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New Techniques of Weighted Sum Method for Solving Multi-Objective Geometric Programming Problems

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تقنيات جديدة لطريقة مجموع الموزون لحل مشاكل البرمجة الهندسية متعددة الاهداف

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ABSTRACT

Multi-objective geometric programming problem is a type of optimization problem that wildly used in engineering problems. Until now there are not many optimization techniques that can easily compute this type of optimization problem. In this paper, we proposed two new techniques with algorithms to optimize multi-objective geometric programming problems. We created the first technique by using the weighted sum method and Arithmetic mean, and by using the weighted sum method and geometric mean we produced the second technique. These two methods are used to convert multi-objective geometric optimization problems to single-objective geometric optimization problems Some examples are considered to illustrate the results. The results were compared with other common techniques used in solving multi-objective engineering optimization problems.

Keyword:

Geometric Programming Problem, Multi-Objective Geometric Programming Problem, Weighted sum method, Arithmetic Mean, Geometric Mean.

الخلاصة

تعد مشكلة البرمجة الهندسية متعددة الأهداف نوعًا من مشاكل التحسين التي تستخدم بشكل كبير في المشاكل الهندسية. حتى الآن لا يوجد الكثير من تقنيات التحسين التي يمكنها حساب هذا النوع من مشاكل التحسين بسهولة. في هذا البحث ، تم تقديم تقنيتين جديدتين مع خوارزميتين لتحسين مشاكل البرمجة الهندسية متعددة الأهداف. التقنية الاولى تم أنشاءها باستخدام طريقة المجموع الموزون والوسط الحسابي والتقنية الثانية باستخدام طريقة المجموع الموزون والمتوسط الهندسي. تم استخدام هاتين الطريقتين لتحويل مشكلة التحسين الهندسي متعدد الأهداف إلى مشكلة التحسين الهندسي ذات الهدف الواحد. تم اخذ بعض الامثلة بالاعتبار لتوضيح النتائج. كذلك تمت مقارنة هذه النتائج مع التقنيات الشائعة الأخرى المستخدمة في حل مشاكل التحسين الهندسي متعدد الأهداف.

الكلمات المفتاحية:

مشكلة البرمجة الهندسية ، مشكلة البرمجة الهندسية متعددة الاهداف ، طريقة المجموع الموزون ، المتوسط الحسابي ، المتوسط الهندسي.

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1. INTRODUCTION

Geometric programming (GP) is a type of mathematical optimization problem with objective, constraint functions, and many useful theoretical and computational properties, which was introduced by [1]. Geometric Programming (GP) has been used in Mechanical and civil engineering, chemical engineering, probability and statistics, finance and economics, control theory, circuit design, information theory, coding and signal processing, wireless networking, and other fields. GP was developed to determine the optimal values of posynomial and signomial functions, most of these GP applications are the posynomial type with zero or few degrees of difficulty. Geometric programming problems whose parameters, and exponents are non-negative integers are called posynomial problems, whereas Geometric programming problems with some negative parameters are referred to as signomial problems. Obtaining the optimal value for geometric programming problems will be very difficult since usually the object function and constraints of this problem are appearing as posynomial or signomial structures [2] [3]. Many studies on Geometric Programming problems have been conducted since the early 1960s. generally, GP problems have been used in engineering design problems and commonly engineering design problems as multiple criteria, so many ways were introduced to solve multi-objective geometric programming as [4] [5] [6] [7] [8] [9] [10] [2] and so on. Generally, the previous research is finding the optimal variable by using the weighted sum method, in this paper, we will introduce new techniques with algorithms that are using the weighted sum method differently.

The goal of this paper is to compare and study the common ways of finding the solutions to multi-objective geometric programming problems which are Chandra Sen`s method [11], Arithmetic Mean method, and weighted Sum method [12] compare to our new algorithms.

2. Some Preliminaries

Definition 2.1: Monomial is an expression in algebra that contains only one term. In mathematics, a monomial can be either a constant, a variable, or the product of some variables and constants. Although, the power of variables can be any real number, generally monomial function for $x_1, x_2, x_3, \dots, x_n$ can be written as:

$$f(x) = c\, x_1^{a_1} x_2^{a_2} \dots x_n^{a_n}$$

where $c \in \boldsymbol{R}^+$ and $a_i \in \boldsymbol{R}$, $i = 1,2, \dots, n$.

