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Some characteristics of picture fuzzy subgroups via cut set of picture fuzzy set

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Abstract

Given any picture fuzzy set Q of a universe Y, the set $C_{r,s,t}(Q)$ called the (r, s, t)-cut set of Q was studied. In this paper, some characteristics of picture fuzzy subgroup of a group are obtained via the cut sets of picture fuzzy set.

Keywords: fuzzy set, picture fuzzy set, picture fuzzy subgroup, cut set

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1 Introduction

Zadeh [18] introduced the notion of fuzzy sets (FSs). This theory has been generalised by many researchers. Atanassov [1] initiated the concept of intuitionistic fuzzy sets (IFSs). For more on intuitionistic fuzzy set, also see [2, 3, 4]. The application of these fuzzy sets have taken different directions such as optimization and decision-making [15] control theory [13] and many others. One particular application is the work of Cuong and Kreinovich [5] who put forward the notion of picture fuzzy sets (PFSs) as a generalisation of fuzzy sets and intuitionistic fuzzy sets. Picture fuzzy set has been extensively studied and applied (see [6, 7, 8, 9, 10, 11, 12, 14, 17] for details).

Rosenfeld [16] generalised fuzzy sets to fuzzy groups (FGs). The idea of cut set of picture fuzzy sets was initiated by Dutta and Ganju [12] and they obtained some of its properties. Dogra and Pal [11] corrected the definition of cut set that was given by Dutta and Ganju [12] and they initiated the notion of picture fuzzy subgroups (PFSGs) of a group.

In this paper, motivated by the work of Dogra and Pal [11], investigation of the characteristics of picture fuzzy subgroups of a group via the cut sets of picture fuzzy sets was done. The organisation of the paper is as follows: Section 2 gives the basic definitions and preliminary ideas of PFSs; Section 3 investigates some properties of picture fuzzy subgroups of a group via cut sets of picture fuzzy sets.

2 Preliminaries

Definition 2.1. *[18] Let Y be a nonempty set. A FS Q of Y is an object of the form*

$$Q = \{\langle y, \sigma_Q(y)\rangle | y \in Y\}$$

with a membership function

$$\sigma_Q : Y \longrightarrow [0, 1]$$

where the function $\sigma_Q(y)$ denotes the degree of membership of $y \in Q$.

Definition 2.2. *[1] Let a nonempty set Y be fixed. An IFS Q of Y is an object of the form*

$$Q = \{\langle y, \sigma_Q(y), \tau_Q(y)\rangle | y \in Y\}$$

where the functions

$$\sigma_Q : Y \to [0, 1] \text{ and } \tau_Q : Y \to [0, 1]$$

are called the membership and non-membership degrees of $y \in Q$, respectively, and for every $y \in Y$,

$$0 \leq \sigma_Q(y) + \tau_Q(y) \leq 1.$$

Definition 2.3. *[5] A picture fuzzy set Q of Y is defined as*

$$Q = \{(y, \sigma_Q(y), \tau_Q(y), \gamma_Q(y))|y \in Y\},$$

where the functions

$$\sigma_Q : Y \to [0,1],\ \tau_Q : Y \to [0,1] \text{ and } \gamma_Q : Y \to [0,1]$$

are called the positive, neutral and negative membership degrees of $y \in Q$, respectively, and $\sigma_Q, \tau_Q, \gamma_Q$ satisfy

$$0 \leq \sigma_Q(y) + \tau_Q(y) + \gamma_Q(y) \leq 1,\ \forall y \in Y.$$

For each $y \in Y$, $S_Q(y) = 1 - (\sigma_Q(y) + \tau_Q(y) + \gamma_Q(y))$ is called the refusal membership degree of $y \in Q$.

Definition 2.4. *[5] Let Q and R be two PFSs. Then, the inclusion, equality, union, intersection and complement are defined as follow:*

(i) $Q \subseteq R$ *if and only if for all* $y \in Y$, $\sigma_Q(y) \leq \sigma_R(y)$, $\tau_Q(y) \leq \tau_R(y)$ *and* $\gamma_Q(y) \geq \gamma_R(y)$.

