ASYMPTOTIC TAIL PROBABILITY FOR THE DISCOUNTED AGGREGATE SUMS IN A TIME DEPENDENT RENEWAL RISK MODEL

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Summary: This paper presents an extension of the classical compound Poisson risk model in which the inter-claim time arrivals and the claim amounts are structurally dependent. We derive the corresponding asymptotic tail probabilities for the discounted aggregate claims in a finite insurance contract under constant force of interest. The dependence assumption between the inter-claim times and the claim amounts is well suited for insurance contracts during extreme and catastrophic events. Based on the existing literature, we use heavy-tailed distributions for the discounted aggregate claims and derive the extreme value at risk (minimum capital requirement). Our results, based on a case study of ten million simulations, show that the independence assumption between the inter-claim times and the claim amounts lead to underestimating the minimum capital requirement proposed by the regulatory authorities.

1. Introduction

The distribution of the discounted aggregate renewal sums is crucial for risk modelling. Its applications spread across various fields such as actuarial science, credit risk management, and health economics. The moments of this distribution are often generated to have a

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glimpse of the expected average claim amounts, the inter-claim time arrivals, their volatility as well as the ruin probability. Under constant force of interest rate, the classical and compound Poisson risk models always assume that the inter-claim time arrivals and the forthcoming claim amount are two independent random variables.
In the literature, the asymptotic tail probability of the discounted aggregate claim process over a finite time horizon is obtained under the assumption of independence between the inter-claim time arrivals and the forthcoming claim amount. The asymptotic tail probability based risk models are more suitable in situations where the inter-claim time arrivals are very small and the magnitude of the claim amount increasingly large. This situation occurs in insurance contracts involving earthquakes, catastrophic events, financial crises etc. The traditional Poisson risk model often fails to generate accurate risk measures without the use of asymptotic tail probabilities. Tang (2005, 2007) and Wang (2008) derive asymptotic tail probability results of the discounted aggregate claim process over a finite time horizon for both the classical compound Poisson and the renewal risk models.

Similar studies that assume independence between the inter-claim time arrivals and the forthcoming claim amount include Taylor (1979), Delbaen and Haezendonck (1987), Waters (1989), Sundt and Teugels (1995), Cai (2002) and Yuen, Wang and Wu (2006) who not only assumed independence between inter-claim time arrivals; but also have derived the asymptotic tail probabilities associated with free interest risk model. In addition, Boogaert, Haezendonck and Delbaen (1988), Willmot (1989), Léveillé and Garrido (2001a, 2001b) Léveillé, Garrido and Wang (2009), Léveillé and Adékambi (2011, 2012) have not only assumed the independence between the inter-claim time arrivals and the claim amount, but also derived the generalization of the moments of the aggregate renewal sums.

Although the independence assumption appears to be custom in risk theory, it has recently been subjected to severe criticism due to the fact that it does not provide reliable estimates under extreme events; for examples see Boudreault, Cossette, Landriault and Marceau (2006) and Zhang and Yang (2011). Thus, the use of dependent risk models have exhibited some positive interests in the literature. Albrecher and Boxma (2004, 2005) develop a dependent risk model in which they assume that the inter-claim arrivals and the forthcoming claim amount are governed by a semi-Markov process. Further to that, Albrecher and Teugels (2006) model the dependence between the inter-claim arrivals and the forthcoming claim amount by making use of a random walk model. Their results are more elaborate as they model this dependence using a copula and derive exponential estimates of ruin probabilities in finite and infinite time horizons.

This paper attempts to assess the impact of the dependence between the inter-claim time arrivals and the forthcoming claim amount by also making use of copulas and deriving the asymptotic tail probabilities. The closest study to ours is that of Asimit and Badescu (2010) who consider in a constant force of interest environment a heavy tailed distribution of the discounted aggregate claim in which the dependence structure is modelled using copulas. They derive the asymptotic tail probabilities, the asymptotic finite time ruin probabilities, as well as asymptotic approximations for some common risk measures associated with the discounted aggregate claims distribution.

This paper has not only generalized the results of Asimit and Badescu (2010) but also presents a general case where the distribution of both the inter-claim time arrivals and the claim amount can be of any form. To our best knowledge, this is the first study of its kinds to obtain general results allowing the inter-claim time arrivals and the claim amount to have any theoretical distribution. Our model inputs are set as follows:

1. The claim counting processes $\{N(t), t \geq 0\}$ and $\{N_d(t), t \geq 0\}$ form respectively an ordinary and a delayed renewal process and, for $k \in N = \{1, 2, 3, ...\}$
   - the positive claim occurrence times is given by $\{T_k, k \in N\}$,
   - the positive claim inter-arrival times are given by $W_k = T_k - T_{k-1}, k \in N$, $T_0 = 0$.
2. The claim severities $\{X_k, k \in N\}$ are such that
   - $\{X_k, k \in N\}$ are independent and identically distributed random variables (i.i.d.). $\{X_k, W_k, k \in N - \{1\}$ is independent of $\{X_1, W_1\}$.
3. The aggregate discounted value at time 0 of the claims severity over a finite time horizon $(0, t]$ yields respectively, for the ordinary and the delayed renewal case,

$$S_{\boldsymbol{\delta}}(t) = \sum_{k=1}^{N(t)} e^{-k} X_k,\ S_{\boldsymbol{\delta}}^d(t) = \sum_{k=1}^{N_d(t)} e^{-k} X_k,\ t \geq 0$$

   where $S_{\boldsymbol{\delta}}(t) = S_{\boldsymbol{\delta}}^d(t) = 0$ if $N(t) = N_d(t) = 0$ and δ is the constant force of interest.

