

Cellular Automata Approach to Durability Analysis of Concrete Structures in Aggressive Environments

Fabio Biondini¹; Franco Bontempi²; Dan M. Frangopol, F.ASCE³; and Pier Giorgio Malerba⁴

Abstract: This paper presents a novel approach to the problem of durability analysis and lifetime assessment of concrete structures under the diffusive attack from external aggressive agents. The proposed formulation mainly refers to beams and frames, but it can be easily extended also to other types of structures. The diffusion process is modeled by using cellular automata. The mechanical damage coupled to diffusion is evaluated by introducing suitable material degradation laws. Since the rate of mass diffusion usually depends on the stress state, the interaction between the diffusion process and the mechanical behavior of the damaged structure is also taken into account by a proper modeling of the stochastic effects in the mass transfer. To this aim, the nonlinear structural analyses during time are performed within the framework of the finite element method by means of a deteriorating reinforced concrete beam element. The effectiveness of the proposed methodology in handling complex geometrical and mechanical boundary conditions is demonstrated through some applications. Firstly, a reinforced concrete box girder cross section is considered and the damaging process is described by the corresponding evolution of both bending moment–curvature diagrams and axial force-bending moment resistance domains. Secondly, the durability analysis of a reinforced concrete continuous T-beam is developed. Finally, the proposed approach is applied to the analysis of an existing arch bridge and to the identification of its critical members.

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Introduction

Satisfactory structural performance is usually described with reference to a specified set of limit states, which separate desired states of the structure from the undesired ones. In this context, the main objective of the structural design is to assure an adequate level of structural performance for each specified limit state during the whole service life of the structure.

From a general point of view, a structure is safe when the effects of the applied actions S are no larger than the corresponding resistance R . However, for concrete structures the structural performance must be considered as time dependent, mainly because of the progressive deterioration of the mechanical properties of materials which makes the structural system less able to

withstand the applied actions. As a consequence, both the demand S and the resistance R may vary during time and a durability analysis leading to a reliable assessment of the actual structural lifetime T_a should be able to account for such variability (Sarja and Vesikari 1996; Enright and Frangopol 1998a, 1998b). In this way, the designer can address the conceptual design process or plan the rehabilitation of the structure in order to achieve a prescribed design value T_d of the structural lifetime.

In the following, the attention will be mainly focused on the damaging process induced by the diffusive attack of environmental aggressive agents, like sulfate and chloride, which may lead to deterioration of concrete and corrosion of reinforcement (CEB 1992). Such process involves several factors, including temperature and humidity. Its dynamics is governed by coupled diffusion process of heat, moisture, and various chemical substances. In addition, damage induced by mechanical loading interacts with the environmental factors and accelerates the deterioration process (Saetta et al. 1993, Xi and Bažant 1999; Xi et al. 2000; Kong et al. 2002).

Based on the previous considerations, a durability analysis of concrete structures in aggressive environments should be capable to account for both the diffusion process and the corresponding mechanical damage, as well as for the coupling effects between diffusion, damage and structural behavior. However, the available information about environmental factors and material characteristics is often very limited and the unavoidable uncertainties involved in a detailed and complex modeling may lead to fictitious results. For these reasons, the assessment of the structural lifetime can be more reliably carried out by means of macroscopic models which exploit the power and generality of the basic laws of diffusion to predict the quantitative time-variant response of damaged structural systems.

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This paper presents a novel approach to the durability analysis of concrete structures under the environmental attack of aggressive agents (Bontempi et al. 2000; Ardigò et al. 2002; Biondini et al. 2002a, b). The proposed formulation mainly refers to beams and frames, but it can be easily extended also to other types of structures. The analysis of the diffusion process is developed by using a special class of evolutionary algorithms called cellular automata, which are mathematical idealizations of physical systems in which space and time are discrete and physical quantities are taken from a finite set of discrete values. In principle, any physical system satisfying differential equations may be approximated as a cellular automaton by introducing discrete coordinates and variables, as well as discrete time steps. However, it is worth noting that models based on cellular automata provide an *alternative* approach to physical modeling rather than an approximation. In fact, they show a complex behavior analogous to that associated with differential equations, but by virtue of their simple formulation are potentially adaptable to a more detailed and complete analysis, giving to the whole system some emergent properties, self-induced only by its local dynamics (von Neumann 1966; Margolus and Toffoli 1987; Wolfram 1994, 2002; Adami 1998). Noteworthy examples of cellular automata modeling of typical physical processes in concrete can be found in the field of cement composites (Bentz and Garboczi 1992; Bentz et al. 1992, 1994).

