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# Air Force Specialty Code Assignment Optimization

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THESIS

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**DEPARTMENT OF THE AIR FORCE**
**AIR UNIVERSITY**

***AIR FORCE INSTITUTE OF TECHNOLOGY***

**Wright-Patterson Air Force Base, Ohio**

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THESIS

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## Abstract

Each year, the Air Force Personnel Center determines the career fields in which newly commissioned officers will serve during their time in the Air Force. The career fields are assigned while considering five priorities dictated by Headquarters Air Force, Manpower and Personnel: target number of cadets, education requirements, average cadet percentile, cadet source of commissioning, and cadet preference. A mixed-integer linear program with elasticized constraints is developed to generate cadet assignments according to these priorities. Each elasticized constraint carries an associated reward and penalty, which is used to dictate the importance of the constraint within the model. A subsequent analysis is conducted on historical data to display the interaction of the constraints and the impact of the rewards and penalties on the model results. The new formulation can generate a feasible set of assignments using the elasticized constraints in instances where the cadet and career field data would cause infeasibility in the original assignments model. Moreover, such a solution can be identified within minutes, utilizing a leading commercial solver. It also provides users and decision makers with the ability to identify trade-offs between goals and prioritize each constraint.

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Rebecca L. Reynolds

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AIR FORCE SPECIALTY CODE ASSIGNMENT OPTIMIZATION

# I. Introduction

The Air Force Personnel Center (AFPC) annually decides the career field assignments of soon-to-commission cadets from both the United States Air Force Academy (USAFA) and the Air Force Reserve Officer Training Corps (ROTC) programs. The matches are assigned while considering the desired target number of cadets for each career field, the balance of cadet merit and source of commissioning across large career fields, degree requirements for each career field, and cadet preference.

Cadets at USAFA and in Air Force ROTC programs complete educational requirements for a bachelor's degree concurrently with commissioning requirements. When cadets reach their second-to-last year in their programs, they submit a list of preferences for career fields to be assigned to upon commissioning. These preferences, along with factors such as cadet merit and degree, are considered by AFPC in assigning cadets to career fields.

The career field assignment each cadet receives is designated by an Air Force Specialty Code (AFSC). These alphanumeric codes have six components – a numeric character to indicate career group, a numeric character to indicate career field, an alpha character to indicate functional area, a numeric character to indicate qualification level, an alpha prefix to indicate a skill, special qualification, or system designator not restricted to any single AFSC, and an alpha suffix to indicate positions associated with specific equipment or functions within a single specialty [1]. For example, the AFSC 11B3A indicates Operations, Pilot, Bomber Pilot, qualified, B-1. The first three components - career group, career field, and functional area - are most often referenced. Thus, the AFSC for a bomber pilot would commonly be referenced as 11B.

Cadets reference the Air Force Officer Classification Directory (AFOCD) to learn about each AFSC and determine their preferences. This document provides a summary of each AFSC, also detailing the duties, responsibilities, and specialty qualifications associated with each career field. The document includes a table for each AFSC indicating which undergraduate degrees are mandatory, desirable, or permitted for entering the career field and associated target percentages for each degree tier [2]. Mandatory degrees are most preferred, then desired, and then permitted. Not all AFSCs have all three tiers of degree requirements. Some AFSCs permit any degree, and some AFSCs only permit specific degrees. The degree requirements for each AFSC are determined by career field managers and described in the AFOCD. Cadets have the option to apply for rated (flying) or non-rated (non-flying) career fields. Cadets who wish to serve in a rated career field must apply to do so and are selected via the Undergraduate Flying Training Selection Board [1]. Cadets who wish to apply to non-rated career fields, and those who are not selected for rated positions, submit their job preferences to AFPC, which matches the cadets to assignments. The focus of this research is the assignment of cadets to non-rated career fields.

AFPC is tasked with assigning the non-rated cadets to career fields. They must balance four priorities in the assignments: meeting education requirements, balancing the distribution of cadet quality within AFSCs, balancing the source of commissioning within AFSCs, and accounting for cadet preference. AFPC uses the cadet's degree, class rank, commissioning source, and AFSC preferences to determine their assignment. These priorities are dictated by Headquarters Air Force, Manpower and Personnel (HAF/A1). The degree to which they are prioritized has an effect on the resulting assignments of cadets to AFSCs. The requirements for assignments vary from year to year. Priorities may shift to reflect these changes, or new priorities may be considered. Thus, the method for assignments must be flexible and easily adapted to new requirements.

AFPC currently assigns cadets to non-rated AFSCs using an integer program. An integer program is an optimization model in which the variables are discrete [3]. For AFPC's cadet-career field matching problem, all discrete variables are restricted to having a value of 0 or 1, so the model is a binary integer program [4]. The integer program maximizes the utility of cadet-AFSC assignments while being constrained by education and manning requirements for each career field. The objective of this research is to improve the matching process for cadet-career field assignments. The current process presents difficulty in finding a feasible solution that is also desirable, which often leads to manual adjustments after a solution is found.

A simplified version of the original integer program, which is presented in Appendix A, includes constraints adapted from the priorities from HAF/A1 and constraints from AFOCD requirements. The constraints are:

- Each cadet can only be assigned to one job.
- The number of cadets assigned to a career field must equal, or be greater than, the target for that career field.
- Each career field may only be overclassified by a set amount.
- The number of cadets with mandatory degrees must meet or exceed the requirement for each AFSC.
- There must be a balance of USAFA and ROTC cadets across large career fields.
- There must be a balance of cadet merit in terms of percentile across large career fields.

The objective of AFPC's integer program is to maximize the utility of assigning cadets to career fields. The utility function, also presented in Appendix A, assigns a score to each cadet-AFSC match. Degree requirements are listed in the AFOCD and are used to determine the educational eligibility of each cadet for each AFSC. The objective function assigns higher scores to cadets who prefer AFSCs and meet their degree requirements. Lower scores are given to matches where a cadet does not prefer an AFSC, even if they meet degree requirements. If a cadet does not meet the degree requirements for an AFSC, the match is given a large negative score to prevent the integer program from making the match. Matching cadets to AFSCs for which they are ineligible is avoided.

While in most years the integer program provides a desirable and mathematically optimal set of matches, some years it results in an infeasible solution when constraints cannot be met. For example, there may be cases where the mandatory education constraint cannot be met because there are not enough cadets with mandatory degrees for the chemistry, physics, or engineering AFSCs. Additionally, the integer program may achieve a mathematically optimal solution which is not considered "desirable" to AFPC or to the specific career field managers. Some AFSCs are not often selected as preferences. When matches are made, assigning cadets to AFSCs they do not prefer results in a lower individual utility score than desired. This lowers the overall utility score achieved and results in an undesirable solution.

The objective of this research is to develop a new mathematical programming formulation which assigns cadets to AFSCs while balancing the priorities from AFPC. Elastic variables are included in the new formulation to address the concerns of the original model. One set of these variables allows for the matches to violate selected constraints but penalizes the objective function value for deviations. The other rewards the objective function value when constraints are exceeded and incentivizes being further above or below a constraint's limit, depending on the type of constraint. The use of the elastic variables ensures feasibility of the cadet-AFSC problem even when constraints cannot be met and increases the potential for a more "desirable" solution to the integer program. They allow flexibility in the constraints and a tool for prioritization of model objectives to achieve a desirable solution for AFPC. A sensitivity analysis is conducted on the penalties and rewards of the variables to determine the correct weights for achieving the stated goals.

The remainder of this thesis is structured as follows. Chapter II provides a literature review which further introduces the career field matching problem, along with the assignment problem and associated solution methods. Chapter III details the methodology for the construction of a new formulation, and highlights further analysis conducted with the new formulation. Chapter IV presents the results of the analysis of the relative efficacy of the new and old formulations. Chapter V provides a conclusion for the thesis and highlights the research impact and potential future research.

# II. Literature Review

The task of assigning cadets to their future career fields is a large one, and it is important to develop a solution method that adequately represents the goals of AFPC and HAF/A1 while considering cadet preferences. As such, it is crucial to understand the problem at its core and to identify a variety of solution methods which can be implemented to determine the best possible assignments.

The cadet-AFSC problem is an assignment problem, more specifically, a matching problem. The goal of the problem is to match cadets to AFSCs while adhering to the requirements of the career field managers and the preferences of the cadets.

## 2.1 The Assignment and Matching Problems

The generalized assignment problem is defined as follows [5]. Suppose the set $I$ indicates a set of agent indices, and the set $J$ indicates a set of task indices. Let $c_{ij}$ be the cost incurred when assigning agent $i \in I$ to task $j \in J$. Let $r_{ij}$ be the required resources for agent $i$ to complete task $j$, and let $b_i$ be the amount of resource available for agent $i$. If agent $i$ is assigned to task $j$, the decision variable $X_{ij}$ is assigned a value of 1, and a value of 0 otherwise. The problem is then formulated as:

$$\min \quad \sum_{i \in I} \sum_{j \in J} c_{ij} X_{ij} \tag{1a}$$

$$\text{s.t.:} \quad \sum_{j \in J} r_{ij} X_{ij} \leq b_i, \ \forall i \in I \tag{1b}$$

$$\sum_{i \in I} X_{ij} = 1, \ \forall j \in J \tag{1c}$$

$$0 \leq X_{ij} \leq 1, \ \forall i \in I, j \in J \tag{1d}$$

The objective of the model is to minimize the cost incurred in assigning the agent-task pairs. Constraint (1b) ensures that the task each agent is assigned to does not require more resources than they have available. Constraint (1c) ensures each agent is only assigned to one task and constraint (1d) defines the binary domain of the decision variable, $X_{ij}$.

In practice, the generalized assignment problem has been adapted to account for dynamic formulations, as well as stochasticity and constraint violations [6], and it is applicable in a variety of optimization scenarios. The matching problem is an assignment problem which involves "assigning a set of agents to another set of agents; based on the preferences of the agents, and some problem-specific constraints" [7], [8]. Research has been conducted regarding assignment problems and matching problems in a number of environments, including education, medicine, and the military. The next three sections provide background on the assignment problem through examples. The remaining sections explore common formulation and solution methods for this type of problem, and discuss their applicability to the cadet-AFSC problem.

### 2.1.1 Education Examples

Gale and Shapley [9] introduce the matching problem with a study of the college admissions process and the stable marriage algorithm. The authors apply the stable marriage algorithm to a set of applicants and colleges. The procedure involves students applying to their first choice college, and then the college submitting their preferences of the applicants. If a student is rejected, they apply to their next choice, and so on. The authors state that the process results in stable matches of students with colleges. The stable marriage algorithm is detailed in Section 2.2.3.

Krauss, Lee, and Newman [10] discuss the assignment of students to classes while balancing recommendations and requests of teachers and parents. To complete the assignments, a binary integer program is developed. This integer program assigns students to four classes, while constraining social and educational requirements for each student. The integer program designed by the authors does not include an objective function, and instead focuses on meeting the constraints of the model. After a feasible solution is found by the integer program, the authors use a genetic algorithm, detailed in Section 2.2.4, to improve upon the solution. The authors' genetic algorithm uses the feasible solution found by the integer program as a starting solution, and the algorithm generates candidate solutions which are scored by how well they meet the desires of parents and teachers. The candidate solutions lose points if constraints met by the integer program are violated. From these candidate solutions, a "best" solution was implemented for the school.

Srinivas et al. [11] discuss the assignment of students from the Holy Family Academy to teams and work-study positions. The authors use a goal programming model to minimize deviations between decision maker goals and achieved results. Goal programming models are explained in more detail in Section 2.2.5. The first model developed is used to match students to teams. The model includes hard constraints that may not be violated, such as assigning each student to exactly one work-study team without violating the work assignment days, and soft constraints that align to goals from which the solution may be permitted to deviate. The objective of the model is to minimize the sum of weighted goal deviations. These goals include balancing students' team membership based on race, gender, and rating; minimizing team assignments that may cause conflict; and maximizing team assignment requests. The second model is used to assign students in teams to work-study positions. The second model has the same objective as the first. The goals for the second model include balancing student potential across funding and non-funding organizations, balancing race and gender across organizations with three or more available jobs, and maximizing student and organization job requests.

### 2.1.2 Medical Examples

The Hospitals/Residents (HR) problem is an example of the matching problem which focuses on the assignment of junior doctors to hospitals for their residency. This problem, like the college admissions problem by Gale and Shapley, seeks a stable pairing of junior doctors to hospitals. The problem was originally solved using a linear programming formulation, as described by Gusfield and Irving [12] and Vande Vate [13]. Integer programming formulations have also been created for the HR problem [14] and the Hospitals/Residents with ties (HRT) problem [15]. The HRT problem is an extension of the HR problem in which it is possible for a junior doctor to prefer two or more hospitals equally. Constraint programming formulations, which are presented in more detail in Section 2.2.1, have been developed for both the HR problem [16] and the HRT problem [17]. Additional extensions of the HR problem exist, but for the purposes of this research, only some are presented.

The Kidney Exchange Problem focuses on instances where a patient needing a kidney transplant and their volunteer donor are medically incompatible. In this problem, the objective is to take two mismatched pairs, in which the recipient of each pair matches the donor of the other pair, and match them for a successful donation. Carvalho et. al [18] explore three solution approaches for matching pairs of patients and donors while limiting the emotional and material costs of swapping the donors. The base model focuses on a directed graph of nodes and arcs, with arcs connecting incompatible pairs. The objective function is to maximize the benefit of selecting a chain, where each node, or incompatible pair, can only be involved in at most one exchange. The base model is first adapted to consider costs of planned transplants that do not occur, and then adapted to allow for the use of alternative pairs if a match were to fail. The authors present a network model for the assignments, which is detailed in Section 2.2.2.

### 2.1.3 Military Examples

The assignment problem has been presented in all branches of the United States military for service member assignments. These include both job assignments, like the cadet-AFSC problem, and duty location assignments. While the basis of the problem is similar across branches, the employed solution methods vary.

Shrimpton and Newman [19] discuss the assignment of United States Army officers to career fields using a network-optimization model. The objective of the model is to minimize the sum of utility scores while meeting flow balance constraints. These constraints ensure that the number of officers designated to each career field meets the required amount. Sets of nodes are used to designate officers and career fields. An arc is present between an officer and career field node if the officer has listed the career field as a preference. Each arc is weighted by a utility score, which takes both officer preference and qualification for the job into consideration. The decision variable on whether to use each arc is given value 1 if the match is made, and 0 otherwise.

Garrett et al. [20] discuss the United States Navy's sailor assignment problem. The objective of the problem is to maximize the "fitness" of sailor-job assignments, while ensuring necessary constraints are met. Previously, the Navy used the stable marriage algorithm to match sailors to jobs. In this case, the two sets are sailors applying for jobs and the commanders in charge of the jobs. The match occurs when a sailor has applied to a job for which they are qualified and interested in, and a commander selects that sailor as a preferred candidate. The stable marriage algorithm cannot take into account Permanent Change of Station (PCS) costs associated with the matches, so a genetic algorithm is considered as a solution method. The algorithm considers constraints that ensure sailors are not assigned to a job for which they are not qualified. The goal is to produce matches which meet the desires of sailors and commanders, while minimizing the costs of said matches.

Seipel [21] explores methods for matching United States Marine Corps sergeants and newly commissioned officers to one of seventeen regions for the Regional, Culture, and Language Familiarization Program. This program aims to educate Marines in regards to the culture and language of the different regions. The first two methods relate to those of this research. The first model used for matching is a simple integer program. The objective of the model is to assign Marines based on their region preference. This is done by minimizing the sum of the preference ranking across the applicants, as a lower number indicates a higher preference. The model is constrained by the number of Marines required in each region and that each Marine may only be assigned to one region. The second model is a continuation of the first. In this model, the language knowledge of each Marine is taken into account. This is done by modeling the region's preference for a Marine using binary variables: if a Marine is preferred, the binary variable receives a value of 1. If a Marine speaks the language of the specified region, the match is more likely to occur. This model determines whether the Marine and the region are first choices for each other, i.e., if the Marine speaks the specified language, and adjusts the objective function accordingly.

Hooper and Ostrin [22] examine the assignment PCS orders to United States Marines. They use a binary integer program to assign the matches. The decision variable indicates if a Marine will be assigned to a station. The objective of the model is to minimize the total cost to send Marines to assignments. The key constraint is that each billet must be assigned to a Marine. The authors conduct analysis to determine the total cost of matching 15 Marines to assignments using the integer program. This integer program takes the preference of each Marine into account by creating a weighted parameter of duty preference using time in grade. The authors discuss how the preference of each Marine can be tightened or loosened by the constraints in the model, resulting in higher or lower costs, respectively. The authors note that prioritizing Marine preference could potentially lead to higher PCS costs. Preference is an important factor to consider, but in this case, aligning to preferences may be detrimental to the objective function value. This opposes the objective function for the AFPC's formulation of the cadet-AFSC problem, in which aligning to cadet preference may have a positive impact on the objective function value.

Kleeman and Lamont [23] discuss the airman assignment problem. The assignment problem uses binary integer programming. The authors discuss the use of hard and soft constraints within a constrained assignment problem. Hard constraints are those which cannot be violated; soft constraints may be violated, but they incur an associated penalty. The authors explore this idea in the airman assignment problem, which determines personnel assignments to duty stations. The problem has the following three objectives: ensuring that Air Force needs are met, considering individuals' preferences, and minimizing the cost associated with the new location assignments. The objective function considers how well the individual Airman meets the requirements for the assignment, as well as their preferences for the assignment.

Armacost and Lowe [24] explore the process of assigning USAFA cadets to career fields. Their methodology focuses on developing an optimization model that maximizes the value of cadet assignments, while meeting career field targets for the number of cadets assigned. They address deviations from these targets using slack variables which penalize the objective function value by a certain cost when the number of assignments are over or under a target. The model contains binary variables that account for cadet AFSC assignments. The deviation costs are integer-valued. The model can be mapped directly to a network flow problem. The authors' use of slack variables motivated the use of elastic variables to allow for constraint deviations in the cadet-AFSC problem in this research, as described in Chapter III.

## 2.2 Modeling and Solution Methods

### 2.2.1 Constraint Programming

Constraint programming is a solution method for problems in which the objective is to satisfy all constraints. In this context, a constraint "can be viewed as a requirement that states which combinations of values from the variable domains are permitted" [25]. The problem to be solved is referred to as a constraint satisfaction problem (CSP), which is solved using constraint solvers. Ultimately, the goal of a CSP is to find a valid set of values given the constraints on the domains of these values and their relations. If a problem is determined to be over-constrained and a solution cannot be found, it may be necessary to use soft constraints. In [26], the authors describe "weighted constraints," which reflect the idea of the constraints and elastic variables described in Chapter I, as a kind of soft constraint. In a constraint problem with weighted constraints, an optimal solution is one in which all variable assignments are made at a minimal cost.

Apt states that the problems that are best suited for constraint programming are "usually those that can be naturally formulated in terms of requirements, general properties, or laws, and for which domain specific methods lead to overly complex formalizations" [25]. Constraint programming is used in certain optimization problems; however, for the cadet-AFSC matching problem, the generation and solving of the CSP may be difficult and time-consuming when other methods may be applied that have been shown to perform well in assignment problems.

### 2.2.2 Network Optimization Model

The assignment problem can be modeled as a minimum-cost network flow problem (MCNFP), which is a specific network optimization model [27]. A MCNFP begins with a set of nodes and connecting arcs. The objective of the MCNFP is to minimize the cost of sending units of flow between nodes, while respecting the lower and upper bounds on flow within arcs. Network problems are totally unimodular, and thus feasible integer solutions are obtained without enforcing variable integrality [28]. This means the problem may be solved with its linear relaxation, and integer programming solution methods need not be employed.

While a network model may be suitable for assignment problems, the constraints of the cadet-AFSC assignment problem suggest it may be difficult to model the subsets of cadets and AFSCs that may match. The constraint matrix of the cadet-AFSC assignment problem is not totally unimodular, and thus solving via its linear relaxation does not guarantee an integer solution.

### 2.2.3 The Stable Marriage Algorithm

Gale and Shapley [9] provide an example of stable assignments within a marriage problem. The basis of the marriage problem is to find a way of marrying all members of a group of $n$ men and $n$ women. It is desired to find a stable set of marriages in which no pair who are not married prefer each other to their actual partners.

While a useful method for assigning matches, the stable marriage algorithm would not easily consider the constraints involved in the cadet-AFSC assignment problem. The education requirements and desired source of commissioning balance for each AFSC would be difficult to account for within the stable marriage algorithm, as each AFSC has unique requirements. This added intricacy suggests the stable marriage algorithm may not be the correct route for the cadet-AFSC assignment problem.

### 2.2.4 Genetic Algorithm

Genetic algorithms are used in optimization for modeling and solving a problem "using processes that mimic the process of natural evolution" [10]. Yang [29] explains that the genetic algorithm consists of five steps: (1) encoding of the objective function; (2) defining a selection criterion for individuals; (3) providing or creating individuals to be evaluated; (4) completing the evolution cycle by evaluating individuals, creating a new population by crossover and mutation of selected individuals, replacing the old population and iterating again; and (5) determining the solution to the problem by the results of the iterations. Yang also describes the advantages and disadvantages of genetic algorithms. Benefits of using genetic algorithms include their ability to handle complex problems and the ability of the algorithm to parallelize; or computing multiple subproblems at the same time. A disadvantage to using genetic algorithms is that they are sensitive to parameter values and the objective function, so the algorithm may struggle to converge or provide meaningful results.

A potential use of genetic algorithms in the cadet-AFSC problem could reflect that of Krauss, Lee and Newman [10]. The problem could be solved without an objective function, and then the results could be inputs for a genetic algorithm to evaluate and generate new solutions. A concern of using genetic algorithms is that the model is complex and the algorithm may falter with any changes in the model from year to year.

