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# ELECTROKINETIC-POTENTIAL FLUCTUATIONS GENERATED BY JET IMPINGEMENT

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**Abstract**—Electrokinetic-potential fluctuations produced by a two-dimensional submerged water jet impinging on a plate have been measured. The potential fluctuations appear to be approximately proportional to the longitudinal-velocity fluctuations  $u_x'$  in the neighborhood of the boundary.

Normalized frequency distributions of potential-fluctuation measurements agree with velocity-fluctuation data taken by Klebanoff and Laufer with a hot-wire anemometer at a dimensionless distance  $y/\delta \approx 10^{-3}$  from the wall. Assumptions made concerning the relationship between potential and velocity fluctuations give a possible explanation of the change in the shape of the potential-fluctuation spectrum with the flow velocity and the electrical conductivity of the water. Further analysis is required to establish a definite relationship between electrokinetic-potential fluctuations and velocity fluctuations occurring near a solid boundary.

## NOMENCLATURE

$d$ ,	double-layer thickness, m;
$e_r$ ,	RMS value of the net amplified signal, $m^2ks^{-3}A^{-1}$ ;
$e_{r0}$ ,	value of $e_r$ at the stagnation point;
$e_w$ ,	net voltage read on the wave analyser;
$f$ ,	frequency, $s^{-1}$ ;
$f_m$ ,	maximum frequency of velocity fluctuation considered, $s^{-1}$ ;
$g$ ,	acceleration of gravity, $ms^{-2}$ ;
$k_1$ ,	wave number in data of others, $m^{-1}$ ;
$n$ ,	wave number, $m^{-1}$ ;
$p_0$ ,	stagnation pressure, $m^{-1}ks^{-2}$ ;
$u$ ,	velocity, $ms^{-1}$ ;
$u_x$ ,	velocity component along $ox$ , $ms^{-1}$ ;
$x$ ,	longitudinal co-ordinate, m;
$x_2$ ,	origin of established mean flow in $x$ -direction, m;
$y$ ,	vertical co-ordinate, m;
$A$ ,	amplification factor;
$B_0$ ,	band width, $s^{-1}$ ;
$E_u$ ,	normalized spectral distribution of $u_x'$ , m or s;

$E'_{\Delta\psi}(f)$ ,	spectral distribution of the potential difference fluctuations, $(mV)^2 s$ , $m^4k^2s^{-5}A^{-2}$ ;
$E'_{\Delta\psi}(n)$ ,	spectral distribution of the potential-difference fluctuations, $(mV)^2 cm$ , $m^5k^2s^{-6}A^{-2}$ ;
$E_{n\Delta\psi}$ ,	normalized value of $E'_{\Delta\psi}(f)$ or $E'_{\Delta\psi}(n)$ , m or s;
$H$ ,	hydrostatic head, m;
$L$ ,	orifice-plate distance, m;
$U$ ,	velocity at the slot $(2gH)^{1/2}$ , $ms^{-1}$ .

## Greek symbols

$\delta$ ,	boundary-layer thickness, m;
$\epsilon$ ,	permittivity of water, $F/m$ , $m^{-3}k^{-1}s^4A^2$ ;
$\nu$ ,	kinematic viscosity of water, $m^2s^{-1}$ ;
$\rho$ ,	mass density of water, $m^{-3}k$ ;
$\sigma$ ,	conductivity, $m^{-3}k^{-1}s^3A$ ;
$\tau$ ,	relaxation time, $\epsilon\sigma^{-1}$ , s;
$\psi$ ,	electrical potential, $m^2ks^{-3}A^{-1}$ ;
$( )'$ ,	time-fluctuating component;
$( )$ ,	time-mean value.

## REVIEW OF THE PROBLEM

### Introduction

IN GENERAL, an electrical double layer arises at the interphase boundary between a liquid and a solid; it is composed of ions of a certain sign

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attached to the solid wall and of a diffuse layer of the opposite sign which can move with the liquid. This double layer is a few hundred ångströms thick. When the charge distribution in the double layer is disturbed by velocity fluctuations, potential-difference fluctuations arise, and may be picked up by electrodes along the wall.

The present investigation was aimed at relating the potential fluctuations to the velocity field close to the wall where existing turbulence-measurement techniques, such as the hot-wire anemometer, become very difficult to use. A two-dimensional jet impinging on a perpendicular plate was the turbulent flow field chosen for this study. The high level of turbulence of such a jet flow produced a readily measured potential fluctuation. In addition, this investigation represents an attempt to extend the results found in pipe flow by Binder [1] to a non-uniform flow case.