Based on such an evolutionary model, the mechanical damage coupled to diffusion is then evaluated by introducing a degradation law of the effective resistant area of both the concrete matrix and steel bars in terms of suitable damage indices. Since the rate of mass diffusion usually depends on the stress state, the interaction between the diffusion process and the mechanical behavior of the damaged structure is also taken into account by a proper modeling of the stochastic effects in the mass transfer. To this aim, the nonlinear structural analyses during time are performed within the framework of the finite element method by means of a deteriorating reinforced concrete beam element (Bontempi et al. 1995; Malerba 1998; Biondini 2000).

The effectiveness of the proposed methodology in handling complex geometrical and mechanical boundary conditions is demonstrated through some applications. Firstly, a reinforced concrete box girder cross-section is considered and the damaging process is described by the corresponding evolution of both bending moment–curvature diagrams and axial force–bending moment resistance domains. Secondly, the durability analysis of a reinforced concrete continuous T-beam is developed. Finally, the proposed approach is applied to the analysis of an existing arch bridge and to the identification of its critical members.

Diffusion Processes and Cellular Automata

Modeling of Diffusion Processes

The kinetic process of diffusion of chemical components in solids is usually described by mathematical relationships that relate the rate of mass diffusion to the concentration gradients responsible for the net mass transfer (Glicksman 2000). The simplest model is represented by the Fick's first law, which assumes a linear relationship between the mass flux and the diffusion gradient. The combination of the Fick's model with the mass conservation principle leads to Fick's second law which, in the case of a single component diffusion in isotropic media, can be written as follows:

$$-\nabla \cdot (-D \nabla C) = \frac{\partial C}{\partial t} \quad (1)$$

where $C=C(\mathbf{x},t)$ =mass concentration of the component and $D=D(\mathbf{x},t)$ =diffusivity coefficient, both evaluated at point $\mathbf{x}=(x,y,z)$ and time t , and where $\nabla C=\mathbf{grad} C$.

Complexities leading to modifications of this simple model may arise from anisotropy, multicomponent diffusion, chemical reactions, external stress fields, memory and stochastic effects. In the case of concrete structures, for example, the diffusivity coefficient depends on several parameters, such as relative humidity, temperature, and mechanical stress, and the Fick's equations must be coupled with the governing equations of both heat and moisture flows, as well as with the constitutive laws of the mechanical problem (CEB 1992; Saetta et al. 1993; Xi and Bažant 1999; Xi et al. 2000). However, as mentioned, due to the uncertainties involved in the calibration of such complex models, the structural lifetime can be more conveniently assessed by using a macroscopic approach which exploits the power and generality of the basic Fick's laws to predict the quantitative response of systems undergoing diffusion. In particular, if the diffusivity coefficient D is assumed to be a constant, the second order partial differential nonlinear Eq. (1) is simplified in the following linear form:

$$D \nabla^2 C = \frac{\partial C}{\partial t} \quad (2)$$

where $\nabla^2 = \nabla \cdot \nabla$. Despite of its linearity, analytical solutions of such an equation exist only for a limited number of simple classical problems. Thus, a general approach dealing with complex geometrical and mechanical boundary conditions usually requires the use of numerical methods. In this study, the diffusion equation is effectively solved by using a special class of evolutionary algorithms called cellular automata.