(ii) $Q = R$ *if and only if* $Q \subseteq R$ *and* $R \subseteq Q$.

(iii) $Q \cup R = \{(y, \sigma_Q(y) \vee \sigma_R(y), \tau_Q(y) \vee \tau_R(y)), \gamma_Q(y) \wedge \gamma_R(y))|y \in Y\}$.

(iv) $Q \cap R = \{(y, \sigma_Q(y) \wedge \sigma_R(y), \tau_Q(y) \wedge \tau_R(y)), \gamma_Q(y) \vee \gamma_R(y))|y \in Y\}$.

Definition 2.5. *[11] Let $(G, *)$ be a crisp group and $Q = \{(y, \sigma_Q(y), \tau_Q(y), \eta_Q(y)) \mid y \in G\}$ be a PFS in G. Then, Q is called picture fuzzy subgroup of G (PFSG) if*
(i) $\sigma_Q(a*b) \geq \sigma_Q(a) \wedge \sigma_Q(b),\ \tau_Q(a*b) \geq \tau_Q(a) \wedge \tau_Q(b),\ \eta_Q(a*b) \leq \eta_Q(a) \vee \eta_Q(b)$

(ii) $\sigma_Q(a^{-1}) \geq \sigma_Q(a),\ \tau_Q(a^{-1}) \geq \tau_Q(a),\ \eta_Q(a^{-1}) \leq \eta_Q(a)$ *for all* $a, b \in G$.
Notice that a^{-1} is the inverse of $a \in G$,

or equivalently, Q is a PFSG of G if and only if
$\sigma_Q(a * b^{-1}) \geq \sigma_Q(a) \wedge \sigma_Q(b),\ \tau_Q(a * b^{-1}) \geq \tau_Q(a) \wedge \tau_Q(b),\ \eta_Q(a * b^{-1}) \leq \eta_Q(a) \vee \eta_Q(b)$.

Definition 2.6. *[11] Let $(G, *)$ be a crisp group and $Q = (\sigma_Q, \tau_Q, \eta_Q)$ be a PFSG of G. Then, for $a \in G$ the picture fuzzy left coset of $Q \in G$ is the PFS $aQ = (\sigma_{aQ},\ \tau_{aQ},\ \eta_{aQ})$ defined by*

$$\sigma_{aQ}(u) = \sigma_Q(a^{-1} * u),\ \tau_{aQ}(u) = \tau_Q(a^{-1} * u) \text{ and } \eta_{aQ}(u) = \eta_Q(a^{-1} * u)$$

for all $u \in G$.

Definition 2.7. *[11] Let $(G,*)$ be a crisp group and $Q=(\sigma_Q,\tau_Q,\eta_Q)$ be a PFSG of G. Then, for $a\in G$ the picture fuzzy right coset of $Q\in G$ is the PFS $Qa=(\sigma_{Qa},\ \tau_{Qa},\ \eta_{Qa})$ defined by*

$$\sigma_{Qa}(u)=\sigma_Q(u*a^{-1}),\ \tau_{Qa}(y)=\tau_Q(u*a^{-1}) \text{ and } \eta_{Qa}(u)=\eta_Q(u*a^{-1})$$

for all $u\in G$.

Definition 2.8. *[11] Let $(G,*)$ be a crisp group and $Q=(\sigma_Q,\tau_Q,\eta_Q)$ be a PFSG of G. Then, Q is called a picture fuzzy normal subgroup (PFNSG) of G if*

$$\sigma_{Qa}(y)=\sigma_{aQ}(y),\ \tau_{Qa}(y)=\tau_{aQ}(y),\ \eta_{Qa}(y)=\eta_{aQ}(y)$$

for all $a,\ y\in G$.

Remark 2.1. *Dogra and Pal established that PFSG of G is normal if and only if*
(i) $\sigma_Q(u^{-1}*a*u)=\sigma_Q(a)$
(ii) $\tau_Q(u^{-1}*a*u)=\tau_Q(a)$
(ii) $\eta_Q(u^{-1}*a*u)=\eta_Q(a)$. *For all $a\in Q$ and $u\in G$.*

Cut set of picture fuzzy sets was introduced by Dutta and Ganju **?** but the definition did not capture the cut set very well. Thus, Dogra and Pal **?** corrected it in their paper.