The remainder of the paper is organized as follows. Section 2 describes in more details the type of dependence structure used in the study and an overview of well-known results. Section 3 provides the asymptotic forms of the tail of the probability distribution of the discounted aggregate claims and their corresponding finite time ruin probabilities. In Section 4, we derive asymptotic formulas for several risk measures associated with the discounted aggregate claim distribution. Numerical illustrations are given at the end of this section.

2. Preliminaries

Assumptions 2.1 The bivariate random vectors (W_i, X_i), $i = 1, 2, \cdots$ are mutually independent and identically distributed. Moreover, there exists a positive and locally bounded function $g(\ )$ such that the relation

$$Pr\big(X_1 > x | W_1 = w\big) \sim Pr\big(X_1 > x\big) g(w)$$

holds uniformly for all $w \in (0, T]$ as $x \to \infty$ where T is the time horizon.

Here uniformly is understood as

$$\lim_{x\to\infty} \sup_{w\in(0,T]} \left| \frac{Pr\big(X_i > x | W_i = w\big)}{Pr\big(X_1 > x\big) g(w)} - 1 \right|.$$

The motivation behind the above assumption is given by the fact that under its premises we can study in a unified way a wide class of dependence structures given in terms of a copula. A two-dimensional copula is a bivariate distribution function defined on $[0,1]^2$ with uniformly distributed marginals. Due to Sklar's Theorem (Sklar, 1959), if G is a joint density function with continuous marginals G_1 and G_2, there exists a unique copula, C, given by

$$C\big(G_1(x), G_2(y)\big) = G(x,y) \in Domain(G).$$

Similarly, the survival copula is defined as the copula relative to the joint survival function and is given by

$$\widehat{C}(u,v) = u + v - 1 + C(1-u, 1-v),\ (u,v) \in [0,1]^2.$$

A more formal definition and examples of copulas are given in Nelsen (1999). Provided that $\widehat{C}_2(u,v) = \frac{\partial \widehat{C}(u,v)}{\partial v}$, $(u,v) \in [0,1]^2$ exists, Assumption 2.1 can be rewritten as

$$\lim_{u\downarrow 0} \sup_{v\in\left[e^{-\lambda T},1\right)} \left| \frac{\widehat{C}(u,v)}{u h(v)} - 1 \right| = 0,$$

where $h(v) = g\big(-\frac{\log v}{\lambda}\big)$. An example of a copula that satisfies Assumption 2.1 is given below:

Example 1 *Ali-Mikhail-Haq*

$$C(u,v) = \frac{uv}{1 - \phi(1-u)(1-v)},$$

with $g(w) = 1 + \phi(1 - 2e^{-\lambda w})$.

There are many characterizations of heavy-tailed distributions, but one of the most popular families is the class $\wp$ of subexponential distributions. By definition, a non-negative random variable X with density function F belongs to $\wp$ and we write $F \in \wp$ if

$$\lim_{x\to\infty} \frac{Pr\big(X_1 + X_2 > x\big)}{Pr\big(X > x\big)} = 2,$$

where X_1 and X_2 are independent copies of X. A well-known subclass of $\wp$ is the set of regularly varying density functions with parameter α denoted by $RV_{-\alpha}$. By definition, a density function F belongs to $RV_{-\alpha}$ family of distribution such that a random variable X belongs to $RV_{-\alpha}$ if

$$\lim_{x\to\infty}\frac{Pr\big(X>xy\big)}{Pr\big(X>x\big)}=y^{-\alpha},\ \alpha>0.$$

For more details on heavy-tailed distributions, we refer the reader to Bingham, Goldie and Teugels (1987) and Embrechts, Klüppelberg and Mikosch (1997).

Lemma 2.1 Consider an ordinary or a delayed renewal counting process, such as given in Section 1. Then, for $0=x_0<x_1<x_2<...<x_k\le t$, $i_0=0,1=i_1<i_2<...<i_k\le n$ and $1\le k\le n$, the conditional joint density probability functions of $T_{i_1},T_{i_2},...,T_{i_k}\big|N(t)=n$ and $T_{i_1},T_{i_2},...,T_{i_k}\big|N_d(t)=n$ are given, for the renewal case, by:

$$f_{T_{i_1},T_{i_2},...,T_{i_k}\big|N(t)}\big(x_1,x_2,...,x_k|n\big)=\frac{P\big(N(t-x_k)=n-i_k\big)\prod_{j=1}^{k}f_{T_{ij}-T_{ij-1}}\big(x_j-x_{j-1}\big)}{P\big(N(t)=n\big)},\quad (1)$$

and, for the delayed case, by:

$$f_{T_{i_1},T_{i_2},...,T_{i_k}\big|N_d(t)}\big(x_1,x_2,...,x_k|n\big)=\frac{P\big(N(t-x_k)=n-i_k\big)\prod_{j=1}^{k}f_{T_{ij}-T_{ij-1}}\big(x_j-x_{j-1}\big)}{P\big(N_d(t)=n\big)},$$

where $T_{i_j}, i_0=0,1=i_1<i_2<...<i_j\le n$ denotes the i_j^{th} claim occurrence time. For the proof, see Léveillé and Adékambi (2012). For the particular case where $W_i\rightsquigarrow$ Exponential(λ), called the homogeneous Poisson process, the probability density function of the inter-claim times, $W=(W_1,...,W_n)$, conditioned on the number of events by time t is

$$f_{W_1,W_2,...,W_k\big|N(t)=n}\big(x_1,x_2,...,x_k|n\big)=\frac{n!}{t^n},$$

on $D_n:=\big\{\big(W_1,...,W_n\big)\in(0,t)^n:\sum_{i=1}^{n}W_i<t\big\}$.