Short History, Formal Definition, and Emerging Properties of Cellular Automata

Cellular automata were firstly introduced by von Neumann and Ulam in 1948–1950 (von Neumann 1966) and subsequently developed by other researchers in many fields of science (see for reviews: Margolus and Toffoli 1987; Adami 1998; Wolfram 2002). Originally related to the study of self-replication problems on the Turing's machine, cellular automata left laboratories in the 1970s and became popular in the academic circles with the now famous Game of Life invented by Conway (Gardner 1970). Basically, they represent simple mathematical idealizations of physical systems in which space and time are discrete, and physical quantities are taken from a finite set of discrete values. In fact, as already mentioned, any physical system satisfying differential equations may be approximated as a cellular automaton by introducing discrete coordinates and variables, as well as discrete time steps. Properly speaking, therefore, models based on cellular automata provide an alternative and more general approach to physical modeling rather than an approximation; they show a complex behavior analogous to that associated with complex differential equations, but in this case complexity emerges from the interaction of simple entities following simple rules.

In its basic form, a cellular automaton consists of a regular uniform grid of sites or cells, theoretically having infinite extension, with a discrete variable in each cell which can take on a finite number of states. The state of the cellular automaton is then completely specified by the values $s_i = s_i(t)$ of the variables at each cell i . During time, cellular automata evolve in discrete time steps

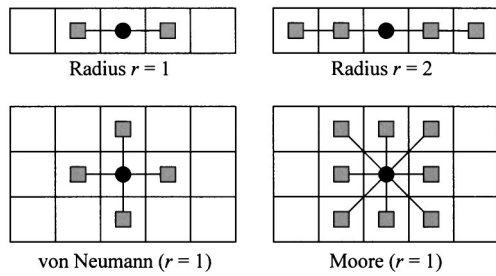


Fig. 1. Typical neighborhoods for one- and two-dimensional cellular automata

according to a parallel state transition determined by a set of local rules: the variables $s_i^{k+1} = s_i(t_{k+1})$ at each site i at time t_{k+1} are updated synchronously based on the values of the variables s_n^k in their “neighborhood” n at the preceding time instant t_k . The neighborhood n of a cell i is typically taken to be the cell itself and a set of adjacent cells within a given radius r , or $i-r \leq n \leq i+r$. Thus, the dynamics of a cellular automaton can be formally represented as

$$s_i^{k+1} = \phi(s_i^k; s_n^k), \quad i-r \leq n \leq i+r \quad (3)$$

where the function ϕ is the evolutionary rule of the automaton. Clearly, a proper choice of the neighborhood plays a crucial role in determining the effectiveness of such a rule. Fig. 1 shows an example of typical neighborhoods for one- and two-dimensional cellular automata, but patterns of higher complexity can be also proposed. Since the actual extension of the automaton cannot be infinite as required by the theory, special attention has to be paid to neighborhoods along the sides of the finite grid, which may be defined in many different ways. The more frequent assumptions refer to the hypotheses of periodic boundaries, in which opposite cells are considered neighbors, or of absorbing boundaries, where the cells at the borders are assumed to have no neighbors beyond the limits of the grid.

As an example of one-dimensional cellular automaton, consider a line of cells with the variable $s = s(t)$ at each cell i and time instant t_k which can take only the value $s_i^k = 0$ or $s_i^k = 1$. Let the evolutionary rule be defined on the base of a neighborhood with radius $r=1$ as follows:

$$s_i^{k+1} = (s_{i-1}^k + s_{i+1}^k) \bmod 2 \quad (4)$$

where $\bmod 2$ indicates that the variable s_i^{k+1} takes the 0 or 1 remainder after division by 2. Even with this very simple rule, complex behaviors can be nevertheless found. In fact, by assuming for example an initial state of the automaton consisting of a single cell with value 1 and all other cells having value 0, after 256 time steps the pattern shown in Fig. 2 appears, where the white/gray cells denote the state $s_i^k = 0$ and the black ones the state $s_i^k = 1$. The geometry of such pattern is characterized by the property of self-similarity, since some of its regions, when magnified, are indistinguishable from the whole. In particular, it can be shown that such a self-similar pattern represents a fractal (Mandelbrot 1982) and can be characterized by a fractal dimension $\log_2 3 \approx 1.59$ (see also Wolfram 1983).