### 2.2.5 Goal Programming

Salunkhe et al. [30] discuss the need for goal programming in multi-objective decision making processes, commenting that often, multi-criterion decision making processes are not sufficient without mathematical programming to help optimize solutions. Tamiz and Azmi [31] support the use of goal programming in multi-objective decision making because of its flexibility in constraints and focus on satisficing as opposed to optimization of objectives. Goal programming provides the best solution possible, given goals and their order of importance, and it is commonly used in situations where it may not be possible to achieve all desired goals. In cases where the user is able to rank objectives in order of priority prior to solving the problem, preemptive goal programming is used. Non-preemptive goal programming is used in cases where the importance of each goal can vary, and is relayed through its assigned weight [32].

Rerkjirattikal et al. [33] apply goal programming to the Nurse Scheduling Problem (NSP) with individual preference satisfaction. The authors' model develops a proposed nurse schedule while considering work restrictions and constraints. There are three goals the model is designed to achieve: balance the number of shifts assigned to nurses, account for nurse preference in shift, and account for nurse preference in days off. The objective of the goal programming formulation is to minimize deviations from the targets of each of the three goals. The deviations are normalized within the objective function and thus considered equally.

Tamiz and Azmi [31] discuss the use of goal programming in stock portfolio selection. The authors focus on achieving goals for seven stock-related factors, including risk, return, debt, earnings per share, price, operating cash flow ratio, and dividend yield. Target and penalization values are set for each of the factors, and the focus of the goal programming model is to minimize the normalized, weighted deviations from each of the target values. This example differs from the NSP because the deviations are weighted, whereas in the NSP example, the goals were viewed as equal.

## 2.3 Summary

The chosen solution method for the cadet-AFSC problem is rooted in goal programming and considers the need for flexibility in the requirements of the problem.

The elastic variables included in the new formulation carry associated weights in the form of penalties and rewards in the objective function. By adjusting these weights, different solutions may be achieved that focus on meeting different constraints. Through non-preemptive goal programming, analysis may be performed to determine a best solution by adjusting goal importance. The resulting formulation, detailed in Chapter III, is a mixed-integer linear programming problem.

# III. Methodology

For this research, AFPC provided their original integer programming formulation, found in Appendix A, and their pre-processing and integer program code. AFPC's pre-processing code is replicated in Python 3 [34], and code is developed for both a reformulation of AFPC's model and a new model using the Pyomo optimization package [35], [36]. Python is the chosen programming language because it is open source and there are many resources available for support and troubleshooting. This allows the code to be easily shared and updated as new personnel are tasked with making the assignments for AFPC.

## 3.1 Original Model Reformulation

The original integer program and the code provided by AFPC are used to reformulate the model. The reformulation, denoted model ($\mathcal{N}$), contains updated sets, indices and parameters for improved readability and performance. This formulation adopts the notation described in [37], such as the use of subscripts to denote indices and superscripts to designate differences in similar variables. The reformulation is provided below.

**Sets and Indices**

| | |
|---|---|
| $a \in \mathcal{A}$ | AFSCs, $1, ..., A$ |
| $c \in \mathcal{C}$ | Cadets, $1, ..., C$ |

**Subsets**

| | |
|---|---|
| $\mathcal{A}^{\mathrm{F}} \subseteq \mathcal{A}$ | AFSCs with overclassification limits |
| $\mathcal{A}^{\mathrm{M}} \subseteq \mathcal{A}$ | AFSCs with mandatory education requirements |
| $\mathcal{A}^{\mathrm{D}} \subseteq \mathcal{A}$ | AFSCs with desired education requirements |
| $\mathcal{A}^{\mathrm{P}} \subseteq \mathcal{A}$ | AFSCs with permitted education requirements |

| | |
|---|---|
| $\mathcal{A}^{\mathrm{I}} \subseteq \mathcal{A}$ | AFSCs for which a cadet's degree may make them ineligible |
| $\mathcal{A}^{\underline{\mathrm{U}}} \subseteq \mathcal{A}$ | AFSCs with lower bound for USAFA cadets |
| $\mathcal{A}^{\overline{\mathrm{U}}} \subseteq \mathcal{A}$ | AFSCs with upper bound for USAFA cadets |
| $\mathcal{A}^{\underline{\mathrm{R}}} \subseteq \mathcal{A}$ | AFSCs with lower bound for cadet percentile |
| $\mathcal{A}^{\overline{\mathrm{R}}} \subseteq \mathcal{A}$ | AFSCs with upper bound for cadet percentile |
| $\mathcal{C}^{\mathrm{U}} \subseteq \mathcal{C}$ | USAFA cadets |

**Indexed Sets**

| | |
|---|---|
| $\mathcal{C}_a^{\mathrm{M}} \in \mathcal{C}$ | Cadets that have degrees mandatory for AFSC $a \in \mathcal{A}^{\mathrm{M}}$ |
| $\mathcal{C}_a^{\mathrm{D}} \in \mathcal{C}$ | Cadets that have degrees desired for AFSC $a \in \mathcal{A}^{\mathrm{D}}$ |
| $\mathcal{C}_a^{\mathrm{P}} \in \mathcal{C}$ | Cadets that have degrees permitted for AFSC $a \in \mathcal{A}^{\mathrm{P}}$ |
| $\mathcal{C}_a^{\mathrm{I}} \in \mathcal{C}$ | Cadets that have ineligible degrees for AFSC $a \in \mathcal{A}^{\mathrm{I}}$ |
| $\mathcal{C}_a^{\mathrm{W}} \in \mathcal{C}$ | Cadets who have AFSC $a \in \mathcal{A}$ as a preference |

| **Parameters** | | **Units** |
|---|---|---|
| $g_{ca}$ | utility of assigning cadet $c$ to AFSC $a$ | [fraction] |
| $t_a$ | target for AFSC $a$ | [number] |
| $f_a$ | factor by which AFSC $a$ can be overclassified | [fraction] |
| $d_a^{\mathrm{M}}$ | target accession rate for mandatory degrees for AFSC $a$ | [fraction] |
| $\underline{u}_a, \overline{u}_a$ | lower, upper limit for USAFA cadets | [fraction] |
| $r_c$ | ranking, using percentile, for cadet $c$ | [fraction] |
| $\underline{r}_a, \overline{r}_a$ | lower, upper limit for cadet percentile | [fraction] |
| $w_{ca}$ | weight cadet $c$ assigns to AFSC $a$ | [fraction] |

| **Decision Variables** | | **Units** |
|---|---|---|
| $X_{ca}$ | 1 if cadet $c$ is assigned to AFSC $a$, and 0 otherwise | [-] |

**Formulation ($\mathcal{N}$)**

$$\text{maximize} \quad \sum_{c\in\mathcal{C}} \sum_{a\in\mathcal{A}} g_{ca} X_{ca} \tag{2a}$$

$$\text{subject to:} \quad \sum_{a\in\mathcal{A}} X_{ca} = 1,\ \forall c \in \mathcal{C} \tag{2b}$$

$$\sum_{c\in\mathcal{C}} X_{ca} \geq t_a,\ \forall a \in \mathcal{A} \tag{2c}$$

$$\sum_{c\in\mathcal{C}} X_{ca} \leq f_a t_a,\ \forall a \in \mathcal{A}^{\mathrm{F}} \tag{2d}$$

$$\sum_{c\in\mathcal{C}_a^{\mathrm{M}}} X_{ca} \geq d_a^M t_a,\ \forall a \in \mathcal{A}^{\mathrm{M}} \tag{2e}$$

$$\sum_{c\in\mathcal{C}^{\mathrm{U}}} X_{ca} \geq \underline{u}_a t_a,\ \forall a \in \mathcal{A}^{\underline{\mathrm{U}}} \tag{2f}$$

$$\sum_{c\in\mathcal{C}^{\mathrm{U}}} X_{ca} \leq \overline{u}_a t_a,\ \forall a \in \mathcal{A}^{\overline{\mathrm{U}}} \tag{2g}$$

$$\sum_{c\in\mathcal{C}} r_c X_{ca} \geq \underline{r}_a \sum_{c\in\mathcal{C}} X_{ca},\ \forall a \in \mathcal{A}^{\underline{\mathrm{R}}} \tag{2h}$$

$$\sum_{c\in\mathcal{C}} r_c X_{ca} \leq \overline{r}_a \sum_{c\in\mathcal{C}} X_{ca},\ \forall a \in \mathcal{A}^{\overline{\mathrm{R}}} \tag{2i}$$

$$X_{ca} \in \{0,1\},\ \forall c \in \mathcal{C},\ a \in \mathcal{A} \tag{2j}$$

The utility function applied in the objective function in the integer program reformulation is as follows:

$$g_{ca} = \begin{cases} (10r_c w_{ca}) + 250 & \text{if } c \in \mathcal{C}_a^{\mathrm{M}} \cap \mathcal{C}_a^{\mathrm{W}} \\ (10r_c w_{ca}) + 150 & \text{if } c \in \mathcal{C}_a^{\mathrm{D}} \cap \mathcal{C}_a^{\mathrm{W}} \\ 10r_c w_{ca} & \text{if } c \in \mathcal{C}_a^{\mathrm{P}} \cap \mathcal{C}_a^{\mathrm{W}} \\ 100r_c & \text{if } c \in \mathcal{C}_a^{\mathrm{M}} \setminus \mathcal{C}_a^{\mathrm{W}} \\ 50r_c & \text{if } c \in \mathcal{C}_a^{\mathrm{D}} \setminus \mathcal{C}_a^{\mathrm{W}} \\ 0 & \text{if } c \in \mathcal{C}_a^{\mathrm{P}} \setminus \mathcal{C}_a^{\mathrm{W}} \\ -50000 & \text{if } c \in \mathcal{C}_a^{\mathrm{I}} \end{cases} \tag{3}$$

The formulation is evaluated over the sets of cadets ($\mathcal{C}$) and AFSCs ($\mathcal{A}$). These sets are broken into subsets of cadets and AFSCs to evaluate certain constraints. The subset $\mathcal{C}^{\mathrm{U}}$ is defined as the set of USAFA cadets. The subsets $\mathcal{A}^{\mathrm{M}}, \mathcal{A}^{\mathrm{D}}, \mathcal{A}^{\mathrm{P}}$ and $\mathcal{A}^{\mathrm{I}}$ specify which AFSCs have mandatory, desired, permitted, and ineligibility constraints, respectively. The subsets $\mathcal{A}^{\underline{\mathrm{U}}}$ and $\mathcal{A}^{\overline{\mathrm{U}}}$ designate AFSCs which have restrictions for the assignment of USAFA cadets. The subsets $\mathcal{A}^{\underline{\mathrm{R}}}$ and $\mathcal{A}^{\overline{\mathrm{R}}}$ contain AFSCs which have order of merit constraints. The subset $\mathcal{A}^{\mathrm{F}}$ designates the AFSCs which are limited in how much they can exceed their target amount.

The indexed sets $\mathcal{C}_a^{\mathrm{M}}, \mathcal{C}_a^{\mathrm{D}}, \mathcal{C}_a^{\mathrm{P}}$, and $\mathcal{C}_a^{\mathrm{I}}$ contain cadets who have degrees that are mandatory, desired, permitted, and ineligible for AFSC $a$ in the subsets $\mathcal{A}^{\mathrm{M}}, \mathcal{A}^{\mathrm{D}}, \mathcal{A}^{\mathrm{P}}$ and $\mathcal{A}^{\mathrm{I}}$, respectively. The indexed set $\mathcal{C}_a^{\mathrm{W}}$ contains cadets who submit AFSC $a \in \mathcal{A}$ as a preference.

The objective function, denoted by (2a) and (3), scores the cadet-AFSC pairs in regards to education requirements, cadet ranking, and cadet preference. Mandatory education requirements achieve higher scores than desired education requirements, which achieve higher scores than permitted education requirements. As an example, consider a cadet who is top rated in terms of percentile and has selected the Intelligence (14N) career field as their top choice with a weight of 100. If the cadet majored in mathematics, which is a mandatory degree for the 14N career field, the utility of the match is:

$$(10 \cdot r_c \cdot w_{ca}) + 250 = (10 \cdot 1 \cdot 100) + 250 = 1000 + 250 = 1250$$

However, if the cadet majored in psychology, which is a desired degree for the 14N career field, the utility of the match is:

$$(10 \cdot r_c \cdot w_{ca}) + 150 = (10 \cdot 1 \cdot 100) + 150 = 1000 + 150 = 1150$$

Since any degree is permitted for the 14N career field, if the cadet majored in anything other than the mandatory or desired degrees, the utility of the match is:

$$10 \cdot r_c \cdot w_{ca} = 10 \cdot 1 \cdot 100 = 1000$$

If the cadet did not select the 14N career field as a preference, but majored in mathematics, a mandatory degree, the utility of the match is:

$$100 \cdot r_c = 100 \cdot 1 = 100$$

However, if the cadet majored in psychology, a desired degree for the 14N career field, the utility of the match is:

$$50 \cdot r_c = 50 \cdot 1 = 50$$

If the cadet majored in any degree that is not considered mandatory or desired for the 14N career field, the utility of the match is 0. In cases where a cadet's degree is not mandatory, desired, or permitted for an AFSC, the utility of the match is -50,000 to discourage the match. The scoring of the objective function suggests degree requirements and cadet preference are of high priority in assigning the matches. However, in the reformulation, only mandatory education requirements are considered in the model constraints, while the remaining education requirements and cadet preferences are addressed in the objective function.

Constraint (2b) permits each cadet to only be assigned to one AFSC. Constraint (2c) ensures that the targets for each AFSC are met and constraint (2d) allows certain AFSCs to be overclassified by a certain factor. Constraint (2e) specifies the mandatory education constraints for the assignment of cadets to AFSCs. Constraints (2f) and (2g) force a specified distribution of USAFA cadets within certain AFSCs. Constraints (2h) and (2i) balance cadet merit across certain AFSCs. Constraint (2j) specifies that the decision variable is binary.

The balance of merit constraints (2h) and (2i) are updated from those in the original formulation (5h) and (5i). The two sets of constraints state the the average cadet merit must be greater than the lower bound and less than the upper bound in certain career fields, respectively. The original set of constraints account for the average merit of the assigned cadets by subtracting 0.5 from each cadet's merit. The updated constraints compare the average merit of all cadets assigned to the specified AFSCs to a target value for average merit.

As an example, consider the Missilier (13N) career field. Suppose the 13N career field has a desired lower bound for average percentile of 0.35 and a desired upper bound of 0.65. Suppose there are five cadets being assigned to the 13N career field with the following percentiles: 0.25, 0.4, 0.15, 0.8, and 0.95. A nonlinear interpretation of the constraints would simply compare the average percentile of the assigned cadets to the lower bound, 0.35, and the upper bound, 0.65 with the following:

$$0.35 \leq \frac{0.25+0.4+0.15+0.8+0.95}{5} \leq 0.65$$

which simplifies to:

$$0.35 \leq 0.51 \leq 0.65$$

Constraints (2h) and (2i) provide linear interpretations of the constraints described above. Substituting the information into constraint (2h) yields

$$(0.25 \cdot 1) + (0.4 \cdot 1) + (0.15 \cdot 1) + (0.8 \cdot 1) + (0.95 \cdot 1) \geq 0.35 \cdot (1 + 1 + 1 + 1 + 1)$$

which can be simplified to

$$2.55 \geq 1.75$$

and thus the assigned cadets meet the lower bound for average percentile.

Substituting the information into constraint (2i) yields

$$(0.25 \cdot 1) + (0.4 \cdot 1) + (0.15 \cdot 1) + (0.8 \cdot 1) + (0.95 \cdot 1) \leq 0.65 \cdot (1 + 1 + 1 + 1 + 1)$$

which can be simplified to

$$2.55 \leq 3.25$$

and thus the assigned cadets meet the upper bound for average percentile. These constraints achieve the goals of a desired lower and upper bound for average percentile while maintaining the linearity of the constraint, and thus the overall formulation.

A notable update within this formulation includes the addition of subsets in place of binary parameters. Many constraints within the provided code only apply to a subset of cadets or AFSCs, as opposed to the sets and subsets in the formulation in Appendix A. Adjustments are made to the formulation to account for differences in the code. The subsets reduce the number of constraints. For example, instead of using the full set of cadets $\mathcal{C}$, the subset $\mathcal{C}^{\mathrm{U}} \subseteq \mathcal{C}$ is included to indicate the set of USAFA cadets as a subset of the set of all cadets. The addition of this subset eliminates the evaluation of ROTC cadets for the lower and upper USAFA limit constraints (2f) and (2g), and removes the binary parameter $u_i$ from the formulation.

Indexed sets are generated to reduce the number of constraints and to simplify the objective function. Three indexed sets are introduced to indicate which cadets meet degree requirements for each AFSC. The set $\mathcal{C}_a^{\mathrm{M}}$ contains cadets who meet mandatory degree requirements for each AFSC in the set of AFSCs with mandatory requirements. Thus, for constraint (2e), only those cadets with mandatory degrees are evaluated, as opposed to evaluating all cadets using the binary parameter $m_{ij}$ as in constraint (5e). Similar sets are generated for cadets who meet desired and permitted education requirements for use in the objective function.

The indexed set $\mathcal{C}_a^{\mathrm{W}}$ contains cadets who have AFSC $a$ as a preference. The education and cadet preference sets are used in evaluating the objective function. The union and set difference of the indexed sets for education and cadet preference simplify the evaluation of the overall utility of assigning cadets to AFSCs. For example, if cadet $c$ prefers AFSC $a$ and has a degree which is mandatory for AFSC $a$, then that cadet would fall in both $\mathcal{C}_a^{\mathrm{M}}$ and $\mathcal{C}_a^{\mathrm{W}}$. Since the cadet falls into the union of these sets, $\mathcal{C}_a^{\mathrm{M}} \cap \mathcal{C}_a^{\mathrm{W}}$, the first condition would be used to evaluate the utility of the cadet-AFSC pair in the objective function. If cadet $c$ does not prefer AFSC $a$, but has a mandatory degree for AFSC $a$, then the cadet would fall in $\mathcal{C}_a^{\mathrm{M}}$, but not $\mathcal{C}_a^{\mathrm{W}}$. Since the cadet falls into the set difference of these sets, $\mathcal{C}_a^{\mathrm{M}} \setminus \mathcal{C}_a^{\mathrm{W}}$, the fourth condition would be used to evaluate the utility of the match in objective function (3).

### 3.2 New Model Formulation

While the reformulation reduces the size of the original model through the introduction of new subsets and indexed sets, it does not address the issues of infeasibility in the original model, or the desired and permitted education requirements outlined in the AFOCD. The priority of desired and permitted degrees are referenced in the objective function of model ($\mathcal{N}$), but this does not address the AFOCD requirements for degree distribution within the AFSCs. A new model ($\hat{\mathcal{N}}$) presents a method for countering these problems by using the elastic variables discussed in Chapter I. The new formulation includes constraints to account for the desired and permitted education requirements of the AFOCD. The objective function also accounts for the weight each cadet assigns to their preferred AFSCs to attempt to assign cadets to their top preferences. The reformulation of AFPC's original model serves as the base of the new formulation. The new formulation maintains a majority of the sets and parameters of the reformulation, model ($\mathcal{N}$), and thus only new aspects of the model are introduced here.

