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A NEW METHOD FOR PREDICTING TRANSIENT STATES OF SALINITY INTRUSION INTO THE SACRAMENTO-SAN JOAQUIN DELTA

Robert E. Glover

Abstract--The mechanism by which the tidal changes propagate salinity into the Delta channels and the manner in which this propagation is opposed by the fresh water stream flows are expressed in mathematical form. Solutions of the basic differential equation are given which are suitable for computation of salinity changes and a method of computation is described. As an example of the use of these methods they are applied to the historical records to determine a depletion curve for the Delta by using the observed salinities as an indicator and finding the flows out of the Delta which were necessary to hold the salinities to the observed amounts. While the primary usefulness of this depletion curve is to form a basis for further estimates of ocean salinity encroachment, it is compared to use curves derived from independent sources.

The Delta area of California extends eastward form the head of Suisun Bay for a distance of about 25 mi. Its extent in the north and south direction is about twice as much. In primitive times it was a vast marsh with a network of channels which carried the flow of the Sacramento, San Joaquin, and the North and South Mokelumne Rivers and other smaller streams. It has been reclaimed by raising the naturally existing dikes to enclose areas which could be pumped out and farmed. The reclaimed areas are commonly called islands even though their land surface may be as much as 15 ft below sea level. Salinity encroachment has always been a matter of concern because there is a continual seepage of water into the islands. So long as this seepage water is fresh it may be useful to growing crops, but if it is saline it may be detrimental. An added interest in salinity propagation has arisen as a result of the construction of the Tracy Pumping Plant, which is supplied by water flowing southwardly through the Delta channel from the Sacramento River.

Encroachment of salinity in the Delta channels is a result of tidal ebb and flow. Tidal changes coming in from the Pacific Ocean through the Golden Gate are propagated through the San Francisco, San Pablo, and Suisun Bays to the junction of the Sacramento and San Joaquin Rivers on the western edge of the Delta. From this point they spread out to traverse the channels throughout its area. Due to the size of the channels, velocities due to river discharge are generally less than the tidal velocities, with the result that the direction of flow in them reverses during each tidal cycle. Since there are roughly two tides per day, this reversal occurs frequently. Because the flow velocities in the middle of a channel are greater than they are near the sides, the incoming tidal flow may carry a tongue of saline water form Suisun Bay into the Delta channels, where turbulence mixes it with the fresher waters in them. On the outgoing tide the process is reversed, and a tongue of fresh water enters and mixes with the more saline waters. The tidal ebb and flow thereby creates a mechanism by which salinity may be propagated into the Delta channels and, in the absence of stream flow, would ultimately establish ocean salinity in all the Delta channels. The salinity propagation by tidal action is opposed by the fresh water stream flow which tends to sweep the salinity out of the channels into the bay. In times of flood the salinity disappears from the channels, but in times of low stream flow the salinity creeps back. All of the Delta channels appear to have sufficiently turbulent flow conditions to prevent the development of a salt water wedge.

A method of computing salinity changes may be based upon the reasonable assumption that the rate of propagation of salinity by tidal action, at any point in a channel, is proportional to the salinity gradient at that point. When the tendency of the stream flow to carry the salinity toward the bay is also taken into account, this assumption leads to the differential equation.

$$\partial S/\partial t = K\,\partial^2 S/\partial X^2 + V\,\partial S/\partial X \; \dots\dots\dots\dots\dots \; (1)$$

where

S represents the salinity expressed in units such as parts of chloride per 100,000 parts of water

t, time in seconds
x, distance measured along the bank of a channel, positive upstream in feet
V, the velocity in the channel due to stream flow, positive if the flow is downstream, in ft/sec
K, a diffusion constant expressing the capacity of the channel for diffusing salt by tidal action in ft^2/sec

It is to be understood that the diffusion constant K for a channel is to be determined empirically from measurements of salinity changes which occur in the channel under the influence of tidal action and stream flow. A solution for the ultimate steady state is given by an expression of the type

$$S = S_o \exp(-Vx/K) \quad \ldots \ldots \ldots \ldots \quad (2)$$

where S_o represents a maintained value of salinity at the point where $x = 0$. A solution useful for treating transient salinity states may be obtained in the following way.