Definition 2.9. *[11] Let $Q=\{(x,\sigma_Q,\tau_Q,\gamma_Q)|a\in Y\}$ be PFS over the universe Y. Then, (r,s,t)-cut set of Q is the crisp set in Q, denoted by $C_{r,s,t}(Q)$ and is defined by*

$$C_{r,s,t}(Q)=\{a\in Y|\sigma_Q(a)\geq r,\ \tau_Q(a)\geq s,\gamma_Q(a)\leq t\}$$

$r,s,t\in[0,1]$ *with the condition* $0\leq r+s+t\leq 1$.

Theorem 2.1. *[11] Let $(G,*)$ be a crisp group and $Q=(\sigma_Q,\tau_Q,\eta_Q)$ be a PFSG of G. Then, Q is a PFSG if and only if $C_{r,s,t}(Q)$ is a crisp subgroup of G.*

Theorem 2.2. *[12] If Q and R are two PFSs of a universe Y, then the following results hold*

(i) $C_{r,s,t}(Q)\subseteq C_{u,v,w}(Q)$ *if* $r\geq u,\ s\leq v,\ t\leq w$.

(ii) $C_{1-s-t,s,t}(Q)\subseteq C_{r,s,t}(Q)\subseteq C_{r,1-r-t,t}(Q)$.

(iii) $Q\subseteq R$ *implies* $C_{r,s,t}(Q)\subseteq C_{r,s,t}(R)$.

(iv) $C_{r,s,t}(Q \cap R) = C_{r,s,t}(Q) \cap C_{r,s,t}(R)$.

(v) $C_{r,s,t}(Q \cup R) \supseteq C_{r,s,t}(Q) \cup C_{r,s,t}(R)$.

(vi) $C_{r,s,t}(\cap Q_i) = \cap C_{r,s,t}(Q_i)$.

(vii) $C_{1,0,0}(Q) = Y$.

3 Main Results

Remark 3.1. *It is important to note that [12] misquoted the result of [5]. Hence, the results built on this foundation cannot be or at all correct. The counter example in Example (3.1) shows that the claims in Theorem 2.2 (i), (ii) and (vii) are wrong. The correct version of Theorem (2.2) is given in Theorem (3.1).*

Example 3.1. *Let*

$$Y = \{y_1, y_2, y_3, y_4\},$$

$$Q = \{(y_1, 0, 0.1, 0.8), (y_2, 0.2, 0.5, 0.3), (y_3, 0.4, 0.2, 0.1), (y_4, 0.5, 0.3, 0.2)\},$$

and

$$R = \{(y_1, 0.1, 0.4, 0.5), (y_2, 0.3, 0.6, 0.1), (y_3, 0.5, 0.3, 0), (y_4, 0.5, 0.4, 0.1)\}$$

be PFSs, taking $r = 0.1$, $s = 0.3$, $t = 0.5$. *Thus, the* $(0.1, 0.3, 0.5)$*-cut set of* Q *are*

$$C_{0.1,0.3,0.5}(Q) = \{y_2, y_4\}$$

and

$$C_{0.1,0.3,0.5}(R) = \{y_1, y_2, y_3, y_4\}.$$

Hence,

$$C_{1-s-t,s,t}(Q) = C_{0.2,0.3,0.5}(Q) = \{y_2, y_4\},$$
$$C_{r,s,t}(Q) = C_{0.1,0.3,0.5}(Q) = \{y_2, y_4\},$$
$$C_{0.1,0.2,0.3}(Q) = \{y_3\}$$

and

$$C_{r,1-r-t,t}(Q) = C_{0.1,0.4,0.5}(Q) = \{y_2\}.$$

Thus,

$$C_{1-s-t,s,t}(R) \subseteq C_{r,s,t}(R) \subsetneq C_{r,1-r-t,t}(R), \text{ which means (i) and (ii) fail.}$$

Note that the left side of the inclusion which holds obey the condition (i) of our Theorem 3.1