**Sets and Indices**

| | |
|---|---|
| $\mathcal{A}^{\underline{\mathrm{D}}}$, $\mathcal{A}^{\overline{\mathrm{D}}} \subseteq \mathcal{A}$ | AFSCs with a lower, upper bound for desired education requirements, respectively |
| $\mathcal{A}^{\mathrm{P}} \subseteq \mathcal{A}$ | AFSCs with permitted education requirements |
| $\mathcal{A}^{\tilde{\mathrm{U}}} \subseteq \mathcal{A}$ | AFSCs which have a USAFA cadet assignment limit |
| $\mathcal{A}^{\mathrm{I}}_c \in \mathcal{A}$ | AFSCs that cadet $c \in \mathcal{C}$ is ineligible for |

| **Parameters** | | **Units** |
|---|---|---|
| $\mu^{\mathrm{T}}$, $\lambda^{\mathrm{T}}$ | penalty, reward associated with the target constraint | [fraction] |
| $\mu^{\mathrm{F}}$, $\lambda^{\mathrm{F}}$ | penalty, reward associated with the overclassification constraint | [fraction] |
| $\mu^{\mathrm{M}}$, $\lambda^{\mathrm{M}}$ | penalty, reward associated with the mandatory education constraint | [fraction] |
| $\mu^{\underline{\mathrm{D}}}$, $\lambda^{\underline{\mathrm{D}}}$ | penalty, reward associated with the lower target desired education constraint | [fraction] |
| $\mu^{\overline{\mathrm{D}}}$, $\lambda^{\overline{\mathrm{D}}}$ | penalty, reward associated with the upper target desired education constraint | [fraction] |
| $\mu^{\mathrm{P}}$, $\lambda^{\mathrm{P}}$ | penalty, reward associated with the permitted education constraint | [fraction] |

| | | |
|---|---|---|
| $\mu^{\underline{U}}$, $\lambda^{\underline{U}}$ | penalty, reward associated with the USAFA lower limit constraint | [fraction] |
| $\mu^{\overline{U}}$, $\lambda^{\overline{U}}$ | penalty, reward associated with the USAFA upper limit constraint | [fraction] |
| $\mu^{\underline{R}}$, $\lambda^{\underline{R}}$ | penalty, reward associated with the percentile lower limit constraint | [fraction] |
| $\mu^{\overline{R}}$, $\lambda^{\overline{R}}$ | penalty, reward associated with the percentile upper limit constraint | [fraction] |
| $\mu^{W}$, $\lambda^{W}$ | penalty, reward associated with the cadet preference constraint | [fraction] |
| $\lambda^{S}$ | reward for assigning cadets in accordance with their preference order | [fraction] |
| $w$ | target for meeting cadet preference | [fraction] |
| $\tilde{u}$ | limit for USAFA cadets in AFSCs within $\mathcal{A}^{\tilde{U}}$ | [fraction] |
| $M$ | a sufficiently large number | [-] |

| **Elastic Variables** | | **Units** |
|---|---|---|
| $Y_a^{T}, Z_a^{T}$ | the amount by which the target constraint is not met, or is exceeded, for AFSC $a \in \mathcal{A}$ | [cadets] |
| $Y_a^{F}, Z_a^{F}$ | the amount by which the overclassification constraint is not met, or is exceeded, for AFSC $a \in \mathcal{A}^{F}$ | [cadets] |
| $Y_a^{M}, Z_a^{M}$ | the amount by which the mandatory education constraint is not met, or is exceeded, for AFSC $a \in \mathcal{A}^{M}$ | [cadets] |
| $Y_a^{\underline{D}}, Z_a^{\underline{D}}$ | the amount by which the lower target desired education constraint is not met, or is exceeded, for AFSC $a \in \mathcal{A}^{\underline{D}}$ | [cadets] |
| $Y_a^{\overline{D}}, Z_a^{\overline{D}}$ | the amount by which the upper target desired education constraint is not met, or is exceeded, for AFSC $a \in \mathcal{A}^{\overline{D}}$ | [cadets] |

| | | |
|---|---|---|
| $Y_a^{\mathrm{P}}, Z_a^{\mathrm{P}}$ | the amount by which the permitted education constraint is not met, or is exceeded for AFSC $a \in \mathcal{A}^{\mathrm{P}}$ | [cadets] |
| $Y_a^{\mathrm{W}}, Z_a^{\mathrm{W}}$ | the amount by which the cadet preference constraint is not met, or is exceeded for AFSC $a \in \mathcal{A}$ | [cadets] |
| $Y_a^{\underline{\mathrm{U}}}, Z_a^{\underline{\mathrm{U}}}$ | the amount by which the USAFA lower limit is not met, or exceeded for AFSC $a \in \mathcal{A}^{\underline{\mathrm{U}}}$ | [cadets] |
| $Y_a^{\overline{\mathrm{U}}}, Z_a^{\overline{\mathrm{U}}}$ | the amount by which the the USAFA upper limit is not met, or exceeded, for AFSC $a \in \mathcal{A}^{\overline{\mathrm{U}}}$ | [cadets] |
| $Y_a^{\underline{\mathrm{R}}}, Z_a^{\underline{\mathrm{R}}}$ | the amount by which the percentile lower limit is not met, or exceeded, for AFSC $a \in \mathcal{A}^{\underline{\mathrm{R}}}$ | [fraction] |
| $Y_a^{\overline{\mathrm{R}}}, Z_a^{\overline{\mathrm{R}}}$ | the amount by which the percentile upper limit is not met, or exceeded, for AFSC $a \in \mathcal{A}^{\underline{\mathrm{R}}}$ | [fraction] |

| **Auxiliary Variables** | | **Units** |
|---|---|---|
| $\alpha_a^{\mathrm{T}}$ | 1 if $Y_a^{\mathrm{T}}$ is used, and 0 otherwise, for $a \in \mathcal{A}$ | [-] |
| $\alpha_a^{\mathrm{F}}$ | 1 if $Y_a^{\mathrm{F}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\mathrm{F}}$ | [-] |
| $\alpha_a^{\mathrm{M}}$ | 1 if $Y_a^{\mathrm{M}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\mathrm{M}}$ | [-] |
| $\alpha_a^{\underline{\mathrm{D}}}$ | 1 if $Y_a^{\underline{\mathrm{D}}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\underline{\mathrm{D}}}$ | [-] |
| $\alpha_a^{\overline{\mathrm{D}}}$ | 1 if $Y_a^{\overline{\mathrm{D}}}$ if used, and 0 otherwise, for $a \in \mathcal{A}^{\overline{\mathrm{D}}}$ | [-] |
| $\alpha_a^{\mathrm{P}}$ | 1 if $Y_a^{\mathrm{P}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\mathrm{P}}$ | [-] |
| $\alpha_a^{\mathrm{W}}$ | 1 if $Y_a^{\mathrm{W}}$ is used, and 0 otherwise, for $a \in \mathcal{A}$ | [-] |
| $\alpha_a^{\underline{\mathrm{U}}}$ | 1 if $Y_a^{\underline{\mathrm{U}}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\underline{\mathrm{U}}}$ | [-] |
| $\alpha_a^{\overline{\mathrm{U}}}$ | 1 if $Y_a^{\overline{\mathrm{U}}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\overline{\mathrm{U}}}$ | [-] |
| $\alpha_a^{\underline{\mathrm{R}}}$ | 1 if $Y_a^{\underline{\mathrm{R}}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\underline{\mathrm{R}}}$ | [-] |
| $\alpha_a^{\overline{\mathrm{R}}}$ | 1 if $Y_a^{\overline{\mathrm{R}}}$ is used, and 0 otherwise, for $a \in \mathcal{A}^{\overline{\mathrm{R}}}$ | [-] |

**Formulation ($\hat{\mathcal{N}}$)**

$$
\begin{aligned}
\text{maximize} \quad & \underbrace{\sum_{a \in \mathcal{A}} (\lambda^{\mathrm{T}} Z_a^{\mathrm{T}} - \mu^{\mathrm{T}} Y_a^{\mathrm{T}})}_{\text{AFSC Target Goal}} + \underbrace{\sum_{a \in \mathcal{A}^{\mathrm{F}}} (\lambda^{\mathrm{F}} Z_a^{\mathrm{F}} - \mu^{\mathrm{F}} Y_a^{\mathrm{F}})}_{\text{AFSC Overclassification Goal}} \\
& + \underbrace{\sum_{a \in \mathcal{A}^{\mathrm{M}}} (\lambda^{\mathrm{M}} Z_a^{\mathrm{M}} - \mu^{\mathrm{M}} Y_a^{\mathrm{M}})}_{\text{Mandatory Education Goals}} + \underbrace{\sum_{a \in \mathcal{A}^{\mathrm{P}}} (\lambda^{\mathrm{P}} Z_a^{\mathrm{P}} - \mu^{\mathrm{P}} Y_a^{\mathrm{P}})}_{\text{Permitted Education Goals}} \\
& + \underbrace{\sum_{a \in \mathcal{A}^{\underline{\mathrm{D}}}} (\lambda^{\underline{\mathrm{D}}} Z_a^{\underline{\mathrm{D}}} - \mu^{\underline{\mathrm{D}}} Y_a^{\underline{\mathrm{D}}}) + \sum_{a \in \mathcal{A}^{\overline{\mathrm{D}}}} (\lambda^{\overline{\mathrm{D}}} Z_a^{\overline{\mathrm{D}}} - \lambda^{\overline{\mathrm{D}}} Y_a^{\overline{\mathrm{D}}})}_{\text{Desired Education Goals}} \\
& + \underbrace{\sum_{a \in \mathcal{A}^{\underline{\mathrm{U}}}} (\lambda^{\underline{\mathrm{U}}} \, Z_a^{\underline{\mathrm{U}}} - \mu^{\underline{\mathrm{U}}} \, Y_a^{\underline{\mathrm{U}}}) + \sum_{a \in \mathcal{A}^{\overline{\mathrm{U}}}} (\lambda^{\overline{\mathrm{U}}} \, Z_a^{\overline{\mathrm{U}}} - \mu^{\overline{\mathrm{U}}} \, Y_a^{\overline{\mathrm{U}}})}_{\text{Source of Commissioning Goals}} \\
& + \underbrace{\sum_{a \in \mathcal{A}^{\underline{\mathrm{R}}}} (\lambda^{\underline{\mathrm{R}}} \, Z_a^{\underline{\mathrm{R}}} - \mu^{\underline{\mathrm{R}}} \, Y_a^{\underline{\mathrm{R}}}) + \sum_{a \in \mathcal{A}^{\overline{\mathrm{R}}}} (\lambda^{\overline{\mathrm{R}}} \, Z_a^{\overline{\mathrm{R}}} - \mu^{\overline{\mathrm{R}}} \, Y_a^{\overline{\mathrm{R}}})}_{\text{Cadet Ranking Goals}} \\
& + \underbrace{\sum_{a \in \mathcal{A}} (\lambda^{\mathrm{W}} Z_a^{\mathrm{W}} - \mu^{\mathrm{W}} \, Y_a^{\mathrm{W}})}_{\text{Cadet Preference Goal}} + \underbrace{\lambda^{\mathrm{S}} \sum_{a \in \mathcal{A}} \sum_{c \in \mathcal{C}} (w_{ca} X_{ca})}_{\text{Cadet Preference Order}}
\end{aligned}
\tag{4a}
$$

$$\text{subject to:} \quad \sum_{a \in \mathcal{A} \setminus \mathcal{A}_c^{\mathrm{I}}} X_{ca} = 1,\ \forall c \in \mathcal{C} \tag{4b}$$

$$\sum_{c \in \mathcal{C}} X_{ca} = t_a - Y_a^{\mathrm{T}} + Z_a^{\mathrm{T}},\ \forall a \in \mathcal{A} \tag{4c}$$

$$\sum_{c \in \mathcal{C}} X_{ca} = f_a t_a + Y_a^{\mathrm{F}} - Z_a^{\mathrm{F}},\ \forall a \in \mathcal{A}^{\mathrm{F}} \tag{4d}$$

$$\sum_{c \in \mathcal{C}_a^{\mathrm{M}}} X_{ca} = d_a^{\mathrm{M}} t_a - Y_a^{\mathrm{M}} + Z_a^{\mathrm{M}},\ \forall a \in \mathcal{A}^{\mathrm{M}} \tag{4e}$$

$$\sum_{c \in \mathcal{C}_a^{\underline{\mathrm{D}}}} X_{ca} = d_a^{\underline{\mathrm{D}}} t_a - Y_a^{\underline{\mathrm{D}}} + Z_a^{\underline{\mathrm{D}}},\ \forall a \in \mathcal{A}^{\underline{\mathrm{D}}} \tag{4f}$$

$$\sum_{c \in \mathcal{C}_a^{\overline{\mathrm{D}}}} X_{ca} = d_a^{\overline{\mathrm{D}}} t_a + Y_a^{\overline{\mathrm{D}}} - Z_a^{\overline{\mathrm{D}}},\ \forall a \in \mathcal{A}^{\overline{\mathrm{D}}} \tag{4g}$$

$$\sum_{c \in \mathcal{C}_a^{\mathrm{P}}} X_{ca} = d_a^{\mathrm{P}} t_a + Y_a^{\mathrm{P}} - Z_a^{\mathrm{P}},\ \forall a \in \mathcal{A}^{\mathrm{P}} \tag{4h}$$

$$\sum_{c \in \mathcal{C}_a^{\mathrm{W}}} X_{ca} = w \sum_{c \in C} (X_{ca}) - Y_a^{\mathrm{W}} + Z_a^{\mathrm{W}},\ \forall a \in \mathcal{A} \tag{4i}$$

$$\sum_{c \in \mathcal{C}^{\mathrm{U}}} X_{ca} = \underline{u}_a t_a - Y_a^{\underline{\mathrm{U}}} + Z_a^{\underline{\mathrm{U}}},\ \forall a \in \mathcal{A}^{\underline{\mathrm{U}}} \tag{4j}$$

$$\sum_{c \in \mathcal{C}^{\mathrm{U}}} X_{ca} = \overline{u}_a t_a + Y_a^{\overline{\mathrm{U}}} - Z_a^{\overline{\mathrm{U}}},\ \forall a \in \mathcal{A}^{\overline{\mathrm{U}}} \tag{4k}$$

$$\sum_{c \in \mathcal{C}^{\mathrm{U}}} \sum_{a \in \mathcal{A}^{\tilde{\mathrm{U}}}} X_{ca} \leq \tilde{u} \sum_{c \in \mathcal{C}} \sum_{a \in \mathcal{A}^{\tilde{\mathrm{U}}}} X_{ca} \tag{4l}$$

$$\sum_{c \in \mathcal{C}} r_c X_{ca} = \underline{r}_a \sum_{c \in \mathcal{C}} (X_{ca}) - Y_a^{\underline{\mathrm{R}}} + Z_a^{\underline{\mathrm{R}}},\ \forall a \in \mathcal{A}^{\underline{\mathrm{R}}} \tag{4m}$$

$$\sum_{c \in \mathcal{C}} r_c X_{ca} = \overline{r}_a \sum_{c \in \mathcal{C}} (X_{ca}) + Y_a^{\overline{\mathrm{R}}} - Z_a^{\overline{\mathrm{R}}},\ \forall a \in \mathcal{A}^{\overline{\mathrm{R}}} \tag{4n}$$

$$
\begin{aligned}
&Y_a^{\mathrm{T}} \leq M\alpha_a^{\mathrm{T}},\ \forall a \in \mathcal{A};\quad Z_a^{\mathrm{T}} \leq M(1-\alpha_a^{\mathrm{T}}),\ \forall a \in \mathcal{A}\\
&Y_a^{\mathrm{F}} \leq M\alpha_a^{\mathrm{F}},\ \forall a \in \mathcal{A}^{\mathrm{F}};\quad Z_a^{\mathrm{F}} \leq M(1-\alpha_a^{\mathrm{F}}),\ \forall a \in \mathcal{A}^{\mathrm{F}}\\
&Y_a^{\mathrm{M}} \leq M\alpha_a^{\mathrm{M}},\ \forall a \in \mathcal{A}^{\mathrm{M}};\quad Z_a^{\mathrm{M}} \leq M(1-\alpha_a^{\mathrm{M}}),\ \forall a \in \mathcal{A}^{\mathrm{M}}\\
&Y_a^{\underline{\mathrm{D}}} \leq M\alpha_a^{\underline{\mathrm{D}}},\ \forall a \in \mathcal{A}^{\underline{\mathrm{D}}};\quad Z_a^{\underline{\mathrm{D}}} \leq M(1-\alpha_a^{\underline{\mathrm{D}}}),\ \forall a \in \mathcal{A}^{\underline{\mathrm{D}}}\\
&Y_a^{\overline{\mathrm{D}}} \leq M\alpha_a^{\overline{\mathrm{D}}},\ \forall a \in \mathcal{A}^{\overline{\mathrm{D}}};\quad Z_a^{\overline{\mathrm{D}}} \leq M(1-\alpha_a^{\overline{\mathrm{D}}}),\ \forall a \in \mathcal{A}^{\overline{\mathrm{D}}}\\
&Y_a^{\mathrm{P}} \leq M\alpha_a^{\mathrm{P}},\ \forall a \in \mathcal{A}^{\mathrm{P}};\quad Z_a^{\mathrm{P}} \leq M(1-\alpha_a^{\mathrm{P}}),\ \forall a \in \mathcal{A}^{\mathrm{P}} \qquad (4o)\\
&Y_a^{\mathrm{W}} \leq M\alpha_a^{\mathrm{W}},\ \forall a \in \mathcal{A};\quad Z_a^{\mathrm{W}} \leq M(1-\alpha_a^{\mathrm{W}}),\ \forall a \in \mathcal{A}\\
&Y_a^{\underline{\mathrm{U}}} \leq M\alpha_a^{\underline{\mathrm{U}}},\ \forall a \in \mathcal{A}^{\underline{\mathrm{U}}};\quad Z_a^{\underline{\mathrm{U}}} \leq M(1-\alpha_a^{\underline{\mathrm{U}}}),\ \forall a \in \mathcal{A}^{\underline{\mathrm{U}}}\\
&Y_a^{\overline{\mathrm{U}}} \leq M\alpha_a^{\overline{\mathrm{U}}},\ \forall a \in \mathcal{A}^{\overline{\mathrm{U}}};\quad Z_a^{\overline{\mathrm{U}}} \leq M(1-\alpha_a^{\overline{\mathrm{U}}}),\ \forall a \in \mathcal{A}^{\overline{\mathrm{U}}}\\
&Y_a^{\underline{\mathrm{R}}} \leq M\alpha_a^{\underline{\mathrm{R}}},\ \forall a \in \mathcal{A}^{\underline{\mathrm{R}}};\quad Z_a^{\underline{\mathrm{R}}} \leq M(1-\alpha_a^{\underline{\mathrm{R}}}),\ \forall a \in \mathcal{A}^{\underline{\mathrm{R}}}\\
&Y_a^{\overline{\mathrm{R}}} \leq M\alpha_a^{\overline{\mathrm{R}}},\ \forall a \in \mathcal{A}^{\overline{\mathrm{R}}};\quad Z_a^{\overline{\mathrm{R}}} \leq M(1-\alpha_a^{\overline{\mathrm{R}}}),\ \forall a \in \mathcal{A}^{\overline{\mathrm{R}}}\\
&X_{ca} \in \{0,1\},\ \forall c \in \mathcal{C},\ a \in \mathcal{A} \setminus \mathcal{A}_c^{\mathrm{I}}\\
&\alpha_a^{\mathrm{T}}, \alpha_a^{\mathrm{F}}, \alpha_a^{\mathrm{M}}, \alpha_a^{\underline{\mathrm{D}}}, \alpha_a^{\overline{\mathrm{D}}}, \alpha_a^{\mathrm{P}}, \alpha_a^{\mathrm{W}}, \alpha_a^{\underline{\mathrm{U}}}, \alpha_a^{\overline{\mathrm{U}}}, \alpha_a^{\underline{\mathrm{R}}}, \alpha_a^{\overline{\mathrm{R}}}, \in \{0,1\},\\
&\qquad \forall a \in \mathcal{A}, \mathcal{A}^{\mathrm{F}}, \mathcal{A}^{\mathrm{M}}, \mathcal{A}^{\underline{\mathrm{D}}}, \mathcal{A}^{\overline{\mathrm{D}}}, \mathcal{A}^{\mathrm{P}}, \mathcal{A}, \mathcal{A}^{\underline{\mathrm{U}}}, \mathcal{A}^{\overline{\mathrm{U}}}, \mathcal{A}^{\underline{\mathrm{R}}}, \mathcal{A}^{\overline{\mathrm{R}}}\\
&Y_a^{\mathrm{T}}, Y_a^{\mathrm{F}}, Y_a^{\mathrm{M}}, Y_a^{\underline{\mathrm{D}}}, Y_a^{\overline{\mathrm{D}}}, Y_a^{\mathrm{P}}, Y_a^{\mathrm{W}}, Y_a^{\underline{\mathrm{U}}}, Y_a^{\overline{\mathrm{U}}}, Y_a^{\underline{\mathrm{R}}}, Y_a^{\overline{\mathrm{R}}} \in \mathbb{R}^+, \qquad (4p)\\
&\qquad \forall a \in \mathcal{A}, \mathcal{A}^{\mathrm{F}}, \mathcal{A}^{\mathrm{M}}, \mathcal{A}^{\underline{\mathrm{D}}}, \mathcal{A}^{\overline{\mathrm{D}}}, \mathcal{A}^{\mathrm{P}}, \mathcal{A}, \mathcal{A}^{\underline{\mathrm{U}}}, \mathcal{A}^{\overline{\mathrm{U}}}, \mathcal{A}^{\underline{\mathrm{R}}}, \mathcal{A}^{\overline{\mathrm{R}}}\\
&Z_a^{\mathrm{T}}, Z_a^{\mathrm{F}}, Z_a^{\mathrm{M}}, Z_a^{\underline{\mathrm{D}}}, Z_a^{\overline{\mathrm{D}}}, Z_a^{\mathrm{P}}, Z_a^{\mathrm{W}}, Z_a^{\underline{\mathrm{U}}}, Z_a^{\overline{\mathrm{U}}}, Z_a^{\underline{\mathrm{R}}}, Z_a^{\overline{\mathrm{R}}} \in \mathbb{R}^+,\\
&\qquad \forall a \in \mathcal{A}, \mathcal{A}^{\mathrm{F}},\ \mathcal{A}^{\mathrm{M}}, \mathcal{A}^{\underline{\mathrm{D}}}, \mathcal{A}^{\overline{\mathrm{D}}}, \mathcal{A}^{\mathrm{P}}, \mathcal{A}, \mathcal{A}^{\underline{\mathrm{U}}}, \mathcal{A}^{\overline{\mathrm{U}}}, \mathcal{A}^{\underline{\mathrm{R}}}, \mathcal{A}^{\overline{\mathrm{R}}}
\end{aligned}
$$

### 3.2.1 Updates to Constraints

Constraint (4b) ensures each cadet is only assigned to one AFSC. The addition of the indexed set $\mathcal{A}_c^{\mathrm{I}}$, which contains AFSCs for which cadet $c$ is ineligible, evaluates this constraint only for those AFSCs each cadet meets the educational requirements. Thus, no ineligible cadets may be assigned to AFSCs in this formulation.

Constraints are added to reflect desired and permitted education requirements, as well as cadet preference. There are two sets of desired educational constraints; one for AFSCs that have a lower limit on cadets with desired degrees, and one with an upper limit. These requirements are reflected in constraints (4f) and (4g), respectively. The permitted educational constraint specifies upper limits for cadets with permitted degrees in certain AFSCs, and is displayed in constraint (4h). The constraint for cadet preference ensures at least a percentage of cadets receive an AFSC match that is in their preferences for each AFSC. This is shown in constraint (4i). Unlike in model ($\mathcal{N}$), desired and permitted degree requirements and cadet preference are considered as constraints to be met by the cadet-AFSC assignments, as prescribed by the AFOCD. Constraint (4l) reflects a policy stating that only a certain percentage of cadets assigned to a specified set of AFSCs are permitted to be USAFA graduates.

The target and overclassification constraints (4c) and (4d), mandatory education constraint (4e), source of commissioning balance constraints (4j) and (4k), and the order of merit balance constraints (4m) and (4n), are those of the reformulation ($\mathcal{N}$), but contain elastic variables. Constraints (4p) define the domains of the variables.

### 3.2.2 Elastic Variables

As stated in Chapter I, elastic variables are included in a constraint to permit the violation of constraints at a cost to the objective function, or to reward the objective function when a constraint is exceeded. In the cadet-AFSC problem, these variables are used in instances where there may not be enough cadets meeting conditions of certain requirements. If a constraint cannot be met, the $Y$ variables are used to overcome infeasibility.