Lemma 2.2 Let $X,Y_1,Y_2,...$ be a sequence of independent non-negative random variables, with G the density function of X, $G\in\wp$. In addition, it is assumed that there exists a constant M such that $Pr\big(Y_1>x\big)\le MPr\big(X>x\big)$ holds for all $x>0$ and any $i=1,2,....$ Then, for any $\varepsilon>0$ there exists a constant $A<\infty$ such that

$$S_n:=\sup_{x>0}\frac{Pr\bigg(\sum_{i=1}^{n}Y_i>x\bigg)}{Pr\big(X>x\big)}\le A(1+\varepsilon)^n,$$

holds for any integer n. For the proof, see Asimit and Badescu (2010).

3. Main results

Lemma 2.2 will now help us to state the main result for the model without interest.

Theorem 3.1 Consider the model without interest ($\delta = 0$) compound renewal model such that $F \in \wp$. If Assumption 2.1 is satisfied for any $t \in (0,T]$, where T is the time horizon, then

$$Pr\big(S_0(T) > x\big) \sim K_0(T)Pr\big(X_1 > x\big), x \to \infty,$$

where

$$K_0(T) = \sum_{i=1}^{\infty} \int_0^T \int_{w_1}^T \dots \int_{w_{n-1}}^T ng(w_1)Pr\big(N(T-w_n)=0\big) \prod_{j=1}^n f_{W_j}(w_j - w_{j-1})\,dw.$$

Proof. By conditioning on the number of claims and inter-claim times by time T and using Equation (1), the cumulative distribution functions of $S_0(T)$ follows the integrals equations

$$Pr\big(S_0(T) > x\big) = \\ \sum_{n=1}^{\infty} Pr\big(N(T)=n\big) \int_{D_n} Pr\bigg(\sum_{k=1}^n X_k > x|W=w, N(T)=n\bigg) Pr\big(W=w|N(T)=n\big)dw.$$

From Lemma 2.1 we have,

$$f_{W_1,W_2,\dots,W_n|N(T)=n}(w_1,w_2,\dots,w_n|n) = \frac{Pr\big(N(T-w_n)=0\big)\prod_{j=1}^n f_{W_j}\big(w_j - w_{j-1}\big)dw}{Pr\big(N(T)=n\big)}.$$

Then,

$$\begin{aligned} Pr\big(S_0(T) > x\big) &= \sum_{n=1}^{\infty} Pr\big(N(T)=n\big) \int_{D_n} Pr\bigg(\sum_{k=1}^n X_k > x|W=w, N(T)=n\bigg) \\ &\qquad \times \frac{Pr\big(N(T-w_n)=0\big)\prod_{j=1}^n f_{W_j}\big(w_j - w_{j-1}\big)}{Pr\big(N(T)=n\big)} dw \\ &= \sum_{n=1}^{\infty} \int_{D_n} Pr\bigg(\sum_{k=1}^n X_k > x|W=w\bigg) Pr\big(N(T-w_n)=0\big) \prod_{j=1}^n f_{W_j}\big(w_j - w_{j-1}\big). \end{aligned} \tag{2}$$

Using the fact that $X_i|W_i = w_i$ are independent and $F \in \wp$, we can apply Lemma 2.2. This is true, since Assumption 2.1 implies that there exists some constant $M > 0$ such that for all $x > 0$ and $w \in (0,T]$

$$Pr\big(X_1 > x|W_1 = w_1\big) \le MPr\big(X_1 > x\big).$$

Due to Lemma 2.1, it follows that for any $\varepsilon > 0$ there exists $A > 0$ such that

$$Pr\Big(\sum_{k=1}^{n} X_k > x|W = w\Big) \leq A(1+\varepsilon)^n Pr\big(X_1 > x\big)$$

holds uniformly for all $x > 0$, $w \in (0,T]^n$ and $n = 1,2,\ldots$. Furthermore,

$$\begin{aligned}
&\sum_{n=1}^{\infty} Pr\big(N(T)=n\big)\int_{D_n} Pr\Big(\sum_{k=1}^{n} X_k > x|W=w,N(T)=n\Big)Pr\big(W=w|N(T)=n\big)dw\\
&\quad\leq \sum_{n=1}^{\infty} Pr\big(N(T)=n\big)\int_{D_n} A(1+\varepsilon)^n Pr\big(W=w|N(T)=n\big)dw\\
&\quad\leq \sum_{n=1}^{\infty} Pr\big(N(T)=n\big)A(1+\varepsilon)^n = AP_{N(T)}(1+\varepsilon) < \infty
\end{aligned}$$

since $P_{N(t)}$ exists for any $t > 0$. This allows us to apply the Dominated Convergence Theorem in Equation (2) which, together with Assumption 2.1, Theorem 3.1 from Cline (1986) and the relation in Equation (1), yield

$$\lim_{x\to\infty}\frac{Pr\big(S_0(T)>x\big)}{Pr\big(X_1>x\big)} = K_0(T) = \sum_{n=1}^{\infty}\int_{D_n}\sum_{k=1}^{n} g(w_k)Pr\big(N(T-w_n)=0\big)\prod_{j=1}^{n} f_{W_j}\big(w_j-w_{j-1}\big)dw. \tag{3}$$

■

Example 2
When $f_{W_1}(t) = \lambda_1 e^{-\lambda t}$, $f_{W_j}(t) = \lambda e^{-\lambda t}$, $j = 2,3,\ldots$ we have

$$\begin{aligned}
K_0(T) =& \frac{\lambda^{\star 2}e^{-\lambda_1 T}}{\lambda_1}\int_0^T g(w)\big(e^{\lambda^\star T} - e^{\lambda^\star w_1}\big)dw\\
&+\lambda e^{-\lambda^\star T}\Big\{1+\frac{\lambda^\star}{\lambda_1}\Big\}\int_0^T g(w)(1+\lambda(T-w))e^{-\lambda w}dw.
\end{aligned} \tag{4}$$

Proof.