Let ξ be a length variable measured along the axis of x from the origin of x so that we can use the variable ξ to describe the location of an element of salinity at time zero and x to indicate the point at which the salinity is to be computed at some subsequent time. If the regimen of a channel has been disturbed by a change of flow, or by some other change, we know that if the new conditions persist long enough and the salinity at $x = 0$ is maintained at some value S_o, the salinity distribution will ultimately come to the steady state configuration given by Eq. (2). Suppose we designate the departure from this ultimate steady state at time zero by $F(\xi)$. Then to calculate the salinity changes between the initial configuration and the ultimate steady state, we need a solution of (1) which satisfies the conditions.

$$\left.\begin{aligned} S &= 0 \text{ when } x = 0 \text{ for } t > 0 \\ S &= F(\xi) \text{ when } t = 0 \text{ for } \xi > 0 \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \quad (3)$$

It is to be understood that this solution represents the departure from the ultimate steady state, and that the salinity at any time is to be computed by adding this solution to (2).

The required solution is

$$S = \frac{\exp(-V^2 t/4K)}{\sqrt{\pi}} \int_o^L \frac{F(\xi)\exp[-V(x-\xi)/2K]\{\exp[-(x-\xi)^2/4Kt] - \exp[-(x+\xi)^2/4Kt]\}}{\sqrt{4Kt}} d\xi \quad \ldots (4)$$

Computation methods--A graphical procedure has been devised for evaluation of (4). This process may be described as follows. Eq. (4) can be put in the form

$$S = \frac{1}{\sqrt{\pi}} \int_o^L \frac{F(\xi)[1 - \exp(-x\xi/Kt)]\exp\left\{(-1/4Kt)[(x-\xi)+Vt]^2\right\}}{\sqrt{4Kt}} \, d\xi$$

Let

$$u = [(x-\xi)+Vt]/\sqrt{4Kt} \qquad du = -d\xi/\sqrt{4Kt}$$

Then if we let I represent the integral

$$I = \frac{1}{\sqrt{\pi}} \int_o^{\xi} \frac{\exp\left\{(-1/4Kt)[(x-\xi)+Vt]^2\right\}}{\sqrt{4Kt}} \, d\xi$$

The above substitution of variable will reduce the integral to the form

$$I = \frac{1}{\sqrt{\pi}} \int_o^u e^{-u^2} du \qquad = 0.5 \; E(u)$$

where

$$E(u) = (2/\sqrt{\pi})\int_{0}^{u} e^{-u^2}\,du$$

The function E (u) is the probability integral for which tables are available.

With this notation we may then write

$$S = \int_{I_1}^{I_2} F(\xi)\,[\,1 - e^{-x\xi/Kt}\,]\,dI$$

or

$$S = \int_{E_1}^{E_2} 0.5\,F(\xi)\,[\,1 - e^{-x\xi/Kt}\,]\,d\xi$$

It will now be possible to arrange a graph on which

$$0.5\,F(\xi)\,[\,1 - e^{-x\xi/Kt}\,]$$

is plotted as abscissa against the variable E. Corresponding ordinates and abscissas will have the same value of ξ. Evaluation of the integral may then be accomplished by means of a planimeter. A chart of this kind is shown in Figure 1. The computations used in the preparation of this chart are shown in Table 1.