The $Y$ variables are either added or subtracted from the right-hand side of the constraint, based on the form of the constraint. For example, the mandatory education constraint (4e) states that the total number of cadets assigned that meet mandatory education requirements for an AFSC must be greater than or equal to a target value. In cases where this constraint cannot be met, the number of cadets assigned is less than the required value. The elastic variable, $Y_a^{\mathrm{M}}$ is then subtracted from the required value so the total number of cadets assigned is equal to the new value. In the permitted education constraint (4h), the opposite occurs: the number of cadets assigned is greater than the required value. In this case, the elastic variable $Y_a^{\mathrm{P}}$ is added to the required value. In both cases, the use of the elastic variable is penalized in the objective function, which is described in detail in Section 3.2.4.

As an algebraic example, consider the mandatory education constraint (4e). Assume the Operations Research (15A) career field has a target of 20 cadets, and needs 65% of those cadets to have mandatory degrees. Assume, however, there are only 5 cadets available to be matched that have mandatory degrees for the 15A career field. If all 5 cadets were matched, the constraint would be

$$5 = (0.65 \cdot 20) - Y_{15A}^{\mathrm{M}} + Z_{15A}^{\mathrm{M}}$$

The value of $Z_{15A}^{\mathrm{M}}$ would be zero, since 5 is less than the required 13 cadets. This would then simplify to

$$Y_{15A}^{\mathrm{M}} = 13 - 5$$

The elastic variable, $Y_{15A}^{\mathrm{M}}$, would then have to equal 8 to make up for the missing cadets. This variable carries an associated penalty ($\mu^{\mathrm{M}}$) that would lower the objective function value for violating the constraint.

If a constraint is exceeded, the $Z$ elastic variables are used to reward the objective function. The $Z$ variable value is added to or subtracted from the constraint based on its goal. The value is calculated differently for cases in which the goal is to be greater than the required value and when it is to be less than the required value. For example, with the mandatory education constraint, the value of $Z_a^{\mathrm{M}}$ is the difference of the number of cadets assigned and the required number of cadets. For the permitted education constraint, the value of $Z_a^{\mathrm{P}}$ is the difference of the required value and the number of cadets assigned. Each elastic $Z$ variable has an associated reward in the objective function. The values of the rewards are described further in Section 3.2.4.

As an algebraic example, consider the permitted education constraint (4h). Assume the Operations Research (15A) career field has a target of 20 cadets, and requires less than 10% of those cadets have permitted degrees. Assume that only 1 cadet is assigned with a permitted degree for the 15A AFSC. Then the constraint would be

$$1 = (0.10 \cdot 20) + Y_{15A}^{\mathrm{P}} - Z_{15A}^{\mathrm{P}}$$

and since this constraint is met without the use of $Y_{15A}^{\mathrm{P}}$, this would simplify to

$$1 = 2 - Z_{15A}^{\mathrm{P}}$$

Since the constraint is successfully met without the use of the elastic variable, it would have a value of 0. The value of $Z_{15A}^{\mathrm{P}}$ is equal to the difference between the left- and right-hand sides of the constraint, and thus $Z_{15A}^{\mathrm{P}} = 2 - 1 = 1$. An associated reward ($\lambda^{\mathrm{P}}$) increases the objective function value for exceeding the desired difference between the left- and right-hand sides of the constraint.

Constraints that are often difficult to meet and those of importance to decision makers contain elastic variables in the updated formulation. These include the target and overclassification constraints (4c) and (4d), the mandatory, desired and permitted education constraints (4e)-(4h), the cadet preference constraint (4i), the source of commissioning balance constraints (4j) and (4k), and the cadet merit constraints (4m) and (4n).

### 3.2.3 Auxiliary Variables

The values of the $Y$ and $Z$ variables directly influence the objective function. If $Y$ and $Z$ are allowed to be greater than zero, the variables could be increased further and further to generate an unbounded objective value. Auxiliary $\alpha$ variables are used in constraints (4o) to restrict the value of the $Y$ and $Z$ elastic variables. These binary variables are implemented to ensure that only one of the two variables may be greater than zero.

As an algebraic example, consider the target constraint (4c). The constraint contains two elastic variables, denoted $Y_a^{\mathrm{T}}$ and $Z_a^{\mathrm{T}}$. Suppose the target for the 14N AFSC is 100 cadets, and there are 50 cadets assigned to the AFSC. Then, the constraints associated with AFSC targets are the following:

$$
\begin{gathered}
50 = 100 - Y_{14N}^{\mathrm{T}} + Z_{14N}^{\mathrm{T}} \\
Y_{14N}^{\mathrm{T}} \leq M \cdot \alpha_{14N}^{\mathrm{T}} \\
Z_{14N}^{\mathrm{T}} \leq M \cdot (1 - \alpha_{14N}^{\mathrm{T}})
\end{gathered}
$$

Since there are fewer cadets than are required to meet the target, the elastic variable $Y_{14N}^{\mathrm{T}}$ must be greater than zero. This forces $\alpha_{14N}^{\mathrm{T}} = 1$, which subsequently forces $Z_{14N}^{\mathrm{T}}$ to be less than or equal to zero. Then the constraints simplify to the following:

$$
\begin{gathered}
50 = 100 - Y_{14N}^{\mathrm{T}} + Z_{14N}^{\mathrm{T}} \\
Y_{14N}^{\mathrm{T}} \leq M \\
Z_{14N}^{\mathrm{T}} \leq 0
\end{gathered}
$$

Since $Y_{14N}^{\mathrm{T}}$ and $Z_{14N}^{\mathrm{T}}$ are restricted to be non-negative, the value of $Z_{14N}^{\mathrm{T}}$ must be 0, and the value of $Y_{14N}^{\mathrm{T}}$ must be 50. If the constraint were met exactly, neither $Y_{14N}^{\mathrm{T}}$ nor $Z_{14N}^{\mathrm{T}}$ would be used and would have value 0.

As an additional example, consider the case where the target for the 13N AFSC is 10 cadets, and there are 15 cadets assigned to the AFSC. Then, the constraints associated with AFSC targets are the following:

$$15 = 10 - Y^{\mathrm{T}}_{13N} + Z^{\mathrm{T}}_{13N}$$
$$Y^{\mathrm{T}}_{13N} \leq M \cdot \alpha^{\mathrm{T}}_{13N}$$
$$Z^{\mathrm{T}}_{13N} \leq M \cdot (1 - \alpha^{\mathrm{T}}_{13N})$$

Since there are more cadets than are required to meet the target, the elastic variable $Y^{\mathrm{T}}_{13N}$ is equal to zero. This forces $\alpha^{T}_{13N} = 0$. Then the constraints simplify to the following:

$$15 = 10 - Y^{\mathrm{T}}_{13N} + Z^{\mathrm{T}}_{13N}$$
$$Y^{\mathrm{T}}_{13N} \leq 0$$
$$Z^{\mathrm{T}}_{13N} \leq M$$

Since $Y^{\mathrm{T}}_{13N}$ and $Z^{\mathrm{T}}_{13N}$ are restricted to be non-negative, the value of $Y^{\mathrm{T}}_{13N}$ must be 0, and the value of $Z^{\mathrm{T}}_{13N}$ must be $15 - 10 = 5$. If the constraint were met exactly, neither $Y^{\mathrm{T}}_{13N}$ nor $Z^{\mathrm{T}}_{13N}$ would be used and would have value 0.

### 3.2.4 Objective Function

The goals of AFPC and HAF/A1 have the potential to vary from year to year, and thus the importance of each constraint can vary as well. A new objective function (4a) is introduced to reflect the importance of each goal. This objective function consists of the penalties and rewards from using the elastic variables, along with an additional objective to assign cadets to their higher preferences. The values of the penalties and rewards reflect the priority of each goal, as well as the relationships between goals. Higher penalty and reward values indicate greater importance, while lower values reflect lower importance. These values can be adjusted yearly to reflect shifting priorities and to achieve a desirable matching of cadets and AFSCs. The objective function also considers the cadet's order of AFSC preferences. A reward is introduced to incentivize the objective function to assign cadets to their more preferred AFSCs over their less preferred.

The values of the rewards and penalties play an important role in finding a desirable solution for the cadet-AFSC problem. They must be considered carefully to adequately reflect the goals of the problem for each year. The impact of the values is explored in a sensitivity analysis, presented in Chapter IV.

## 3.3 Solution Methodology

The penalty and reward values of the objective function are updated iteratively to determine the best possible weights which achieve the goals dictated by AFPC. The penalties and rewards are assigned initial values, which ensure the elastic variables have the same initial impact on the objective function value. Then, the values are adjusted to account for constraint importance, assigning a higher penalty value to those elastic variables associated with constraints that have a higher priority of being satisfied. Once the run results have been analyzed, the weights are updated to reflect the new balance of priority for the constraints. A "best" solution is determined in terms of the model performance across all constraints.

# IV. Analysis and Results

As stated in Chapter III, the penalties and rewards assigned to each constraint affect model performance and the resulting assignments. Thus, it is necessary to evaluate various weighting schemes to determine the best possible performance given a set of cadet and AFSC data. One year of data is evaluated using the reformulation of AFPC's model ($\mathcal{N}$) and the new model ($\hat{\mathcal{N}}$). Within the analysis and discussion, the AFSCs are labeled as O1 – O44, and A1 – A44. The labels correspond to the size of the AFSC target, with AFSC O1 having the largest original target value, and O44 having the smallest original target value. AFSC A1 has the largest target value out of the adjusted data, and AFSC A44 has the smallest target of the adjusted data. The adjustments made to the data are discussed further in Section 4.2. The resulting assignments from the two models are compared and a "best" weighting method is identified for a prioritization of the constraints and the given cadet and AFSC data. All runs are completed using the Gurobi optimization solver [38] in Python with the Pyomo package on a Lenovo Ideapad S340 with a 2.1GHz processor and 8GB of RAM. The solver's default relative gap terminaion criterion of 0.01% is used.

## 4.1 Reformulation Analysis

The reformulation of AFPC's model ($\mathcal{N}$) is analyzed to determine its performance against the new model ($\hat{\mathcal{N}}$). The target constraint is met for all AFSCs in the reformulated model, but AFSCs O28, O37, O39, and O44 are greatly overclassified. These AFSCs do not have overclassification limits, which explains why the number of cadets assigned greatly exceeds the target. Figure 1 provides a visual representation of these AFSC classifications.

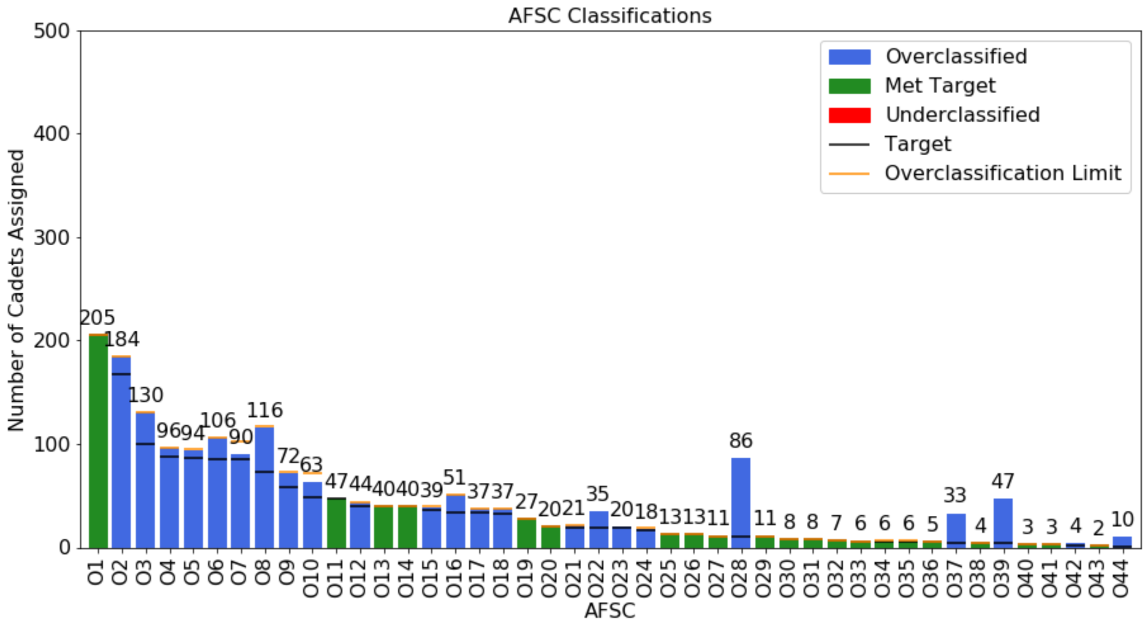


**Figure 1. Target and overclassification results for model ($\mathcal{N}$) on the original dataset. The black lines on each bar indicate the target value for that AFSC and the orange lines indicate the overclassification limit for the AFSC. The target and overclassification constraints are met for all AFSCs, but those AFSCs without overclassifcation limits are overfilled.**

Figures 2 and 3 present the percentile and source of commissioning balance results for large AFSCs, respectively. Out of the 15 large AFSCs, 14 fall within desired range for average cadet merit. AFSC O15 achieves an average merit of 0.68, which is 0.03 more than the desired lower bound, but is not a poor result, because a higher average merit within an AFSC is better. For the source of commissioning distribution, 6 of the 15 large AFSCs violate the desired bounds. This is because the cadet percentile and source of commissioning balance constraints focus on a subset of AFSCs, not all of which are large AFSCs. If the average cadet merit constraints are reformulated to be evaluated over the large AFSCs, the model returns a feasible solution, but the reformulation of the source of commissioning constraints results in an infeasible model.

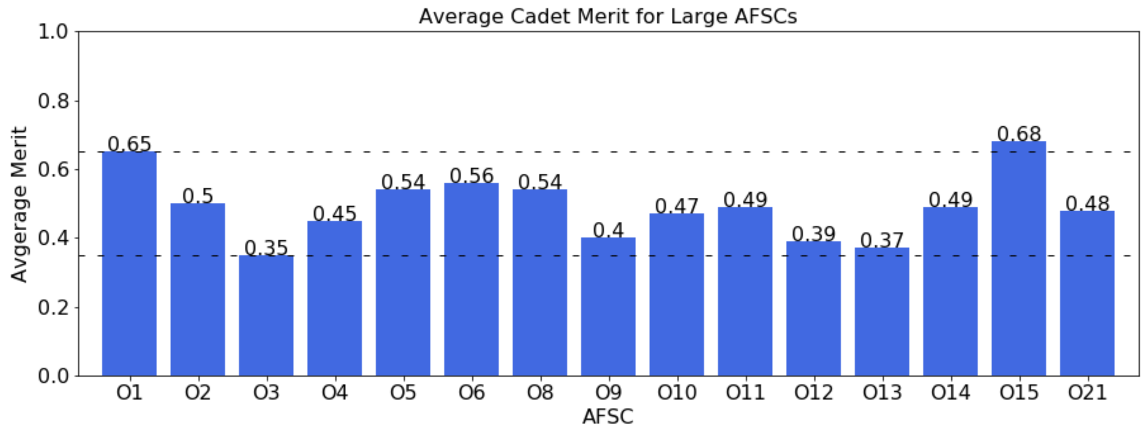


**Figure 2. Cadet percentile balance results for large AFSCs for model ($\mathcal{N}$) on the original dataset. The dashed lines indicate the desired bounds for the average cadet merit to fall within. For this solution, the desired upper bound for average cadet merit is violated by AFSC O15, but the high merit value is still desirable.**

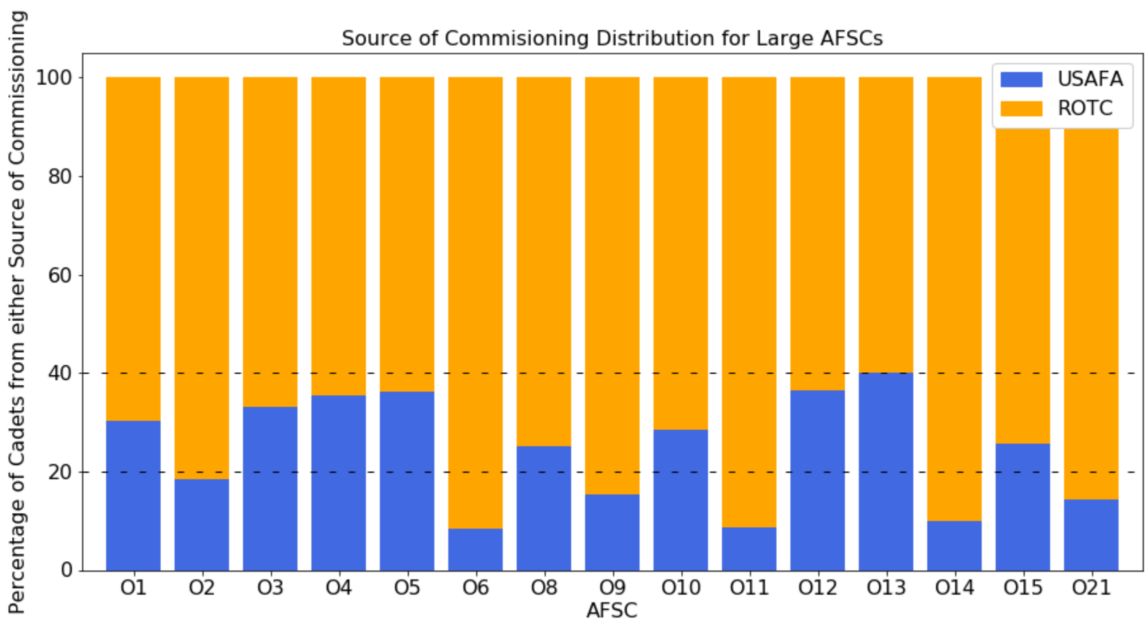


**Figure 3. Source of commissioning balance results for model ($\mathcal{N}$) on the original dataset. The dashed lines indicate the desired bounds for the source of commissioning split to fall within. Out of the 15 large AFSCs, 6 do not fall within the desired bounds for the source of commissioning split.**

For cadet preference, out of the 44 AFSCs, 36% are completely filled by cadets who listed them as a preference and 4.5% are completely filled by cadets who do not have them as a preference. One of these two AFSCs, O34, does not have any cadets with it as a preference. AFSC O34 is the only AFSC which no cadets list as a preference. AFSC O40, is only preferred by 4 cadets, and the model does not assign them to the AFSC. Three of the cadets are assigned to AFSCs for which they have mandatory degrees, but they did not list as a preference. The fourth cadet is assigned to their top preference, AFSC O1. The results of the cadet preference constraint are presented in Figure 4.

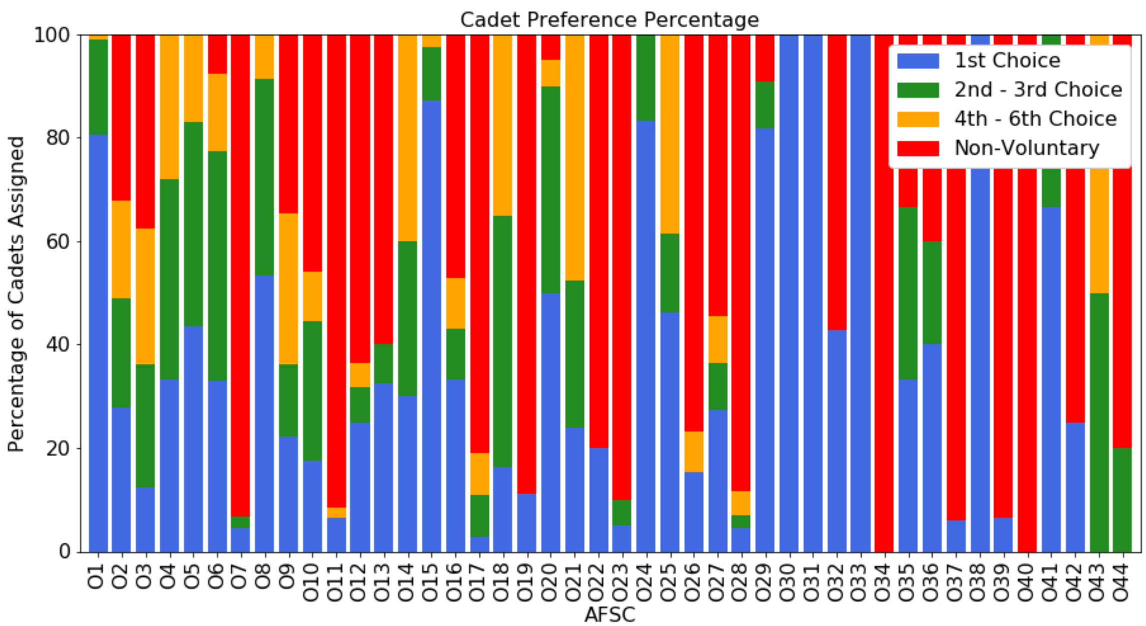


**Figure 4. Cadet preference results for model ($\mathcal{N}$) on the original dataset. Out of the 44 AFSCs, 2 are completely assigned to cadets non-voluntarily, while 16 are filled with cadets that prefer them.**

Model ($\mathcal{N}$) performs fairly well in all areas except the cadet preference and education constraints. It assigns 125 cadets to AFSCs for which they are ineligible. These AFSCs include many engineering career fields and physics. It is not reasonable to assign ineligible cadets within these career fields due to the education required. A cadet who does not have a background in engineering would struggle greatly in an engineering-based career field. These assignments would cost the Air Force time and money to give the cadets the background needed to succeed. Though the model is successful across the remaining constraints, the faulty assignments for 6.5% of the total cadets is a performance concern. Within the model result, the ineligible cadets are only assigned to AFSCs that only have mandatory degree requirements. This is because the mandatory education constraint does not include those AFSCs that only permit cadets with mandatory degrees. If ineligible cadets are not permitted to be assigned, the model is infeasible. The model also does not consider the desired and permitted education constraints of each of the AFSCs, except as part of the objective function. This may lead to an imbalance of the preferred education background as listed in the AFOCD. The resulting mandatory, desired and permitted degree distribution for each AFSC is presented in Figure 5.