$$\begin{aligned}
K_0(T) &= \sum_{n=1}^{\infty}\int_{D_n}\sum_{k=1}^{n} g(w_k)Pr\big(N(T-w_n)=0\big)\prod_{j=1}^{n} f_{W_j}\big(w_j-w_{j-1}\big)d\underline{w}\\
&= \sum_{n=1}^{\infty}\int_0^T\int_{w_1}^T\cdots\int_{w_{n-1}}^T ng(w_1)Pr\big(N(T-w_n)=0\big)\prod_{j=1}^{n} f_{W_j}\big(w_j-w_{j-1}\big)d\underline{w}
\end{aligned} \tag{5}$$

$$\begin{aligned}
&Pr\big(N(T-w_n)=0\big)\prod_{j=1}^{n} f_{W_j}\big(w_j-w_{j-1}\big)\\
&\qquad=e^{-\lambda_1(T-w_n)}\lambda_1 e^{-\lambda_1 w_1}\lambda e^{-\lambda(w_2-w_1)}\ldots\lambda e^{-\lambda(w_{n-1}-w_{n-2})}\lambda e^{-\lambda(w_n-w_{n-1})}\\
&\qquad=\lambda_1\lambda^{n-1}e^{-\lambda_1 T}e^{-(\lambda_1-\lambda)w_1}e^{-(\lambda-\lambda_1)w_n}.
\end{aligned} \tag{6}$$

Replacing the left hand side of Equation (6) in (5), we have

$$\begin{aligned}K_0(T) =& \sum_{n=1}^{\infty} \int_0^T \int_{w_1}^T \ldots \int_{w_{n-1}}^T n g(w_1)\lambda_1 \lambda^{n-1} e^{-\lambda_1 T} e^{-(\lambda_1-\lambda)w_1} e^{-(\lambda-\lambda_1)w_n} dw_1 \ldots dw_n \\ =& \lambda_1 e^{-\lambda_1 T} \sum_{n=1}^{\infty} n\lambda^{n-1} \int_0^T g(w_1) e^{-(\lambda_1-\lambda)w_1} dw_1 \int_{w_1}^T \ldots \int_{w_{n-1}}^T e^{-(\lambda-\lambda_1)w_n} dw_2 \ldots dw_n.\end{aligned}$$

Rescaling, let $\lambda^\star = \lambda_1 - \lambda$, we then have

$$\begin{aligned}K_0(T) =& \lambda_1 e^{-\lambda_1 T} \sum_{n=1}^{\infty} n\lambda^{n-1} \int_0^T \int_{w_1}^T \ldots \int_{w_{n-1}}^T g(w_1) e^{-(\lambda_1-\lambda)w_1} e^{-(\lambda-\lambda_1)w_n} dw_1 \ldots dw_n \\ =& \lambda_1 e^{-\lambda_1 T} \sum_{n=1}^{\infty} n \int_0^T \lambda^{n-1} g(w_1) e^{-\lambda^\star w_1} dw_1 \int_{w_1}^T \ldots \int_{w_{n-1}}^T e^{\lambda^\star w_n} dw_2 \ldots dw_n. \qquad (7)\end{aligned}$$

■

Result 3.1

$$\begin{aligned}&\int_{w_1}^T \ldots \int_{w_{n-1}}^T e^{\lambda^\star w_n} dw_2 \ldots dw_n \\ &= \sum_{j=2}^{n-1} \frac{(-1)^j e^{\lambda^\star T}}{\lambda^{\star(j-1)}} \int_{w_1}^T \ldots \int_{w_{n-1}}^T dw_2 \ldots dw_{n-j+1} + (-1)^n \left(\frac{e^{\lambda^\star T} - e^{\lambda^\star w_1}}{\lambda^{\star(j-1)}} \right) \\ &= \sum_{j=1}^{n-2} \frac{(-1)^{j+1} e^{\lambda^\star T}}{\lambda^{\star j}} \int_{w_1}^T \ldots \int_{w_{n-j-1}}^T dw_2 \ldots dw_{n-j} + (-1)^n \left(\frac{e^{\lambda^\star T} - e^{\lambda^\star w_1}}{\lambda^{\star(j-1)}} \right).\end{aligned}$$

Proof. When $n = 2$, we have

$$\int_{w_1}^T e^{\lambda^\star w_n} dw_2 = \left(\frac{e^{\lambda^\star T} - e^{\lambda^\star w_1}}{\lambda^\star} \right).$$

The result is then true for $n = 2$.
We have:

$$\begin{aligned}\int_{w_1}^T \cdots \int_{w_{n-1}}^T e^{\lambda^\star w_n} dw_2 \ldots dw_n =& \frac{e^{\lambda^\star T}}{\lambda^\star} \int_{w_1}^T \cdots \int_{w_{n-2}}^T dw_2 \ldots dw_{n-1} \\ &- \frac{1}{\lambda^\star} \int_{w_1}^T \cdots \int_{w_{n-2}}^T e^{\lambda^\star w_{n-1}} dw_2 \ldots dw_{n-1}. \qquad (8)\end{aligned}$$

Let say the result is true for $n-1$, then

$$\begin{aligned}\int_{w_1}^T \cdots \int_{w_{n-2}}^T e^{\lambda^\star w_{n-1}} dw_2 \cdots dw_{n-1} =& \sum_{j=1}^{n-3} \frac{(-1)^{j+1} e^{\lambda^\star T}}{\lambda^{\star j}} \int_{w_1}^T \cdots \int_{w_{n-j-1}}^T dw_2 \cdots dw_{n-j-1} \\ &+ (-1)^{n-1} \left(\frac{e^{\lambda^\star T} - e^{\lambda^\star w_1}}{\lambda^\star (n-2)} \right). \qquad (9)\end{aligned}$$

