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Competence and autonomous motivation as motivational predictors of college students' mathematics achievement: from the perspective of self-determination theory

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Abstract

Background: Applied Calculus courses serve hundreds of thousands of undergraduates as quantitative preparation and gatekeepers across diverse fields of study. The current study investigated how motivational factors are associated with students' learning outcomes in Applied Calculus courses from the perspective of self-determination theory—a sound comprehensive motivation theory that has been supported by considerable research in psychology and education. In order to have a nuanced understanding of students' motivation and learning in Applied Calculus courses, we used three different types of learning measures to investigate students' mathematics achievement, including course grades, a standardized knowledge exam, and students' perceived knowledge transferability.

Results: We tested the relationships between motivational factors and learning outcomes with a multi-semester sample of 3226 undergraduates from 188 Applied Calculus classrooms. To increase the precision of our analysis, we controlled for three demographic variables that are suggested to be relevant to mathematics achievement: gender, minority group status, and socioeconomic status. With a series of multilevel modeling analyses, the results reveal that: (1) competence satisfaction predicts college students' mathematics achievement over and above the satisfaction of needs for autonomy and relatedness; and (2) autonomous motivation is a more powerful predictor of college students' mathematics achievement than controlled motivation and amotivation. These findings are consistent across different types of learning outcomes.

Conclusions: Self-determination theory provides an effective framework for understanding college students' motivation and learning in Applied Calculus courses. This study extends self-determination theory in the field of mathematics education and contributes to the dialogue on advancing undergraduate science, technology, engineering, and mathematics (STEM) education by providing evidence to understand how motivational factors are associated with students' learning outcomes in undergraduate mathematics courses.

Keywords: Calculus, Competence, Autonomous motivation, Mathematics learning, Learning outcomes, Higher education

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Introduction

Applied Calculus (AC) courses serve as mathematical preparation for undergraduate students who do not require further study in mathematics (such as in differential equations). In response to calls for a more “lean and lively” calculus in the late 1980s (Steen, 1987), these courses, and course sequences, have been developed and scaled over the last three decades to meet discipline-specific needs outside of engineering, mathematics, and physical sciences. Taught at both 2- and 4-year institutions, fall enrollments have grown from 163,000 in 1990 to over 600,000 in 2015 (Blair et al., 2018; Loftsgaarden et al., 1997). While overall national success rates in these courses can be difficult to estimate, the foundational position of these courses in students’ curricula along with long-studied difficulties in the teaching and learning of calculus at all levels (NRC, 2013) establishes the importance of understanding how and why students learn (or fail to learn) in undergraduate calculus learning environments. While “mainstream” calculus courses have a somewhat standardized curriculum, generally aligned with the Advanced Placement Calculus content, with a target audience of undergraduates in engineering, mathematics, and physical sciences, AC courses are highly diverse with students from many disciplines (including some STEM fields). The complexity of this calculus pipeline necessitates the careful, contextually relevant study of both cognitive and non-cognitive factors associated with success while accounting for individual differences. Here, we examined the particular non-cognitive factor of motivation using a comprehensive motivation theory, self-determination theory (SDT), as a framework for comprehending how students’ success, specifically in AC courses, may be explained in terms of motivational variables.

SDT asserts that people grow in conditions that satisfy the basic psychological needs of autonomy, competence, and relatedness (Deci & Ryan, 1985, 2002; Ryan & Deci, 2017). According to SDT, satisfying students’ basic psychological needs improves their autonomous motivation, which produces well-being and achievement. Although previous research demonstrated that autonomous motivation has played a crucial role in mathematics learning (e.g., Hagger et al., 2015; León et al., 2015; Reindl et al., 2015), most studies were conducted in K-12 settings and relied on a single learning outcome variable related math achievement, such as mathematics scores or course grades. Because any single variable is limited in representing students’ learning achievement, in this study, we assessed students’ learning outcomes using three different types of learning measures, including course grades, a standardized knowledge exam, and students’ perceived knowledge transferability, to have a more nuanced understanding of students’ learning outcomes in undergraduate AC courses.

Autonomous motivation and learning outcomes in self-determination studies

To understand how motivational factors influence different types of students’ learning outcomes, SDT was used as a theoretical framework in the current study. SDT (Deci & Ryan, 2002; Ryan & Deci, 2017) usually features the basic psychological needs as follows: *autonomy* means the volition to self-regulate an individual’s experiences and behaviors; *competence* denotes the perceptions of ability to effectively complete tasks or gain certain skills; and *relatedness* involves the feelings of being connected with others (Ryan & Deci, 2017). People tend to demonstrate autonomous motivation and behaviors when their three basic psychological needs are satisfied by the external environments (Ryan & Deci, 2017).

Based on SDT, motivation can be subdivided into six dimensions of regulatory styles, which sit along a continuum that reveals the degree of autonomy (Ryan & Deci, 2000). Intrinsic motivation represents the prototype of autonomous motivation, which indicates the state of doing an activity out of interest and pleasure (Deci & Ryan, 2008). *Integration* is a form of autonomous extrinsic motivation (Deci & Ryan, 2008). Individuals with integration behave because the behavior is an expression of who they are and is congruous with other aspects of their beliefs, goals, values, and needs (Schreiber, 2016). People with integration act with a full sense of volition. *Identification* is another type of autonomous extrinsic motivation. That is, people with this type of motivation identify the meaning and consequence of the behaviors and accept them as their own (Ryan & Deci, 2017). *Introjection* is a controlled form of motivation that people behave to deflect blame or to please others (Ryan & Deci, 2000, 2017). People with *external regulation*, take action to gratify external interest and requests, such as rewards and punishments (Ryan & Deci, 2000, 2017). Lastly, *amotivation* refers to an absence of motivation (Ryan & Deci, 2000). Based on the level of autonomy, these six types of motivation can be grouped into three: autonomous motivation, controlled motivation, and amotivation. SDT suggests that autonomous motivation includes intrinsic motivation, integration, and identification, whereas controlled motivation involves introjection and extrinsic regulation (Ryan & Deci, 2017).

Abundant research has demonstrated that students who perceive the satisfaction of their basic psychological needs and who have developed more autonomous motivation are more likely to benefit from numerous positive personal and academic outcomes (e.g., Ryan & Deci, 2017). For example, the satisfaction of basic psychological needs was found to be associated with students’ psychosocial well-being, positive affect, academic engagement, and better learning outcomes (Cho &

Jeon, 2019; Garn et al., 2019; Karimi & Sotoodeh, 2020; Milyavskaya & Koestner, 2011; Tjin A Tsoi et al., 2018). Autonomous motivation was found to have a significant association with desirable learning outcomes such as self-regulation, academic achievement, perseverance, and quality of learning, study strategy, and effort (Burton et al., 2006; Chirkov et al., 2007; Kusurkar et al., 2013; Thøgersen-Ntoumani & Ntoumanis, 2006). Intrinsic motivation and identification were significant predictors of students' academic achievement (e.g., Burton et al., 2006; Taylor et al., 2014). Identification especially was found to promote individuals' engagement and conceptual learning, and the long-term orientation toward tasks (Jang, 2008; Thøgersen-Ntoumani & Ntoumanis, 2006). A recent study focusing on U.S. college students' learning experience showed that identification and competence satisfaction play dominant roles in predicting students' perceived learning in the context of higher education (Wang et al., 2020).

