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Input-output (IO) and Life Cycle Assessment (LCA) modeling share a strikingly similar mathematical foundation and formulation, yet melding the two frameworks is not completely straightforward. In particular, the boundaries of the two systems must be brought into alignment carefully, so as to avoid double counting. Below, the two frameworks and their mathematical foundations are presented in brief, and the salient issues that enter into procedures integrating the two are highlighted.

Input-Output Models of Economic Systems

In input-output (IO) modeling, a technology matrix A represents the dollar amounts of row inputs needed to produce one dollar of column industry outputs. By convention, output is notated by $x_i \in X$, for industries i, and dollar flows of inputs between industries are denoted by $z_{ij} \in Z$, so technical coefficients are computed as shown below.

$$a_{ij} = z_{ij} / x_j \qquad (1)$$

The amounts of output delivered to final consumers or exported from the region are called final demand, and represented by $y_i \in Y$. The output balance equations relating inputs and outputs from an IO system are given by:

$$X - Ax = Y \qquad (2)$$

$$(I - A)^{-1} Y = X \qquad (3)$$

Here, final demands are the remainder once intermediate inputs are subtracted from total output produced.

More recent data compilation and publication for input-output systems have as their own foundations a commodity-by-industry (CxI) framework comprising two sets of information. The first depicts the commodities used by industries and is called the Use table, U, and the second depicts the commodities produced (made and supplied) by industries called the Make (or Supply) table conventionally annotated by V. These two tables can be combined to create the equivalent of an interindustry IO table. The example below illustrates the mathematical foundation based on an industry technology assumption in which industry shares of commodity production are invariant.

By definition,

$$Ui + E \equiv q \qquad (4)$$

Where E is commodity final demand and q is commodity output. Then

$$Vi = g \tag{5}$$

where g is industry output (denoted X in conventional industry-by-industry frameworks), and

$$V'i = q \tag{6}$$

By assumption,

$$B = U\hat{g}^{-1} \tag{7}$$

$$U = B\hat{g} \tag{8}$$

By substitution,

$$q = Bg + E \tag{9}$$

By assumption,

$$V = D\hat{q} \tag{10}$$

$$D = V\hat{q}^{-1} \rightarrow d_{ij} = v_{ij} / q_j \tag{11}$$

Post-multiplying both sides of equation (10) by q yields $Dq = V\hat{q}^{-1}q = Vi = g$. So

$$g = Dq \tag{12}$$

and matrix D serves as a transformation from commodity to industry space and its inverse transforms industry to commodity, so

$$Y = DE \tag{13}$$

Where Y is final demand in industry rather than commodity space. Substituting equation (12) into equation (9) yields

$$q = BDq + E \tag{14}$$

so

$$\left(I - BD\right)^{-1} E = q \tag{15}$$

$$E = (I - BD)D^{-1}g \tag{16}$$

$$D\left(I - BD\right)^{-1} E = g \tag{17}$$

From equation (16)

$$E = (D^{-1} - B)g \tag{18}$$

$$DE = \left(I - DB\right)g \tag{19}$$

$$\left(I - DB\right)^{-1} Y = g \qquad (20)$$

Equation (20) is the commodity-by-industry equivalent of equation(3).

Life Cycle Assessment

Following the Heijungs and Suh [HEI02] description of the life cycle inventory problem, the processes within the system boundary – called the technosphere – are represented by a non-singular technology matrix, A, which is inverted and then post-multiplied by f, a demand vector that represents all that the entire system should ultimately deliver, to solve for s, a scaling vector *that represents* the total amount of each process needed to produce and deliver the specified demand:

$$A^{-1} f = s \qquad (21)$$

Next, s is used to scale the environmental process flows embodied in a matrix B that has columns corresponding to processes in A and rows for inputs from and outputs to the environment (e.g., crude oil and carbon dioxide emissions):

$$Bs = g \qquad (22)$$

such that the inventory result in vector g summarizes the life cycle resource use and emissions.

Comparisons

In the IO framework, the elements of the technology matrix are non-negative by definition, since the inputs are taken from the economic system and there cannot be negative amounts of industry output. I.e., all inputs are physical products produced by other industries and absorbed by the consuming industries in their production processes.

By contrast, the LCA is designed to capture extractions from and contributions to the technosphere and its environment, so for that reason, the technology (process) matrix will have negative and positive values corresponding to these extractions and contributions. The LCA matrix will have a unit value (1.0) to represent the process output, where the process output is not explicitly a part of the IO technical matrix. Hence, in effect, the LCA A matrix is more closely aligned with the $(I - A)$ matrix (called the Leontief matrix) of IO. These two matrices share nearly identical interpretations. Positive values in each are contributions to the technosphere and environment, and negative values are extractions.

