DISCUSSION OF

"RIVER DEPLETION RESULTING FROM PUMPING A WELL NEAR A RIVER"

by R. E. Glover and G. G. Balmer

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Mahdi S. Hantush (Geophysics Department, University of Utah, Salt Lake City, Utah)--In addition to the notation used by the authors, the following is also used:

b = the thickness of the artesian aquifer, ft
b' = the thickness of the confining bed through which vertical leakage takes place in proportion to the drawdown, ft
K' = permeability of the confining bed, ft/sec
T = transmissibility of the aquifer which is equal to Kb in confined flow systems, and equals KD in unconfined flow systems, ft/sec
S = storage coefficient which, in artesian aquifers, is the volume of water that a unit decline of head releases from storage in a vertical prism of the aquifer of unit cross-section; its ultimate value in water table aquifers is the specific yield of the aquifer, which is designated by V in the authors' paper, demensionless
B = leakage factor, which equals $\sqrt{Tb'/K'}$ ft [HANTUSH and JACOB, 1954]
$\text{erf}(Z)$ = the error function or the probability integral = $(2/\sqrt{\pi})\int_0^Z e^{-v^2}\,dv$
$\text{erfc}(Z)$ = the complementary error function = $1 - \text{erf}(Z)$

This paper is a valuable piece of work in the field of hydrology. The authors did not discuss fully the result of their valuable work. The following points are therefore to be observed:

(1) The authors' Eq. (4) is basically for artesian aquifers, although it is used in water-table aquifers with certain limitations. Thus the result obtained by the authors is basically for artesian aquifers and, therefore, can be used without limitations provided, of course, the assumptions made in deriving (4) are met.

(2) Under water-table conditions, (4) is valid if $s/D \leqq 0.02$ [JACOB, 1950, pp. 376-377]. It can be shown that if the left side of (4) is replaced by $(s-s^2/2D)$, then the resulting expression is valid for $s/D \leqq 0.25$, provided that the storage coefficient remains constant [JACOB, 1950]. If the authors' analysis, or a slightly different one, is applied to the new modified equation, the final result will be identical to that of the authors' (12). Hence, under water-table conditions, (12) is valid for $s/D \leqq 0.25$ provided the storage coefficient is constant.

(3) To maintain a uniform head over the river, the authors applied the method of images on the equation for discharge (11) rather than on the equation for drawdown (4). While this approach is absolutely correct, it not only leaves the potential distribution in the aquifer unknown, but it necessitates also reducing the resulting complicated expression to a form suitable for computation, a process that may be avoided if the potential distribution for a particular problem is known. In the present problem the expression resulting from applying the method of images on the discharge equation is simple and needs no further reduction, in which case no particular advantage is obtained by basing the analysis on an already known potential distribution. However, one may be confused as to why the well should be imaged by the river bank when the total discharge across the river is independent of the position of the recharging well so long as the well is located on a line x_1 distant from and parallel to the river. The drawdown at the line x_1 remains zero only if the velocity induced by the recharging well at every point of the line x_1 is equal to that induced by the discharging well.

Potential distributions in several flow systems of different boundary conditions have been worked out [THEIS, 1941; HANTUSH and JACOB, 1954, 1955abc]. These developed solutions afford ready expressions which when combined with the generalized Darcy's Law make possible the computation of the quantity of flow through any section of the flow system.

(4) It may be of interest to mention here that a more general solution that takes into consideration the effect of leakage into the aquifer and of which (12) is a special case (leakage is zero) can be obtained by the same or a slightly different procedure from that followed by the authors.

The drawdown produced by the flow to a steadily discharging well located near a fairly long straight stretch of a river that cuts through an artesian leaky aquifer is given by HANTUSH and JACOB [1955b].

$$s = (Q/4\pi T)\int_1^{\infty} (e^{-u_1 v} - e^{-u_2 v})\,(e^{-w/v}/v)\,dv \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

where $u_1 = [(x-x_1)^2 + y^2]\,S/4Tt$, $u_2 = [(x+x_2)^2 + y^2]\,S/4Tt$, and $w = Tt/SB^2$. Leakage into or out of the artesian aquifer is assumed vertical and in proportion to the drawdown [JACOB, 1946].

If (13) is combined with the generalized equation of Darcy, namely, $q = K\int_A (\partial s/\partial x)\,dA$, the quantity of water drawn from the river will be given, after mathematical reduction, by the following relation

$$q = (Q/2)\,[e^{x_1/B}\,\mathrm{erfc}\,(\sqrt{x_1^2 S/4Tt} + \sqrt{Tt/SB^2}) + e^{-x_1/B}\,\mathrm{erfc}\,(\sqrt{x_1^2 S/4Tt} - \sqrt{Tt/SB^2})] \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

When there is no leakage $[(1/B) = 0]$, (14) will reduce to (12).

Although (13) and (14) are developed for artesian condition, they can be used also for water-table conditions provided that the limitations discussed under item (2) above are met.

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Robert E. Glover and Glenn G. Balmer (U. S. Bureau of Reclamation, Denver Federal Center, Denver 2, Colo., Authors' closure)--It is gratifying that Hantush has been able to develop a more general solution than that obtained by the authors, and to confirm the special case treated by them.

In regard to his question about the position of the image well, this well may be placed anywhere along the line he describes, and it will ultimately cause the total well flow to cross the line representing the position of the stream. However, the only line along which the drawdown will remain at zero is a straight line drawn midway between the pump well and the recharge image well and at right angles to the line joining them. Since the stream maintains the condition of zero drawdown along the near bank, the line of zero drawdown, described above, must coincide with it, and this fixes the position of the image well.