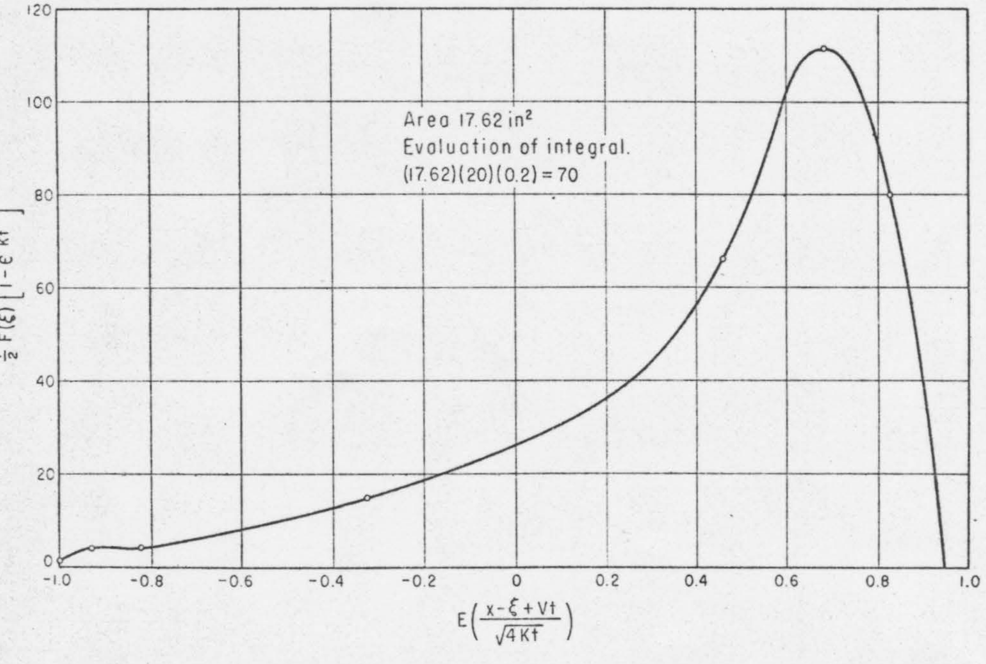

Fig. 1--Chart of computation of salinity at Bulls' Head Point, June 20, 1929

In the computations in Table 1

K = 2,900 ft^2 / sec
V/2K = 0.000019913
x/Kt = (36.239) (10)$^{-6}$
4Kt = (5011.2) (10)^{6}
Area 17.62
Correction: (20) (0.2) (17.62) = 70

V = 0.1155 ft/sec
x = 45,400 ft
t = 432,000 sec (5 days)
x + Vt = 45,400 + 49,896 = 95,296
4Kt = 70,780
Salinity on June 15 is 300

Then the total salinity at Bulls' Head Point on June 20 is 300 + 70 = 370.

Table 1--Computation of ordinates and abscissas, salinity at Bulls' Head point, June 20, 1929

ξ	$F(\xi)$	$x\xi/Kt$	$1-e^{-x\xi/Kt}$	$0.5F(\xi)[1-e^{-x\xi/Kt}]$	$(x-\xi)+Vt/4Kt$	E
ft						
0	0	0	0	0	1.346	0.9430
26,700	250	0.968	0.6202	77.5	0.969	0.8294
45,400	275	1.645	0.8070	111.0	0.705	0.6812
64,700	145	2.344	0.9041	65.5	0.432	0.4588
116,100	30	4.207	0.9852	14.8	-0.294	-0.3224
162,500	7	5.889	0.9972	3.5	-0.949	-0.8204
186,900	8	6.773	0.9988	4.0	-1.294	-0.9328
289,300	3	10.483	1	1.5	-2.741	-0.9999
435,000	0	15.763	1	0		-1.0000

Preliminary studies--Before the methods described in the previous paragraphs could be used for the actual estimation of salinity changes in the Delta, it was necessary to develop an idealization and to find values for the essential constants which would make the computations yield results in conformity with observations made during past years. The years 1924, 1929, and 1931 were particularly useful for these purposes because they were years of low runoff which produced well-developed salinity patterns in the Delta. Among the factors to be developed by trial were those of an equivalent channel cross section, channel lengths, and a corresponding diffusion constant. Since these choices are not independent, it was decided to choose a channel of arbitrary cross section, to seek corresponding distances for the stations at which salinity was to be computed, and a value for a diffusion constant. It was also found to be advantageous to work from a hypothetical location for the point $x = 0$ such that the salinity at this point will always be at the concentration of ocean salinity. An idealization of this kind may seem to be over-simplified, but if the equations described in the preceding paragraphs of this memorandum are studied it will be realized that it is equivalent to, and at the same time much simpler to use, than other choices which could be made even though these alternative choices would appear to be more appropriate at first sight. The choices finally arrived at are shown in Table 2. In making these choices, use was made of information developed several year ago by operation of an hydraulic model of the Delta, in addition to the historical situations in the Delta. Separate studies were made to evaluate the effects of surface storage and bank storage on the flows past Collinsville. It was found that these factors were negligible.