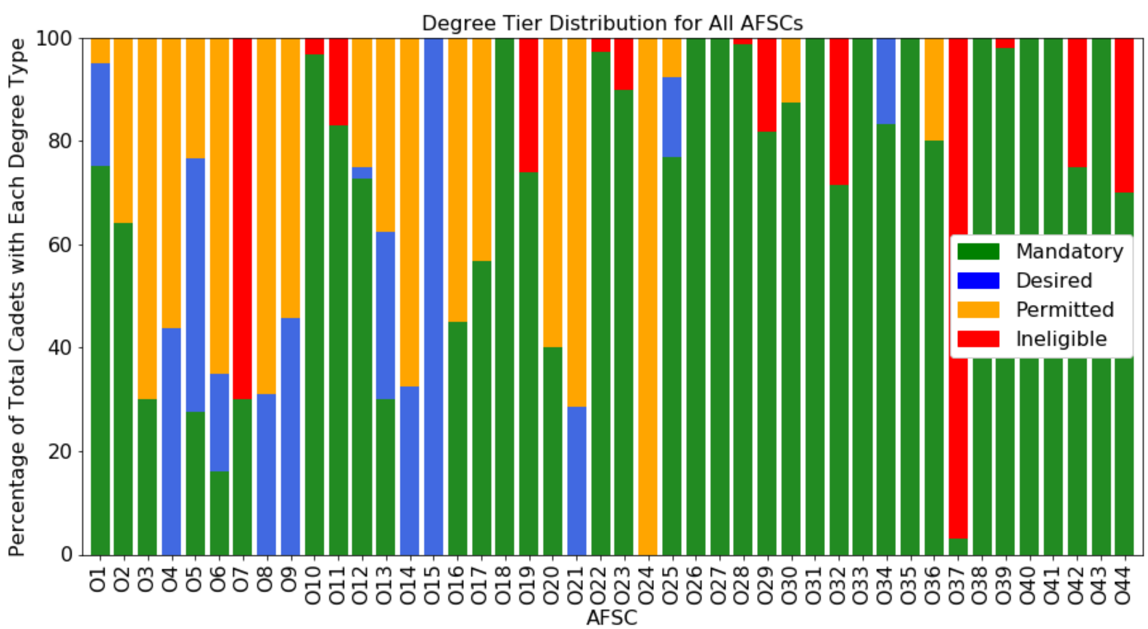


**Figure 5. Education distribution results for model ($\mathcal{N}$) on the original dataset. The ineligible assignments are undesirable.**

As mentioned previously, the model does not contain constraints for the desired and permitted education requirements. Out of the 18 AFSCs with a lower bound for desired degrees, only 2 AFSCs meet the requirements. The requirements are met for all AFSCs with an upper bound for desired degrees. The permitted education requirements are only met by 11 of the 21 AFSCs with those requirements. The poor performance for these requirements is understandable due to the absence of these constraints.

## 4.2 New Formulation Analysis

As discussed in previous sections, elastic variables are important to the success of the cadet-AFSC assignment model. The elastic $Y$ variables allow for constraints to be violated, but penalize the objective function for these violations. These variables prevent the model from reaching an infeasible result. The elastic $Z$ variables are used to reward the model for exceeding constraint requirements and incentivize the model to focus on particular constraints. The penalties and rewards are assigned to restrict or permit the violation of each constraint and are easily adjusted to account for changes in model objectives.

The penalty and reward values must be scaled to properly evaluate their impact on the model's assignments and the objective function. For both the penalty and reward weights, the maximum achievable sum for the $Y$ and $Z$ variables are found. This is done by changing the model objective function to maximize the sum of each of the sets of elastic variables. For example, to find the maximum sum of the target $Y$ variable, the model is solved to maximize $\sum_{a \in \mathcal{A}} Y_a^{\mathrm{T}}$. This finds the maximum total deviation from the AFSC target values the model may achieve. Similarly, the maximum sum for the target $Z$ variables is found by maximizing $\sum_{a \in \mathcal{A}} Z_a^{\mathrm{T}}$. This finds the maximum amount the model can exceed the AFSC target values. Each of the elastic variable sets are maximized in this manner. This is done to ensure that a variable set which can achieve a large maximum value does not have a greater impact on the objective function than a variable set with a smaller maximum value simply because more cadets and AFSCs are present within the former constraint. This makes it so that the variable sets which can achieve the largest values and those which can achieve the smallest values can be considered equal in the objective function. The normalized penalties and rewards are multiplied by the $Y$ and $Z$ variables in the objective function as starting weights, and are displayed in Table 1.

**Table 1. Normalized penalties and weights for each elasticized constraint**

| Constraint | Penalty ($\mu$) | Reward ($\lambda$) |
|---|---|---|
| Target (4c) | 0.057 | 0.007 |
| Overclassification (4d) | 0.046 | 0.008 |
| Mandatory Education (4e) | 0.123 | 0.009 |
| Desired Education Lower (4f) | 0.175 | 0.007 |
| Desired Education Upper Bound (4g) | 1 | 1 |
| Permitted Education (4h) | 0.046 | 0.062 |
| Cadet Preference (4i) | 0.091 | 0.014 |
| Source of Commissioning Lower Bound (4j) | 0.670 | 0.026 |
| Source of Commissioning Upper Bound (4k) | 0.173 | 0.081 |
| Cadet Percentile Lower Bound (4m) | 0.575 | 0.028 |
| Cadet Percentile Upper Bound (4n) | 0.747 | 0.033 |
| Cadet Preference Order (4a) | - | 0.00007 |

The normalized penalties and rewards can be used to consider all of the elasticized constraints equally. When this is done, the resulting solution for model ($\hat{\mathcal{N}}$) violates the target values for 3 AFSCs, as shown in Figure 6. This run of the new model is solved in about 50 seconds. The run requires more time since there are more trade-offs when all constraints are considered equally.

Out of the 44 AFSCs, 9% are filled entirely by cadets who do not prefer the AFSC, and 27% of the AFSCs are filled entirely by cadets who prefer them. The same two AFSCs make up the completely non-voluntary assignments as in the reformulation solution. The remaining AFSCs are filled by both cadets that prefer them and cadets that do not. This result is displayed in Figure 7.

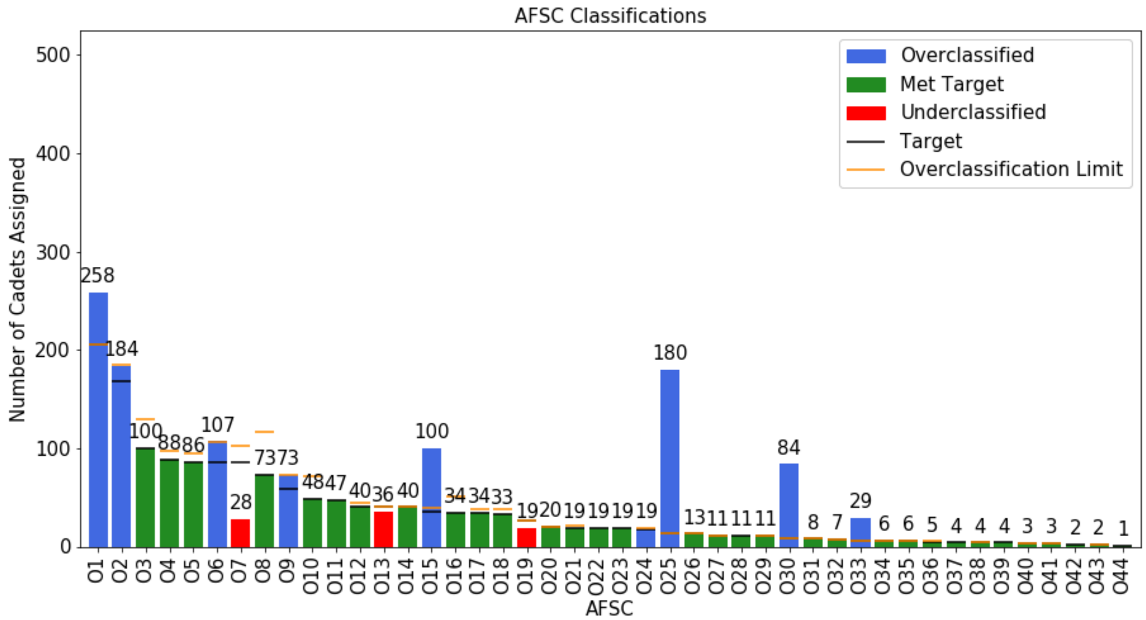


**Figure 6. Target and overclassification results for model ($\hat{\mathcal{N}}$) when all elasticized constraints are considered equally. The black lines on each bar indicate the target value for that AFSC and the orange lines indicate the overclassification limit for that AFSC. The targets for AFSCs O7, O13 and O19 are violated.**

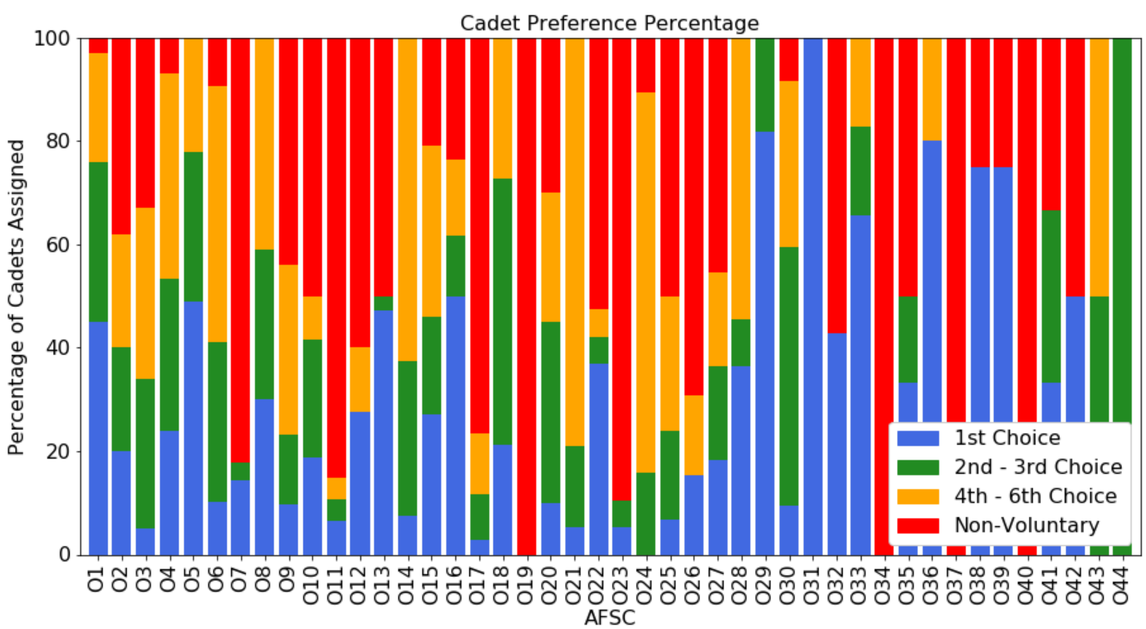


**Figure 7. Cadet preference results for model ($\hat{\mathcal{N}}$) on the original dataset when all elasticized constraints are considered equally. Out of the 44 AFSCs, 4 are completely filled by cadets who do not prefer them, and 12 are completely filled by cadets who do prefer them.**

The assignments result in all 15 large AFSCs falling within the desired bounds for cadet percentile for large AFSCs, and 6 of the 15 large of AFSCs falling within the desired bounds for the source of commissioning balance for large AFSCs, as shown in Figures 8 and 9, respectively. AFSC O13 has a higher percentage of USAFA cadets assigned to the AFSC than required, while the remaining violations are below the desired range.

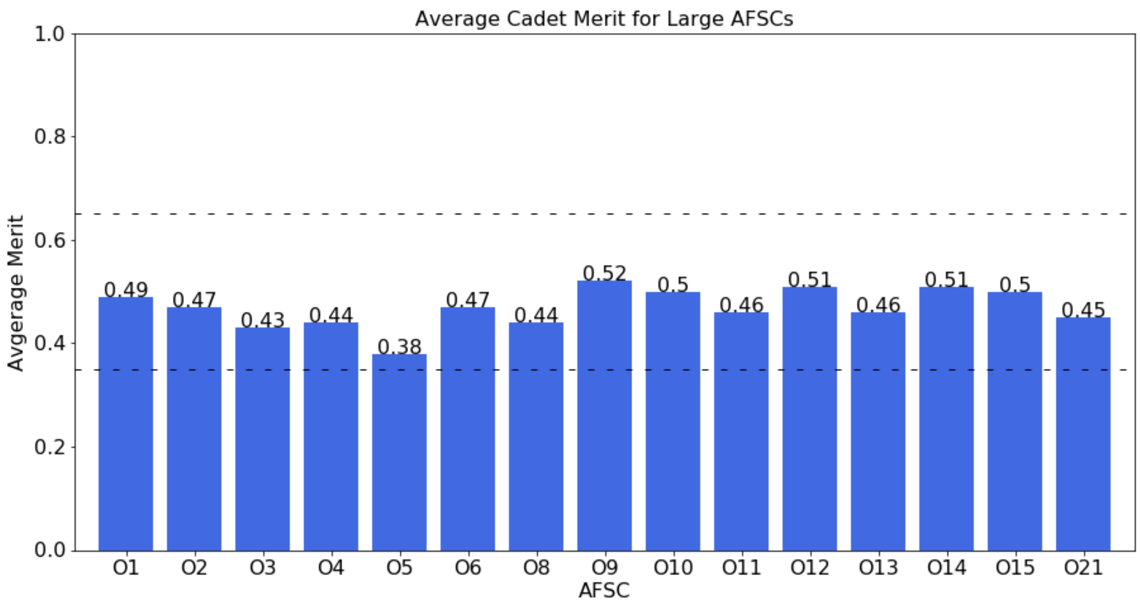


**Figure 8. Cadet percentile balance results for model ($\hat{\mathcal{N}}$) when all elasticized constraints are considered equally. The dashed lines indicate the desired bounds for the average cadet merit to fall within. All large AFSCs meet the desired average cadet merit result.**

The mandatory education constraint is met for 30 of the 36 AFSCs with mandatory education requirements. AFSC O7 only has mandatory constraints and does not meet its target value or the mandatory education requirement by 57 cadets because there are only 28 cadets available with the required degree for this AFSC. AFSC O19 does not meet its desired target value or the mandatory education requirement by 8 cadets. This is because there are only 70 cadets with mandatory degrees for both AFSCs O19 and O11, but these AFSCs have a combined target of 74 cadets, all of

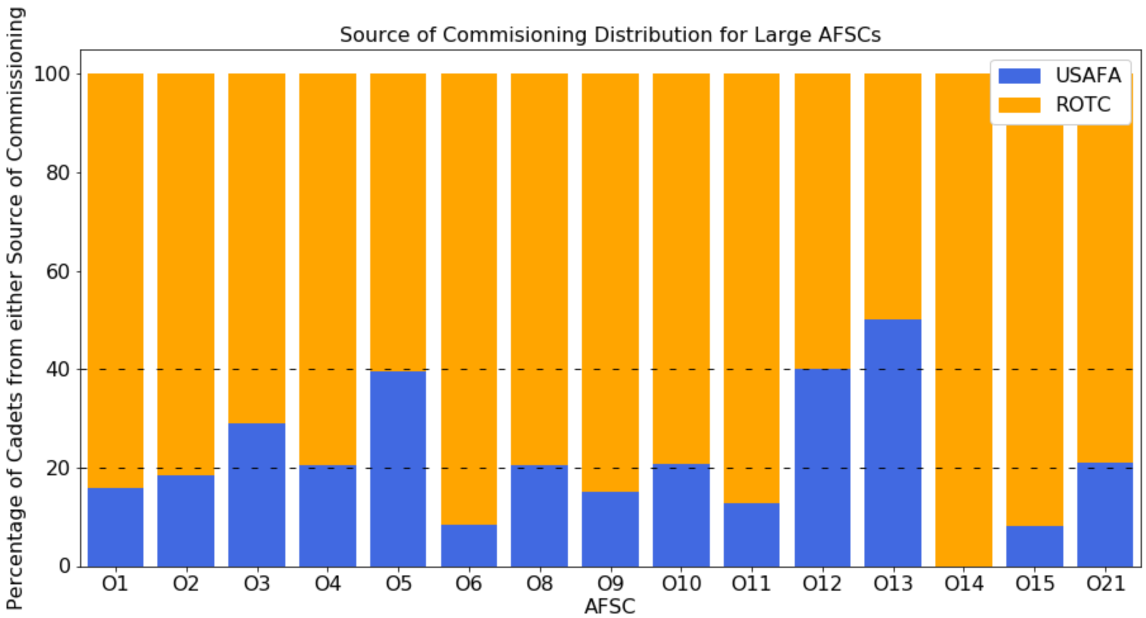


**Figure 9. Source of commissioning balance results for model ($\hat{\mathcal{N}}$) when all elasticized constraints are considered equally. The dashed lines indicate the desired bounds for the source of commissioning split to fall within. AFSC O14 is not assigned any USAFA cadets, which is not a desirable result.**

whom must have mandatory degrees. The remaining AFSCs only miss their requirements by a fractional amount. The desired education lower bound requirement is violated by one cadet for five AFSCs. The desired education upper bound is met by all three of the AFSCs with those requirements. The permitted education constraint is met for 14 of the 21 AFSCs with permitted education requirements. Most of the violations are by only a few cadets, but AFSC O8 violates the limit by 29 cadets. It is not possible to meet all of the education requirements at the same time. This is discussed further in Section 4.2.1. This weighting does achieve positive performance with some constraints, such as the mandatory and desired education constraints, but is not successful in other constraints, such as cadet preference, target, and overclassification. It is not reasonable to consider each constraint equally when attempting to find the best possible set of assignments – constraint importance must be factored into the weightings for the rewards and penalties of the elastic variables.

The solution in which all constraints are considered equally does not meet one of the major constraints of the cadet-AFSC problem – the AFSC target constraint. Thus, this solution cannot be considered reasonable for assigning cadets to AFSCs. The following sections detail the process of developing the ideal penalty and reward weights for the given cadet-AFSC data set. Greedy solution methods are implemented to determine the best possible cadet assignments when focusing on particular sets of constraints. The results of the greedy solutions, along with the priorities of HAF/A1 and AFPC, are utilized to generate penalty and reward weights to optimize the assignments while considering the restrictions of the dataset. The various weighting schemes are compared and "best" solutions, provided the cadet and AFSC data, are suggested.

### 4.2.1 Greedy Solutions

Greedy solution methods are explored to identify the best possible solutions when focusing on particular constraints. These solutions are beneficial in determining the relationships between constraints and to determine the quality of the cadet-AFSC assignments. The majority of constraints within the model rely on the target value of the AFSCs being evaluated. As such, the target constraint is included as part of each of the greedy solutions. The constraints each greedy solution focuses on are as follows:

(a) the target constraint, (4c)

(b) the target and overclassification constraints, (4c) and (4d)

(c) the target and mandatory education constraints, (4c) and (4e)

(d) the target and desired education constraints, (4c), (4f), and (4g)

(e) the target and permitted education constraints, (4c) and (4h)

(f) the target and cadet preference constraints, (4c) and (4i)

(g) the target and source of commissioning constraints, (4c), (4j) and (4k)

(h) the target and cadet percentile constraints, (4c), (4m) and (4n)

Table 2 provides the penalty and reward values assigned to each of the constraints for each greedy solution. These values are based on the normalization described in the previous section so that the constraints within the greedy solutions may be considered equally. Within the greedy solutions, the model focus is directed towards the model penalties, and not rewards. This prevents the model incentivizing any solution which may detract from the "best" possible performance of the greedy solutions.