Replacing Equation (9) in (8), we have:

$$
\begin{aligned}
&\frac{e^{\lambda^\star T}}{\lambda^\star}\int_{w_1}^{T}\cdots\int_{w_{n-2}}^{T} dw_2\cdots dw_{n-1} - \frac{1}{\lambda^\star}\int_{w_1}^{T}\cdots\int_{w_{n-2}}^{T} e^{\lambda^\star w_{n-1}} dw_2\cdots dw_{n-1}\\
&= \frac{e^{\lambda^\star T}}{\lambda^\star}\int_{w_1}^{T}\dots\int_{w_{n-2}}^{T} dw_2\cdots dw_{n-1} - \frac{1}{\lambda^\star}\left[\sum_{j=1}^{n-3}\frac{(-1)^{j+1}e^{\lambda^\star T}}{\lambda^{\star j}}\int_{w_1}^{T}\cdots\int_{w_{n-j-1}}^{T} dw_2\cdots dw_{n-j-1}\right.\\
&\qquad \left. + (-1)^{n-1}\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-2)}}\right)\right]\\
&= \frac{e^{\lambda^\star T}}{\lambda^\star}\int_{w_1}^{T}\dots\int_{w_{n-2}}^{T} dw_2\cdots dw_{n-1} + \sum_{j=1}^{n-3}\frac{(-1)^{j}e^{\lambda^\star T}}{\lambda^{\star(j+1)}}\int_{w_1}^{T}\dots\int_{w_{n-j-2}}^{T} dw_2\cdots dw_{n-j-1}\\
&\qquad + (-1)^{n}\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-1)}}\right)\\
&= \frac{e^{\lambda^\star T}}{\lambda^\star}\int_{w_1}^{T}\cdots\int_{w_{n-2}}^{T} dw_2\cdots dw_{n-1}\\
&\qquad + \sum_{k=2}^{n-2}\frac{(-1)^{(k-1)}e^{\lambda^\star T}}{\lambda^{\star k}}\int_{w_1}^{T}\cdots\int_{w_{n-k-1}}^{T} dw_2\cdots dw_{n-k} + (-1)^{n}\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-1)}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{e^{\lambda^\star T}}{\lambda^\star}\int_{w_1}^{T}\cdots\int_{w_{n-2}}^{T} dw_2\cdots dw_{n-1} &= \sum_{k=1}^{n-2}\frac{(-1)^{(k-1)}e^{\lambda^\star T}}{\lambda^{\star k}}\int_{w_1}^{T}\cdots\int_{w_{n-k-1}}^{T} dw_2\cdots dw_{n-k}\\
&\qquad + (-1)^{n}\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-1)}}\right). \qquad (10)
\end{aligned}
$$

We also have

$$
\int_{w_1}^{T}\cdots\int_{w_{m-1}}^{T} dw_2\cdots dw_m = \frac{(T-w_1)^{m-1}}{(m-1)!} \qquad (11)
$$

which can be proved easily. Replacing Equation (11) in (10), we have:

$$
\begin{aligned}
&\int_{w_1}^{T}\dots\int_{w_{n-1}}^{T} e^{\lambda^\star w_n} dw_2\dots dw_n\\
&\quad = \sum_{k=1}^{n-2}\frac{(-1)^{k-1}e^{\lambda^\star T}}{\lambda^{\star k}}\int_{w_1}^{T}\dots\int_{w_{n-k-1}}^{T} dw_2\dots dw_{n-k} + (-1)^{n}\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-1)}}\right)\\
&\quad = \sum_{k=1}^{n-2}\frac{(-1)^{k-1}e^{\lambda^\star T}}{\lambda^{\star k}}\frac{(T-w_1)^{n-k-1}}{(n-k-1)!} + (-1)^{n}\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-1)}}\right). \qquad (12)
\end{aligned}
$$

Replacing Equation (12) in (7)