#### Analytical considerations

A simplified representation of Stern's concept of the electrical double layer at a solid-liquid interface was used to interpret the results obtained in this study [2, 3, 4]. The thickness of the double layer  $d$  is defined as the distance between the solid surface and the "Gouy plane", which has a charge opposite to the wall charge and is electrically equivalent to the diffuse layer. The thickness of the double layer  $d$  can be calculated theoretically in the static case [3] or in the laminar streaming current case [2, 5]; the value of  $d$  decreases as the electrical conductivity of the water is increased.

As suggested by experimental results [1, 6] it was assumed that the double-layer thickness  $d$  is approximately the same in a turbulent flow case as in the static case, that is  $d \approx 5 \times 10^{-6}$  cm for the present experimental conditions; further, that the value of amplified signal  $A\psi'$  is proportional to the value of the longitudinal-velocity fluctuation  $u'_x$  at a distance of order  $d$  from the boundary [4]. The consequences of these two assumptions are not in contradiction with the experimental results found in the present study or in Chuang's investigations [7].

#### Application to the present investigation

From the latter assumption it would follow that the RMS value of the amplified signal  $e_r$ , which is proportional to  $\sqrt{\psi'^2}$ , varies as  $\sqrt{u_x'^2}$ . According to Klebanoff [8], Laufer [9], and Hinze [10], the RMS value of the velocity fluctuations  $\sqrt{u_x'^2}$  is proportional to the local-mean velocity  $u_x$  very close to the wall. On the other hand, the local mean velocity varies as the wall shear. Wall jet measurements by Myers and others [11], in agreement with data taken by previous investigators, show that the wall shear is inversely proportional to the longitudinal distance  $x$  (see definition sketch in Fig. 1, which includes the symbol key for all subsequent figures).

As the plane jet impinging upon a plate is similar to a wall jet for  $x > x_2$ , it follows that  $e_r$  should vary as  $x^{-1}$  for  $x > x_2$ .

For  $x < x_2$ , dimensional considerations [4] show that either one of the parameters:

$$\frac{e_r^2 \sigma}{\rho U^4} f_m \quad \text{or} \quad \frac{e_r^2 \sigma}{\rho U^4 \tau} \quad (1)$$

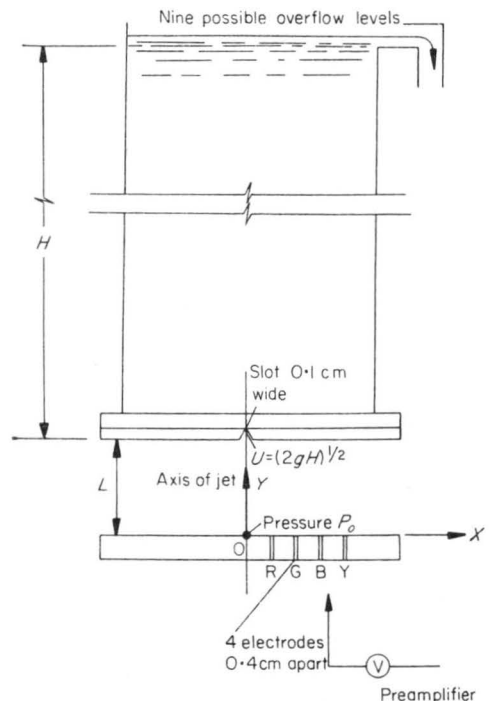


FIG. 1. Definition sketch.

should be a constant of the flow. The distance  $x_2$  must be determined by experiment.

### EQUIPMENT AND PROCEDURE

#### The flow system

A schematic of the constant-head system with overflow at various heights  $H$ , built for this investigation, is shown in Fig. 2. Only materials which do not contaminate distilled water such as Lucite, brass, and platinum, were used. The elevation of the diffuser could be changed so that the flow may be undisturbed just before the outlet. The orifice itself ( $0.1 \times 2.65$  cm) was brass; the jet impinged upon a Lucite plate fixed in a Lucite tank, which could be moved in the  $x$  and  $y$  direction. The water in the tank was kept at a constant level; it was then recirculated through a weir box and polyethylene bottles, by means of an ECO "Teflon" and stainless steel pump (type 22U2 Allchem). The conductivity of the equilibrium water introduced into the system stabilized itself around  $4.5 \mu(\Omega\text{cm})^{-1}$  after several days.