Definition 2.2: Posynomial function is a sum of two or more monomials it can represent as:

$$f(x) = \sum_{i=1}^{K} c_i\, x_1^{a_{1i}} x_2^{a_{2i}} \dots x_n^{a_{ni}}$$

For n variables, where $c_i \in \boldsymbol{R}^+$, $i = 1,2, \dots, K$ and $a_{ni} \in \boldsymbol{R}^{K\times n}$ [13]

Definition 2.3: Signomail function can be obtained from posynomials without the condition that the coefficient needs to be positive. Signomials are closed under addition, subtraction, multiplication, and scaling [14].

Definition 2.4: Geometric programming problem is the process of minimizing or maximizing a posynomial under posynomial upper bound inequality constraints and monomial equality constraints. In standard form:

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$$\begin{aligned} & max/\min f_0(x) \\ & \text{Subject to:} \\ & f_i(x) \leq 1 \\ & g_i(x) = 1 \\ & x_i > 0 \end{aligned} \qquad (1)$$

where f_0 is posynomial or signomail, f_i are posynomials or monomial functions $i = 0,1,\dots,n$ and $g_i(x)$ is monomial $i = 1,2,\dots,m$.

Definition 2.5: The Arithmetic Mean (AM) is determined by adding all available data sets over the total number of items in a data. Arithmetic Mean for $x_1, x_2, x_3, \dots, x_n$ calculated as:

$$AM = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Definition 2.6: The Geometric Mean

(GM) is an average value or mean that represents the central tendency of a group of numbers by calculating the product of their values. Geometric Mean for $x_1, x_2, x_3, \dots, x_n$ can be calculated as below:

$$GM = \sqrt[n]{x_1 \times x_2 \times x_3 \times \dots \times x_n}$$

3. Formulation of Multi-Objective Geometric Programming

Multi geometric programming problems can be formulated as:

$$Minf_k(x) = \sum_{t=1}^{T_0^k} C_{0t}^k \prod_{j=1}^{n} x_j^{a_{0tj}^k} \quad k = 1,2,\dots,p$$

$$\text{Subject to:} \qquad (2)$$

$$g_i(x) = \sum_{t=1}^{T_i} C_{it} \prod_{j=1}^{n} x_j^{aitj} \leq 1, \qquad i = 1,2,\dots,m$$

$$x_j > 0\,, j = 1,2,\dots,n$$

where C_{0t}^k ,$\forall\, k$ and t is a positive real numbers, and $aitj$ and a_{0tj}^k are real numbers $\forall\, k, i, t,$ and j. Also T_0^k represents the number of terms in k^{th} objective functions, and T_i is the number of terms in i^{th} constraints. The above is p numbers of the objective function, m number of constraints, and n number of strictly positive variables.

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4. New Techniques

We obtained two algorithms to solve multi-geometric programming problems that can be summarized as follows:

4.1 First Algorithm (using the Arithmetic mean)

Step 1: Put the given problem to a standard form of multi-geometric programming problem as the problem (2).

Step 2: Solve each of the k^{th} objective functions to find $S_i, i = 1,2,\dots,k$, by using the dual programming technique [1].

Step 3: Determine Arithmetic Mean (AM) for $(S_1, S_2, \dots, S_k)$ so we get:

$$AM = \frac{S_1 + S_2 + \dots + S_k}{k} \tag{3}$$

Step 4: Use the weighted sum method to create a single objective programming problem as follows:

$$Min\ AMF(x) = \frac{w_1 f_1(x) + w_2 f_2(x) + \dots + w_k f_k(x)}{AM}$$

Subject to:

$$g_i(x) = \sum_{t=1}^{T_i} C_{it} \prod_{j=1}^{n} x_j^{aitj} \le 1, \qquad i = 1,2,\dots,m \tag{4}$$

$$x_j > 0 \,, j = 1,2,\dots,n$$

where w_i are positive weights, $i = 1,2,\dots,k$ and $w_1 + w_2 + \dots + w_k = 1$, $C_{0t}^k \in R^+$,$\forall$ k and t is a positive real numbers, and $aitj$ and a_{0tj}^k are real numbers $\forall$ $k, i, t,$ and j. Also T_0^k represents the number of terms in k^{th} objective functions, and T_i is the number of terms in i^{th} constraints.

Step 5: Find a solution to the problem (4) by using dual programming.