**Table 2. Penalty and reward scaling for greedy solutions. The bold values indicate the rewards and penalties for the applicable constraints across each run.**

| | Greedy Solution Focus | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Penalty/ Reward | (a) | (b) | (c) | (d) | (e) | (f) | (g) | (h) |
| $\mu^{\mathrm{T}}$ | **100** | **100** | **100** | **100** | **100** | **100** | **100** | **100** |
| $\lambda^{\mathrm{T}}$ | **0.007** | **0.007** | **0.007** | **0.007** | **0.007** | **0.007** | **0.007** | **0.007** |
| $\mu^{\mathrm{F}}$ | 0.046 | **50** | 0.046 | 0.046 | 0.046 | 0.046 | 0.046 | 0.046 |
| $\lambda^{\mathrm{F}}$ | 0.008 | **0.008** | 0.008 | 0.008 | 0.008 | 0.008 | 0.008 | 0.008 |
| $\mu^{\mathrm{M}}$ | 0.123 | 0.123 | **50** | 0.123 | 0.123 | 0.123 | 0.123 | 0.123 |
| $\lambda^{\mathrm{M}}$ | 0.009 | 0.009 | **0.009** | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 |
| $\mu^{\underline{\mathrm{D}}}$ | 0.175 | 0.175 | 0.175 | **50** | 0.175 | 0.175 | 0.175 | 0.175 |
| $\lambda^{\underline{\mathrm{D}}}$ | 0.007 | 0.007 | 0.007 | **0.007** | 0.007 | 0.007 | 0.007 | 0.007 |
| $\mu^{\overline{\mathrm{D}}}$ | 1 | 1 | 1 | **50** | 1 | 1 | 1 | 1 |
| $\lambda^{\overline{\mathrm{D}}}$ | 1 | 1 | 1 | **1** | 1 | 1 | 1 | 1 |
| $\mu^{\mathrm{P}}$ | 0.046 | 0.046 | 0.046 | 0.046 | **50** | 0.046 | 0.046 | 0.046 |
| $\lambda^{\mathrm{P}}$ | 0.062 | 0.062 | 0.062 | 0.062 | **0.062** | 0.062 | 0.062 | 0.062 |
| $\mu^{\mathrm{W}}$ | 0.091 | 0.091 | 0.091 | 0.091 | 0.091 | **50** | 0.091 | 0.091 |
| $\lambda^{\mathrm{W}}$ | 0.014 | 0.014 | 0.014 | 0.014 | 0.014 | **0.014** | 0.014 | 0.014 |
| $\mu^{\underline{\mathrm{U}}}$ | 0.670 | 0.670 | 0.670 | 0.670 | 0.670 | 0.670 | **50** | 0.670 |
| $\lambda^{\underline{\mathrm{U}}}$ | 0.026 | 0.026 | 0.026 | 0.026 | 0.026 | 0.026 | **0.026** | 0.026 |
| $\mu^{\overline{\mathrm{U}}}$ | 0.173 | 0.173 | 0.173 | 0.173 | 0.173 | 0.173 | **50** | 0.173 |
| $\lambda^{\overline{\mathrm{U}}}$ | 0.081 | 0.081 | 0.081 | 0.081 | 0.081 | 0.081 | **0.081** | 0.081 |
| $\mu^{\underline{\mathrm{R}}}$ | 0.575 | 0.575 | 0.575 | 0.575 | 0.575 | 0.575 | 0.575 | **50** |
| $\lambda^{\underline{\mathrm{R}}}$ | 0.028 | 0.028 | 0.028 | 0.028 | 0.028 | 0.028 | 0.028 | **0.028** |
| $\mu^{\overline{\mathrm{R}}}$ | 0.747 | 0.747 | 0.747 | 0.747 | 0.747 | 0.747 | 0.747 | **50** |
| $\lambda^{\overline{\mathrm{R}}}$ | 0.033 | 0.033 | 0.033 | 0.033 | 0.033 | 0.033 | 0.033 | **0.033** |
| $\lambda^{\mathrm{S}}$ | 7E-05 | 7E-05 | 7E-05 | 7E-05 | 7E-05 | **0.05** | 7E-05 | 7E-05 |

Greedy solution (a) focuses the model on meeting the target constraint. This goal is not achieved, however. The target constraint is not met for two AFSCs - O7 and O19, which can be seen in Figure 10. The target for AFSC O7 is missed by 57 cadets and the target for AFSC O19 is missed by 8 cadets. The two AFSCs have mandatory education requirements, which suggests there may not be enough cadets with mandatory degrees to meet the AFSC targets. The results of this greedy solution show that it is not possible to meet the target constraints given the cadet and AFSC data and this result is expected across the remaining runs. This is confirmed by removing the elastic variables from the target constraint and achieving an infeasible result. Since many of the remaining constraints rely on the target values of each AFSC, the target values are adjusted in the data so this constraint is met for all AFSCs. The remaining solutions are evaluated using the updated AFSC target data. This allows for sets of weights to be developed which achieve the various goals of AFPC. If the data was not adjusted, the results of all of the runs would be inadequate in meeting the aforementioned priorities. The adjusted AFSC data is used, and the AFSCs will be referenced as A1–A44, for the remainder of the analysis.

Greedy solution (b) focuses on the target and overclassification constraints. The AFSC assignments are given in Figure 11. The overclassification constraint is not met by three AFSCs in the solution. AFSCs A24, A30 and A33 violate their overclassification limits by 47, 37, and 19 cadets, respectively. Additionally, AFSCs A27, A39 and A44 are filled a large amount over their target values. However, these AFSCs do not have overclassification limits. There are 235 more cadets than there are slots for assignments between the overclassification limits and targets. Regardless of the penalty and reward weights, this results in either the AFSCs without overclassification constraints being largely overfilled or violations of the overclassification constraints for the remaining AFSCs. It is unreasonable to accept model results which overfill any

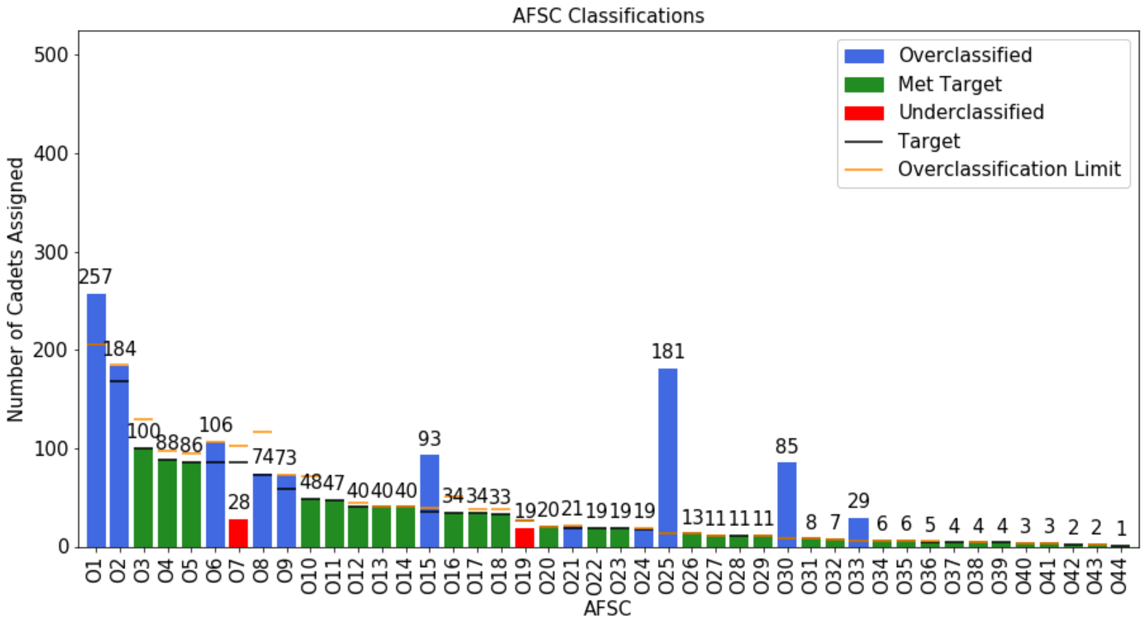


**Figure 10. Target and overclassification results for greedy solution (a). The black lines on each bar indicate the target value for that AFSC. Two AFSCs violate their desired target values.**

AFSC excessively. Alternative methods for accounting for AFSC overclassification are explored later in the analysis.

Greedy solution (c) changes the model focus to the mandatory education constraint. Table 3 displays the results of the solution for the mandatory education constraint. Two of the AFSCs, A16 and A34 violate their mandatory education requirements by one cadet. When the mandatory education requirements are changed from an elasticized constraint to a hard constraint, the model does find a feasible solution for the cadet-AFSC assignments. This shows that it is possible to meet all of the mandatory education requirements, but that higher reward and penalty values may need to be used. Too high of a reward or penalty value may detract from the model performance across other constraints, so caution must be taken with weight assingments. Greedy solution (d) focuses the model on the desired education constraints. The desired education requirements are met by all AFSCs with the requirements.

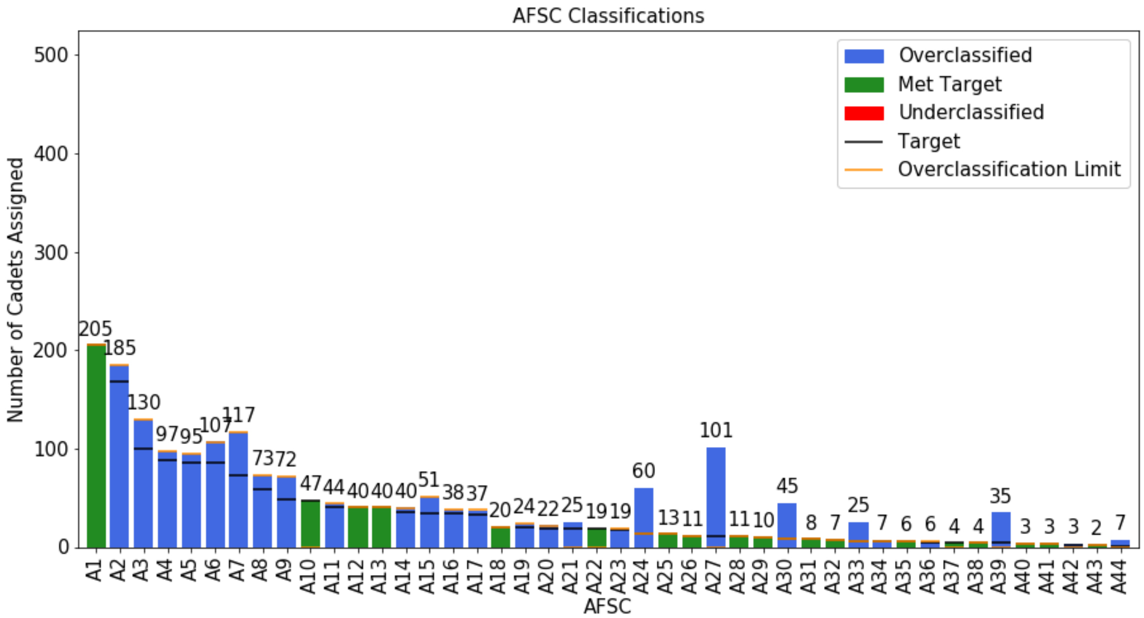


**Figure 11. Target and overclassification results for greedy solution (b). The black lines on each bar indicate the target value for that AFSC. The orange lines indicate the overclassification limit for that AFSC. Three of the AFSCs with overclassification limits violate said limits.**

Thus, it is possible to meet all of the desired education requirements within future solutions. Greedy solution (e) directs the model focus to the permitted education constraint. The permitted education limits are violated by four AFSCs. As with the mandatory education constraint, the model provides a feasible solution when the permitted education constraint is modeled as a hard constraint. This shows that it is possible to meet all of the permitted education requirements, but that higher reward and penalty values may need to be used. For each of the education greedy solutions the primary constraint is met, but one or both of the others is violated, showing that it is not possible to meet all of the education requirements concurrently.

Greedy solution (f) focuses on achieving cadet preference within the assignments, while still meeting the target goal for each AFSC. Specifically, the constraint forces that each cadet be assigned to an AFSC on their preference list. Figure 12 displays

**Table 3. Mandatory education results for greedy solution (c). The "Total Available" column indicates the number of cadets with mandatory degrees for each AFSC. The "Minimum Required" column indicates the minimum number of cadets with mandatory degrees required to fill each AFSC according to the mandatory education constraint. The "Difference" column indicates the difference between the required number of cadets and the number of cadets with mandatory degrees assigned by the model. The red rows indicate that the mandatory education constraint was violated for that AFSC. Bold rows indicate instances where the AFSC is assigned more than double the minimum required cadets with mandatory degrees. AFSCs with a difference of zero are omitted.**

| AFSC | Total Available | Minimum Required | Assigned | Difference |
|---|---|---|---|---|
| A1 | 1139 | 154 | 303 | 149 |
| **A3** | **671** | **10** | **79** | **69** |
| A5 | 532 | 18 | 23 | 5 |
| A15 | 68 | 23 | 34 | 11 |
| A16 | 35 | 28 | 27 | -1 |
| A25 | 47 | 8 | 13 | 5 |
| A33 | 122 | 5 | 6 | 1 |
| A34 | 300 | 5 | 4 | -1 |

the results of the solution in terms of cadet preference. Only 2 of the 44 AFSCs are completely filled by cadets who do not prefer them, while 21 of the AFSCs are completely filled by cadets who prefer them. The two AFSCs filled by cadets who do not prefer them are assigned 9 cadets total. Overall, 83% of cadets received an assignment that was in their top three preferences. If this constraint is changed to a hard constraint, the model is infeasible, and thus it is not possible to meet this constraint in further solutions.

Greedy solution (g) is concerned with the balance of USAFA and ROTC cadets within large AFSCs. The results for the source of commissioning balance within large AFSCs are presented in Figure 13. The desired source of commissioning balance is not achieved for 4 of the 15 large AFSCs. When the source of commissioning balance constraints are transformed from elasticized constraints to hard constraints, the model is no longer feasible; so it is not possible to achieve this goal for the given set of data.

Greedy solution (h) seeks to balance the average cadet merit within large AFSCs. Figure 14 displays the results for average cadet percentile within large AFSCs. For the greedy greedy solution, the constraint is met for all 15 of the large AFSCs.

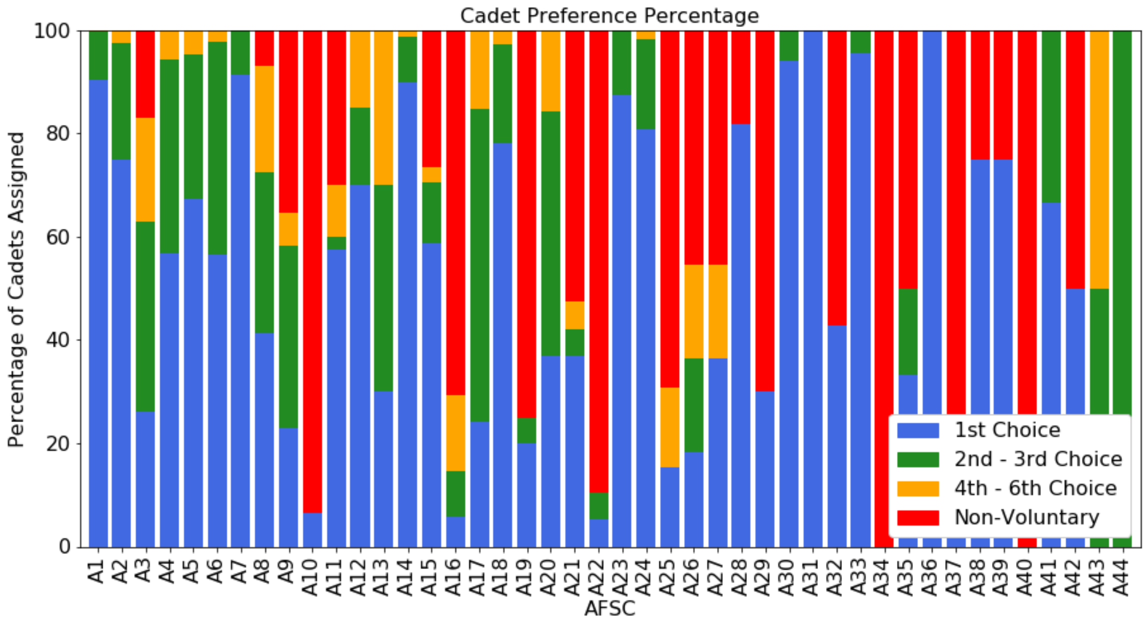


**Figure 12. Cadet preference results for greedy solution (f). Out of the 44 AFSCs, 2 are completely filled by cadets who do not prefer them and 21 are completely filled by cadets who do.**

Most of the greedy solutions require between 10 and 50 seconds to run. The greedy solution for the source of commissioning distribution, (g) requires 245 seconds to run, which is due to the inability to satisfy the constraints. The greedy solutions generally require more time to run than the solutions in which all are within the model focus. The greedy solutions provide decision makers with a reference for the best possible outcomes for each of the different priorities by focusing on a subset of constraints. This is important to developing a set of weights that meets the goals of AFPC and HAF/A1. The desired source of commissioning bounds cannot be met for large AFSCs given cadet and AFSC data, so this should be considered when evaluating the final solutions. The overclassification constraints are also not met, but should still be considered in the ideal weight formulation to spread out the assignment of extra cadets across the AFSCs. The remaining constraints are able to be met with their greedy solutions, but this does not mean they will be met when other constraints

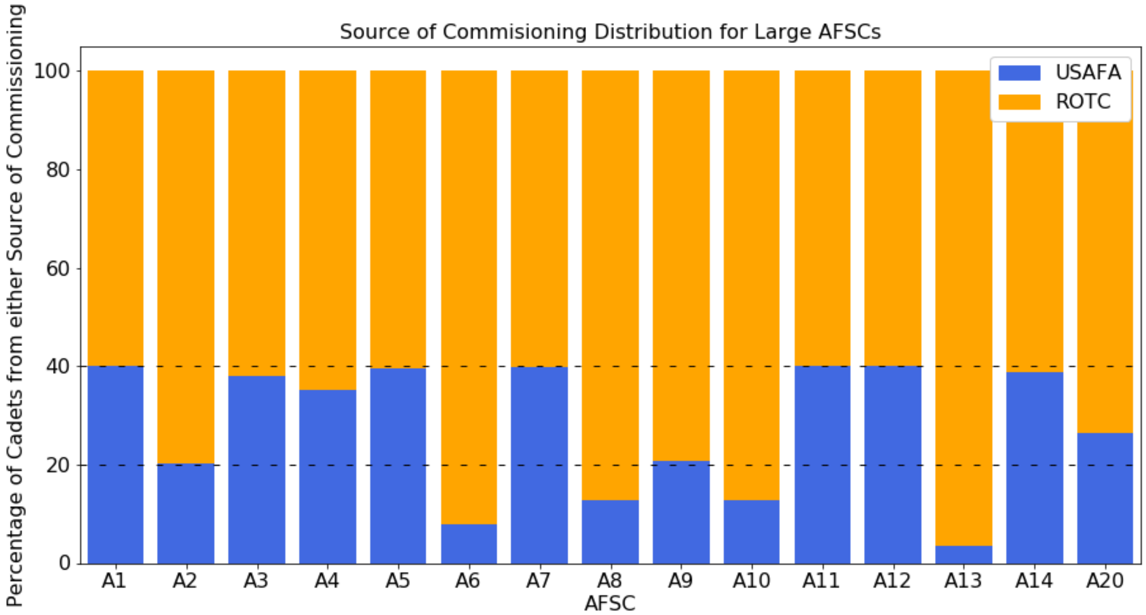


**Figure 13. Cadet source of commissioning results for large AFSCs for greedy solution run (g). Out of the 15 large AFSCs, 4 do not meet the desired source of commissioning bounds when the model focus is on the target and source of commissioning constraints.**

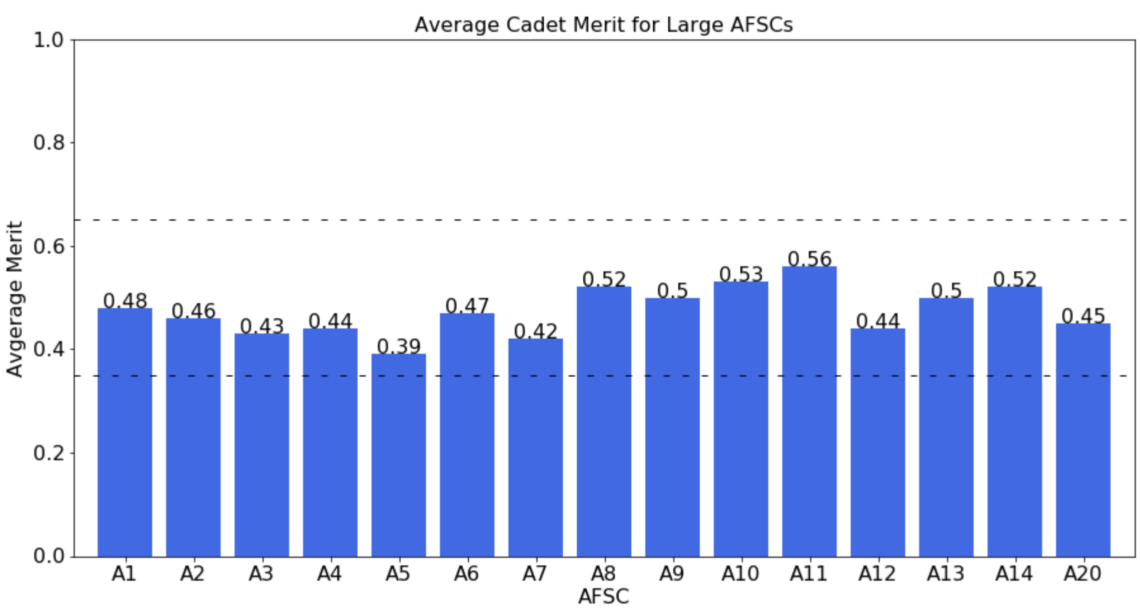


**Figure 14. Cadet percentile results for large AFSCs for greedy solution run (h). For this run, the desired cadet percentile bounds are achieved by all 15 large AFSCs.**

are considered. The interactions between constraints can cause the solution to worsen in performance for individual constraints, but will provide a better overall solution which prioritizes the goals of the model.

### 4.2.2 Additional Solutions

Additional solutions are developed to test the capability of the model to generate assignments that account for the prioritization of each of the goals. Table 4 contains the penalties and rewards assigned for two weighting scenarios: Run 1 and Run 2. The priorities of the two runs are based on the previous model and information from the AFOCD. The target and overclassification constraints must be met to satisfy the manning requirements for each AFSC. The AFOCD [2] states, "When distributing officer accessions across the education needs of all AFSs, [mandatory degree] requirements are considered first until the pool of available accessions with matching education has been exhausted. [Desired degree] accessions will then be considered, then [permitted], and so on". Thus, the mandatory education constraints are assigned a higher priority than the desired and permitted. The source of commissioning distribution and average cadet merit are prioritized towards the middle of the group of constraints because they are requested by AFPC and were considered in the original formulation, but are not dictated by policy. The desired and permitted education and cadet preference constraints receive the lowest priority because they are not included as constraints in the original model. These runs are two examples of possible weighting scenarios; ultimately, the weights are selected by the decision maker to achieve the desired prioritizations.

Run 1 attempts to reflect the importance of each goal as they are represented in the original model of AFPC, shown in Appendix A. In this run, the target constraint receives the highest penalty, followed by the overclassification and mandatory education constraints. The balance constraints for the source of commissioning and average cadet merit within large AFSCs follow. The constraints for desired and permitted education requirements, along with cadet preference, are given the lowest priority, since they are only present in the model's objective function, and are not constraints. Rewards are not assigned within the first run, with the exception of the cadet preference order. The lack of rewards forces the model to focus on not violating the elasticized constraints.

Run 2 tests a different weighting of priorities, where the mandatory education and overclassification constraints are considered to be as important as the target constraints. The source of commissioning distribution and average cadet percentile balance constraints are prioritized next, and then the remaining constraints. The run also tests the impact of the inclusion of rewards for the source of commissioning and average cadet merit, desired and permitted education, and cadet preference constraints. Both Run 1 and Run 2 require less than 30 seconds to find a feasible set of assignments. This is a similar result to the reformulated model ($\mathcal{N}$). Assigning larger weights across each of the constraints allows the model to come to a solution in less time.

