBL with STEM approach, particularly in the experimental class, which scored better than the control class. Based on the results obtained, we suggest that future researchers develop more detailed learning materials related to combinatorial thinking skills and apply RBL-STEM by assessing the different thinking skills.

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