

# Bayesian Shape Optimisation of Complex Structures under Stability Criteria Applied to Brake Systems

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## 論文内容要約

The main objective of this thesis is to work on new methodologies to design complex structures, in particular shape optimization. Within the context of aerospace engineering, the studied structure is a simplified brake system. Indeed, the dynamic behaviour of aircraft brakes is very important and difficult to understand. Aircraft braking systems are subjected to friction-induced vibrations, which leads to mechanical vibrations and unstable behaviours. The aim of the present PhD work is to develop shape optimization strategy to prevent unstable behaviours. The developed strategy is applied to a simplified brake systems to demonstrate the validity of the method, but are intended to extend aerospace engineers to design complex brake systems. It must be noticed that the simplified braking system considered in this PhD can reproduce the unstable phenomena of complex aircraft braking systems. Hence, the main objective of this thesis is to provide design methodologies for shape optimization to obtain optimal braking system designs for which unstable behaviours are avoided. The following section is thus devoted to the explanation of the academic and scientific context, based on the optimization methods used during this PhD.

In structural dynamics, the characteristics of a system can be largely described by its mass and stiffness properties which are in turn defined by its underlying material and geometric properties. Hence, to optimise a system from the perspective of structural dynamics requires optimisation of its material and geometric properties. Optimising geometric properties can be largely defined by shape optimisation or topology optimisation, where if a geometry is parameterised, parametric optimisation can also be achieved. The main objective of this thesis is to provide methodologies for shape optimization methods to obtain optimal braking system designs for which unstable behaviour is avoided. Hence, we expose an efficient strategy to deal with shape optimization of dynamical systems exhibiting flutter-type instability induced by friction, such as the considered disc-pad system. The stability of such systems can be analysed through Complex-Eigenvalue Analysis, through which we present a squeal noise criterion to be minimized as a computationally expensive black-box function. The computational domain is discretized through Isogeometric formulation for its advantages in optimization and superior approximation properties which are well studied in structural dynamics. Finite element method is principally based on approximation of solution in finite dimensional function space which is classically defined by Lagrange polynomials that parameterises the domain. This brings the idea of Isogeometric approach, where instead of replacing CAD description of a domain parameterised by splines with completely

different representation through Lagrange polynomials to define function space for approximation, the basis functions of splines can be directly defined for approximation given that sufficient refinement is made over initial CAD parameterisation to curtail approximation error, where refinement over initial CAD parameterisation to define analysis-suitable parameterisation can be easily achieved in the same parametric space of CAD description. This completely preserves the geometry. The definition of function space with splines is mainly possible because the underlying polynomials of splines are well-suited to define function space as subset of Sobolev space for energy functionals. Further, the advantages of splines over Lagrange polynomials in CAD are also reflected in approximation properties of solution, such as reduced oscillation in interpolation and providing higher solution continuity between knots (Knots can be interpreted as equivalent to elements of classical FEM). This also means that optimisation can be defined directly over the control points of analysis-suitable parameterisation in relation to solution at the control points, where with higher-order continuity, it can avoid non-smooth shape definition of boundary that classical finite element methods with position continuity suffer from.

To be computationally efficient with the expensive black-box function, we defined the optimization based on Efficient Global Optimization scheme in the context of multi-objective optimization, with the integration of Isogeometric design-through-analysis methodology. As gradient information is hard to access for such black-box functions, in addition to the presence of constraints, we relied on meta-heuristic approach as a more generic strategy for realizing optimization of such functions in multi-objective context.

The idea of gradient-descent in multi-objective optimisation can be even more limiting for the definition of Pareto-optimal solutions, where it demands a multi-objective optimisation problem to be posed as multiple single-objective optimisation problems which can be optimised with gradient descent. The expression of multi-objective optimisation problem as set of single-objective optimisation problems loses the explicit definition of optimality in multi-objective context, where Pareto-optimal solutions are implicitly achieved with single-objective optimisations. This implicit nature of optimising for Pareto-optimal solutions typically demands prior idea of the Pareto-front to select parameters for expressing a multi-objective optimisation problem as single-objective optimisation problems, especially for diversity of Pareto-optimal solutions. The single-objective optimisation problems can also be solved by evolutionary algorithms which can help for global convergence, but nevertheless, it still preserves the limitations owing to the implicit nature of optimising for Pareto-optimal solutions. But instead of using evolutionary algorithms in solving for Pareto-optimal solutions implicitly with multiple single objective optimisations, evolutionary algorithms can be defined to explicitly solve for Pareto-optimal solutions. This is mainly because the population based nature of evolutionary algorithms can handle set of solutions, where the fitness of individuals can be selected to favour Pareto optimality in multi-objective context. We consider NSGA-2 in this thesis as the evolutionary algorithm for multi-objective optimisation. The major limitation with evolutionary algorithms is the stochastic nature of genetic operators to advance solutions towards optimum at least in the context of single objective optimisation, rather than a more mathematically sounding

approach of gradient-descent.