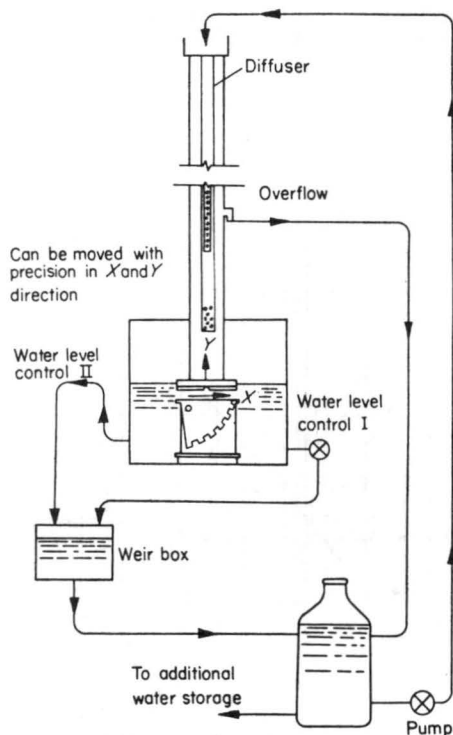


FIG. 2. Flow system.

#### The electrical system

In order to eliminate electrical pick-up, especially at 60 c/s, a grounded enclosure in sheet metal was built around the experimental set-up (Fig. 3). The pump was located outside of this shielding and two grounded brass rings were incorporated into the plastic tubing, downstream and upstream of the pump, respectively.

The results presented here were taken with 0.081 cm diameter platinum electrodes, placed and connected as indicated in Figs. 1 and 3, and sealed by a rubber adhesive. Data taken with 0.081 cm diameter copper wire showed that dimensionless ratios, such as normalized spectra, were not affected by the material of the electrodes, as hypothesized by Binder [1]. Platinum was used in order to obtain reliable results when the conductivity of the water was increased.

From the electrodes, the signal was fed into a Tektronix type 122 low-level preamplifier. The

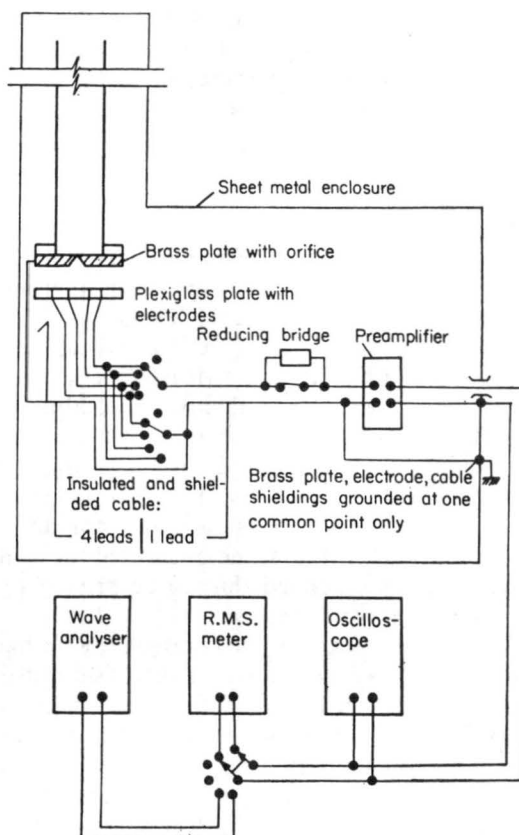


FIG. 3. Electrical system.

60 c/s pick-up was reduced to a negligible quantity by grounding the brass orifice, the reference electrode and the cable shieldings at one point only, thus eliminating ground currents. The amplification factor of the preamplifier was 1000 and input impedance was  $10^7 \Omega$ . Since the impedance between two electrodes was normally less than  $10^5 \Omega$ , no appreciable distortion of the signal was to be expected. The output of the preamplifier was fed into a wave analyser (Hewlett-Packard, Model 300A) or a sinusoidal RMS meter (Hewlett-Packard, Model 400H). An oscilloscope (Tektronix type 502) was always connected in parallel with either the wave analyser or the RMS meter, so that the signal could be observed.

The wave analyser had a constant band width, the value of which was  $B_o = 14.55$  c/s, if the definition given for the band width by Klebanoff [8]  $B_o$  is the rectangular band width having the same area as the actual band shape, was adopted. The difference between the true RMS value of the signal, found by graphical integration of the frequency spectral distribution, and the value given by the RMS meter, was about 5 per cent.