Table 2--Constants required for salinity computations: diffusion constant, $K = 2900\ ft^2/sec$; area of cross section of idealized channel, $132,500\ ft^2$;

Point	Distances from origin, x	Salinity factors (assumed ratio of mean tidal cycle surface zone salinity to observed salinities[a])
	ft	
Point Davis	26,700	0.94
Bulls' Head Point	45,400	0.86
Bay Point	64,700	0.90
O and A Ferry	116,100	0.84
Collinsville	162,500	0.83
Antioch	186,900	0.82
Jersey	289,300	0.76
Rio Vista	435,000	0.74
Central Landing	485,000	

[a] Estimated from comparison of data in Tables 25 and 33, Bulletin 27, CALIFORNIA, STATE OF [1931a]

Flow depletion in the Delta--A knowledge of the amount of water lost or diverted from the Delta channels is of importance because it is one of the factors which helps to determine the amounts of water which must enter the Delta if the salinity encroachment is to be held in check. Because much of the water supply to the islands is supplied by seepage, a determination of Delta depletion is attended with unusual difficulties. Efforts at evaluation have generally taken the form of studies of the use of water by crops, of evaporation losses, and the like from which the use and losses could be estimated with the aid of crop survey and water surface area data. For salinity propagation studies, such data may not be suitable because the part used by plants may have been diverted from the channels some time earlier to be stored temporarily in the soil. We will therefore distinguish between the terms depletion and use, using the former to designate water diverted from the channels and the latter to indicate water quantities as determined by studies of the type described above. It is, of course, the depletion data which are needed for studies of salinity propagation and an independent method for determining a curve of depletions, such as is shown in Figure 2, will now be described.

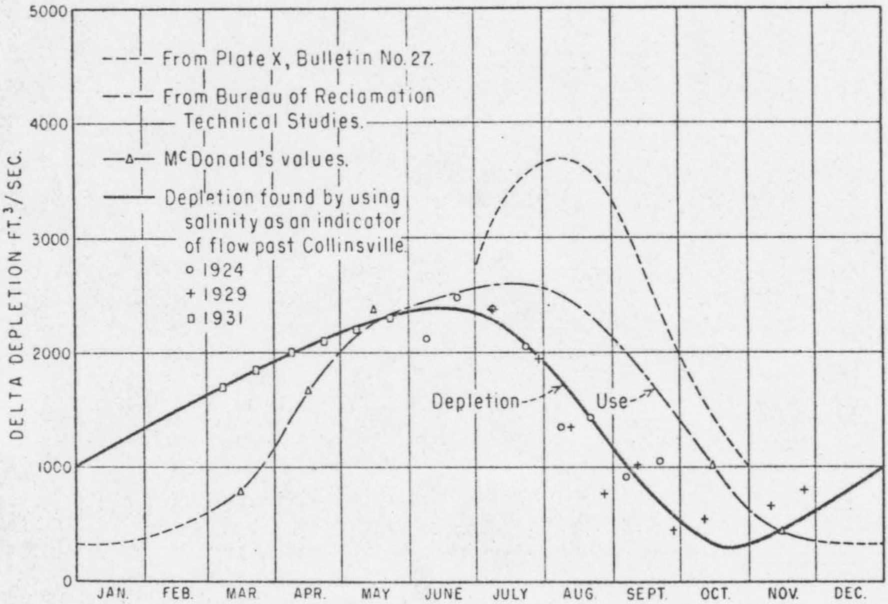

Fig. 2--Comparison of various Delta depletion curves

Having obtained, by trial, a group of constants which give a satisfactory correlation with experience, it becomes possible to derive a Delta depletion curve by using the salinity as an indicator and working back to the flows which would be required to pass out of the Delta at Collinsville to hold the salinity to the observed values. The process was carried out in the following way. In a year of record, such as 1924, for example, a beginning time was chosen at which a usable salinity pattern had developed. A trial outflow was then chosen and the changes in salinity computed for some subsequent time. The usual interval was 15 days. The computed salinities were then checked against the observed salinities, a new trial value was chosen, and the computation repeated. When a satisfactory correlation was found, the process was applied to the succeeding 15-day interval. The result of these computations was a series of computed values for the flow past Collinsville. The Delta depletion was then computed by deducting the flow past Collinsville from the inflow to the Delta. Delta inflows were obtained, when necessary, by applying a formula essentially similar to the one described on pages 424 to 427, inclusive, of Bulletin 27 (a) of the State of California. The year 1924 provided data from which outflows for the months June through September could be computed; the year 1931 provided data useful through the months of March to May. Values obtained for the year 1931 subsequent to May indicated depletions decidedly lower than were obtained from 1924 and 1929. It seems reasonable that this should be the case since 1931 was a year of heavy salinity encroachment which interfered with the normal use of water. For this reason the computations for the year 1931 were not carried past the months of early summer even though a usable salinity pattern was available. The records of the years 1924, 1929, and 1931 permitted calculation of the outflow past Collinsville for all the months except December, January, and February. The depletion curve obtained from these computations is shown on Figure 2. Use curves obtained from Bureau of Reclamation technical studies and from other sources are also shown for purposes of comparison.