The stochastic nature of genetic operators typically crave more iterations in evolutionary algorithms to converge, which can be unrealistic with expensive functions, for which the idea of Bayesian optimisation can be helpful. Bayesian optimisation is essentially based on Bayes probability of prior and posterior probabilistic inference, where the idea is that a function can be approximated with some prior knowledge and given the observed data on the function, posterior knowledge can be inferred over the function with Bayes theorem. With the inference of posterior knowledge, a new infill point can be sampled to move towards optimum. The new evaluation is then used to update the prior belief to infer a new posterior, where eventually with several iterations, the idea is to find global optimum. As one such scheme with its own advantages was observed to provide lack of resolution to define Expected Improvement (EI) with a single reference value, we propose a multi-reference acquisition strategy which can be defined through a fast and efficient algorithm with fewer adaptation to the existing scheme. Results show the efficiency of this approach for our applicative example, which can be extended to other such applications as well.

Following introduction, in Chapter 2 we focus on sensitivity analysis and optimisation of classical shapes with classical finite element method. Subjectively, we define classical shapes to be the realisation of shapes with simple geometric parameters such as radius, thickness and angle, where the design space is largely restricted. The parameterisation through simple geometric parameters enables use of more robust structured mesh with classical finite element method which could be otherwise for arbitrary shape definition where structured mesh strategy can not be defined to be generic. The definition of Node-to-Node contact formulation provides further constraints in meshing which only increases the problem. This problem could be avoided with contact formulation which does not require the definition of conforming mesh at the interface. This is the motivation of Chapter 3 and 4, where we define optimisation scheme with contact formulation which can handle non-conforming mesh at the interface with focus on Isogeometric approach for discretisation in Chapter 3, followed by more detailed explanation of achieving finite element numerical approximation of the contact and friction integrals defined through weak form in Chapter 4. In Chapter 2 and 3, we also introduce a black-box function to optimise for squeal noise arising as flutter-type dynamic instability, where we provide a parallelisation strategy through Craig-Bampton method of model reduction. This helps to reduce the computation time for the evaluation of the expensive black-box function.

Contrary to optimisation of classical shapes in Chapter 2, in Chapter 3, we focus on achieving unconventional shapes with optimisation. Shape optimisation defines functional optimisation where the design space to explore is considerably large. But nevertheless, shapes can be parameterized with control points of NURBS such that the parameterisation can represent sufficient variation of shapes to be defined for optimisation. This provides direct extension for Isogeometric approach to use the approximation bases of NURBS as the function space for approximation of solution in the context of finite element methods, which means that no classical meshing step is required. This is advantageous because robust meshing for any possible shape definition in optimisation is idealistic. Though Isogeometric approach circumnavigates the meshing problem, it brings the problem

of parameterising the geometry suitable for analysis at least in the context of body-fitted parameterisation, where it requires complete injective mapping of the physical domain from reference domain. For this thesis, we mainly use linear method of parameterisation to focus on frame work where more advanced parameterisation can be developed in future to consider large design space. Considering the optimisation of black-box function and large design space, we used the idea of Bayesian optimisation to be more computationally efficient, where the idea is to approximate the function with a meta-model through which prediction can be inferred with its uncertainty. We focus on multi-objective optimisation where we optimise for a set of solutions called Pareto-optimal solutions through direct extension of concepts from single-objective Bayesian optimisation. This brings the idea of multi-objective Bayesian optimisation, where the goal of Chapter 3 is also to make the optimisation strategy to be generic which was achieved through Evolutionary algorithms. The intrinsic nature of Evolutionary algorithms which are population-based makes it suitable to consider set of solutions to obtain Pareto-optimal solutions.

In Chapter 4, we define detailed explanation of contact and friction formulation for the application of modelling friction induced flutter-type instability with focus on Isogeometric approach. The idea is to define weak form of contact and friction for approximation in the context of finite element methods, where we use the NURBS bases functions for its intrinsic advantages provided on continuity between knots which are equivalent of element boundaries in classical finite element context. We show the comparison between solving for weak form and collocation approach where the strong form of the equation is satisfied at discrete points. The most difficult problem is with solving the integrals of weak form for contact is the independent discretisation of two domains, where the contact and friction integral involves the presence of terms from two domains. In this context, no integration scheme can be defined at the already defined discretisation unless the contact interface has conforming mesh. The common alternative is to define integral with defining new elements at the interface based on common span from the discretisation of domains in contact. This method can be computationally expensive and also difficult to implement, where in some cases, it has also been shown to be less robust. As an alternative to avoid definition of integration scheme specific for the contact interface, we define the projection of numerical integration defined on one of the domains to the other, where in this case higher solution continuity across the knots from NURBS leads to more smooth interpolation between non-conforming mesh at the interface. This is shown by less pressure oscillation at the interface with the weak form compared to the collocation scheme. Finally, we provide conclusion as Chapter 5.