4.2 Second Algorithm (using Geometric mean)

Step 1: Put the given problem to a standard form of multi-geometric programming problem as the problem (2).

Step 2: Solve each of the k^{th}objective functions to find $S_i, i = 1,2,\dots,k$, by using the dual programming technique [1].

Step 3: Determine the Geometric Mean for $(S_1, S_2, \dots, S_k)$ so we get:

$$GM = \sqrt[k]{S_1 \times S_2 \times \dots \times S_k} \tag{5}$$

Step 4: Use the weighted sum method to create a single objective programming problem as follows:

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$$Min\ GMF(x) = \frac{w_1 f_1(x) + w_2 f_2(x) + \cdots + w_k f_k(x)}{GM}$$

Subject to:

$$g_i(x) = \sum_{t=1}^{T_i} C_{it} \prod_{j=1}^{n} x_j^{aitj} \le 1, \qquad i = 1,2,\dots,m \tag{6}$$

$$x_j > 0\,, j = 1,2,\dots,n$$

where w_i are positive weights, $i = 1,2,\dots,k\ and\ w_1 + w_2 + \cdots + w_k = 1$, $C_{0t}^k \in R^+$,$\forall\ k$ and t is a positive real numbers, and $aitj$ and a_{0tj}^k are real numbers $\forall\ k, i, t$, and j. Also T_0^k represents the number of terms in k^{th} objective functions, and T_i is the number of terms in i^{th} constraints.

Step 5: Find a solution to the problem (6) by using dual programming.

5. Numerical Examples

We constructed three examples to apply and compare the new algorithms and preview techniques.

Example 5.1:

Find x_1, and x_2 for the following multi geometric optimization:

$$\min f_1 = 10x_1^{-1} + 12x_2^{-1}$$
$$\min f_2 = 8x_1^{-2} + 6x_2^{-1}$$
$$Subject\ to:$$
$$\frac{1}{4}x_1x_2^{-1} + \frac{3}{4}x_2 \le 1$$
$$x_1, x_2 > 0$$

Solution 5.1:

By using the dual programming technique in [1] we can get the optimal solution which $f_1 = 10.673$ at (0.9369,1.1243), and $f_2 = 6.0525$ at (1.626,1.9829).

- **Using the first Algorithm:**

$$AM\ (f_1, f_2) = \frac{minf_1 + minf_2}{2} = \frac{10.673\ + 6.0525}{2} = 13.69575$$

$$Min\ AMF(x) = \frac{w_1 f_1 + w_2 f_2}{AM} \tag{7}$$
$$Subject\ to:$$
$$\frac{1}{4}x_1x_2^{-1} + \frac{3}{4}x_2 \le 1$$
$$x_1, x_2 > 0$$

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where $w_1 + w_2 = 1$

After solving problem (7) by using the dual programming technique we get the following results:

Table 1: Solution of Example 5.1 by using the first algorithm

w_1	w_2	x_1	x_2	$AMF(x)$
0.1	0.9	2.237458952	14.766420400	0.130539781
0.2	0.8	3.905160218	14.058576780	0.149577169
0.3	0.7	5.546419928	14.420691810	0.157972931
0.4	9.6	5.159754354	15.835484140	0.160099557
0.5	0.5	9.288088632	16.718559530	0.157223632
0.6	0.4	11.715478660	18.744769050	0.149577169
0.7	0.3	14.990344000	21.795130250	0.136683117
0.8	0.2	19.961401530	26.947892070	0.117050460
0.9	0.1	30.422867540	38.535632210	0.086400555

- **Using the second Algorithm:**

$$GM\ (f_1, f_2) = \sqrt{minf_1 \times minf_2} = \sqrt{10.673 \times 6.0525}$$
$$= 11.36393462$$

$$\text{Min}\ GMF(x) = \frac{w_1 f_1 + w_2 f_2}{GM} \qquad (8)$$
$$Subject\ to:$$
$$\frac{1}{4} x_1 x_2^{-1} + \frac{3}{4} x_2 \leq 1$$
$$x_1, x_2 > 0$$

where $w_1 + w_2 = 1$

Determining Table 2 after using the dual programming technique for the problem (8):