Furthermore,

$$C_{1-s-t,s,t}(Q) \subseteq C_{r,s,t}(Q) \subsetneq C_{r,1-r-t,t}(Q), \text{ which means (i) and (ii) fail.}$$

Note that the left side of the inclusion which holds obey the condition (i) of our Theorem 3.1

In addition,

$$C_{0.1,0.2,0.3}(Q) = \{y_3\} \subsetneq= \{y_2, y_4\} = C_{0.1,0.3,0.5}(Q) \text{ which means (i) and (ii) fail.}$$

Also, $C_{1,0,0}(Q) = \emptyset \neq Y$, *which means (vii) fails.*

Theorem 3.1. *Let Q and R be two PFSs of a universe Y. Then, the following assertions hold:*

(i) $C_{r,s,t}(Q) \subseteq C_{u,v,w}(Q)$ *if* $r \geq u,\ s \geq v,\ t \leq w$.

(ii) $C_{1-s-t,s,t}(Q) \subseteq C_{r,s,t}(Q) \subseteq C_{r,s,1-r-s}(Q)$,

(iii) $C_{1-s-t,1-r-t,t}(Q) \subseteq C_{r,s,t}(Q) \subseteq C_{r,s,1-r-s}(Q)$,

(iv) $C_{r,1-r-t,t}(Q) \subseteq C_{r,s,t}(Q) \subseteq C_{r,s,1-r-s}(Q)$,

(v) $C_{0,0,1}(Q) = Y$.

Proof. (i) Let $x \in C_{r,s,t}(Q)$. Using Definition 2.9, $\sigma_Q(x) \geq r \geq u,\ \tau_Q(x) \geq s \geq v, \gamma_Q(x) \leq t \leq w$. Thus, $x \in C_{u,v,w}(Q)$ and, as such, $C_{r,s,t}(Q) \subseteq C_{u,v,w}(Q)$.

(ii) Since $r+s+t \leq 1$ implies that $1-s-t \geq r,\ s \geq s$ and $t \leq t$, and $r \geq r,\ s \geq s$ and $t \leq 1-r-s$, by Theorem 3.1 (i), the result holds.

(iii) Since $r+s+t \leq 1$ implies that $1-s-t \geq r,\ 1-r-t \geq s$ and $t \leq t$, and $r \geq r,\ s \geq s$ and $t \leq 1-s-r$, by Theorem 3.1 (i), the result holds.

(iv) Since $r+s+t \leq 1$ implies that $r \geq r,\ 1-r-t \geq s$ and $t \leq t$, and $r \geq r,\ s \geq s$ and $t \leq 1-s-r$, by Theorem 3.1 (i), the result holds.

(v) Since $\forall\, y \in Y,\ \sigma(y) \geq 0,\ \tau(y) \geq 0,\ \gamma(y) \leq 1,\ C_{0,0,1}(Q) = Y$,

□

Proposition 3.1. *If Q is PFSG of G, then $C_{r,s,t}(Q)$ is a subgroup of G, where $\sigma_Q(e) \geq r,\ \tau_Q(e) \geq s,$ and $\eta_Q(e) \leq t$ and e is the identity element of G.*

Proof. Clearly $C_{r,s,t}(Q) \neq \emptyset$ as $e \in C_{r,s,t}(Q)$. Let $a, b \in C_{r,s,t}(Q)$ be any two elements. Then,

$$\sigma_Q(a) \geq r,\ \tau_Q(a) \geq s,\ \eta_Q(a) \leq t \text{ and } \sigma_Q(b) \geq r,\ \tau_Q(b) \geq s,\ \eta_Q(b) \leq t$$

$$\sigma_Q(a) \wedge \sigma_Q(b) \geq r,\ \tau_Q(a) \wedge \tau_Q(b) \geq s \text{ and } \eta_Q(a) \vee \eta_Q(b) \leq t$$