#### Procedure

The electrodes were submerged for several hours in water before each run. When a steady flow was obtained at the desired head (Figs. 1, 2), the instruments were switched on and allowed to warm up for half an hour. Measurements were then taken, at least twice; finally, the total noise caused by the pump, diffuser, pick-up and instruments were measured by covering the electrodes with a Plexiglass plate. The value of the noise found by this method did not differ appreciably from the value measured in still water. The noise seemed thus to be principally caused by the instruments.

When the influence of the conductivity on the signal was studied, constant amounts of saturated KCl solution were added to the water at given intervals of time. Data were taken half an hour after the conductivity was found to remain constant, that is, about every two hours. After the run, the system was flushed several times with distilled water.

## RESULTS AND DISCUSSION

### Reproducibility of data

The signal amplitude taken with a given pair of electrodes was reproducible within experimental error for a few days. Then, as noticed by most investigators, the signal amplitude changed progressively, presumably because the electrical double layer was modified. The conductivity seemed to play an important role in this drift. The total error in taking data, which included random error due to fluctuations of the instrument's indicator during a given experiment and systematic error, due to personal factors or day-to-day changes, was estimated at less than 10 per cent.

The results found for the RMS value of the signal, taken with one electrode connected to the amplifier and another electrode grounded, are presented first. The spectral density distribution of the potential fluctuations are presented next, and compared to velocity-fluctuation data.

### Root-mean-square of the potential fluctuation

The RMS value of the signal  $e_{ro}$  at the stagnation point is proportional to the stagnation pressure  $p_o$  (Fig. 4). The variation of the ratio

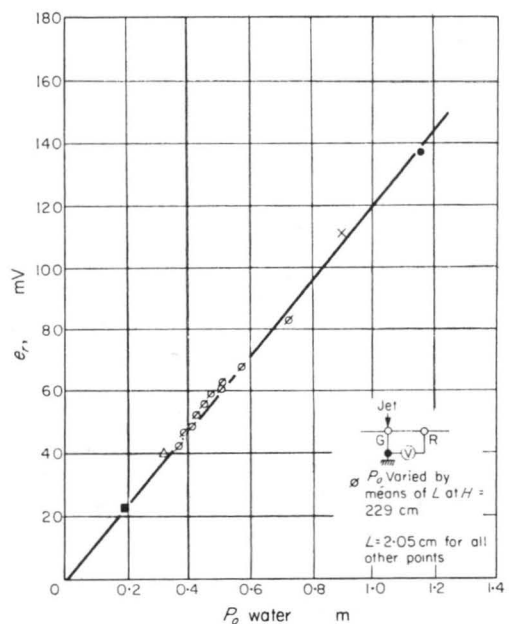


FIG. 4. Variation of amplified potential-fluctuations with stagnation pressure.

$e_r/e_{r0}$  with the longitudinal distance  $x$  is shown in Fig. 5:  $e_r = e_{r0}$  for  $x < 0.35$  cm and  $e_r/e_{r0} = 0.35/x$  for  $x > 0.35$  cm. This variation is independent of the location of the grounded electrode. These results are in agreement with the assumptions made in the section Review of the Problem regarding the proportionality between  $e_r^2$  and  $\overline{u_x^2}$  and the remarks presented in the same section about the expected variation of  $\overline{u_x}$  with  $p_0$  (or  $U$ ) and  $x$  from wall-jet measurements made by others. In particular, the value  $x_2$  mentioned in connection with (1) may be taken as 0.35 cm for the configuration used. For a study of the influence of the conductivity  $\sigma$  upon the value of  $e_r$ , a distinction had to be made between continuous runs and separate runs.

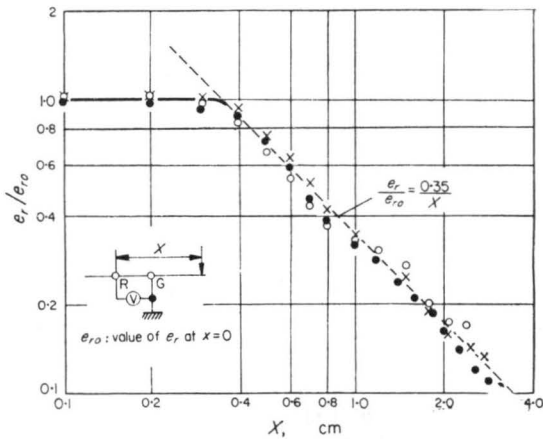


FIG. 5. Variation of amplified potential-fluctuations at the wall as a function of distance from stagnation point.