Autonomous motivation in mathematics

Students' early math experiences have an impact on students' math affect, motivation, and pursuit of a career in math (John et al., 2020). Nevertheless, it is suggested that students' motivation in mathematics decreased over a period of early adolescence (Reindl et al., 2015).

A certain level of mathematical knowledge is crucial for those who plan to major in relevant fields and apply mathematics as a professional skill to their career fields (Nortvedt & Siqveland, 2019). Even though mathematics plays a key role in various career opportunities, some students struggle with a lack of competence in their mathematics abilities or understand the value of mathematical learning (Hourigan & O'Donoghue, 2007; John et al., 2020; Nortvedt & Siqveland, 2019). As a number of students struggle in their mathematics courses, concerns regarding motivation become salient (George, 2012). To support students who are struggling with mathematics, some universities attempt teaching initiative efforts or intervention programs (e.g., Hieb et al., 2015). However, to improve the quality of mathematics education through various teaching efforts, we may first identify what affects students' motivation to learn and how mathematics learning is associated with students' motivation (Goldin et al., 2016; León et al., 2015). That is, attention to the role of motivational processes in mathematics learning should be paid in mathematics teaching and learning (Goldin et al., 2016).

As autonomous motivation has been highlighted in educational settings, extant studies showed that autonomous motivation has played a crucial role in mathematics learning (e.g., Hagger et al., 2015; León et al., 2015; Reindl et al., 2015). The study of Hagger et al. (2015) demonstrated that high school students' autonomous motivation about mathematics activities in school was associated with their motivation in doing mathematics homework outside of school. León et al. (2015) found that autonomous motivation led to students' deeper learning and enabled them to apply their efforts in mathematics studies that predicted math achievement. Froiland and Davison (2016) also suggested that when students felt intrinsically motivated for mathematics, they were likely to attempt challenging and higher-level mathematics courses. Existing literature suggested that autonomous motivation matters in mathematics learning; however, most of the studies have focused on K-12 students. More studies should be conducted in higher education mathematics with respect to undergraduate students' motivational factors and their learning experiences (Matthews et al., 2013).

Other student factors

Prior literature suggests a number of other student factors associated with success in undergraduate mathematics courses, such as students' socioeconomic status (e.g., Barnett et al., 2014; Sonnert et al., 2016), gender (e.g., George, 2019; Harwell et al., 2016), and minority group status (e.g., Leyva et al., 2021; Van Sickle et al., 2020). For instance, in a national study of more than 10,000 college students at 134 U.S. institutions, Sonnert et al. (2016) found that students' racial/ethnic backgrounds and socioeconomic status were significantly associated with their calculus performance. Data about the gender effect on mathematics performance are mixed. For example, in a large-scale study with more than 7500 college students, George (2019) found that female students had significantly higher course grades than male students in calculus. Nevertheless, in another large-scale study with 11,324 students, Harwell et al. (2016) found nonsignificant differences between males and females in terms of college mathematics achievement. Because the focus of the current study is to test the likelihood that motivational constructs help understand college students' learning outcomes in AC courses, we included students' gender, minority group status, and socioeconomic status as controlled variables to increase the precision of our analysis.

Hypotheses

Based on the literature concerning SDT and prior empirical research in higher education (Levesque-Bristol et al., 2020; Wang et al., 2020), it was hypothesized that after controlling for students' demographic information, students' learning outcomes would be positively related to their perceptions of basic psychological needs satisfaction and autonomous motivation, and negatively associated with controlled motivation and amotivation. Moreover, as has been previously reported (Wang et al., 2020), we expected to see that competence and autonomous motivation are more effective in predicting learning outcomes. Since the students' data were nested in a hierarchical structure way in classes, the multilevel modeling approach was applied to investigate the relationships. Our findings would provide a holistic picture of college students' motivation and learning in AC classes.

Methods

Study context

AC I is an introductory calculus course that is required for students in many management, technology, and health sciences-related disciplines. The course enrolls around 2000 students in the fall term and is primarily taught in sections with 30–40 students by Graduate Teaching Assistants (GTAs) under the direction of a permanent course coordinator. The GTAs undergo a departmental screening for teaching ability and have typically assisted with recitation sections associated with other large faculty-taught courses. Before teaching the AC course, the GTAs will have also completed various departmental orientation and training programs. Course logistics, curriculum, schedule, exams, and homework are determined by the course coordinator. GTAs write and grade quizzes for their individual sections. The majority of student grades are determined by common, multiple-choice midterm and final exams. Online and flipped versions of the course are available for a small percentage of the overall population; however, the digital course materials are open educational resources and are available for all students through the learning management system. In this study, student-level factors including Pell Grants eligibility status, gender, and underrepresented minority (URM) status were controlled as covariates.

Participants

A total of 3226 students (1,812 females, 1,414 males) in the AC course participated in the current study. The average age was 19.01 years (SD = 1.22). A majority of the participants ($n = 2119$; 65.7%) were white, followed by international students ($n = 387$; 12%), Asian American ($n = 263$; 8.2%), Hispanic ($n = 166$; 5.1%), African American ($n = 111$; 3.4%), two or more races ($n = 96$; 3.0%), unknown ($n = 77$; 2.4%), and American Indian or Native Hawaiian or Other Pacific Islander ($n = 7$; 0.2%). The majority of the students were non-URM ($n = 2893$, 89.7%) and non-Pell-eligible students ($n = 2593$, 80.5%) while 10.3% ($n = 333$) were URM students and about 19.5% ($n = 629$) were Pell-eligible students. Most participants represented ten colleges on the university campus with 20.1% in the School of Management ($n = 650$), 15.3% in the Polytechnic Institute ($n = 494$), 12.3% in the College of Agriculture ($n = 397$), 11.8% in the College of Health and Human Sciences ($n = 381$), 9.8% in Exploratory Studies ($n = 317$), 8.6% in the College of Science ($n = 279$), 7.6% in the Pre-Pharmacy ($n = 244$), 5.4% in the College of Liberal Arts ($n = 175$), 4.3% in the College of Technology ($n = 138$), 4.0% in the College of Pharmacy ($n = 129$), and 0.8% in other majors such as College of Education or First Year Engineering ($n = 22$). The average of participants' SAT mathematics score was 591.50 (SD = 72.81) out of 800 max. Students were offered nominal course participation credit (less than 3% of the total number of points in this course) for completing the survey. This project was approved by the institution's Institutional Review Boards (IRB) Human Subjects Committee.

Measures

The student perception survey consists of three discrete measures that address the perceptions of students' basic psychological needs, motivational regulation and perceived knowledge transferability.