Since Q is a PFSG of G,

$$\sigma_Q(a * b^{-1}) \geq \sigma_Q(a) \wedge \sigma_Q(b) \geq r,\ \tau_Q(a * b^{-1}) \geq \tau_Q(a) \wedge \tau_Q(b) \geq s$$

$$\text{and } \eta_Q(a * b^{-1}) \leq \eta_Q(a) \vee \eta_Q(b) \leq t.$$

Hence, $a * b^{-1} \in C_{r,s,t}(Q)$ and $C_{r,s,t}(Q)$ is a subgroup of G. □

Proposition 3.2. *If Q is PFNSG of G, then $C_{r,s,t}(Q)$ is a normal subgroup of G, where $\sigma_Q(e) \geq r,\ \tau_Q(e) \geq s,$ and $\eta_Q(e) \leq t$ and e is the identity element of G.*

Proof. Let $a \in C_{r,s,t}(Q)$ and $u \in G$ be any element. Then,

$$\sigma_Q(a) \geq r,\ \tau_Q(a) \geq s,\ \eta_Q(a) \leq t.$$

Also, since Q is a PFNSG of G,

$$\sigma_Q(u^{-1}*a*u) = \sigma_Q(a) \geq r,\ \tau_Q(u^{-1}*a*u) = \tau_Q(a) \geq s, \text{ and } \eta_Q(u^{-1}*a*u) = \eta_Q(a) \leq t$$

$\forall\ a \in Q$ and $u \in G$. Therefore, $u^{-1} * a * u \in C_{r,s,t}(Q),$ so $a * u \in u * C_{r,s,t}(Q)$ which implies $C_{r,s,t}(Q)*u \subseteq u*C_{r,s,t}(Q)$. Also, $u*a*u^{-1} \in C_{r,s,t}(Q),$ so $u*a \in C_{r,s,t}(Q) * u$ which implies $u * C_{r,s,t}(Q) \subseteq C_{r,s,t}(Q) * u$. Hence, $u * C_{r,s,t}(Q) = C_{r,s,t}(Q) * u$. Thus, $C_{r,s,t}(Q)$ is a normal subgroup of G. □

Proposition 3.3. *Given two PFSGs Q and R of a group $(G, *)$. Then, $Q \cap R$ is a PFSG of G.*

This proposition has been proved by Dogra and Pal [11] by using PFSG definition. But, in this paper, a rather simpler approach of cut set of PFS is used to prove it.

Proof. By Theorem 2.1, $Q \cap R$ is a PFSG of G if and only if $C_{r,s,t}(Q \cap R)$ is a crisp subgroup of G. Since $C_{r,s,t}(Q \cap R) = C_{r,s,t}(Q) \cap C_{r,s,t}(R)$ (Theorem 2.2, iv) and both $C_{r,s,t}(Q)$ and $C_{r,s,t}(R)$ are subgroups of G and intersection of two subgroups of a group is its subgroup, $C_{r,s,t}(Q \cap R)$ is a subgroup of G. Therefore, $Q \cap R$ is a PFSG of G. □

Notice that the union of two PFSGs of G need not be a PFSG of G [11].

Proposition 3.4. *Let Q be a PFSG of G and a be any fixed element of G. Then,*

(i.) $a * C_{r,s,t}(Q) = C_{r,s,t}(a * Q)$.

(ii.) $C_{r,s,t}(Q) * a = C_{r,s,t}(Q * a),\ \forall\, r, s, t\ [0, 1]$ *with* $r + s + t \leq 1$.

Proof. (i.)

$$C_{r,s,t}(a * Q) = \{u \in G | \sigma_{a*Q}(u) \geq r,\ \tau_{a*Q}(u) \geq s, \eta_{a*Q}(u) \leq t\}$$

with the condition $0 \leq r + s + t \leq 1$.
Also

$$\begin{aligned} a * C_{r,s,t}(Q) &= a * \{b \in G | \sigma_Q(b) \geq r,\ \tau_Q(b) \geq s, \eta_Q(b) \leq t\} \\ &= \{a * b \in G | \sigma_Q(b) \geq r,\ \tau_Q(b) \geq s, \eta_Q(b) \leq t\} \end{aligned}$$

Set $a * b = u$, so that $b = a^{-1} * u$.