Cellular Automata Solution of Diffusion Equations

One of the most effective applications of cellular automata is the simulation of diffusion processes, since their dynamics can accu-

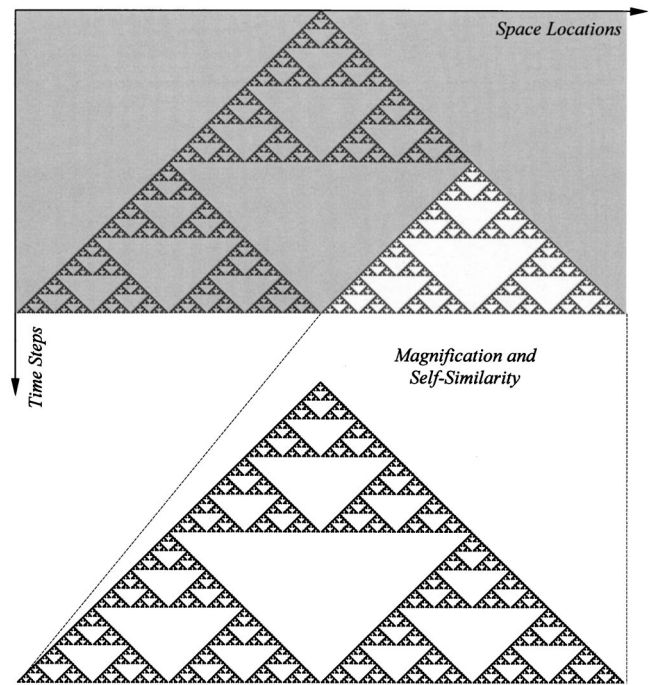


Fig. 2. One-dimensional cellular automaton leading to geometrical pattern characterized by property of self-similarity and having fractal dimension

rately reproduce linear or nonlinear flows with complex boundary conditions (Whitney 1990). This result is easily achieved through a proper selection of both the neighborhood n and the rule ϕ . For example, the diffusion process described by Fick’s laws in d dimensions ($d=1, 2, 3$) can be effectively simulated by adopting a von Neumann neighborhood with radius $r=1$ and the following rule of evolution (Biondini et al. 2002a):

$$C_i^{k+1} = \phi_0 C_i^k + \sum_{j=1}^d (\phi_j^- C_{i-1,j}^k + \phi_j^+ C_{i+1,j}^k) \quad (5)$$

where the discrete variable $s_i^k = C_i^k = C(\mathbf{x}_i, t_k)$ represents the concentration of the component in the cell i at time t_k . The values of the evolutionary coefficients ϕ_0 , ϕ_j^- , and ϕ_j^+ ($j=1, \dots, d$) must verify the following normality rule:

$$\phi_0 + \sum_{j=1}^d (\phi_j^- + \phi_j^+) = 1 \quad (6)$$

as required by the mass conservation law. Clearly, for isotropic media the symmetry condition $\phi_j^- = \phi_j^+ = \phi_1$ ($j=1, \dots, d$) must be adopted in order to avoid directionality effects. Moreover, to regulate the process according to a given diffusivity D , a proper discretization in space and time should be chosen in such a way that the grid dimension Δx and the time step Δt satisfy the following relationship:

$$D = \frac{1 - \phi_0}{2d} \frac{\Delta x^2}{\Delta t} = \phi_1 \frac{\Delta x^2}{\Delta t} \quad (7)$$

The demonstration of this result can be derived by a proper manipulation of the Fick’s second law and by its subsequent integration over the whole d -dimensional space. It can be also proven