Several hours were necessary for the double layer to reach a new equilibrium after new ions had been introduced into the water. The variation of  $e_r^2$  with  $\sigma$  for separate runs, which represents the most useful case, is shown in Fig. 6.  $e_r^2$  varies approximately as  $\sigma^{-1}$  in the range of conductivities considered.

The dimensionless parameter  $(e_r^2 \sigma f_m) / (\rho U^4)$ , proposed earlier, appears as a possible constant of the flow; one should note that by definition  $p_0$  is proportional to  $\rho U^2$  for a given value of  $L$  (Fig. 1).

*Spectral distribution*

If  $e_w$  is the net quantity read on the wave

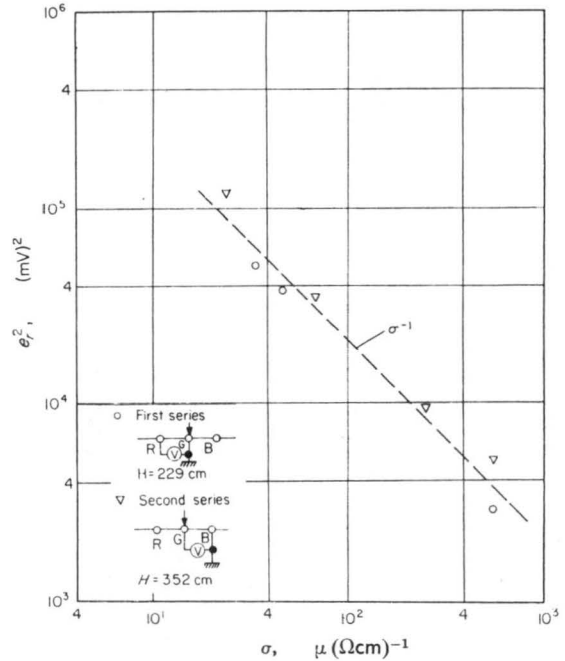


FIG. 6. Mean-square of amplified wall potential-fluctuations with electrical conductivity of the water.

analyser, the frequency spectral distribution of the amplified potential-difference fluctuations is by definition:

$$E'_{\Delta\psi}(f) = \frac{e_w^2}{B_0} (\text{mV})^2 \text{ s.} \quad (2)$$

It is convenient to define a wave-number spectrum as

$$E'_{\Delta\psi}(n) = \frac{e_w^2}{B_0} \frac{U}{2\pi} (\text{mV})^2 \text{ cm,}$$

$$\text{with } n = \frac{2\pi f}{U} = \frac{2\pi f}{(2gH)^{1/2}} \text{ cm}^{-1}. \quad (3)$$

Finally, the normalized spectra are obtained by dividing  $E'_{\Delta\psi}(f)$  and  $E'_{\Delta\psi}(n)$  by the mean-square value of the signal  $e_r^2$ .

The change of the shape of the normalized spectrum at the stagnation point as the head is varied is shown in Figs. 7 and 8. The frequency power spectrum rotates clockwise, which means that the relative amount of low-frequency fluctuation increases and the relative amount of high-frequency fluctuation decreases, as the head is decreased. The wave number spectrum

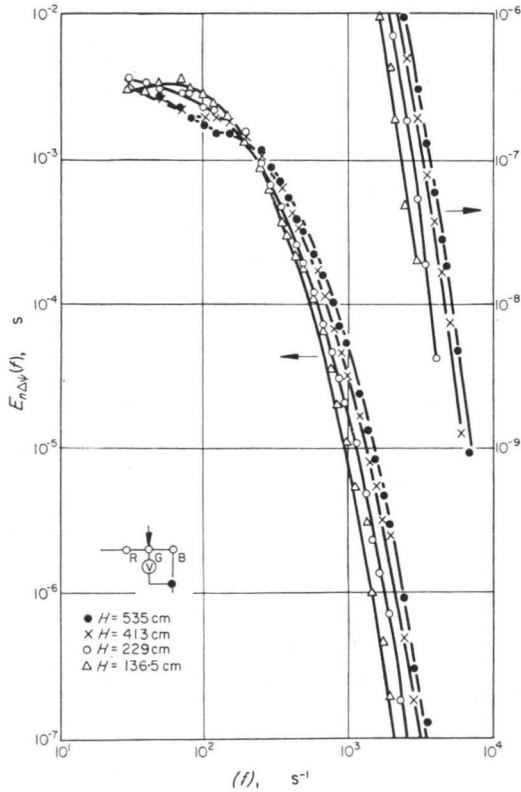


FIG. 7. Frequency spectrum at the stagnation point.