The greedy solutions identified that the overclassification constraint will always be violated within the solutions, which is expected due to a higher number of cadets than available assignments. Additionally, those AFSCs without overclassification limits will be assigned the majority of the extra cadets. To account for these problems, two changes are made to the model inputs: the penalty for overclassification is scaled based on the target size of each AFSC and overclassification limits are established for those AFSCs that did not originally have them. These changes are made in an attempt to spread out the assignment of the extra cadets across AFSCs, as this is a more realistic solution than the model assigning all extra cadets to a few AFSCs.

**Table 4. Penalty and reward scaling for additional solutions.**

| Penalty, Reward | Run 1 | Run 2 |
|---|---|---|
| $\mu^{\mathrm{T}}, \lambda^{\mathrm{T}}$ | (100, 0) | (100, 0) |
| $\mu^{\mathrm{F}}, \lambda^{\mathrm{F}}$ | (75, 0) | (100, 0) |
| $\mu^{\mathrm{M}}, \lambda^{\mathrm{M}}$ | (75, 0) | (100, 0) |
| $\mu^{\underline{\mathrm{D}}}, \lambda^{\underline{\mathrm{D}}}$ | (25, 0) | (25, 2.5) |
| $\mu^{\overline{\mathrm{D}}}, \lambda^{\underline{\mathrm{D}}}$ | (25, 0) | (25, 2.5) |
| $\mu^{\mathrm{P}}, \lambda^{\mathrm{P}}$ | (25, 0) | (25, 2.5) |
| $\mu^{\mathrm{W}}, \lambda^{\mathrm{W}}$ | (25, 0) | (25, 2.5) |
| $\mu^{\underline{\mathrm{U}}}, \lambda^{\underline{\mathrm{U}}}$ | (50, 0) | (50, 5) |
| $\mu^{\overline{\mathrm{U}}}, \lambda^{\overline{\mathrm{U}}}$ | (50, 0) | (50, 5) |
| $\mu^{\underline{\mathrm{R}}}, \lambda^{\underline{\mathrm{R}}}$ | (50, 0) | (50, 5) |
| $\mu^{\overline{\mathrm{R}}}, \lambda^{\overline{\mathrm{R}}}$ | (50, 0) | (50, 5) |
| $\lambda^{\mathrm{S}}$ | 7E-05 | 7E-05 |

The results of Run 1 meet all target constraints, but violate the overclassification limits for 5 of the 44 AFSCs. AFSC A1 is assigned 297 more cadets than allowed by its overclassification maximum. The remaining violations are smaller in comparison. Figure 15 displays the classifications of each AFSC. The overclassification of cadets to AFSC A1 is an undesirable result and is influenced by the mandatory education requirements for the AFSC and the high number of cadets who prefer the AFSC.

The overall assignments result in 51% of cadets receiving an AFSC that is in their top three preferences. Figure 16 presents the results of Run 1 in terms of cadet preference. Out of the 44 AFSCs, 4 are completely filled by cadets who do not prefer them, and 8 are completely filled by cadets who do prefer them. Of the 4 AFSCs assigned to cadets non-voluntarily, only one does not have any cadets which prefer it. This is not a good result for cadet preference, as greedy solution (f) showed that it is possible to meet the targets for each AFSC and assign a majority of cadets to an AFSC which they prefer. The greedy solution resulted in 14 more AFSCs being filled by cadets who prefer them, and has 32% more cadets assigned to one of their top 3 preferences.

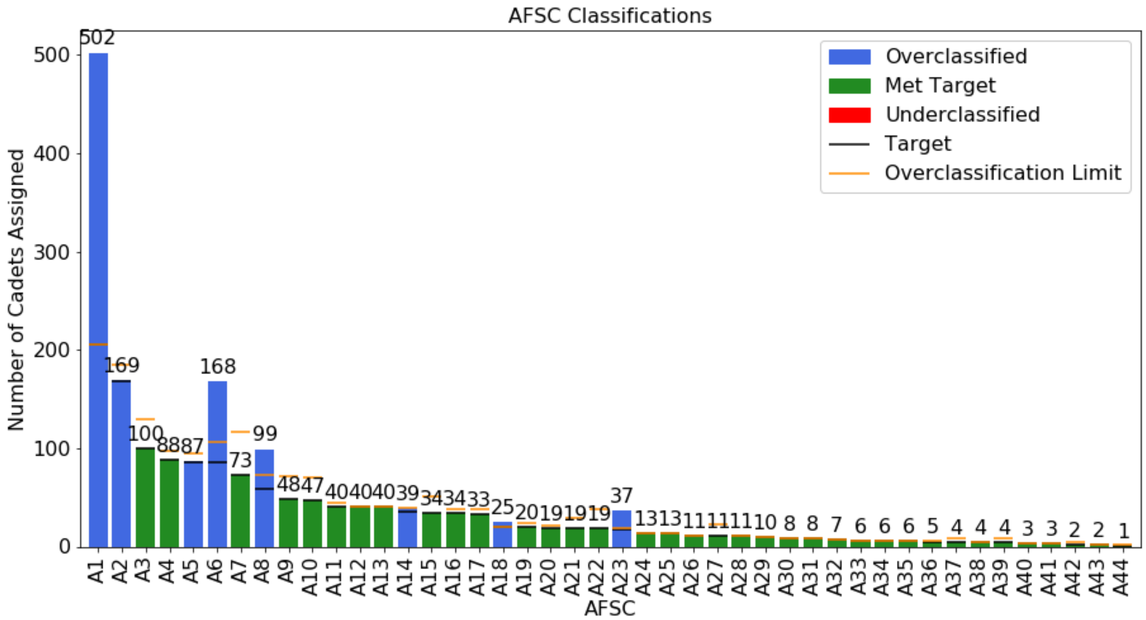


**Figure 15. Target and overclassification results for Run 1. The black lines on each bar indicate the target value for that AFSC. The orange lines indicate the overclassification limit for that AFSC. AFSC A1 is filled more than double its overclassification limit.**

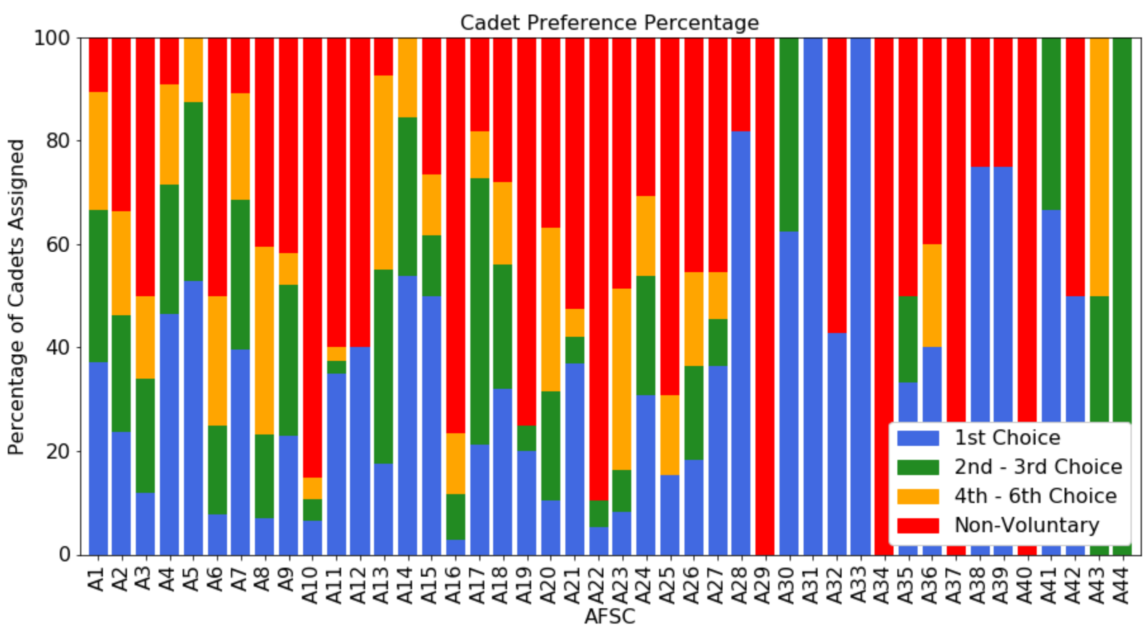


**Figure 16. Cadet preference results for Run 1. Out of the 44 AFSCs, 4 are completely filled by cadets who do not prefer them and 8 are completely filled by cadets who do.**

The average cadet merit for each large AFSC falls within the desired range for Run 1. This is a positive result because it is as good as greedy solution (h). Figure 17 presents these results. Figure 18 presents the source of commissioning distribution for large AFSCs. As expected, the desired bounds are not achieved by all AFSCs. This performance is worse than that of greedy solution (g), however, because 2 AFSCs are not assigned any USAFA cadets and a variety of cadet background is desired.

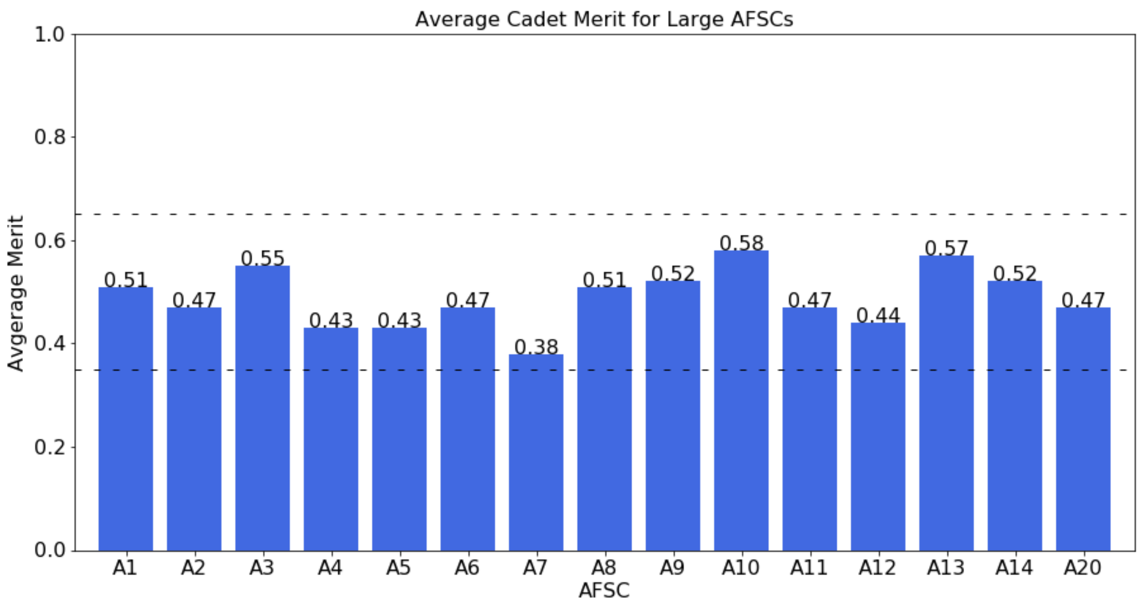


**Figure 17. Cadet percentile results for large AFSCs for Run 1. The dashed lines indicate the desired bounds for the average cadet percentile to fall within. For Run 1, all large AFSCs achieved the desired result for average cadet percentile.**

The mandatory education requirements of AFSCs A16 and A34 are violated, each by one cadet. This aligns with what was seen in greedy solution (c), but it is known from the greedy solution analysis that it is possible to meet all mandatory education requirements. It may be possible to meet these requirements by assigning a higher penalty or reward for the mandatory education constraints. The desired education requirements are achieved for all of the corresponding AFSCs. This follows the result from greedy solution (d). The permitted education requirements are violated for 4

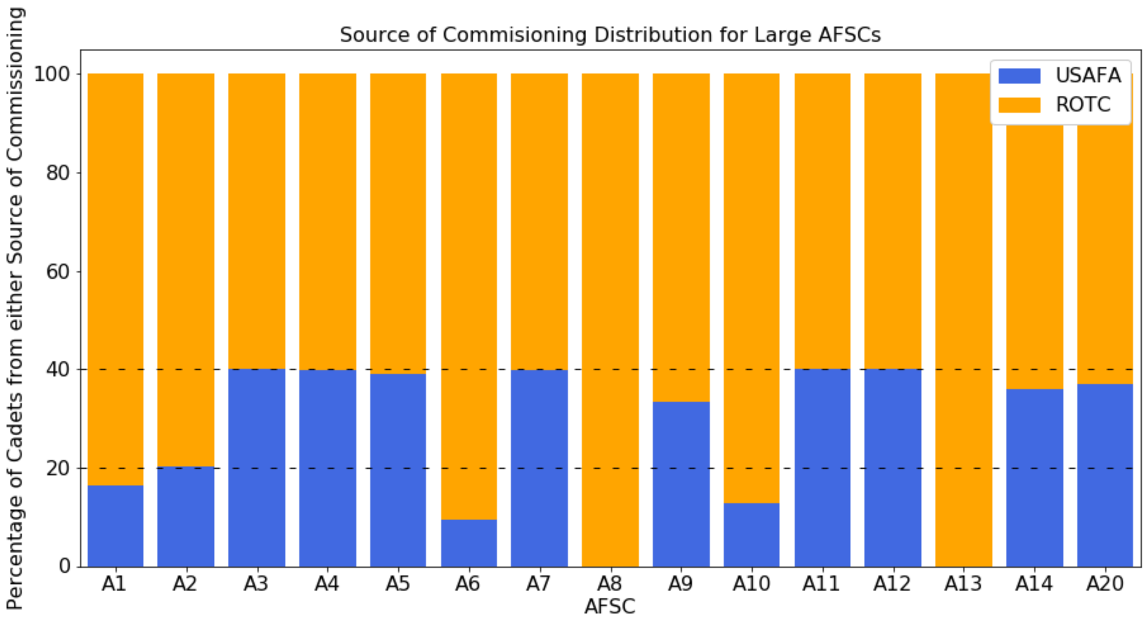


**Figure 18. Cadet source of commissioning results for large AFSCs for Run 1. The dashed lines indicate the desired range for the source of commissioning split to fall within. For Run 1, 5 AFSCs do not achieve the desired source of commissioning distribution, and 2 AFSCs have no USAFA assignments.**

AFSCs. The largest violation comes from AFSC A7, which is assigned 43 more cadets with permitted degrees than allowed by the education requirements. The remaining violations are small in comparison.

While the results from Run 1 are successful in many of the constraints, the overclassification of AFSCs is a major problem for the model. It is not mathematically incorrect for the model to assign the cadets as it is, but the results are not practical for the actual assignment of cadets. The extra cadets should be assigned across many career fields as opposed to a few. AFPC could encourage the model to assign cadets to those career fields willing to accept extra assignments by increasing the target reward, or decreasing the overclassification penalty, for those career fields. Additionally, the overclassification penalties may be raised for those career fields which do not wish to accept additional assignments.

The individual overclassification penalties and rewards assigned to each AFSC for Run 1 are adjusted to encourage the model to spread out the cadet-AFSC assignments. The penalties are raised for AFSCs that are violating their overclassification limits, such as A1, A6, A8, and A23, while the target rewards are raised for AFSCs which are not meeting their limit, such as A3, A4, A5, A9, and A15. The target rewards are scaled in the same way as the overclassification penalties. If the target rewards and the overclassification penalties for the aforementioned AFSCs are raised to 150, the classification result in Figure 19 is achieved. The assignments to AFSCs A1, A6, A8 and A23 decrease, while the assignments to AFSCs A3, A4, A5, A9 and A15 increase. The number of cadets assigned to AFSC A1 is still more than double its overclassification limit, which is undesirable, but the decrease shows that adjusting the individual penalties and rewards has a positive impact on the results of the overclassification constraint. Another remedy to this may be to impose an overall upper limit on assignments for each AFSC.

The resulting set of assignments has 52% of the total cadets assigned to an AFSC which is in their top 3 choices, but only 7 of the 44 AFSCs completely filled by cadets who prefer them. The average cadet merit balance is still achieved for all of the large AFSCs, and the source of commissioning distribution improves for large AFSCs, with only one of the AFSCs not having any USAFA cadet assignments. The performance for the mandatory, desired and permitted education requirements remains the same as before the individual penalties and rewards were introduced.

Run 2 also meets the required target goals, but has problems with overclassification of AFSCs. In this instance, 6 of the 44 AFSCs are overclassified past their upper limit for assignments, but 19 of the 44 AFSCs are assigned fewer cadets than their overclassification limit. The largest violations of the overclassification constraint come from AFSCs A1 and A14, with A1 having 186 more cadets assigned than its

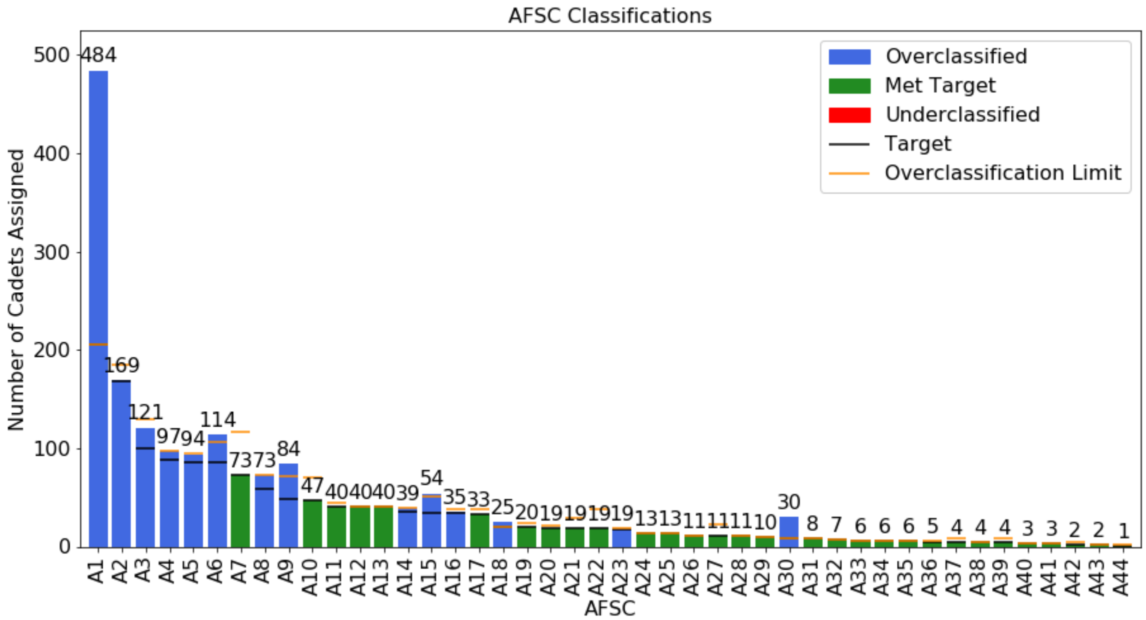


**Figure 19. Target and overclassification results for Run 1 with individual overclassification penalties and target rewards. The black lines on each bar indicate the target value for that AFSC. The orange lines indicate the overclassification limit for that AFSC. The spread of assignments are improved from Figure 15.**

overclassification limit, and A14 having 132 more cadets than its overclassification limit. This solution is an improvement from the first run because the overclassifications are better spread across AFSCs, but measures must still be taken to prevent the large overclassifications seen in AFSCs A1 and A14.

Run 2 results in 48% of the cadets receiving an assignment in their top 3 choices preferences, and 10 of the 44 AFSCs being completely filled by cadets who prefer them. While not all cadets are getting a preference that is in their top 3, more AFSCs are meeting the cadet preference goal in this run over Run 1. Figure 21 displays the preference distribution of the cadets across each AFSC.

As with Run 1, the average cadet merit constraints for large AFSCs are met with Run 2's penalty and reward weights. In this run, the desired source of commissioning distribution is not met for 5 of the 15 large AFSCs, but only 1 AFSC has no USAFA

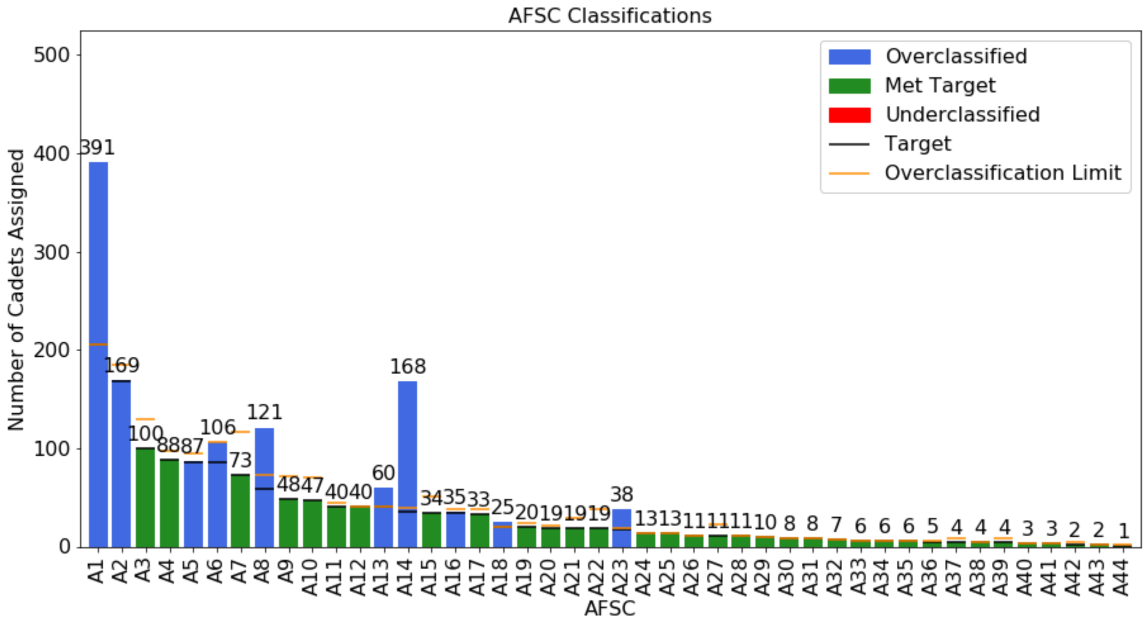


**Figure 20. Target and overclassification results for Run 2. The black lines on each bar indicate the target value for that AFSC. The orange lines indicate the overclassification limit for that AFSC. The targets are met for all AFSCs, but the overclassification limits are violated for 6 AFSCs.**

cadet assignments. This solution is an improvement from the first run, but the result is still undesirable since a variety in cadet background is desired within the large AFSCs. Figures 22 and 23 present the results of the balance constraints. The education requirement results for Run 2 are equivalent to those of Run 1. Assigning individual penalties and rewards to those AFSCs which are violating the constraints may improve their performance, but may cause the cadet assignments to shift.