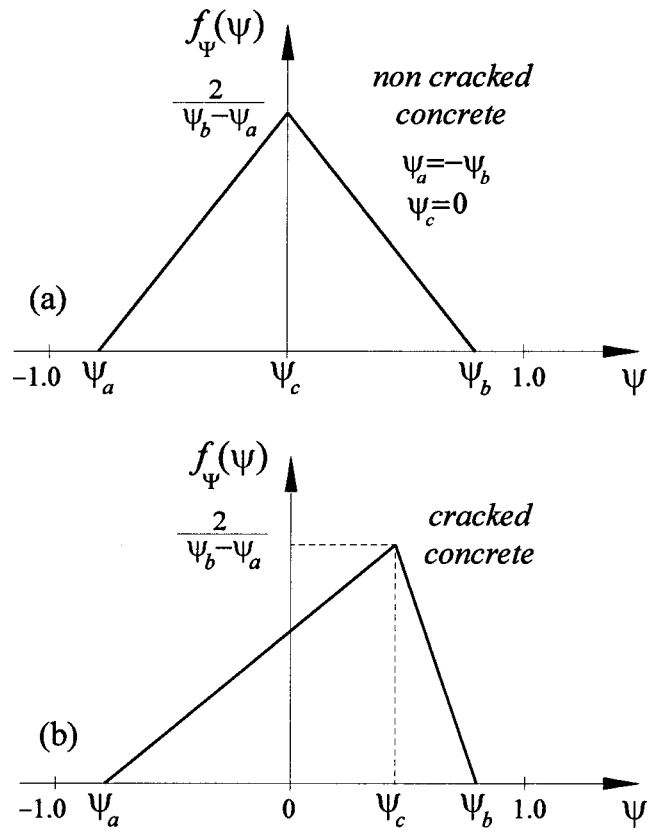


Fig. 3. Triangular density distribution $f_{\psi}(\psi)$: (a) symmetrical (no directionality effects) and (b) skewed toward highest ψ values (suitable, for example, for cracked concrete)

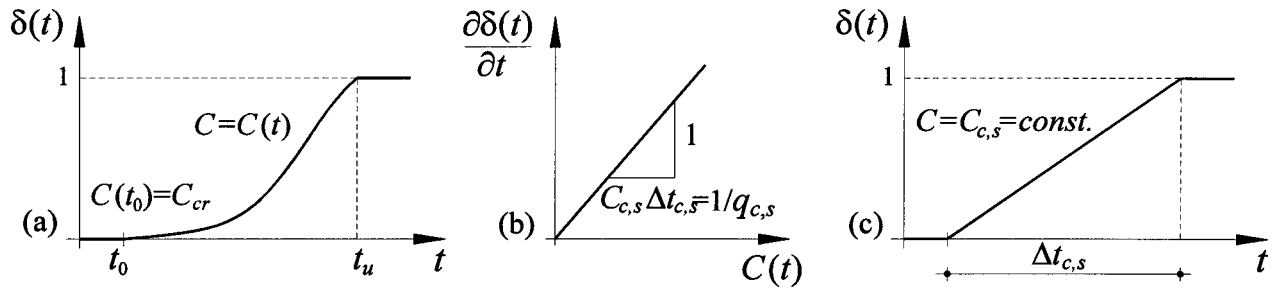


Fig. 4. Modeling of mechanical damage: (a) time evolution of damage indices during diffusion process; (b) linear relationship between rate of damage and concentration of aggressive agent; (c) meaning of material parameters for calibration of damage models

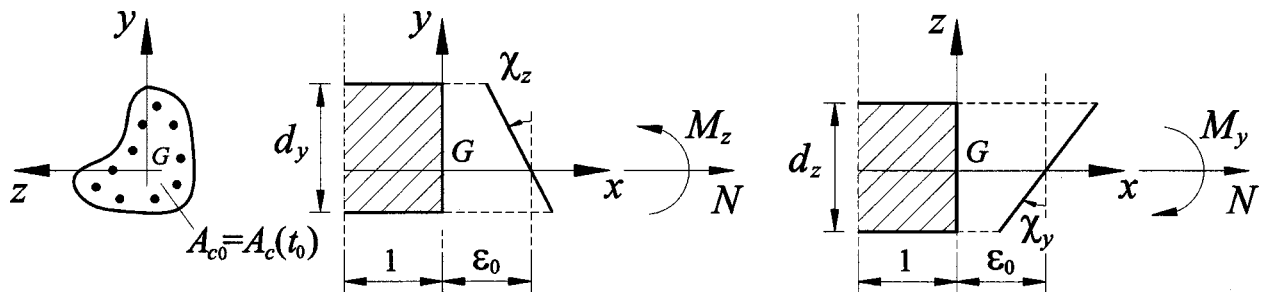


Fig. 5. Main parameters, local reference system, and sign conventions for cross sectional nonlinear analysis

that the values $\phi_0=1/2$ and $\phi_1=1/(4d)$ usually lead to a very good accuracy of the automaton, in one-, two-, and three- dimensional problems (Ardigò and Motta 2002). Clearly, since the diffusivity D can vary over a fairly wide range depending on the diffusive species, a proper balance between grid dimension Δx and time step Δt must be achieved consistently with the required level of accuracy.