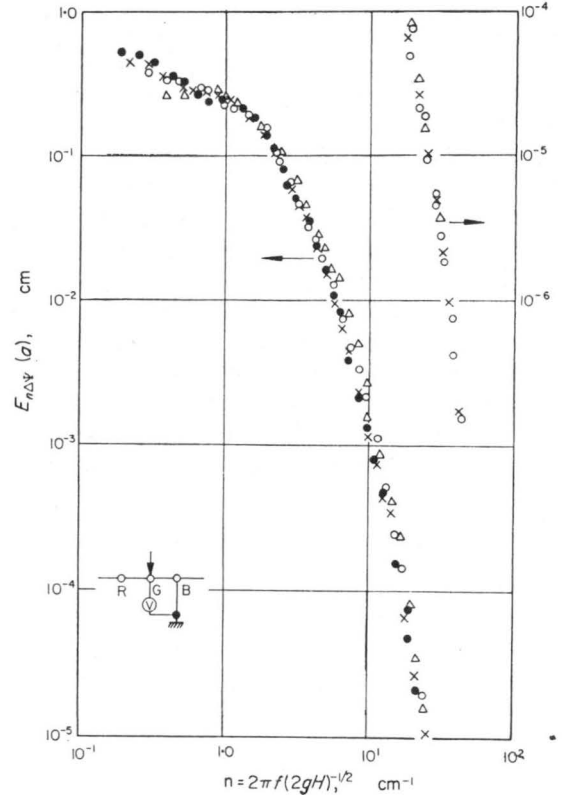


FIG. 8. Wave number spectrum at the stagnation point.

is almost the same for all heads. Similarly, the curve  $E_{n\Delta\psi}(f)$  rotates counter-clockwise when the conductivity  $\sigma$  is increased, as shown in Fig. 9. The relative amount of high-frequency energy seems thus to increase as  $H$  or  $\sigma$  are increased.

According to the assumption presented above, the potential-fluctuation measurements represent the longitudinal-velocity fluctuation at a distance of order  $d$  from the boundary. The value of  $d$  is about  $5 \times 10^{-6}$  cm when the conductivity is  $5 \mu(\Omega\text{cm})^{-1}$  [1]. If the boundary-layer thickness is computed from the formula for a smooth flat plate in turbulent flow [12]:

$$\delta(x) = 0.37 x^{4/5} (\nu/U_\infty)^{1/5}, \tag{4}$$

one finds  $\delta \approx 10^{-2}$  cm for the conditions corresponding to Fig. 10:  $x = 0.2$  cm;  $p_0 \approx 100$  cm of water for  $H = 413$  cm. With these conditions the calculated-dimensionless distance from the wall  $d/\delta$  is approximately  $5 \times 10^{-4}$ . The normalized frequency spectra of potential fluctuations

corresponding to that value of  $d/\delta$  agree well with velocity-fluctuation spectra taken by Klebanoff [8] at  $y/\delta = 1.1 \times 10^{-3}$ , as can be seen in Fig. 10. A similar comparison is made between wave number spectra in Fig. 11. The data seem to fit an  $n^{-7}$  variation at high wave numbers, which could be related to the assumptions of Tchen [13] that the high-frequency velocity fluctuations are not affected by shear.

At any rate, the spectral-distribution curves always shift in a counter-clockwise direction as the calculated value of  $d/\delta$  or experimental value of  $y/\delta$  is decreased. Calculated values of  $d/\delta$  as a function of an initial value  $d_0/\delta_0$  are shown in Fig. 9, where  $\delta$  stays constant ( $\delta = \delta_0$ ) and  $d$  changes according to the theory [3] that is,  $d \approx \sigma^{-1/2}$ , approximately. Similar computations are shown in Fig. 12, but here  $d$  stays constant and  $\delta$  varies according to equation (4); the second moment of the spectral distribution