The results of Run 2 show that increasing the penalties of the overclassification and mandatory education constraints while introducing rewards for all but the most important constraints results in an improved solution from what was achieved in Run 1. The overclassification of AFSCs still negatively impact the model results, as it is not reasonable to assign almost double the number of requested cadets to an AFSC. Increasing the overclassification penalties for AFSCs A1, A8, A14 and A23

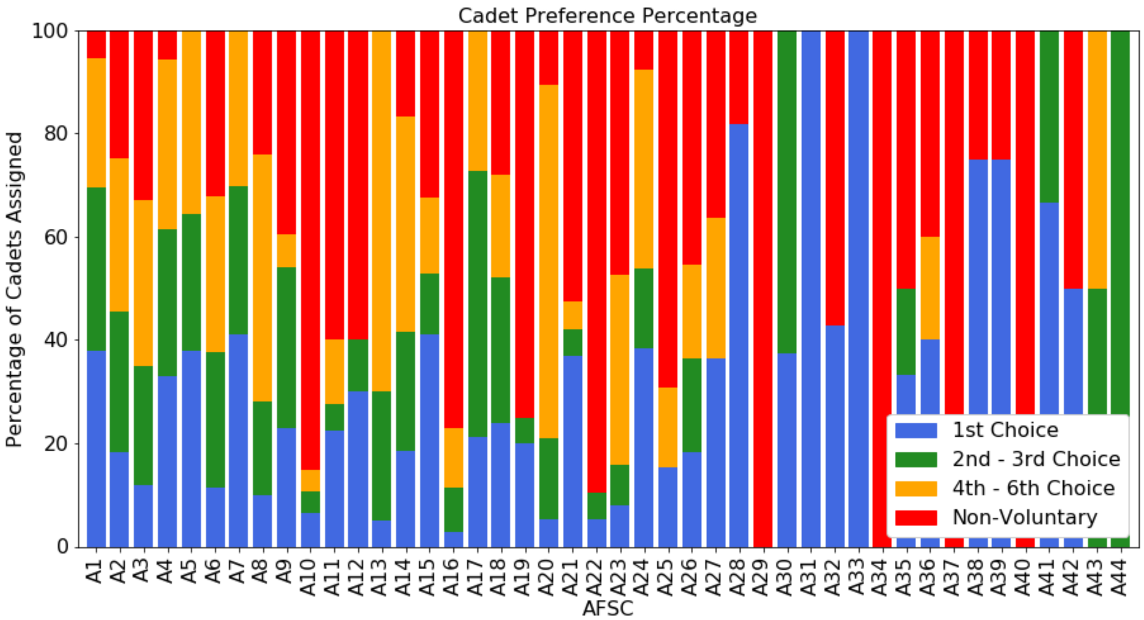


**Figure 21. Cadet preference results for Run 2. Out of the 44 AFSCs, 4 are completely filled by cadets who do not prefer them, and 10 are completely filled by cadets who do prefer them.**

and decreasing the respective penalties for those AFSCs which are not being filled to their overclassification limit results in the classifications shown in Figure 24. The classifications decrease for those AFSCs with increased penalties. However, it can be seen that the assignments from the 4 AFSCs are just shifted to other AFSCs with the penalty change. As more individual penalties are increased or decreased, the assignments continue to shift from AFSC to AFSC until the model reaches a solution which spreads the AFSC assignments enough to say the overclassifications are reasonable. The decision for what assignments are considered reasonable would come from AFPC, considering those career fields which are willing to accept the extra assignments and which are not. It is also important to note that as the model is forced to shift assignments via the penalty changes, the performance of other constraints may decline. Additionally, forcing the model assignments by drastic changes in penalties reduces the impact and functionality of model $(\hat{\mathcal{N}})$. If the penalties and rewards are

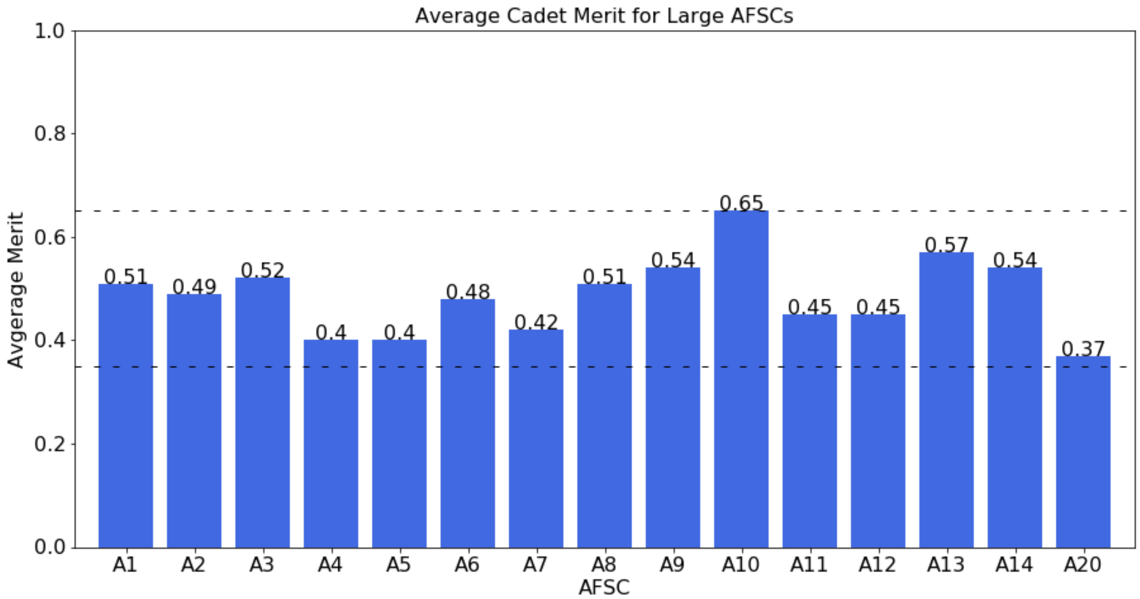


**Figure 22. Cadet percentile results for large AFSCs for Run 2. The dashed lines indicate the desired bounds for the average cadet percentile to fall within. For Run 2, all AFSCs meet the desired bounds for average cadet percentile.**

adjusted too much, the user essentially determines the classification of each AFSC for the model. The purpose of the model is to generate a set of assignments which consider the prioritization of the goals dictated by AFPC and HAF/A1 when the inputs for the model may generate an infeasible result, and excessive user input over the rewards and penalties detracts from this purpose.

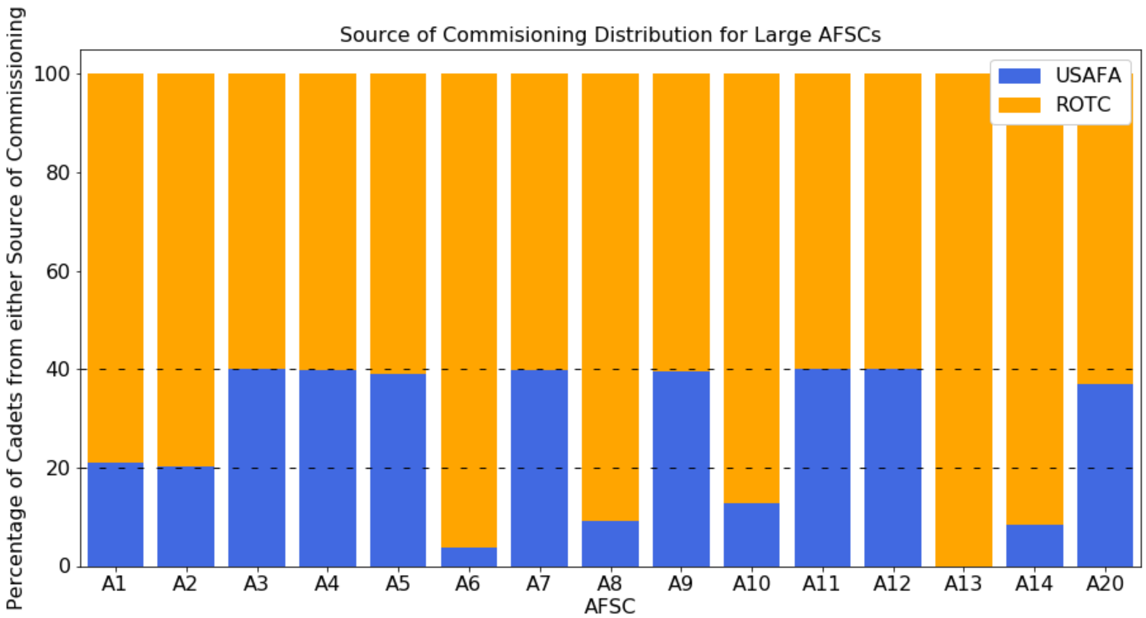


**Figure 23. Cadet source of commissioning results for large AFSCs for Run 2. The dashed lines indicate the desired range for the source of commissioning split to fall within. Out of the 15 large AFSCs, 5 do not meet the desired dstribution of USAFA and ROTC cadets, and one does not have any USAFA assignments.**

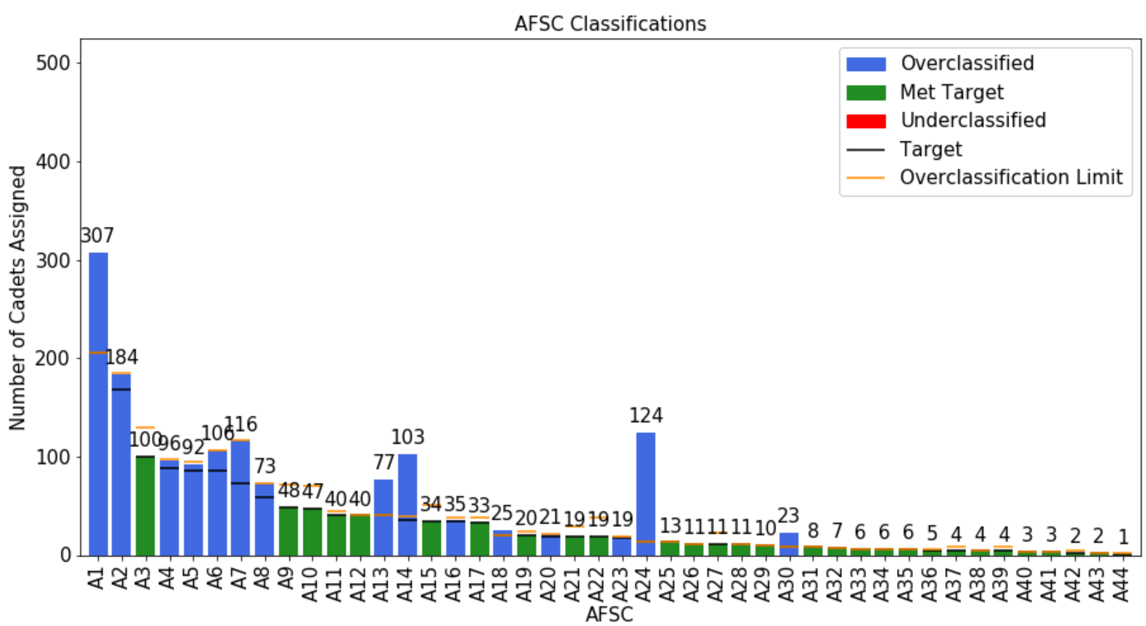


**Figure 24. Target and overclassification results for Run 2 with individual overclassification penalties. The black lines on each bar indicate the target value for that AFSC. The orange lines indicate the overclassification limit for that AFSC. The extra cadets are more spread out than in Figure 20, but there are still AFSCs filled largely past their classification limits.**

### 4.3 Analytic Conclusions

The reformulation of AFPC's model ($\mathcal{N}$) and the new model ($\hat{\mathcal{N}}$) were analyzed to determine an ideal weighting scheme to achieve the goals from AFPC. Model ($\mathcal{N}$) achieves the goals of AFPC, but permits ineligible cadets to be assigned to AFSCs. These assignments are detrimental to both the cadet and the career field, as the training process for a cadet who does not have the required educational background is tedious and costly. Though the model does not provide a desirable solution, its weaknesses highlight areas of improvement for model ($\hat{\mathcal{N}}$). The greedy solutions alone do not meet all of the goals of AFPC, but inform the user of the best possible solutions for each of the elasticized constraints. The overall best model performance is achieved when the model is focused on the target, overclassification, and mandatory education constraints, given the cadet and AFSC data used for analysis. However, it is necessary to receive further input from decision makers to determine which goals are most important in selecting the optimal weighting scheme. The model attempts to balance conflicting priorities, but decision makers must approach the assignments process with a clear prioritization of goals, since it is not likely that all will be achieved.

Individual penalty and reward assignments grant the user control over the model's assignment of cadets to AFSCs and promote a more even spread of extra assignments. However, this approach must be used with caution, as adjusting the rewards and penalties too far leads to a result which is hand-picked by the user and decreases the functionality of the model. Higher overclassification limits should be imposed where possible to spread out cadet assignments to AFSCs.

The guidelines from HAF/A1 determine the model focus. These priorities shift based on retention, officer performance, and other factors. The model rewards and penalties can be updated to address the shifts. The analysis described in this chapter may be performed each year to inform the model weights. The greedy solutions provide a baseline for the performance of each constraint and additional runs determine the necessary trade-offs to find the best weighting option for the year at hand. The presented analysis results are unique to the given cadet and AFSC data, but the techniques may be applied to each year of data to determine the best possible model weightings.

# V. Conclusion and Recommendations

The cadet-AFSC problem is a multidimensional assignment problem that must balance the priorities of HAF/A1 and AFPC with the background and preferences of the cadets being assigned. A mixed integer-linear programming formulation, $\hat{\mathcal{N}}$, was developed as a tool for determining the optimal assignments while accounting for the necessary trade-offs between priorities. The assignment process is not static, with new goals taking precedence each year, and the design of the formulation is intended to account for the dynamic nature of this problem.

## 5.1 Model Impact

The elastic constraints introduced in the new formulation allow certain priorities to be violated while still generating a set of cadet-AFSC assignments. The model can match cadets to AFSCs in instances where the problem would otherwise be infeasible. This is of value to AFPC in the assignments process, as well as the penalties and rewards associated with the elasticized constraints.

The penalties and rewards are adjusted to reflect the importance of each goal to AFPC and HAF/A1. Penalties prevent the model from violating constraints while rewards incentivize the model towards particular solutions. New data and priorities are easily addressed through model modification. The end user of this model can quickly evaluate a variety of solutions in which the weighting of priorities is shifted. The model is quick to adapt to changing data and priorities and allows for a more thorough analysis of the cadet-AFSC assignment solution space.

## 5.2 Future Research and Recommendations

A potential area for improvement within the model is the assignment of individual rewards and penalties for all constraints. This allows the end user to tailor the results of the model towards goals of specific AFSCs and easily achieve desired constraint performance. Additionally, the overclassification, average cadet merit, and desired source of commissioning balance constraints may be applied to all AFSCs. Combining this with the previous suggestion may improve model performance. A nonlinear or piecewise model can be developed to adjust the values of the penalties and rewards as certain constraints are met, to avoid exceeding the overclassification limits of individual AFSCs.

A tiered solution approach may also benefit the cadet-AFSC assignment process with the new model. In this approach, the model would be solved with the target, overclassification and mandatory education requirements as hard constraints. A dummy AFSC would be included for extra cadets to be assigned to once the target and overclassification requirements are met. Then, the results of the model, except for the dummy variable assignments, are fixed so that the cadet-AFSC pairs are not affected when the model is run again. After this is done, the model is run again, with the hard constraints elasticized, so that the additional cadets may be assigned.

The cadet-AFSC problem may also be approached using other solution methods, such as those described in Chapter II. The formulation developed through this research may serve as an entry point for exploring other methods. The penalty and rewards analysis may be used to develop a solution which is comparable to that of other models.

The goals of AFPC and HAF/A1 must be clearly prioritized before generating the cadet-AFSC assignments from the model. The greedy solutions may first be evaluated to determine the best possible performance for each constraint and may inform the decision makers of what can be achieved before assigning the model weights. If the importance of each goal is not expressed prior to generating the assignments, the model weights may be adjusted in a way that finds an attractive solution for the model that does not align with the desires of AFPC and HAF/A1. The goals shift in priority from year to year and clear guidance is necessary to generate the best possible assignments for the cadet-AFSC pairs.

# Appendix A. AFPC's Original Integer Program Formulation

## 1.1 AFPC's Original Formulation

| Sets and Indices | |
|---|---|
| $i$ | Cadets, $\mathcal{I} = 1, \ldots, n$ |
| $j$ | AFSCs , $\mathcal{J} = 1, \ldots, m$ |
| $g$ | Large AFSCs, $\mathcal{G} \subseteq \mathcal{J}$ |

| Parameters | | Units |
|---|---|---|
| $m_{ij}$ | 1 if cadet $i$ is mandatory for AFSC $j, 0$ otherwise | [-] |
| $u_i$ | 1 if cadet $i$ is a USAFA graduate, 0 otherwise | [-] |
| $p_i$ | percentile for cadet $i$ | [fraction] |
| $c_{ij}$ | utility gained by assigning cadet $i$ to AFSC $j$ | [fraction] |
| $t_j$ | target for AFSC $j$ | [fraction] |
| $f_j$ | factor by which we can overclassify AFSC $j$ | [fraction] |
| $d_j$ | mandatory target accession rate for AFSC $j$ | [fraction] |

| Decision Variables | | Units |
|---|---|---|
| $X_{ij}$ | 1 if cadet $i$ is assigned to AFSC $j$, and 0 otherwise | [-] |

### Formulation

$$\text{maximize} \quad \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} c_{ij}X_{ij} \tag{5a}$$

$$\text{subject to:} \quad \sum_{j\in\mathcal{J}} X_{ij} = 1, \ \forall i \in \mathcal{I} \tag{5b}$$

$$\sum_{i\in\mathcal{I}} X_{ij} \geq t_j, \ \forall j \in \mathcal{J} \tag{5c}$$

$$\sum_{i\in\mathcal{I}} X_{ij} \leq f_j t_j, \ \forall j \in \mathcal{J} \tag{5d}$$

$$\sum_{i\in\mathcal{I}} m_{ij}X_{ij} \geq d_j t_j, \ \forall j \in \mathcal{J} \tag{5e}$$

$$\sum_{i\in\mathcal{I}} u_i X_{ij} \geq 0.2 t_j, \ \forall j \in \mathcal{G} \tag{5f}$$

$$\sum_{i\in\mathcal{I}} u_i X_{ij} \leq 0.4 t_j, \ \forall j \in \mathcal{G} \tag{5g}$$

$$\sum_{i\in\mathcal{I}} p_i X_{ij} \geq 0.35 t_j, \ \forall j \in \mathcal{G} \tag{5h}$$

$$\sum_{i\in\mathcal{I}} p_i X_{ij} \leq 0.65 t_j, \ \forall j \in \mathcal{G} \tag{5i}$$

$$X_{ij} \in \{0, 1\}, \ \forall i, j \in \mathcal{I}, \mathcal{J} \tag{5j}$$

Constraint (5b) permits each cadet to only be assigned to one AFSC. Constraint (5c) ensures that the targets for each AFSC are met and constraint (5d) allows certain AFSCs to be overclassified by a certain factor. Constraint (5e) specifies the mandatory education constraints for the assignment of cadets to AFSCs. Constraints (5f) and (5g) force a specified distribution of USAFA cadets within certain AFSCs. Constraints (5h) and (5i) balance cadet merit across certain AFSCs. Constraint (5j) specifies that the decision variable is binary.

## 1.2 AFPC's Original Objective Function

The utility function applied in the objective function in the integer program formulation is as follows:

When AFSC $j$ is a preference for cadet $i$:

$$c_{ij} = \begin{cases} (10p_i \cdot weight) + 250 & \text{if } M \\ (10p_i \cdot weight) + 150 & \text{if } D \\ 10p_i \cdot weight & \text{if } P \\ -50000 & \text{otherwise} \end{cases} \tag{6}$$

When AFSC $j$ is not a preference for cadet $i$:

$$c_{ij} = \begin{cases} 100p_i & \text{if } M \\ 50p_i & \text{if } D \\ 0 & \text{if } P \\ -50000 & \text{otherwise} \end{cases} \tag{7}$$

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**14. ABSTRACT**

Each year, the Air Force Personnel Center determines which career field newly commissioned officers will serve under during their time in the Air Force. The career fields are assigned while considering five priorities, dictated by Headquarters Air Force, Manpower and Personnel: target number of cadets, education requirements, average cadet percentile, cadet source of commissioning, and cadet preference. A mixed-integer linear program with elasticized constraints is developed to generate cadet assignments according to these priorities. Each elasticized constraint carries an associated reward and penalty, which is used to dictate the importance of the constraint within the model. A subsequent analysis is conducted on historical data to display the interaction of the constraints and the impact of the rewards and penalties on the model results. The new formulation can generate a feasible set of assignments using the elasticized constraints in instances where the cadet and AFSC data would cause infeasibility in the original assignments model. It also provides users and decision makers with the ability to identify trade-offs between goals and prioritize each constraint.

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