Basic psychological needs (BPN; Deci & Ryan, 2000; Gagné, 2003)

Basic Psychological Needs Satisfaction Scale (Gagné, 2003) consisting of 21 items was applied for assessing the extent to which individuals discern their needs for autonomy, competence, and relatedness are satisfied. We adapted and modified the items for an educational setting. The modified scale was proven to be a valid and reliable measure to assess college students' basic psychological needs in higher education settings (e.g., Fedesco et al., 2019; Wang et al., 2019; Yu & Levesque-Bristol, 2020). Autonomy included seven items, such as "My feelings are taken into consideration in this course" and "There is not much opportunity for me to decide for myself how to go about my coursework (reverse item)." Competence included six items, such as "I have been able to learn interesting new skills in this course" and "I do not feel very competent in this course (reverse item)." Relatedness included eight items, such as "People in this course care about me" and "There are not many people in this course that I am close to (reverse item)." Students expressed their level of congruity with the statements using a 7-point Likert-type scale (1 = *strongly disagree*; 7 = *strongly agree*).

Situational Motivation Scale (Guay et al., 2000)

The Situational Motivation Scale (Guay et al., 2000) was utilized to assess students' perceived motives for participating in the course. The 18-item scale appraises the six different types of motivation suggested in SDT for tasks associated with college (Deci & Ryan, 2000). Students were asked to express their agreement or disagreement on a 7-point Likert-type scale with answers to the overarching question of why they are taking this course. Sample items included: "I take this course because I really enjoy it" (intrinsic motivation), "I take this course because learning all I can about academic work is really essential for me" (integration), "I take this course because it allows me to develop skills that are important to me" (identification), "I take this course because I would feel guilty if I didn't" (introjection), "I take this course because I feel I have to" (external regulation), and "I don't know. I have the impression I'm wasting my time" (amotivation). The autonomous motivation was calculated by averaging all items of intrinsic motivation, integration, and identification. The controlled motivation was calculated by averaging all items of introjection and extrinsic regulation.

Perceived knowledge transferability (PKT; Levesque-Bristol et al., 2020)

Perceived Knowledge Transferability consisting of 8 items was measured participants' perception of knowledge transfer (Levesque-Bristol et al., 2020). This scale uses a 7-point scale (1 = *strongly disagree*; 7 = *strongly agree*), with higher scores indicating students' greater likelihood of perceiving knowledge transferability. Sample items included: "I understand how I will use the information learned in this class in my professional life," "I feel as if the material covered in this course is relevant to my future career," and "I feel confident in my ability to apply the course material in other class that I have."

Course grade

Students' course grades consist of homework, quizzes, attendance, and exams. Each letter grade matches the following numerical score: A+/A = 4.0, A − = 3.7, B+ = 3.3, B = 3.0, B − = 2.7, C+ = 2.3, C = 2.0, C − = 1.7, D+ = 1.3, D = 1.0, D − = 0.7, F = 0.0.

Standardized knowledge exam

We used a standardized knowledge exam, the Calculus Concept Inventory (CCI; Epstein, 2013), as one of the measures of students' learning outcomes. The CCI is a widely used concept inventory that focuses on assessing students' conceptual understanding of basic principles of differential calculus rather than computation. Although the psychometric properties of the CCI have been supported by a set of validation studies and cognitive laboratories (Epstein, 2007, 2013), instructors were concerned that the original CCI with 22 multiple-choice items may take a large amount of instructional time to finish. To address this practical concern, we developed a short version of the CCI called the S-CCI. Ten items from the original inventory were selected by a panel of instructors experienced in calculus education based on three criteria: (a) the items were as representative as possible; (b) the items aligned with the canonical assessments for the course as well as the curriculum itself; and (c) the assessment could be completed in under 30 min.

To demonstrate the reliability and validity of this measure, we evaluated the psychometric properties of the S-CCI using item response theory (IRT). Overall, the S-CCI is sufficiently capable of distinguishing students in terms of their conceptual understanding of calculus. No items on the S-CCI were extremely easy or difficult. In terms of the scale reliability, the test information function showed that the test provided significant information between $\theta = 0.4$ and $\theta = 2.0$. That is, this scale was more effective and precise in identifying students with middle to high levels of conceptual understanding rather than in distinguishing students with low levels of conceptual understanding. The current analysis generally supports the satisfactory psychometric properties of the S-CCI. The detailed validation process and the estimates of item parameters are included in the Supplemental Material.

Student-level covariates

Pell-eligibility status, gender, and URM status were gained from the Office of Registrar. Gender identification, the URM indication, Pell-eligibility were re-coded into two dummy variables: male students as the reference group (i.e., Females = 1, Males = 0); non-URM as the reference group (i.e., URM = 1, non-URM = 0); and non-Pell Grant as the reference group (i.e., Pell-eligibility = 1, non-Pell-eligibility = 0).

Data analysis

Six multilevel models were tested: (1) unconditional model; (2) model with covariates added; (3) model with covariates and basic psychological needs added separately; (4) model with covariates and basic psychological needs added together; (5) model with covariates and motivational variables added separately; and (6) model with covariates and motivational variables added together. We investigated the influences of basic psychological needs and motivation separately because based on SDT, these two constructs are strongly correlated and share much variance, which may cause suppression effects. Considering the nested characteristics of the current data, a multilevel modeling analysis was employed in this study.

Results

Correlational analysis

Table 1 shows the bivariate Pearson correlation among the variables. The results revealed that students' basic psychological needs were positively correlated to

Table 1 Correlation among the variables

	Autonomy	Competence	Relatedness	Autonomous	Controlled	Amotivation	SKE	Final Grade	PKT
Autonomy	1								
Competence	0.71**	1							
Relatedness	0.53**	0.51**	1						
Autonomous	0.54**	0.60**	0.41**	1					
Controlled	− 0.13**	− 0.17**	− 0.02	0.14**	1				
Amotivation	− 0.37**	− 0.56**	− 0.22**	− 0.17**	0.38**	1			
SKE	0.13**	0.18**	0.05*	0.13**	0.00	− 0.08**	1		
Final grade	0.35**	0.48**	0.16**	0.33**	− 0.03*	− 0.32**	0.32**	1	
PKT	0.54**	0.61**	0.38**	0.77**	0.05**	− 0.25**	0.15**	0.34**	1

**Correlation is significant at the 0.01 level (2-tailed). *Correlation is significant at the 0.05 level (2-tailed). SKE stands for standardized knowledge exam while PKT refers to perceived knowledge transferability

autonomous motivation, whereas they were negatively related to controlled motivation and amotivation. More importantly, basic psychological needs and autonomous motivation were all correlated with students' final grades, standardized knowledge exam scores, and perceived knowledge transferability while controlled motivation and amotivation were negatively related to learning outcomes except for the relationship between controlled motivation and standardized knowledge exam.

Course grade

The multilevel modeling analysis was conducted to examine how basic psychological needs and situational motivation are associated with students' course grade. Students' course grades were determined by homework, quizzes, attendance, and exams.