$$\begin{aligned}
K_0(T) =&\lambda_1 e^{-\lambda_1 T}\sum_{n=1}^{\infty} n\int_0^T \lambda^{n-1} g(w_1) e^{-\lambda^\star w_1} dw_1 \int_{w_1}^{T}\ldots\int_{w_{n-1}}^{T} e^{\lambda^\star w_n} dw_2 \ldots dw_n\\
=&\lambda_1 e^{-\lambda_1 T}\sum_{n=1}^{\infty} n\int_0^T \lambda^{n-1} g(w_1)\Bigg[\sum_{k=1}^{n-2}\frac{(-1)^{k-1}e^{\lambda^\star T}}{\lambda^{\star k}}\frac{(T-w_1)^{n-k-1}}{(n-k-1)!}\\
&\qquad\qquad + (-1)^n\left(\frac{e^{\lambda^\star T}-e^{\lambda^\star w_1}}{\lambda^{\star(n-1)}}\right)\Bigg] dw_1\\
=&-\lambda_1 e^{-\lambda_1 T}\sum_{n=1}^{\infty} n\int_0^T \lambda^{n-1} g(w_1)\big(e^{\lambda^\star T}-e^{\lambda^\star w_1}\big)\left(\frac{-\lambda}{\lambda^\star}\right)^{n-1} dw_1\\
&+\lambda_1 e^{-\lambda_T}\sum_{n=1}^{\infty}\int_0^T \lambda^k\lambda^{\star(-k)}(-1)^{k-1} g(w_1)\sum_{n=k+1}^{\infty} n\lambda^{n-k-1}\frac{(T-w_1)^{n-k-1}}{(n-k-1)!} dw_1\\
=&\frac{\lambda_1 e^{-\lambda_1 T}}{\left(1+\frac{\lambda}{\lambda^\star}\right)^2}\int_0^T g(w_1)\big(e^{\lambda^\star T}-e^{\lambda^\star w_1}\big)dw_1 - \lambda_1 e^{-\lambda_1 T}\int_0^T g(w_1)\{1+\lambda(T-w_1)+k\}\\
&\times e^{\lambda(T-w_1)} dw_1 \sum_{n=1}^{\infty}\left(-\frac{\lambda}{\lambda^\star}\right)^k\\
=&\frac{\lambda_1 e^{-\lambda_1 T}}{\left(1+\frac{\lambda}{\lambda^\star}\right)^2}\int_0^T g(w_1)\big(e^{\lambda^\star T}-e^{\lambda^\star w_1}\big)dw_1\\
&+\frac{\lambda}{\lambda^\star+\lambda}\lambda_1 e^{\lambda^\star T}\int_0^T g(w_1)(1+\lambda(T-w_1))e^{-\lambda w_1} dw_1\\
&+\frac{\lambda\lambda^\star}{(\lambda^\star+\lambda)^2}\lambda_1 e^{\lambda^\star T}\int_0^T g(w_1)(1+\lambda(T-w_1))e^{-\lambda w_1} dw_1\\
=&\frac{\lambda_1\lambda^{\star 2} e^{-\lambda_1 T}}{(\lambda^\star+\lambda)^2}\int_0^T g(w_1)\big(e^{\lambda^\star T}-e^{\lambda^\star w_1}\big)dw_1\\
&+\frac{\lambda}{\lambda^\star+\lambda}\lambda_1 e^{\lambda^\star T}\left\{1+\frac{\lambda^\star}{\lambda^\star+\lambda}\right\}\int_0^T g(w_1)(1+\lambda(T-w_1))e^{-\lambda w_1} dw_1\\
=&\frac{\lambda^{\star 2} e^{-\lambda_1 T}}{\lambda_1}\int_0^T g(w)\big(e^{\lambda^\star T}-e^{\lambda^\star w_1}\big)dw\\
&+\lambda e^{-\lambda^\star T}\left\{1+\frac{\lambda^\star}{\lambda_1}\right\}\int_0^T g(w)(1+\lambda(T-w))e^{-\lambda w} dw
\end{aligned}$$

which proves the result of Equation (4). ■

When $\lambda_1 = \lambda$ meaning $\lambda^\star = 0$ and taking the limit of the above expression when $\lambda^\star \to 0$, we recover the result of Asimit and Badescu (2010).

Theorem 3.2 Consider the compound renewal model with constant force of interest ($\delta > 0$) such that $F \in RV_{-\infty}$. If Assumption 2.1 is satisfied for any $t \in (0,T]$, then

$$Pr\big(S_\delta(T) > x\big) \sim K_\delta Pr(X > x), x \to \infty,$$

where

$$K_\delta = \sum_{n=1}^{\infty} \int_{D_n} \sum_{i=1}^{n} g(w_i) e^{-\alpha\delta\sum_{j=1}^{i} w_j} Pr\big(N(T-w-n) = 0\big) \prod_{j=1}^{n} f_{W_j}\big(w_j - w_{j-1}\big) dw.$$

When $f_{W_1}(t) = \lambda_1 e^{-\lambda_1 T}$, $f_{W_j}(t) = \lambda e^{-\lambda T}$, $j = 1,2,\ldots$, we have

$$K_\delta = \lambda_1 e^{-\lambda_1 T} \sum_{n=1}^{\infty} \lambda^{n-1} \int_0^T \int_{w_1}^T \ldots \int_{w_{n-1}}^T \sum_{j=1}^{i} w_j e^{-(\lambda_1-\lambda)w_1} e^{-(\lambda-\lambda_1)w_n} dw_1 \ldots dw_n.$$

When $\lambda^\star = \lambda_1 - \lambda = 0$, we find the result of Asimit and Badescu (2010).
Proof. Using the same argument as in the proof for Theorem 3.1, we have

$$\begin{aligned} Pr\big(S_\delta(T) > x\big) &= \sum_{n=1}^{\infty} Pr\big(N(T) = n\big) \int_{D_n} Pr\left(\sum_{i=1}^{n} X_i e^{\delta\sum_{j=1}^{i} w_j} > x | W = w, N(T) = n \right) \\ &\quad \times \frac{Pr\big(N(T-w_n) = 0\big) \prod_{j=1}^{n} f_{W_j}(w_j - w_{j-1})}{Pr\big(N(T) = n\big)} \\ &= \sum_{n=1}^{\infty} \int_{D_n} Pr\left(\sum_{i=1}^{n} X_i e^{\delta\sum_{j=1}^{i} w_j} > x | W = w \right) \prod_{j=1}^{n} f_{W_j}(w_j - w_{j-1}) dw. \end{aligned} \tag{13}$$

Note that

$$Pr\left(\sum_{i=1}^{n} X_i e^{\delta\sum_{j=1}^{i} w_j} > x | W = w \right) \leq Pr\left(\sum_{i=1}^{n} X_i > x | W = w \right).$$

The above inequality will allow us to apply Lemma 2.2 in a similar manner as in the proof of Theorem 3.1. Applying the Dominance Convergence Theorem in Equation (13) completes the proof. ■

3.1. Ruin probability

The calculation of the exact value of the probability of ruin remains an extremely complex problem. For rather simple cases of the collective risk model, exact formulas and approximations were found for the probability of ruin, see Panjer and Willmot (1992). However when one incorporates in the collective risk model, the effect of the force of interest, calculations become more difficult. In the case of the discounted compound Poisson risk model, a differential equation was obtained for the probability of ruin and a solution was even found when the claims amount follows an exponential distribution. For more details, see Sundt and Teugels (1995).

The probability of ruin can also be calculated if we know the distribution of our risk process. Expressions obtained by Léveillé et al. (2009) for the distribution of the discounted renewal aggregate sums, allow us to affirm that it will be very difficult to find an explicit expression of the probability of ruin within the framework defined by the previous authors. For those reasons, authors such as Cai (2002) suggest upper bounds to estimate the probability of ruin.