The procedure used for finding the depletion curve of Figure 2 is an indirect one. This process should be effective, however, for finding a depletion schedule which will work with the computation procedure described. The whole process described herein, of which the depletion curve of Figure 2 is a part, may be considered to be a process of correlation which, used as a whole, may be useful for predicting salinity states in the Delta. While no claim is made that the depletion curve obtained in this way does accurately represent Delta performance, it is of interest that the results indicate a depletion which antedates use, as it should, and also that the maximum rates are not greatly different from those developed on other grounds by the Bureau of Reclamation technical studies.

Another comparison was made by completing the use curve obtained from the technical studies and comparing total yearly use and depletion. Data for the months of March, April, May, October, and November were supplied by Harris McDonald. Data for the months of December, January, and February were taken from Chapter II of Bulletin 27 (a). The total yearly use comes out 1,008,000 ac-ft while the total yearly depletion is 1,051,000 ac ft. These two figures should be equal. The discrepancy is about four per cent.

It seems probable that the depletion pattern changes somewhat from year to year. Changes in the types of crops grown should cause some changes in the depletion pattern. In the past, salinity encroachment has probably reduced the depletions in very dry years such as the year 1931. In years when essentially normal conditions prevail, however, it is proposed to use the curve of Figure 2 to represent the depletions for the purpose of estimating salinity changes in all cases.

It was observed earlier that operation of the Delta model with depletions based on the use data of Plate X of Bulletin 27 (a) would not produce correlation with historical salinity patterns. Although this model is no longer available, it is the opinion of those familiar with its operation that the depletion curve of Figure 2 has magnitudes and a maximum occurring at about the right time of year to correct the discrepancies which were previously found.

Study of 1949 salinities--In order to test these methods against the records of a recent year, they were applied to the data for 1949. This year had a water supply which was about 63 per cent of normal and antedates the opening of the Walnut Grove Cut. The results of this study are shown graphically on Figures 3 and 4. The computations are arranged to yield the mean tidal cycle surface zone salinity. Salinity samples are generally taken about 1-1/2 hours after high-high tide with the intention of obtaining a sample which represents the maximum salinity during the tidal cycle. A comparison of the salinities obtained in this way with the results of observations extending over a complete tidal cycle indicates, however, that maximum salinities are not always obtained by such samples. Before the observed salinities can be compared with the computed values, it is necessary to obtain from them an estimate of the mean tidal cycle surface zone salinity. To do this use was made of the data in Table 25 of Bulletin 27 (a). From a comparison of these data with those of Table 33 (a) a series of factors was derived relating the mean tidal cycle surface zone salinity to the observed salinity. These factors are listed in Table 2.

A plot of the observed values of salinity will show a considerable variation. These variations are probably due to the variation of effectiveness of tidal diffusion of salt by the spring and neap tides. The process of computation described above does not account for this variation of tidal activity and, therefore, the computed values should be compared to an average of the observed salinities after correcting to give the mean tidal cycle surface zone salinity. An example of the process for reducing the observed salinities is shown on Figure 4. The observed salinities are first plotted and then a curve averaging the observed values is drawn by eye. The factors of Table 2 are then applied to yield the mean tidal cycle surface zone salinity. In the plot of Figure 4, data for Martinez were substituted for the Bulls' Head data because data for the latter station are not given in the Report of Sacramento - San Joaquin water supervision for 1949.

Control flow--The unchanging flow past Collinsville required to hold the salinity at Antioch permanently to 100 parts of chlorine per 100,000 parts of water may be estimated from formula 2 to be about 5,900 cu ft per second. A similar value was obtained earlier from operation of the Delta Model. The discrepancy between this figure and those obtained from studies made in the Delta itself caused some concern because these values, even though obtained under difficulties, were based on observation and should not be lightly cast aside. It is believed that the present studies go a long way toward reconciling these differences because they indicate that the salinity repulsion caused by the winter and spring flood flows is effective for sufficiently long periods to reduce the control flow requirements very substantially below the above amount during the low inflow period at the end of the summer. The flow needed to control salinity would approach this figure only if sufficient reservoir capacity were available to completely contain the winter and spring flood flows.