It is worth noting that the hypothesis of absorbing boundaries associated with the finite extension of the grid of the automaton is usually consistent with the nature of the problem investigated in the present study.

Stochastic Effects and External Stress Fields

Diffusion processes always exhibit stochastic effects. Although the deterministic solutions usually provide good approximations (Ardigò and Motta 2002), for the purpose of the present work it is useful to include stochastic effects. Therefore, in order to account for the stochastic nature of the diffusion process, a modification of the previous cellular automata formulation is proposed. The coefficients ϕ , which define the evolutionary rule, are no longer considered as deterministic quantities but are instead assumed as random variables Φ with given probability density distributions $f_\Phi(\phi)$. To this aim, it is convenient to denote $\Phi=(1+\Psi)\phi$, where the coefficients ϕ maintain the original deterministic values, and Ψ are random variables whose outcomes $\psi \in [-1; 1]$ describe the stochastic part of diffusion. Clearly, the new variables Ψ_0, Ψ_j^- and Ψ_j^+ are not independent, but they are related through the mass conservation law as follows ($\phi_0=2d\phi_1=1/2$):

$$\frac{1}{2}\Psi_0 + \frac{1}{4d} \sum_{j=1}^d (\Psi_j^- + \Psi_j^+) = 0 \quad (8)$$

After the probability density functions $f_\Psi(\psi)$ are introduced, the random components of diffusion can be effectively simulated by means of a Monte Carlo method.

The choice of $f_\Psi(\psi)$ does not usually represent a critical point, and several functions are suitable for this purpose. However, it is important to outline that, for isotropic media, symmetrical functions $f_\Psi(\psi)=f_\Psi(-\psi)$ must be adopted in order to avoid directionality effects in the stochastic model. On the other hand, skewed distributions can successfully be applied to simulate local modifications in the rate of mass diffusion induced by mechanical stress states. In concrete structures, for example, cracking usually favors higher gradient of concentration. This phenomenon may be accounted for by adopting in the regions of cracked concrete distributions $f_\Psi(\psi)$ skewed towards the highest ψ values, which on the average tend to increase the local diffusivity, while in the regions of undamaged material symmetrical functions $f_\Psi(\psi)$ still hold. These assumptions are represented in Fig. 3 for the case of a simple, but also very effective, triangular density function.

Durability Analysis of Reinforced Concrete Structures

As previously mentioned, the safety of reinforced concrete structures exposed to aggressive environments needs to be checked during time by means of a durability analysis able to account for both the diffusion process and the corresponding mechanical damage, as well as for the coupling effects between diffusion, damage and structural behavior. To this aim, a general approach to the time-variant nonlinear analysis of damaging reinforced

concrete framed structures is developed in the context of the cellular automata formulation adopted to capture the diffusion process.

Modeling of Mechanical Damage

The damaging processes in concrete structures undergoing diffusion are, in general, very complex. Moreover, the available information about environmental agents and material characteristics is usually not sufficient for a detailed modeling. However, despite such complexities, very simple degradation models can be often successfully adopted.

Structural damage can be viewed as a degradation of the mechanical properties which makes the structural system less able to withstand the applied actions. Such a damage is here modeled by introducing a degradation law of the effective resistant area for both the concrete matrix $A_c=A_c(t)$ and the steel bars $A_s=A_s(t)$

$$dA_c(t) = [1 - \delta_c(t)]dA_{c0}$$

$$dA_s(t) = [1 - \delta_s(t)]dA_{s0} \quad (9)$$

where the symbol “0” denotes the undamaged state at the initial time $t=t_0$, and the dimensionless functions $\delta_c=\delta_c(t)$, $\delta_s=\delta_s(t)$ represent damage indices which give a direct measure of the damage level within the range [0;1] [Fig. 4(a)].