In this section, our intention is to extend the work of Asimit and Badescu (2010). Consider the compound renewal risk model with constant force of interest, for which the evolution of the surplus $U_\delta(t)$ is given by

$$U_\delta(t) = xe^{\delta t} + C_\delta(t) - e^{\delta t} S_\delta(t),$$

where x is the initial capital and $C_\delta(t) = \int_0^t e^{(t-s)} dC(s)$ represents the accumulated amount of premiums at time t. We let $\{C(s)\}_{s \geq 0}$ with $C(0) = 0$ be a non-decreasing and right continuous stochastic process, denoting the total amount of premiums accumulated to time s . Furthermore, we define the time to ruin as

$$\tau(x) = \inf\big\{t > 0 : U_\delta(t) < 0 | U_\delta(0) = x\big\}$$

and the associated finite time ruin probability by

$$\psi_\delta(x;T) = Pr\big(\tau(x) \leq T\big). \tag{14}$$

Clearly,

$$Pr\big(S_\delta(T) > x + e^{-\delta T} C_\delta(T)\big) \leq \psi_\delta(x;T) \leq Pr\big(S_\delta(T) > x\big) \tag{15}$$

holds for $\delta \geq 0$. Since we can use the long-tailed property of subexponential distributions (Embrechts et al., 1997) in Equation (15), this leads to the following corollary of Theorems 3.1 and 3.2.

Corollary 3.1 Consider the compound renewal risk model with constant interest rate such that Assumption 2.1 is satisfied for any $t \in (0,T]$.

In addition if $C_\delta(T) < \infty$, then

1. if $\delta = 0$ and $F \in \wp$, then $\psi_0(x;T) \sim K_0 Pr(X_1 > x), x \to \infty$,

2. if $F \in RV_\infty$, then $\psi_\delta(x;T) \sim K_\delta Pr(X_1 > x), x \to \infty$.

Proof. $\psi_\delta(x;T)$, the insurer's probability of ruin, which is the probability that the discounted aggregate claim process paid out over a finite time horizon $(0,T]$ exceeds the discounted value of the premium received and the insurer's initial capital . So clearly,

$$\psi_\delta(x;T) \le Pr\big(S_\delta(T) > x\big)$$

and

$$\lim_{x\to\infty} \psi_\delta(x;T) \le \lim_{x\to\infty} Pr\big(S_\delta(T) > x\big) \sim \lim_{x\to\infty} K_\delta Pr(X_1 > x), x \to \infty.$$

■

4. Risk Management

The European Solvency II project lay down some new regulatory requirements that every insurance company inside the European Union will have to fulfil. In addition, several other countries outside the European Union (e.g. Canada, Columbia or Mexico) are likely to use similar principles[2]. Insurance companies are required to compute the minimum capital requirement as proposed by the regulatory authorities. The 1-year 99.5% Value at risk (VaR) is the most used measure of such capital requirement. In this section of the paper, we compute the minimum capital requirement using the internal model approach over a 50-year horizon for delayed exponentially distributed inter-claim times and Weibull claim amount distributions. We use Equation (4) in Section 4.2. This section mimics Asimit and Badescu (2010) approach and presents a simulated results for illustration purposes.

4.1. Risk Measures

The *VaR* at a confidence level of p for a loss variable L is the p-quantile, defined as (Jorion, 2001):

$$VaR_p(L) = \inf\big\{x \in \mathbb{R} : Pr(L > x) \le 1 - p\big\}.$$

Theoretically speaking, *VaR* is the alpha quantile of the distribution of the discounted aggregate sum of claims. Using Theorems 3.1 and 3.2, this quantile can be expressed as:

$$VaR_{1-p}\big(S_\delta(T)\big) \sim VaR_{1-p/K_\delta}(X_1), \text{for } p \downarrow 0\,,$$

provided that the density function of $S_\delta(T)$ is continuous close enough in the right tail. It is important to notice that *VaR* and ruin probability give the same asymptotic results. Both risk

[2] In South Africa, insurance companies are required by the regulators to compute the minimum capital requirement based on the SAM (Solvency Assessment and management); a regulatory framework that is similar to the Solvency II used in European Union countries.

measures fail to incorporate the severity of the extreme events. Asimit and Badescu (2010) show that the Expected Shortfall (ES) of a loss variable with continuous density function, at a confidence level p, represents the average loss in the worst $100\times p\%$ cases, and is given by:

$$ES_p\big(S_\delta(T)\big) \sim \frac{\alpha}{\alpha-1} VaR_p\big(S_\delta(T)\big),\, p \uparrow 1.$$

We will be using the same asymptotic result for the ES of the discounted aggregate loss as in Asimit and Badescu (2010).

A density function F is in the maximum domain of attraction of a non-degenerate density function G, written as $F \in MDA(G)$, if

$$\lim_{x\to\infty} F^n(a_n x + b_n) = G(x),$$

where $a_n > 0$ and $b_n > 0$ are real numbers. The connection between ES and *VaR* of the discounted aggregate loss, for high confidence levels, for the case that is given by

$$ES_p\big(S_0(T)\big) \sim VaR_p\big(S_0(T)\big),\, p \uparrow 1,$$

provided that $\alpha > 1$.

For the proof, see Theorem 3.1 of Alink, Löwe and Wüthrich (2005).