First, we tested an unconditional model to partition the total variance in course grade into within- and between-class components (Tabachnick & Fidell, 2007). Then, we added the individual-level covariates in Model 2, and basic psychological needs separately in Model 3. Then, we tested a model with three basic psychological needs together in Model 4. In Model 5, we added autonomous, controlled motivation and amotivation separately, and then three motivational constructs together in Model 6. The final model for basic psychological needs with students-level covariates is Model 4 while the final model for motivation with covariates is Model 6.

In Model 1, the intercept intra-class correlation (ICC) was 0.03, which means that 3% of the variances in course grades could be interpreted by between-class differences. In Model 2, it revealed that URM status ($\beta = -0.22$) and Pell-eligibility ($\beta = -0.41$) were significant predictors of course grades, whereas the effect of gender was not significant. URM or Pell-eligible students had lower course grades than non-URM or non-Pell-eligible students. With three basic psychological needs variables added in Model 4, competence ($\beta = 0.53$), was the most important factor predicting students' course grades. The nonsignificant effect of autonomy ($\beta = 0.05$), and the negative regression coefficient of relatedness ($\beta = -0.16$), were not in line with the theory, which was likely due to the suppression effect. The variances of autonomy and relatedness might be shared by the dominant variable, competence. Regarding the roles of various types of motivation in Model 6, autonomous motivation ($\beta = 0.22$), and amotivation ($\beta = -0.22$), were important predictors of students' course grades. The standardized regression coefficients for course grades are presented in Table 2.

Standardized knowledge exam

Another analysis was conducted to examine how basic psychological needs and situational motivation are linked to students' scores on the standardized knowledge exam. In Model 1, we tested an unconditional model. Intercept ICC was 0.08, which showed that 8% of the variances in the exam could be interpreted by between-class differences. In Model 2, the covariates of gender ($\beta = -0.40$) and URM ($\beta = -0.58$) were significant for the standardized knowledge exam scores, however, Pell-eligibility was not significant. Females had lower scores than male students did and URM students had lower scores than the non-URM students did.

With three basic psychological needs variables added in Model 4, competence ($\beta = 0.35$) was still the most important predictor among the three needs, whereas the effect of relatedness was significantly negative ($\beta = -0.17$). In this model, autonomy was not a significant predictor ($\beta = 0.08$) although it was a salient predictor in Model 3 with separate basic psychological needs ($\beta = 0.29$). In Model 6, autonomous motivation ($\beta = 0.17$) was the most salient factor among motivation and amotivation was a significant negative predictor ($\beta = -0.08$), which is consistent with the findings in course grade. The

Table 2 Standardized regression coefficients for course grade

		Model 1: unconditional model (ICC = 0.03)	Model 2: student-level covariates	Model 3: covariates and BPN			Model 4: Covariates and BPN (all)	Model 5: Covariates and Motivation			Model 6: Covariates and motivation (all)
				Autonomy	Competence	Relatedness		Autonomous	Controlled	Amotivation	
Covariates	PellEligible (yes)		− 0.22*	− 0.20*	− 0.17*	− 0.21*	− 0.19*	− 0.19*	− 0.23*	− 0.23*	− 0.20*
	Gender (female)		0.07	0.04	0.03	0.06	0.03	0.07	0.06	0.01	0.02
	URM (yes)		− 0.41*	− 0.34*	− 0.34*	− 0.37*	− 0.36*	− 0.36*	− 0.41*	− 0.39*	− 0.35*
Basic psychological needs	Autonomy			0.43*			0.05				
	Competence				0.49*		0.53*				
	Relatedness					0.19*	− 0.16*				
Motivation	Autonomous							0.27*			0.22*
	Controlled								− 0.04*		0.03
	Amotivation									− 0.24*	− 0.22*

*$p < 0.05$

Table 3 Standardized regression coefficients for standardized knowledge exam

		Model 1: unconditional model (ICC = 0.08)	Model 2: student-level Covariates	Model 3: covariates and BPN			Model 4: covariates and BPN (all)	Model 5: covariates and Motivation			Model 6: Covariates and Motivation (all)
				Autonomy	Competence	Relatedness		Autonomous	Controlled	Amotivation	
Covariates	PellEligible		− 0.09	− 0.09	− 0.07	− 0.09	− 0.08	− 0.08	− 0.09	− 0.09	− 0.08
	Gender		− 0.40*	− 0.43*	− 0.43*	− 0.41*	− 0.44*	− 0.41*	− 0.41*	− 0.43*	− 0.42*
	URM		− 0.58*	− 0.55*	− 0.55*	− 0.57*	− 0.57*	− 0.56*	− 0.59*	− 0.59*	− 0.56*
Basic psychological needs	Autonomy			0.29*			0.08				
	Competence				0.32*		0.35*				
	Relatedness					0.08	− 0.17*				
Motivation	Autonomous							0.19*			0.17*
	Controlled								− 0.00		0.01
	Amotivation									− 0.11*	− 0.08*

*$p < 0.05$

standardized regression coefficients for the standardized knowledge exam are presented in Table 3.

Perceived knowledge transferability

Finally, the relationships between students' basic psychological needs, motivational regulations, and perceived knowledge transferability were investigated. In Model 1, the intercept ICC was 0.02, which showed that 2% of the variances in perceived knowledge transferability could be interpreted by between-class differences. Model 2 demonstrated that Pell-eligibility ($\beta = -0.15$) and gender ($\beta = -0.14$) were significant predictors while URM was not a significant predictor. Pell-eligible students and female students perceived less knowledge transferability in AC classes than non-Pell-eligible students and male students did. In Model 4, all three needs were significant predictors: autonomy ($\beta = 0.32$), competence ($\beta = 0.62$), and relatedness ($\beta = 0.09$). Unlike the findings for course grade and the standardized knowledge exam, autonomy and relatedness played important roles in perceived knowledge transferability, although competence was still the dominant predictor. In line with the results for course grade and the standardized knowledge exam, in Model 6, autonomous motivation ($\beta = 0.83$) was the most salient factor among motivation while amotivation was a significant negative predictor ($\beta = -0.12$), which is consistent with the findings of other learning outcomes. The standardized regression coefficients for students' perceived knowledge transferability are shown in Table 4.

Discussion

In this study, we examined the relationships between SDT-related variables and multiple learning outcomes, including course grades, standardized knowledge exam scores, and students' perceived knowledge transferability. Both between-classroom and within-classroom variations were considered by using the multilevel modeling technique. The results suggest that the most constructive way to strengthen students' learning outcomes in undergraduate AC courses might be to satisfy students' needs for competence and to foster autonomous motivation. Autonomous motivation is the most predictive motivational factor across the three learning outcomes (i.e., course grade, standardized knowledge exam, and perceived knowledge transferability) while amotivation was negatively associated with students' mathematics learning. These findings provide a holistic picture of college students' motivation and learning in AC courses.