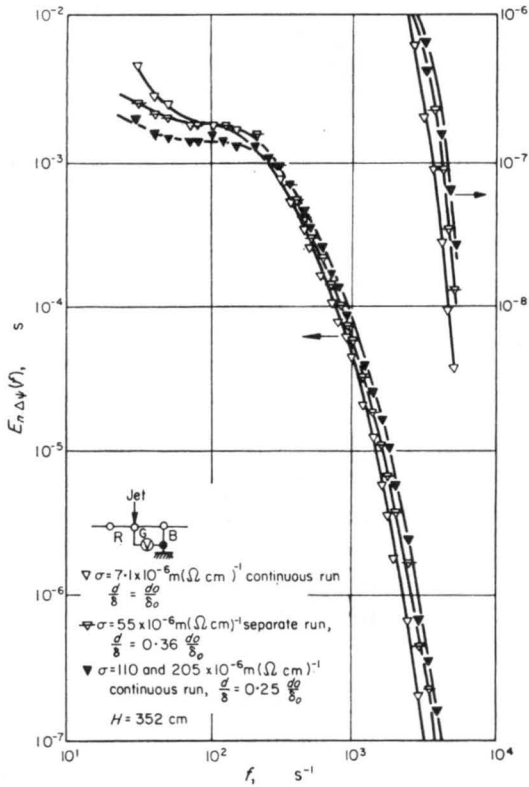


FIG. 9. Variation of the frequency spectrum with the electrical conductivity of the water.

emphasizes the change in the shape of the spectrum at medium frequencies. It can be seen on all three Figs. 10, 11, 12, that the spectral distribution curves always shift in a counter-clockwise direction as the calculated value of  $d/\delta$  or the experimental value of  $y/\delta$  is decreased; in other words, the proportion of high-frequency fluctuations seems to increase as one approaches the wall.

If further theoretical developments and experiments would confirm the assumptions and results presented in this paper, the electrokinetic method could then be used to study turbulent shear flows very close to the wall. The dimensionless distance from the wall  $d/\delta$ , at which the velocity fluctuations occur according to the assumption, could be changed at will by acting either on the conductivity which changes  $d$ , or on the velocity which changes  $\delta$ , depending on the case considered.

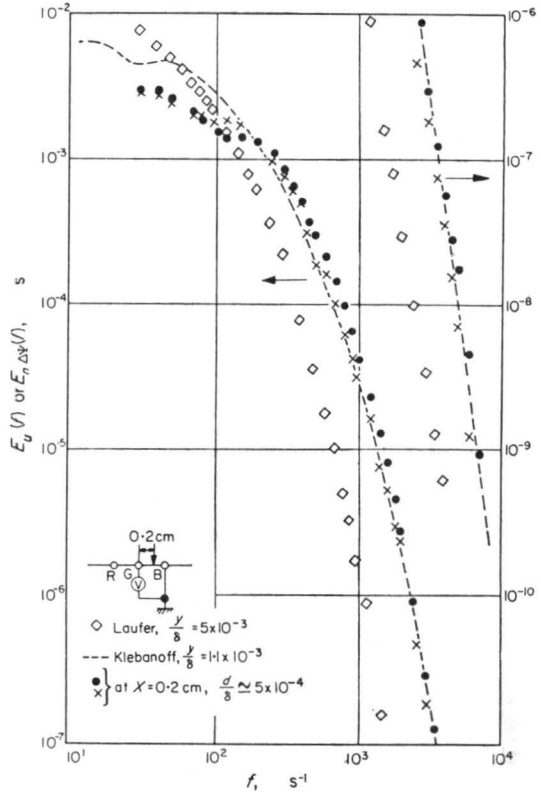


FIG. 10. Normalized spectra for potential and velocity fluctuations.

## CONCLUSIONS

The conclusions derived from this experimental study of the electrokinetic-potential fluctuations created by jet impingement are:

(1) The root-mean-square value of the amplified potential-fluctuation signal at the wall is proportional to the stagnation pressure on the plate. As the stagnation pressure is increased, the normalized frequency spectrum shifts counter-clockwise, which means that the relative amount of high-frequency energy increases with the hydrostatic head. The normalized wave number spectrum, where the wave number is defined as  $2\pi f(2gH)^{-1/2}$ , remains nearly invariant with the stagnation pressure.

(2) Along the longitudinal axis  $ox$ , the RMS value of the signal remains approximately constant near the stagnation point; then it decreases as  $x^{-1}$ .