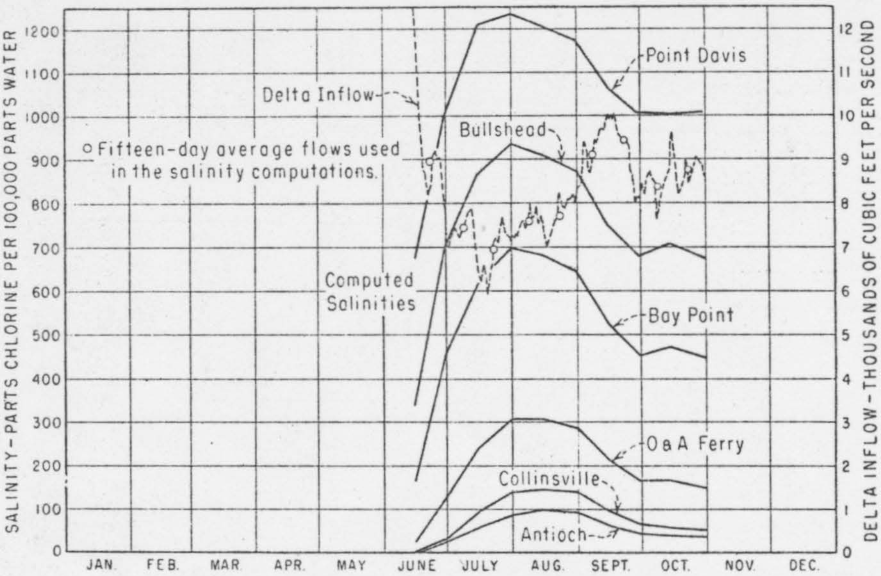

Fig. 3--Computed mean tidal cycle surface zone salinities for 1949

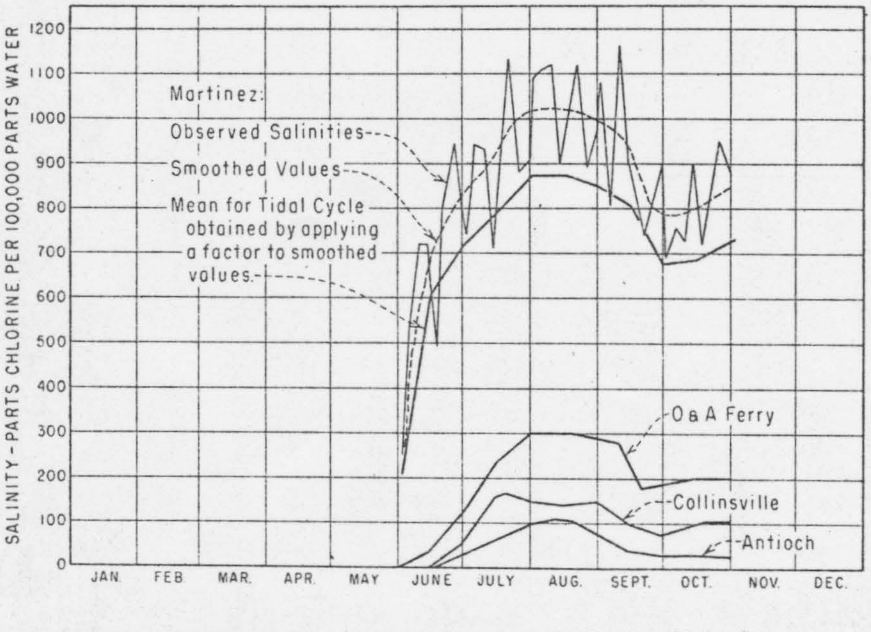

Fig. 4--Mean tidal cycle surface zone salinities as obtained from observations in 1949

Acknowledgments--The formulas presented herein have been checked by Q. L. Florey. A part of the preliminary computations, as well as the computations of depletion for the years 1929 and 1931, was made by him. L. J. Mitchell made a part of the preliminary computations for the year 1924. He also made salinity state computations for the year 1949, as well as a study to evaluate the effect of bank storage in the Delta. Some values used for plotting the use curve, as shown on Figure 2, were contributed by H. R. McDonald. Helpful suggestions have also been received from D. J. Hebert.

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1936 S. Lincoln St.,
Denver 10, Colorado

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