Our analysis shows that students' competence was found to be a significant predictor of all three learning outcome variables. Much of the SDT literature shows that competence is the dominant variable among three types of basic psychological needs (Vazou & Skrade, 2017; Wang et al., 2020; Yu & Levesque-Bristol, 2020). The current findings are in congruence with the previous studies suggesting that improving students' competence is effective to enhance students' learning outcomes in the context of higher education (Wang et al., 2020). As long as students feel competent in the subject content, they are able to learn more and even feel confident applying the knowledge in other contexts.

There were some unexpected results of autonomy and relatedness needs. When testing autonomy, competence, and relatedness simultaneously, autonomy and relatedness failed to show positive associations with students' objective learning outcomes (i.e., course grades and standardized knowledge exam scores). In particular, the satisfaction of the need for relatedness has shown unexpected negative regression coefficients. It is important to note that these unexpected findings do not necessarily suggest that autonomy and relatedness are less important or undesirable in AC courses. Previous research has found that autonomy and relatedness needs contribute to students' motivation and have significant associations with students' engagement, effort, and course satisfaction (Cho et al., 2021; Deci & Ryan, 2000; Froiland et al., 2019). In fact, when running the models with each need satisfaction separately, the needs for autonomy and relatedness were found to be positively associated with course grades. The results of bivariate correlation also indicated positive associations between autonomy, relatedness, and all three types of learning outcomes. The unexpected results of autonomy and relatedness needs may partly be explained by a suppressor phenomenon, which may take place when predictors are highly correlated with each other and one predictor has a much stronger correlation with the outcome variable than other predictors (Maassen & Bakker, 2001; Pandey & Elliott, 2010). In the case of the suppression effect, variables can be found to have unexpected null or negative influences on outcomes (Pandey & Elliott, 2010), which might be the case for autonomy and relatedness in our study. It is credible that the strong relationship between competence and learning outcomes may suppress the effects of autonomy and relatedness. Our results further support the argument that autonomy and relatedness may have indirect impact on students' learning outcomes through competence satisfaction (Hsu et al., 2019; Levesque-Bristol et al., 2020). It could also be beneficial to examine how autonomy and relatedness needs are related to other adaptive student outcomes, such as well-being and course satisfaction, to further understand the roles of autonomy and relatedness in educational settings.

Another important finding reveals that autonomous motivation is the most predictive motivational factor across the three outcomes ($\beta = 0.22$ for course grade,

Table 4 Standardized regression coefficients for perceived knowledge transferability

		Model 1: unconditional model (ICC = 0.02)	Model 2: student-level covariates	Model 3: Covariates and BPN			Model 4: covariates and BPN (all)	Model 5: Covariates and motivation			Model 6: covariates and motivation (all)
				Autonomy	Competence	Relatedness		Autonomous	Controlled	Amotivation	
Covariates	PellEligible (yes)		− 0.15*	− 0.09	− 0.05	− 0.07	− 0.05	− 0.03	− 0.14*	− 0.15*	− 0.03
	Gender (female)		− 0.14*	− 0.21*	− 0.21*	− 0.15*	− 0.22*	− 0.14*	− 0.14*	− 0.20*	− 0.17*
	URM (yes)		− 0.11	0.03	− 0.01	− 0.01	0.02	0.02	− 0.10	− 0.09	0.02
Basic psychological needs	Autonomy			0.90*			0.32*				
	Competence				0.84*		0.62*				
	Relatedness					0.63*	0.09*				
Motivation	Autonomous							0.85*			0.83*
	Controlled								0.06*		− 0.01
	Amotivation									− 0.25*	− 0.12*

*$p < 0.05$

$\beta = 0.17$ for standardized knowledge exam, and $\beta = 0.83$ for PKT) while amotivation was a significant negative predictor of these learning outcomes ($\beta = -0.22$ for course grade, $\beta = -0.08$ for standardized knowledge exam, & $\beta = -0.12$ for PKT). Perceiving the course as interesting and identifying with the value of the course might play crucial roles for student learning in AC classes. The current study corroborated many research findings that autonomous motivation has a strong association with students' academic achievement in mathematics learning (e.g., Van Soom & Donche, 2014). Students' autonomous forms of motivation in mathematics are associated with their academic achievement across various educational stages: among adolescent students (Areepattamannil, 2014), college students (Van Soom & Donche, 2014), and graduate students (Ahmed & Bruinsma, 2006). Previous research has shown that students who are intrinsically motivated to learn tend to show desirable learning behaviors and happiness (Froiland et al., 2012), and better academic performance (Hagger et al., 2015; Ratelle et al., 2007). Given the fact that autonomous motivation plays a significant role in AC courses, instructors should pay more attention to fostering students' autonomous motivation. Over the past two decades, SDT research in education has demonstrated that autonomy-supportive teaching could support students' basic psychological needs and enhance their autonomous motivation (e.g., Jang et al., 2009; Reeve, 2006; Reeve & Jang, 2006; Ryan & Deci, 2017). For example, instructors may let students choose the format of homework or the way to complete the classroom activity (e.g., individual work or group work). Instructors can also provide optional quizzes for students to test themselves on the materials and check their understanding. When students perceived their instructors as supportive and responsive, they were more likely to be autonomously motivated to learn (León et al., 2015). A natural progression of this work is to investigate the effectiveness of SDT-informed interventions for promoting college students' autonomous motivation in mathematics courses.

The present study has been one of the first attempts to thoroughly examine the links between SDT-related variables and student learning outcomes in undergraduate calculus courses. Multiple learning outcomes have been considered, including a standardized knowledge exam, perceived knowledge transferability, and students' final grades. Although the associations between the SDT-related variables and the three types of learning outcomes were generally consistent in terms of statistical significance, the associations between motivation and learning were much stronger when learning was measured by subjective self-assessment (here, perceived knowledge transferability) as opposed to objective measures (here, course grade and standardized knowledge exam). This finding aligns with previous research (e.g., Hsu et al., 2019; Wang et al., 2019). The estimates of the relationships between motivation and learning are likely to be biased if we only use a single indicator to represent student learning outcomes. In fact, the relationships between student background variables and learning also varied across different types of learning outcomes. For example, we found that female students had significantly lower scores in the perceived knowledge transferability and the standardized knowledge exam than male students; however, the differences in course grades between female students and male students were nonsignificant. We suspect that the measures of learning outcomes may play a role here. Female students often reported lower levels of self-efficacy (e.g., Peters, 2013) and higher levels of anxiety (e.g., Primi et al., 2018) in mathematics, which could explain why we see female students scored lower in the perceived knowledge transferability and the standardized knowledge exam than male students. However, prior research also suggests that female students generally demonstrated higher levels of engagement in learning than male students (e.g., Miller et al., 2021). Compared to the other two learning outcomes, course grade includes multiple indicators, such as participation, homework, quizzes, and exams. It is possible that female students outperformed male students in some of these tasks. Therefore, we do not see a significant difference in course grades between males and females. We encourage future research to examine more closely the links between students' demographic background variables and achievement-related learning outcomes by comparing different types of learning outcomes.