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Table 2: Solution of Example 5.1 by using a second algorithm

w_1	w_2	x_1	x_2	$GMF(x)$
0.1	0.9	2.237458952	14.766420400	0.157323654
0.2	0.8	3.905160218	14.058576780	0.180269585
0.3	0.7	5.546419928	14.420691810	0.190388087
0.4	9.6	5.159754354	15.835484140	0.195208209
0.5	0.5	9.288088632	16.718559530	0.189485036
0.6	0.4	11.715478660	18.744769050	0.180269562
0.7	0.3	14.990344000	21.795130250	0.164729722
0.8	0.2	19.961401530	26.947892070	0.141068555
0.9	0.1	30.422867540	38.535632210	0.104129462

Table 3: Comparative between new techniques and the weighted sum method for example 5.1

w_1	w_2	$AMF(x)$	$GMF(x)$	Weighted Sum Method
0.1	0.9	0.130539781	0.157323654	1.787815724
0.2	0.8	0.149577169	0.180269585	2.048571519
0.3	0.7	0.157972931	0.190388087	2.163557782
0.4	9.6	0.160099557	0.195208209	2.192683551
0.5	0.5	0.157223632	0.189485036	2.153295559
0.6	0.4	0.149577169	0.180269562	2.048571519
0.7	0.3	0.136683117	0.164729722	1.871977801
0.8	0.2	0.117050460	0.141068555	1.603093848
0.9	0.1	0.086400555	0.104129462	1.183320407

Example 5.2:

Find x_1, x_2 and x_3 for the given programming problem:

$$\min f_1 = \frac{1}{x_1^2} + \frac{x_2^2}{4x_3}$$

$$\min f_2 = \frac{2}{x_1 x_2 x_3} + 2x_1 x_2$$

$$Subject\ to:$$

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$$\frac{6x_1^2}{x_2^2} + 3x_2x_3^2 \le 8$$
$$x_1, x_2, x_3 > 0$$

Solution 5.2:

The problem can be rewritten as:

$$\min f_1 = x_1^{-2} + \frac{1}{4}x_2^2x_3^{-1}$$
$$\min f_2 = 2x_1^{-1}x_2^{-1}x_3^{-1} + 2x_1x_2$$
$$Subject\ to:$$
$$\frac{3}{4}x_1^2x_2^{-2} + \frac{3}{8}x_2x_3^2 \le 1$$
$$x_1, x_2, x_3 > 0$$

By using the dual programming technique in [1] we can get the optimal solution for each f_1, and f_2 which are 1.17159505 at (1.239502732,1.25233969,0.5543851669), and 3.504278981 at (0.622713283,1.205879087,1.330079126) respectively.

- **Using the first Algorithm:**

$$AM\ (f_1, f_2) = \frac{minf_1 + minf_2}{2} = \frac{1.17159505 + 3.504278981}{2}$$
$$= 2.337937016$$

$$Min\ AMF(x) = \frac{w_1f_1 + w_2f_2}{AM} \tag{9}$$
$$Subject\ to:$$
$$\frac{3}{4}x_1^2x_2^{-2} + \frac{3}{8}x_2x_3^2 \le 1$$
$$x_1, x_2, x_3 > 0$$

where $w_1 + w_2 = 1$

The solution to the problem (9) is shown in Table 4:

Table 4: Solution of Example 5.2 by using the first algorithm

w_1	w_2	x_1	x_2	x_3	$AMF(x)$
0.1	0.9	0.762195547	0.125466641	3.366817639	0.736265376
0.2	0.8	0.950918483	0.145371792	3.127833717	0.946042420
0.3	0.7	1.148318255	0.141500209	3.195607482	0.973040742
0.4	9.6	1.148361731	0.141500209	3.195007482	0.888332233
0.5	0.5	1.643512810	0.112629128	3.553516758	0.791755357
0.6	0.4	1.895322972	0.110156869	3.593171431	0.714417345
0.7	0.3	2.231712481	0.104961991	3.681015944	0.601158162
0.8	0.2	2.671606669	0.104884772	3.682370737	0.479415578
0.9	0.1	3.763252983	0.084434953	4.104144045	0.271820917

- **Using the second Algorithm:**

$$GM\ (f_1, f_2) = \sqrt{minf_1 \times minf_2} = \sqrt{1.17159505 \times 3.504278981}$$
$$= 2.026227013$$

$$\text{Min}\ GMF(x) = \frac{w_1 f_1 + w_2 f_2}{GM} \tag{10}$$

$$Subject\ to:$$
$$\frac{3}{4}x_1^2 x_2^{-2} + \frac{3}{8}x_2 x_3^2 \leq 1$$
$$x_1, x_2, x_3 > 0$$

where $w_1 + w_2 = 1$

The solution to the problem (10) is shown in Table 5:

Table 5: Solution of Example 5.2 by using a second algorithm

w_1	w_2	x_1	x_2	x_3	$GMF(x)$
0.1	0.9	0.762195547	0.125466641	3.366817639	0.849530712
0.2	0.8	0.950918483	0.145371792	3.127833717	0.962609657
0.3	0.7	1.148318255	0.141500209	3.195607482	0.974628489
0.4	9.6	1.148361731	0.141500209	3.195007482	0.950655029
0.5	0.5	1.643512810	0.112629128	3.553516758	0.893959891
0.6	0.4	1.895322972	0.110156869	3.593171431	0.772419797
0.7	0.3	2.231712481	0.104961991	3.681015944	0.661424560
0.8	0.2	2.671606669	0.104884772	3.682370737	0.514308711
0.9	0.1	3.763252983	0.084434953	4.104144045	0.330551231

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Table 6: Comparing the first and second techniques with the weighted sum method for example 5.2.

w_1	w_2	$AMF(x)$	$GMF(x)$	Weighted Sum Method
0.1	0.9	0.736265376	0.849530712	0.714776691
0.2	0.8	0.946042420	0.962609657	1.120844646
0.3	0.7	0.973040742	0.974628489	1.430249217
0.4	9.6	0.888332233	0.950655029	3.228921973
0.5	0.5	0.791755357	0.893959891	3.573881807
0.6	0.4	0.714417345	0.772419797	3.797467398
0.7	0.3	0.601158162	0.661424560	3.880388294
0.8	0.2	0.479415578	0.514308711	3.772612253
0.9	0.1	0.271820917	0.330551231	3.327667598

Example 5.3:
Consider the following multi-objective programming problem find x_1, x_2, and x_3:

$$\min f_1 = x_1^{-1} x_2^{\frac{1}{2}} x_3 + 20 x_1^{-1} x_3$$
$$\min f_2 = x_1^{-1} x_2^{-1} x_3 + x_1^{-1} x_3$$
$$Subject\ to:$$
$$x_1^2 x_2^{-2} + 4 x_2^{\frac{1}{2}} x_3^{\frac{-3}{4}} \leq 3$$
$$x_1, x_2, x_3 > 0$$

Solution 5.3:
Now the problem can be rewritten as:

$$\min f_1 = x_1^{-1} x_2^{\frac{1}{2}} x_3 + 20 x_1^{-1} x_3$$
$$\min f_2 = x_1^{-1} x_2^{-1} x_3 + x_1^{-1} x_3$$
$$Subject\ to:$$
$$\frac{1}{3} x_1^2 x_2^{-2} + \frac{4}{3} x_2^{\frac{1}{2}} x_3^{\frac{-3}{4}} \leq 1$$
$$x_1, x_2, x_3 > 0$$

We can get an optimal solution for each object function with the above constraint by using the dual programming technique which is $\min f_1 = 12.72550201$ at (8.090398353,6.324555321,1.71590634) and $\min f_2 = 4.688115022$ at (8.090398353,6.324555321,1.71590634)

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- **Using the first Algorithm:**

$$AM\ (f_1, f_2) = \frac{minf_1 + minf_2}{2} = \frac{12.72550201 + 4.688115022}{2} = 8.706808516$$

$$Min\ AMF(x) = \frac{w_1 f_1 + w_2 f_2}{AM} \tag{11}$$

$$Subject\ to:$$

$$\frac{1}{3} x_1^2 x_2^{-2} + \frac{4}{3} x_2^{\frac{1}{2}} x_3^{\frac{-3}{4}} \le 1$$

$$x_1, x_2, x_3 > 0$$

where $w_1 + w_2 = 1$

Table 7 shows the solution to the problem (11):

Table 7: Solution of Example 5.3 by using the first algorithm.