Therefore,

$$\begin{aligned} a * C_{r,s,t}(Q) &= \left\{u \in G | \sigma_Q(a^{-1} * u) \geq r,\ \tau_Q(a^{-1} * u) \geq s, \eta_Q(a^{-1} * u) \leq t\right\} \\ &= \{u \in G | \sigma_{a*Q}(u) \geq r,\ \tau_{a*Q}(u) \geq s, \eta_{a*Q}(u) \leq t\} \\ &= C_{r,s,t}(a * Q) \end{aligned}$$

for all $r, s, t\ [0, 1]$ with $r + s + t \leq 1$.

(ii.)

$$C_{r,s,t}(Q * a) = \{u \in G | \sigma_{Q*a}(u) \geq r,\ \tau_{Q*a}(u) \geq s, \eta_{Q*a}(u) \leq t\}$$

with the condition $0 \leq r + s + t \leq 1$.
Also

$$\begin{aligned} C_{r,s,t}(Q * a) &= \{b \in G | \sigma_Q(b) \geq r,\ \tau_Q(b) \geq s, \eta_Q(b) \leq t\} * a \\ &= \{b * a \in G | \sigma_Q(b) \geq r,\ \tau_Q(b) \geq s, \eta_Q(b) \leq t\} \end{aligned}$$

Set $b * a = u$, so that $b = u * a^{-1}$.

Therefore,

$$\begin{aligned} C_{r,s,t}(Q) * a &= \left\{u \in G | \sigma_Q(u * a^{-1}) \geq r,\ \tau_Q(u * a^{-1}) \geq s, \eta_Q(u * a^{-1}) \leq t\right\} \\ &= \{u \in G | \sigma_{Q*a}(u) \geq r,\ \tau_{Q*a}(u) \geq s, \eta_{Q*a}(u) \leq t\} \\ &= C_{r,s,t}(Q * a) \end{aligned}$$

for all r, s, t $[0, 1]$ with $r + s + t \leq 1$.

□

Proposition 3.5. *Let Q be a PFSG of G. Let $a, b \in G$ such that*

$$\sigma_Q(a) \wedge \sigma_Q(b) = r,\ \tau_Q(a) \wedge \tau_Q(b) = s,\ \eta_Q(a) \vee \eta_Q(b) = t.$$

Then,

(i.) $a * Q = b * Q \Leftrightarrow a^{-1} * b \in C_{r,s,t}(Q)$

(ii.) $Q * a = Q * b \Leftrightarrow a * b^{-1} \in C_{r,s,t}(Q)$

Proof. (i.) $a * Q = b * Q \ \Leftrightarrow\ C_{r,s,t}(a * Q) = C_{r,s,t}(b * Q)$

$\Leftrightarrow a * C_{r,s,t}(Q) = b * C_{r,s,t}(Q)$ [by Proposition 3.4]

$\Leftrightarrow a^{-1}{*}b \in C_{r,s,t}(Q)$ [since $C_{r,s,t}(Q)$ is a subgroup of G and $a, b \in C_{r,s,t}(Q)$].

(ii.) $Q * a = Q * b \ \Leftrightarrow\ C_{r,s,t}(Q * a) = C_{r,s,t}(Q * b)$

$\Leftrightarrow\ C_{r,s,t}(Q) * a = C_{r,s,t}(Q) * b$ [by Proposition 3.4]

$\Leftrightarrow\ a{*}b^{-1} \in C_{r,s,t}(Q)$ [since $C_{r,s,t}(Q)$ is a subgroup of G and $a, b \in C_{r,s,t}(Q)$].

□

Corolary 3.1. *If $C_{r,s,t}(Q)$ is PFNSG of G, then Proposition 3.4 (i) and (ii) coincide.*

Proof.

$$C_{r,s,t}(a * Q) = a * C_{r,s,t}(Q) = C_{r,s,t}(Q) * a = C_{r,s,t}(Q * a).$$

□

4 Conclusion

In this paper, some of the existing results in fuzzy group and fuzzy cut groups of a fuzzy group have been extended to picture fuzzy group and cut subgroup of a picture fuzzy subgroup. In further research, it will be of interest to see the relationship between a picture fuzzy set and bipolar fuzzy set and establish which one is a generalisation of the other.

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