Limitations and future studies

This current study should be interpreted in light of several limitations. First, this study is limited in that students' motivational variables were collected with self-report surveys. Classroom observation or qualitative evidence of students' autonomous motivation could serve to triangulate and complement our current study. Additionally, we did not examine how the environment impacts students' learning outcomes. Unlike the traditional lecture, active learning classes offer more opportunities for students to engage in collaborative learning and express their own thinking. It would be valuable to investigate how different instructional approaches or learning environments (e.g., lecture-based sections and active-learning sections) impact students' mathematics learning experience. Investigating how classroom environment or intervention are related to students' motivation and learning outcomes would provide further evidence to support our findings. Third, it is important to note that our data were collected from a single research university that has competitive STEM programs. The findings need to be interpreted with caution because leading universities and/or programs in STEM fields often put excessive emphasis on students' competence and performance (Gesun et al., 2021). Studies similar to this one should be carried out at a variety of different types of institutions. Fourth, to examine students' mathematical motivation in AC courses, investigation from a person-oriented perspective such as a motivational profile would be beneficial. We found that students' autonomous motivation was a significant predictor of three different learning outcomes. However, motivation is not a unidimensional construct but a more complex, multi-dimensional one. A future study of students' motivation in AC courses would benefit from a motivational profile analysis, which is a person-centered approach that can complete the dimension-centered approach (Vansteenkiste et al., 2009). This type of study will complement the quantitative findings from a variable-centered approach. Motivational profiles can be used to see whether differences in academic achievement are associated with these motivational profiles (Van Soom & Donche, 2014). Research that focuses on an in-depth understanding of students' motivation through a motivational profile would enable us to examine students' mathematical motivation and the dynamics in the learning process. Motivational profile analyses would also allow teachers to apply effective teaching strategies to enhance students' motivation (Ng et al., 2016).

Conclusions

This study contributes to the literature on undergraduate mathematics education by examining the relationships between motivation and a variety of students' learning outcomes. Through the lens of SDT, we found that satisfying students' needs for competence and promoting autonomous motivation play critical roles in supporting college students' academic success in AC courses. Our findings suggest that instructors should strive to incorporate ways to satisfy students' basic psychological needs and foster autonomous motivation in mathematics. Sound SDT-based interventions (e.g., autonomy-support, and rationale provision) that developed over the decades could be informative for AC instructors; however, more research is needed to determine the effectiveness of those interventions in undergraduate AC courses.

Abbreviations

AC: Applied Calculus; BPN: Basic psychological needs; CCI: Calculus Concept Inventory; GTAs: Graduate teaching assistants; ICC: Intra-class correlation; IRT: Item response theory; PKT: Perceived knowledge transferability; S-CCI: Short version of the Calculus Concept Inventory; SDT: Self-determination theory; SKE: Standardized knowledge exam; STEM: Science, technology, engineering, and mathematics; URM: Underrepresented minority.

Supplementary Information

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Additional file 1. Validation of the Standardized Knowledge Exam -- S-CCI.

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Author contributions

All authors contributed to the paper. CW and HC: conceptualization, data collection and curation, formal analysis, draft preparation, manuscript revising and editing; BW: conceptualization, draft preparation, manuscript review and editing; JM: data collection, draft preparation, manuscript review and editing; EB: data curation, manuscript review and editing; QL and YL: data interpretation, manuscript review and editing for Study 1; CL: supervision, funding acquisition, manuscript revising and editing. All authors read and approved the final manuscript.

Availability of data and materials

Data will not be shared in public due to the characteristics of the data, but are available from the corresponding author on reasonable request and with permission of Purdue University.

Declarations

Ethical approval and consent to participate

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards. The participants provided their written informed consent to participate in this study.

Competing interests

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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References

Ahmed, W., & Bruinsma, M. (2006). A structural model of self-concept, autonomous motivation and academic performance in cross-cultural perspective. *Electronic Journal of Research in Educational Psychology, 4*(10), 551–576. https://doi.org/10.25115/ejrep.v4i10.1214