w_1	w_2	x_1	x_2	x_3	$AMF(x)$
0.1	0.9	0.031492356	0.102532778	0.080814078	0.589458051
0.2	0.8	0.013996604	0.045570127	0.001877413	0.616223319
0.3	0.7	0.008164685	0.026582574	0.001095157	0.600567913
0.4	9.6	0.004373939	0.014240665	0.000267691	0.562331970
0.5	0.5	0.003499949	0.011395129	0.000154896	0.508183167
0.6	0.4	0.002332767	0.007595021	0.000074626	0.440898788
0.7	0.3	0.001499267	0.004882487	0.000013689	0.361225078
0.8	0.2	0.000874357	0.002848067	0.000012762	0.268228027
0.9	0.1	0.000386995	0.001259979	0.000002938	0.157729225

- **Using the second Algorithm:**

$$GM\ (f_1, f_2) = \sqrt{minf_1 \times minf_2} = \sqrt{12.72550201 \times 4.688115022} = 7.723899087$$

$$\text{Min}\ GMF(x) = \frac{w_1 f_1 + w_2 f_2}{GM} \tag{12}$$

$$Subject\ to:$$

$$\frac{1}{3} x_1^2 x_2^{-2} + \frac{4}{3} x_2^{\frac{1}{2}} x_3^{\frac{-3}{4}} \le 1$$

$$x_1, x_2, x_3 > 0$$

مـجلــة جـــامعــة بــــابـل للعلـــوم الصـــرفــة والتطـــبيقيــة مـجلــة جـــامعــة بــــابـل للعلـــوم الصـــرفــة والتطـــبيقيــة مـجلــة جـــامعــة بــــابـل

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where $w_1 + w_2 = 1$

The solution to the problem (12) is shown in Table 8:

Table 8: Solution of Example 5.3 by using a second algorithm.

w_1	w_2	x_1	x_2	x_3	$GMF(x)$
0.1	0.9	0.031492356	0.102532778	0.080814078	0.664496492
0.2	0.8	0.013996604	0.045570127	0.001877413	0.694642935
0.3	0.7	0.008164685	0.026582574	0.001095157	0.676992299
0.4	9.6	0.004373939	0.014240665	0.000267691	0.633891078
0.5	0.5	0.003499949	0.011395129	0.000154896	0.572865039
0.6	0.4	0.002332767	0.007595021	0.000074626	0.628442089
0.7	0.3	0.001499267	0.004882487	0.000013689	0.408983170
0.8	0.2	0.000874357	0.002848067	0.000012762	0.303691467
0.9	0.1	0.000386995	0.001259979	0.000002938	0.177801034

Table 9: Comparing the first and second techniques with the weighted sum method for example 5.3.

w_1	w_2	$AMF(x)$	$GMF(x)$	Weighted Sum Method
0.1	0.9	0.589458051	0.664496492	4.076685400
0.2	0.8	0.616223319	0.694642935	4.261784003
0.3	0.7	0.600567913	0.676992299	4.153521081
0.4	9.6	0.562331970	0.633891078	3.889081725
0.5	0.5	0.508183167	0.572865039	3.514669326
0.6	0.4	0.440898788	0.628442089	3.649251137
0.7	0.3	0.361225078	0.408983170	2.498228668
0.8	0.2	0.268228027	0.303691467	1.855062091
0.9	0.1	0.157729225	0.177801034	1.090853586

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6. Comparison and Result

The data from table 10 demonstrate how the results of our two algorithms have better outcomes than other ways which are (Chandra Sen's method, Arithmetic mean, and weighted sum methods).

Table 10: Comparison between new techniques and the preview methods

Methods \ Examples	Example 5.1	Example 5.2	Example 5.3
Chandra Sen's method	0.441503938	1.515703667	1.491181596
Arithmetic mean technique	0.314356723	1.581483341	1.019591281
Weighted sum method $(w_1 = 0.9, w_2 = 0.1)$	1.183320407	3.327667598	1.090853586
First algorithm $(w_1 = 0.9, w_2 = 0.1)$	0.086400555	0.271820917	0.157729225
Second algorithm $(w_1 = 0.9, w_2 = 0.1)$	0.104129462	0.330551231	0.177801034

7. Conclusion

In this work, we have presented and discussed two new techniques to solve multi-objective geometric programming problems (MOGPP). We used both techniques to transform MOGPP into a single objective geometric programming problem and then solved a single objective geometric programming problem by dual programming, Although, discussed some techniques that were studied previously which are used to solve MOGPP.

After we applied the two algorithms and other techniques on the numerical examples we conclude that through our new techniques we achieve a better result than the previews algorithms.

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Conflict of interests.

There are non-conflicts of interest.

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