Melding IO and LCA Frameworks

When a new process is added to an IO system, we can generate a corresponding new column of a Use matrix using a modified approach to LCA process table construction.[1] Begin by setting up a conventional LCA process matrix, but partition the process matrix into four quadrants according to New Activities and Conventional Industries (those already present in the IO matrix), as shown in Table 1 below:

Table 1. Hypothetical Biomass Process

	New Activities/Processes Associated with the New Technology							Existing Industries				
	BioMass Processing	Ancillary activity 1	Ancillary activity 2	Ancillary activity 3	Ancillary activity 4	Ancillary activity 5	Ancillary activity 6	Existing Industry 1	Existing Industry 2	Existing Industry 3	Existing Industry 4	
BioMass	1	0	0	0	0	0	0	0	0	0	0	1
New Activities /Processes	-0.7	1	0	0	0	0	0	0	0	0	0	2
	-0.3	0	1	0	0	0	0	0	0	0	0	3
	0	-1	-1	1	0	0	0	0	0	0	0	4
	0	0	0	-0.04125	1	0	0	0	0	0	0	5
	0	0	0	-0.01375	0	1	0	0	0	0	0	6
	0	0	0	-8.25	0	0	1	0	0	0	0	7
Existing Industries	0	0	0	-2.75	0	0	0	1	0	0	0	8
	0	-0.3	0	0	0	0	0	0	1	0	0	9
	0	0	-0.15	-0.05	-0.00042	-0.00007	-0.05	0	0	1	0	10
	0	0	0	-2	0	0	-2	0	0	0	1	11
	1	2	3	4	5	6	7	8	9	10	11	

Table 1 depicts an hypothetical woody biomass process matrix partitioned according to whether the activities and accompanying processes are new or are virtually identical to existing industries (this example corresponds to an IO table with only four industry sectors). The biomass process shown here used inputs from ancillary activities 1 and 2 (AA1 and AA2), both of which use inputs from AA3 and from existing industries. AA3 uses inputs from AA4, AA5, and AA6 along with existing industry inputs, and so on. For the purposes of creating a new column in the IO commodity-by-industry Use matrix, the fact that the only existing-industry column entries are unit values in the rows of the existing industries indicates that additional rounds of exchange lie outside of this process description. Indirect and subsequent rounds of exchange will be captured once the new column has been inserted and the biomass activity becomes an additional industry in the IO matrix.

To complete the Use column construction, post-multiply the inverse of this matrix by a unit final demand vector. The existing-industry rows of the resulting vector will be the values that comprise the new Use table column corresponding to the biomass industry, as shown for the example process in Table 2.

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[1] This procedure was introduced in Cooper, Jackson, and Leigh (2013).

Table 2. Process Inverse -- Unit Final Demand Product

BioMass	1
New Activities /Processes	0.7
	0.3
	1
	0.04125
	0.01375
	8.25
Existing Industry 1	2.75
Existing Industry 2	0.21
Existing Industry 3	0.507518
Existing Industry 4	18.5

To complete the IO accounts, a new industry row corresponding to the Biomass industry also needs to be added to the make table. Whereas the values in the use column correspond to commodities used by the Biomass industry, the values in the new make row correspond to the regional Biomass industry's production of commodities. If the new industry produces commodities that were previously produced by other industries in the IO system, they can simply be entered in this row in the columns corresponding to the pre-existing commodities. If new commodities are produced, new commodity columns corresponding to each new commodity would be added. If these new commodities substitute for inputs other industries use, the Use table would have to be edited to reflect these substitutions.

New Industry Impacts

A number of alternatives exist for impact model "drivers." The most straightforward alternative is simply to allow all new output to enter the production system as replacement for imports. This method reflects the consideration of *avoided* life cycles in LCA (e.g., new sources of these commodities preclude the need to import them from other economic regions). Post-adjustment output, employment and income levels can be compared to pre-adjustment levels to determine impacts. However, should total intra-regional demand for new commodity output be less than total produced, a final demand entry corresponding to exports will be required to balance the accounts, a concept that would be reflected in a well-developed, consequential LCA (Ekval and Andræ 2006). Likewise, other well-founded final demand estimates can be used, including export scenarios. Finally, more elaborate structural decomposition analyses can yield additional insights.

References

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