Areepattamannil, S. (2014). Relationship between academic motivation and mathematics achievement among Indian adolescents in Canada and India. *Journal of General Psychology, 141*(3), 247–262. https://doi.org/10.1080/00221309.2014.897929
Barnett, M. D., Sonnert, G., & Sadler, P. M. (2014). Productive and ineffective efforts: How student effort in high school mathematics relates to college calculus success. *International Journal of Mathematical Education in Science and Technology, 45*(7), 996–1020.
Burton, K. D., Lydon, J. E., D'Alessandro, D. U., & Koestner, R. (2006). The differential effects of intrinsic and identified motivation on well-being and performance: Prospective, experimental, and implicit approaches to self-determination theory. *Journal of Personality and Social Psychology, 91*(4), 750–762. https://doi.org/10.1037/0022-3514.91.4.750
Blair, R. M., Kirkman, E. E., Maxwell, J. W. (2018). Statistical abstract undergraduate programs in the mathematical sciences in the United States: 2018 CBMS survey. *American Mathematical Society.* http://www.ams.org/profession/data/cbms-survey/cbms2015-Report.pdf.
Chirkov, V., Vansteenkiste, M., Tao, R., & Lynch, M. (2007). The role of self-determined motivation and goals for study abroad in the adaptation of international students. *International Journal of Intercultural Relations, 31*(2), 199–222. https://doi.org/10.1016/j.ijintrel.2006.03.002
Cho, E., & Jeon, S. (2019). The role of empathy and psychological need satisfaction in pharmacy students' burnout and well-being. *BMC Medical Education, 19*(1), 1–12. https://doi.org/10.1186/s12909-019-1477-2
Cho, H. J., Melloch, M. R., & Levesque-Bristol, C. (2021). Enhanced student perceptions of learning and performance using concept-point-recovery teaching sessions: A mixed-method approach. *International Journal of STEM Education, 8*(1), 1–17. https://doi.org/10.1186/s40594-021-00276-1
Deci, E. L., & Ryan, R. M. (1985). The general causality orientations scale: Self-determination in personality. *Journal of Research in Personality, 19*, 109–134. https://doi.org/10.1016/0092-6566(85)90023-6
Deci, E. L., & Ryan, R. M. (2000). The "what" and "why" of goal pursuits: Human needs and the self-determination of behavior. *Psychological Inquiry, 11*, 227–268. https://doi.org/10.1207/S15327965PLI1104_01
Deci, E. L., & Ryan, R. M. (2002). *Handbook of self-determination research*. University Rochester Press.
Deci, E. L., & Ryan, R. M. (2008). Facilitating optimal motivation and psychological well-being across life's domains. *Canadian Psychology/psychologie Canadienne, 49*, 14–23. https://doi.org/10.1037/0708-5591.49.1.14
Epstein, J. (2007). September). Development and validation of the Calculus Concept Inventory. *Proceedings of the Ninth International Conference on Mathematics Education in a Global Community, 9*, 165–170.
Epstein, J. (2013). The Calculus Concept Inventory-measurement of the effect of teaching methodology in mathematics. *Notices of the American Mathematical Society, 60*(8), 1018–1026.
Fedesco, H. N., Bonem, E. M., Wang, C., & Henares, R. (2019). Connections in the classroom: Separating the effects of instructor and peer relatedness in the basic needs satisfaction scale. *Motivation and Emotion, 43*(5), 758–770.
Froiland, J. M., & Davison, M. L. (2016). The longitudinal influences of peers, parents, motivation, and mathematics course-taking on high school math achievement. *Learning and Individual Differences, 50*, 252–259. https://doi.org/10.1016/j.lindif.2016.07.012
Froiland, J. M., Oros, E., Smith, L., & Hirchert, T. (2012). Intrinsic motivation to learn: The nexus between psychological health and academic success. *Contemporary School Psychology, 16*, 91–101.
Froiland, J. M., Worrell, F. C., & Oh, H. (2019). Teacher–student relationships, psychological need satisfaction, and happiness among diverse students. *Psychology in the Schools, 56*(5), 856–870. https://doi.org/10.1002/pits.22245
Gagné, M. (2003). Autonomy support and need satisfaction in the motivation and well-being of gymnasts. *Journal of Applied Sport Psychology, 15*, 372–390. https://doi.org/10.1080/714044203
Garn, A. C., Morin, A. J. S., & Lonsdale, C. (2019). Basic psychological need satisfaction toward learning: A longitudinal test of mediation using bifactor exploratory structural equation modeling. *Journal of Educational Psychology, 111*(2), 354–372. https://doi.org/10.1037/edu0000283
George, M. (2012). Autonomy and motivation in remedial mathematics. *Primus, 22*(4), 255–264. https://doi.org/10.1080/10511970.2010.497958
George, R. (2019). Predictors for success in Calculus I. *Proceedings of 2019 ASEE Annual Conference & Exposition*. https://doi.org/10.18260/1-2--33184
Gesun, J. S., Pitman-Gammon, R., Berger, E. J., Froiland, J. M., & Godwin, A. (2021). Developing a consensus model of engineering thriving using a Delphi process. *International Journal of Engineering Education, 37*(4), 939–959.
Goldin, G., Hannula, M., Heyd, E., Jansen, A., Kaasila, R., Lutovac, S., Di Martino, P., Morselli, F., Middeleton, J., Pantziara, M., & Zhang, Q. (2016). Attitudes, beliefs, motivation and identity in mathematics education. *Springer Nature*. https://doi.org/10.1007/978-3-319-32811-9
Guay, F., Vallerand, R., & Blanchard, C. (2000). On the assessment of situational intrinsic and extrinsic motivation: The Situational Motivation Scale (SIMS). *Motivation and Emotion, 24*, 175–213. https://doi.org/10.1023/A:1005614228250
Hagger, M. S., Sultan, S., Hardcastle, S. J., & Chatzisarantis, N. L. D. (2015). Perceived autonomy support and autonomous motivation toward mathematics activities in educational and out-of-school contexts is related to mathematics homework behavior and attainment. *Contemporary Educational Psychology, 41*, 111–123. https://doi.org/10.1016/j.cedpsych.2014.12.002
Harwell, M., Moreno, M., & Post, T. (2016). A study of the relationship between the ACT college mathematics readiness standard and college mathematics achievement. *Journal of Psychoeducational Assessment, 34*(3), 269–281. https://doi.org/10.1177/0734282915594069
Hieb, J. L., Lyle, K. B., Ralston, P. A. S., & Chariker, J. (2015). Predicting performance in a first engineering calculus course: Implications for interventions. *International Journal of Mathematical Education in Science and Technology, 46*(1), 40–55. https://doi.org/10.1080/0020739X.2014.936976
Hourigan, M., & O'Donoghue, J. (2007). Mathematical under-preparedness: The influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in Ireland. *International Journal of Mathematical Education in Science and Technology, 38*(4), 461–476. https://doi.org/10.1080/00207390601129279
Hsu, H. C. K., Wang, C. V., & Levesque-Bristol, C. (2019). Reexamining the impact of self-determination theory on learning outcomes in the online learning environment. *Education and Information Technologies, 24*(3), 2159–2174. https://doi.org/10.1007/s10639-019-09863-w
Jang, H. (2008). Supporting students' motivation, engagement, and learning during an uninteresting activity. *Journal of Educational Psychology, 100*(4), 798–811. https://doi.org/10.1037/a0012841
Jang, H., Reeve, J., Ryan, R. M., & Kim, A. (2009). Can self-determination theory explain what underlies the productive, satisfying learning experiences of collectivistically oriented Korean students? *Journal of Educational Psychology, 101*(3), 644–661. https://doi.org/10.1037/a0014241
John, J. E., Nelson, P. A., Klenczar, B., & Robnett, R. D. (2020). Memories of math: Narrative predictors of math affect, math motivation, and future math plans. *Contemporary Educational Psychology, 60*, 101838. https://doi.org/10.1016/j.cedpsych.2020.101838
Karimi, S., & Sotoodeh, B. (2020). The mediating role of intrinsic motivation in the relationship between basic psychological needs satisfaction and academic engagement in agriculture students. *Teaching in Higher Education, 25*(8), 959–975. https://doi.org/10.1080/13562517.2019.1623775
Kusurkar, R. A., Ten Cate, T. J., Vos, C. M. P., Westers, P., & Croiset, G. (2013). How motivation affects academic performance: A structural equation modelling analysis. *Advances in Health Sciences Education, 18*(1), 57–69. https://doi.org/10.1007/s10459-012-9354-3
León, J., Núñez, J. L., & Liew, J. (2015). Self-determination and STEM education: Effects of autonomy, motivation, and self-regulated learning on high school math achievement. *Learning and Individual Differences, 43*, 156–163. https://doi.org/10.1016/j.lindif.2015.08.017
Levesque-Bristol, C., Richards, K. A. R., Zissimopoulos, A., Wang, C., & Yu, S. (2020). An evaluation of the integrative model for learning and motivation in the college classroom. *Current Psychology*. https://doi.org/10.1007/s12144-020-00671-x
Leyva, L. A., Quea, R., Weber, K., Battey, D., & López, D. (2021). Detailing racialized and gendered mechanisms of undergraduate precalculus and calculus classroom instruction. *Cognition and Instruction, 39*(1), 1–34. https://doi.org/10.1080/07370008.2020.1849218
Loftsgaarden, D. O., Rung, D. C., and Watkins, A. E. (1997). *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 1995 CBMS Survey*, MAA Reports Number 2, The Mathematical Association of America