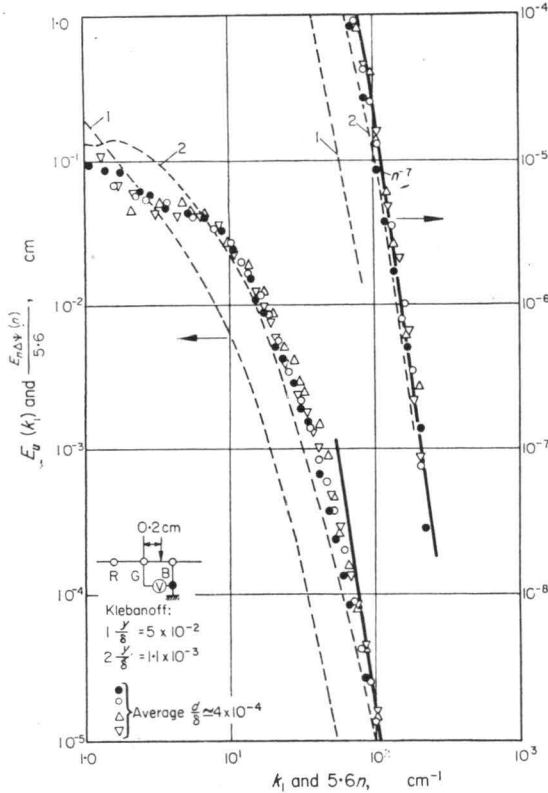


FIG. 11. Normalized spectra for potential and velocity fluctuations.

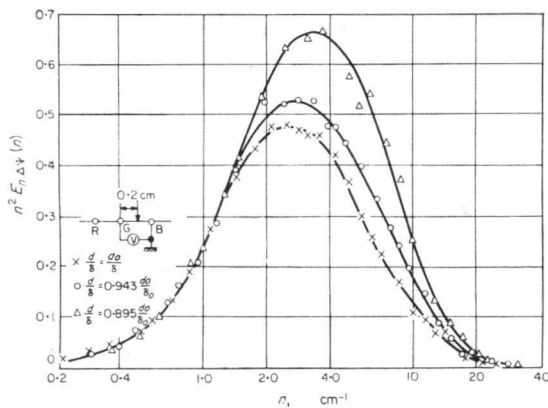


FIG. 12. Second moment of spectral distribution.

(3) The value of the electrical conductivity of the water has a definite influence on the signal. The RMS value of the signal varies approximately as the square-root of the conductivity

for low and medium conductivities. The normalized frequency spectrum shifts in the same manner, when the conductivity is increased, as when the stagnation pressure is increased.

(4) The normalized spectrum of the potential fluctuations compares well with normalized spectrum of longitudinal-velocity fluctuations, measured by Klebanoff with a hot-wire anemometer at a dimensionless distance  $y/\delta \approx 10^{-3}$  from the wall, in a two-dimensional turbulent boundary-layer flow. The estimated value of the dimensionless distance from the wall at which the velocity fluctuations would be measured, according to the hypotheses made in this study, is of the same order of magnitude as  $y/\delta$ .

(5) No directional sensitivity of a pair of electrodes, placed 0.4 cm apart, can be detected with one of the electrodes permanently grounded.

ACKNOWLEDGEMENT

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**Zusammenfassung**—Elektrokinetische Potentialschwankungen, die von einem zweidimensionalen Unterwasserstrahl, der auf eine Platte trifft, herrühren, werden ausgemessen. Die Potentialschwankungen  $u_x'$  nahe der Begrenzung zu sein.

Normalisierte Frequenzverteilungen der Potentialschwankungsmessungen stimmen überein mit Ergebnissen für Geschwindigkeitsschwankungen, die von Klebanoff und Laufer mit einem Hitzdrahtanemometer im dimensionslosen Wandabstand  $y/\delta \approx 10^{-3}$  erhalten wurden. Annahmen für die Beziehung zwischen Potential- und Geschwindigkeitsschwankungen ermöglichen eine Erklärung für die Änderung der Form des Potentialschwankungsspektrums mit der Strömungsgeschwindigkeit und der elektrischen Leitfähigkeit des Wassers. Weitere Analysen sind notwendig, um definierte Beziehungen aufstellen zu können zwischen elektrokinetischen Potentialschwankungen und Geschwindigkeitsschwankungen, wie sie nahe einer festen Begrenzung auftreten.

**Аннотация**—Измерялись пульсации электро-кинетического потенциала, создаваемые у пластины плоскими затопленными струями воды. Повидимому, пульсации потенциала приблизительно пропорциональны пульсациям продольной скорости  $u_x'$  около стенки.

Приведенный спектр частот измерений пульсации потенциала согласуется с данными о пульсациях скорости, полученными Клебановым и Лауфером с помощью термоанемометра на безразмерном расстоянии от стенки  $y/\delta \approx 10^{-3}$ . Допущения, сделанные относительно связи пульсаций потенциала и скорости, удовлетворительно объясняют изменения формы спектра пульсаций потенциала в зависимости от скорости течения и электрической проводимости воды. Необходимо дальнейшее исследование, чтобы установить определенную зависимость между пульсациями электро-кинетического потенциала и пульсациями скорости у твердой стенки.