4.2. Numerical results

In this numerical example reported in Tables 1 and 2, we determine the minimum capital requirement as the 99.5^{th} quantile (VaR 99.5 %) of the distribution of discounted aggregate sums and show how the dependence assumption affects significantly the overall results. For that purpose, we assume that the inter-claim time arrivals are exponentially distributed whereas the forthcoming claim amounts have a Weibull distribution given by:

$$F_{X_1}(x) = 1 - \exp\big(-x^{1/\tau}\big),\, x \geq 0,\, \tau > 1.$$

We then model the dependence structure between the inter-claim times and the claim amounts by making use of the Ali-Mikhail-Haq copula (see Example 1) with parameter values θ equal to -0.9, -0.5, 0, 0.5 and 0.9. Notice that the risk model corresponds to the case where the claim amounts and the inter-claim times are independent. The asymptotic constant of Theorem 3.1 is in that case given by:

$$\begin{aligned} K_0(\theta) =& \frac{\lambda^{\star 2} e^{-\lambda_1 T}}{\lambda_1}\left[(1+\theta)\left(Te^{\lambda^\star T} - \frac{e^{\lambda^\star T}-1}{\lambda_1}\right) + 2\theta\left(\frac{1-e^{-\lambda T}}{\lambda} - \frac{e^{\lambda^\star T}-e^{-\lambda T}}{\lambda_1}\right)\right] \\ &+ \left(1+\frac{\lambda^\star}{\lambda_1}\right)e^{-\lambda^\star T}\left(\lambda T + \frac{\theta}{2}\big(e^{-2\lambda T}-1\big)\right). \end{aligned}$$

When $\lambda_1 = \lambda$, we find the result of Asimit and Badescu (2010). Each analysis consists of 10,000,000 simulations of the delayed Poisson risk process with respectively $f_{W_1}(t) = e^{-t}$, $f_{W_j}(t) = 1/4e^{-t/6}$, $j = 2,3,\dots$ and time horizon $T = 50$. This choice of parameters is arbitrary. We have performed simulation studies for other parameter settings and have found that the conclusion remains the same as the one obtained for this particular choice. For each simulation study, the values of $Pr\big(S_0(t) > x\big)$ are calculated empirically for a threshold x such that $Pr(X_1 > x)$ is 5×10^{-4}, 10^{-4}, and 5×10^{-5} , respectively. The choice of these tail probabilities follows Asimit and Badescu (2010). We assume different values for Spearman's correlation coefficients, ρ and analyse their impact on the *VaR* measure. Note that for this particular dependence structure, the correlation coefficient varies between $(-0.27, 0.48)$ (Nelsen, 1999).

Table 1: Estimated probability ratios $Pr\big(S_0(T) > x\big)/K_0 Pr\big(X_1 > x\big)$.

x	$\theta=-0.9$	$\theta=-0.5$	$\theta=0$	$\theta=0.5$	$\theta=0.9$
$\tau=6$					
$\overline{F}_{X_1}^{-1}(5\times10^{-4})$	1.1513	1.1566	1.1677	1.1738	1.1792
$\overline{F}_{X_1}^{-1}(10^{-4})$	1.0659	1.0746	1.0739	1.0731	1.0757
$\overline{F}_{X_1}^{-1}(5\times10^{-5})$	1.0577	1.0604	1.0731	1.0696	1.0749
$\tau=8$					
$\overline{F}_{X_1}^{-1}(5\times10^{-4})$	1.0404	1.0487	1.0511	1.0567	1.0617
$\overline{F}_{X_1}^{-1}(10^{-4})$	1.0201	1.0228	1.0297	1.0302	1.0338
$\overline{F}_{X_1}^{-1}(5\times10^{-5})$	1.0121	1.0189	1.0289	1.0275	1.0308
$\tau=10$					
$\overline{F}_{X_1}^{-1}(5\times10^{-4})$	1.0131	1.0166	1.0196	1.0240	1.0287
$\overline{F}_{X_1}^{-1}(10^{-4})$	1.0047	1.0090	1.0150	1.0144	1.0181
$\overline{F}_{X_1}^{-1}(5\times10^{-5})$	1.0159	1.0089	1.0195	1.0173	1.0210

Table 2: $VaR_{99,5\%}\big[S_0(50)\big]$ for Weibull claim amounts and Ali-Mikhail-Haq copula.

	$\rho=-0.2$	$\rho=-0.1$	$\rho=0$	$\rho=+0.3$	$\rho=+0.4$
$\tau=6$	373937	370081	366716	361316	357397
$\tau=8$	2.6940004×10^7	2.6570247×10^7	2.6248602×10^7	2.5734561×10^7	2.5363002×10^7
$\tau=10$	1.9408728×10^9	1.9076315×10^9	1.8788094×10^9	1.8329303×10^9	1.7999101×10^9

5. Discussion

From Table 2, two observations are worth being mentioned - the first is that negative correlation induces significantly larger capital requirements (as measured by the *VaR*) than positive correlations. Secondly, when the level of dependence increases sufficiently; the minimum capital requirement increases as well. Furthermore, even in the case where we assume independence between the inter-claim times and the claim amounts; the resulting minimum capital requirement still increases exponentially. These findings show that the minimum capital requirements obtained under the independence assumption might be misleading, especially during catastrophic and extreme events. We therefore recommend that the regulatory authorities set a threshold minimum capital requirement based on the dependence assumption in order to curb the risk of solvency associated with low level of capital observed in many insurance firms during major events such as the Hurricane Katrina, Tsunami earthquake, financial crises, epidemic disease etc. For further research in this field, we intend to prove that the expression of K_0 in Theorem 3.2 can be given by:

$$K_0(T) = \sum_{n=1}^{\infty} \int_0^T \int_0^{w_n} \dots \int_0^{w_2} ng(w_1) Pr\big(N(T-w_n)=0\big) \prod_{j=1}^{n} f_{W_j}\big(w_j - w_{j-1}\big) dw_1 \dots dw_n.$$

This setting is well suited when both the inter-claim times and the claim amounts follow any theoretical distribution process.

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