Maasen, G. H., & Bakker, A. B. (2001). Suppressor variables in path model: Definitions and interpretations. *Sociological Methods & Research, 30*, 241–270. https://doi.org/10.1177/0049124101030002004
Matthews, A. R., Hoessler, C., Jonker, L., & Stockley, D. (2013). Academic motivation in calculus. *Canadian Journal of Science, Mathematics and Technology Education, 13*(1), 1–17. https://doi.org/10.1080/14926156.2013.758328
Miller, C. J., Perera, H. N., & Maghsoudlou, A. (2021). Students' multidimensional profiles of math engagement: Predictors and outcomes from a self-system motivational perspective. *British Journal of Educational Psychology, 91*(1), 261–285. https://doi.org/10.1111/bjep.12358
Milyavskaya, M., & Koestner, R. (2011). Psychological needs, motivation, and well-being: A test of self-determination theory across multiple domains. *Personality and Individual Differences, 50*(3), 387–391. https://doi.org/10.1016/j.paid.2010.10.029
National Research Council of the National Academies. (2013). *The mathematical sciences in 2025*. The National Academies Press.
Ng, B. L. L., Liu, W. C., & Wang, J. C. K. (2016). Student motivation and learning in mathematics and science: A cluster analysis. *International Journal of Science and Mathematics Education, 14*(7), 1359–1376. https://doi.org/10.1007/s10763-015-9654-1
Nortvedt, G. A., & Siqveland, A. (2019). Are beginning calculus and engineering students adequately prepared for higher education? An assessment of students' basic mathematical knowledge. *International Journal of Mathematical Education in Science and Technology, 50*(3), 325–343. https://doi.org/10.1080/0020739X.2018.1501826
Pandey, S., & Elliott, W. (2010). Suppressor variables in social work research: Ways to identify in multiple regression models. *Journal of the Society for Social Work and Research, 1*(1), 28–40. https://doi.org/10.5243/jsswr.2010.2
Peters, M. L. (2013). Examining the relationships among classroom climate, self-efficacy, and achievement in undergraduate mathematics: A multi-level analysis. *International Journal of Science and Mathematics Education, 11*(2), 459–480. https://doi.org/10.1007/s10763-012-9347-y
Primi, C., Donati, M. A., Chiesi, F., & Morsanyi, K. (2018). Are there gender differences in cognitive reflection? Invariance and differences related to mathematics. *Thinking & Reasoning, 24*(2), 258–279. https://doi.org/10.1080/13546783.2017.1387606
Ratelle, C. F., Guay, F., Vallerand, R. J., Larose, S., & Senécal, C. (2007). Autonomous, controlled, and amotivated types of academic motivation: A person-oriented analysis. *Journal of Educational Psychology, 99*(4), 734–746. https://doi.org/10.1037/0022-0663.99.4.734
Reeve, J. (2006). Teachers as facilitators: What autonomy-supportive teachers do and why their students benefit. *The Elementary School Journal, 106*(3), 225–236. https://doi.org/10.1086/501484
Reeve, J., & Jang, H. (2006). What teachers say and do to support students' autonomy during a learning activity. *Journal of Educational Psychology, 98*(1), 209–218. https://doi.org/10.1037/0022-0663.98.1.209
Reindl, M., Berner, V. D., Scheunpflug, A., Zeinz, H., & Dresel, M. (2015). Effect of negative peer climate on the development of autonomous motivation in mathematics. *Learning and Individual Differences, 38*, 68–75. https://doi.org/10.1016/j.lindif.2015.01.017
Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*(1), 68–78. https://doi.org/10.1037/0003-066X.55.1.68
Ryan, R. M., & Deci, E. L. (2017). Self-determination theory: Basic psychological needs in motivation, development, and wellness. Guilford Publications. http://ebookcentral.proquest.com/lib/purdue/detail.action?docID=4773318.
Schreiber, J. B. (2016). *Motivation 101*. Springer.
Sonnert, G., Barnett, M. D., & Sadler, P. M. (2016). Individual and neighborhood effects of race/ethnicity and socioeconomic status on participation and performance in college calculus. *Journal of Women and Minorities in Science and Engineering, 22*(4), 281–307. https://doi.org/10.1615/JWomenMinorScienEng.2016016096
Steen, L. A. (1987). Calculus for a new century: A pump, not a filter [Colloquium presentation]. Mathematical Association of America 1987, Washington D.C. https://files.eric.ed.gov/fulltext/ED300252.pdf.
Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics*. Allyn & Bacon/Pearson Education.
Taylor, G., Jungert, T., Mageau, G. A., Schattke, K., Dedic, H., Rosenfield, S., & Koestner, R. (2014). A self-determination theory approach to predicting school achievement over time: The unique role of intrinsic motivation. *Contemporary Educational Psychology, 39*(4), 342–358. https://doi.org/10.1016/j.cedpsych.2014.08.002
Thøgersen-Ntoumani, C., & Ntoumanis, N. (2006). The role of self-determined motivation in the understanding of exercise-related behaviours, cognitions and physical self-evaluations. *Journal of Sports Sciences, 24*(4), 393–404. https://doi.org/10.1080/02640410500131670
Tjin A Tsoi, S. L. N. M., de Boer, A., Croiset, G., Koster, A. S., van der Burgt, S., & Kusurkar, R. A. (2018). How basic psychological needs and motivation affect vitality and lifelong learning adaptability of pharmacists: a structural equation model. *Advances in Health Sciences Education, 23*(3), 549–566. https://doi.org/10.1007/s10459-018-9812-7
Van Sickle, J., Schuler, K. R., Holcomb, J. P., Carver, S. D., Resnick, A., Quinn, C., Jackson, D. K., Duffy, S. F., & Sridhar, N. (2020). Closing the Achievement gap for underrepresented minority students in STEM: A deep look at a comprehensive intervention. *Journal of STEM Education: Innovations and Research, 21*(2), 5–18.
Van Soom, C., & Donche, V. (2014). Profiling first-year students in stem programs based on autonomous motivation and academic self-concept and relationship with academic achievement. *PLoS ONE*. https://doi.org/10.1371/journal.pone.0112489
Vansteenkiste, M., Sierens, E., Soenens, B., Luyckx, K., & Lens, W. (2009). Motivational profiles from a self-determination perspective: The quality of motivation matters. *Journal of Educational Psychology, 101*(3), 671–688. https://doi.org/10.1037/a0015083
Vazou, S., & Skrade, M. A. (2017). Intervention integrating physical activity with math: Math performance, perceived competence, and need satisfaction. *International Journal of Sport and Exercise Psychology, 15*(5), 508–522. https://doi.org/10.1080/1612197X.2016.1164226
Wang, C., Hsu, H. C. K., Bonem, E. M., Moss, J. D., Yu, S., Nelson, D. B., & Levesque-Bristol, C. (2019). Need satisfaction and need dissatisfaction: A comparative study of online and face-to-face learning contexts. *Computers in Human Behavior, 95*, 114–125. https://doi.org/10.1016/j.chb.2019.01.034
Wang, C., Zhang, Y., Moss, J. D., Bonem, E. M., & Levesque-Bristol, C. (2020). Multilevel factors affecting college students' perceived knowledge transferability: From the perspective of self-determination theory. *Research in Higher Education, 61*(8), 1002–1026. https://doi.org/10.1007/s11162-020-09592-x
Yu, S., & Levesque-Bristol, C. (2020). A cross-classified path analysis of the self-determination theory model on the situational, individual and classroom levels in college education. *Contemporary Educational Psychology, 61*(March), 101857. https://doi.org/10.1016/j.cedpsych.2020.101857

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