In this study, such indices are correlated to the diffusion process by assuming, for both materials, a linear relationship between the rate of damage and the concentration of the aggressive agent [Fig. 4(b)]

$$\frac{\partial \delta_c(t)}{\partial t} = \frac{C(t)}{C_c \Delta t_c} = q_c C(t)$$

$$\frac{\partial \delta_s(t)}{\partial t} = \frac{C(t)}{C_s \Delta t_s} = q_s C(t) \quad (10)$$

where C_c and C_s represent the values of constant concentration $C(\mathbf{x}, t)$ which lead to a complete damage of the materials after the time periods Δt_c and Δt_s , respectively [Fig. 4(c)]. In addition, the initial conditions $\delta_c(\mathbf{x}, t_0)=\delta_s(\mathbf{x}, t_0)=0$ with $t_0=\max\{t | C(\mathbf{x}, t) \leq C_{cr}\}$ are assumed, where C_{cr} =critical threshold of concentration. Clearly, the three parameters $q_c=(C_c \Delta t_c)^{-1}$, $q_s=(C_s \Delta t_s)^{-1}$, and C_{cr} are related to the materials and they must be set according to the actual rates of the damaging process.

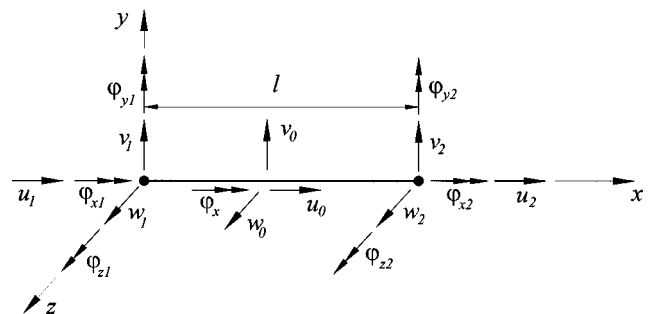


Fig. 6. Local reference system and sign conventions for the finite beam element

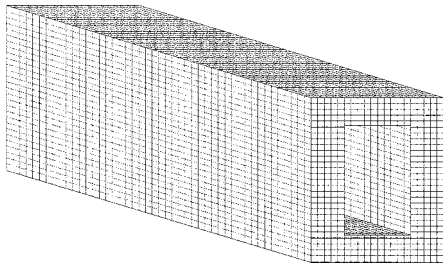


Fig. 7. Example of cellular grid actually used for numerical integration for a beam element with rectangular box cross section

Nonlinear Analysis of Deteriorating Reinforced Concrete Cross Sections

The previous general criteria are now applied to the durability analysis of reinforced concrete cross sections. The formulation assumes the linearity of concrete strain field and neglects the bond-slip of the reinforcement, as shown in Fig. 5 for a cross section with arbitrary shape. Based on such hypotheses, the vectors of the stress resultants $\mathbf{r} = \mathbf{r}(x, t) = [N \ M_z \ M_y]^T$ and of the global strains $\mathbf{e} = \mathbf{e}(x, t) = [\varepsilon_0 \ \chi_z \ \chi_y]^T$ are then related, at each time instant t , as follows (Malerba 1998):

$$\mathbf{r}(x, t) = \mathbf{H}(x, t)\mathbf{e}(x, t) \quad (11)$$

The stiffness matrix $\mathbf{H} = \mathbf{H}(x, t)$ is derived by integration over the area of the composite cross section, or by assembling the contributions of both concrete $\mathbf{H}_c = \mathbf{H}_c(x, t)$ and steel $\mathbf{H}_s = \mathbf{H}_s(x, t)$

$$\mathbf{H}(x, t) = \mathbf{H}_c(x, t) + \mathbf{H}_s(x, t) \quad (12)$$

$$\mathbf{H}_c(x, t) = \int_{A_c(x)} E_c(x, y, z, t) \mathbf{b}(y, z)^T \mathbf{b}(y, z) [1 - \delta_c(x, y, z, t)] dA \quad (13)$$

$$\mathbf{H}_s(x, t) = \sum_m E_{sm}(x, t) \mathbf{b}_m^T \mathbf{b}_m [1 - \delta_{sm}(x, t)] A_{sm} \quad (14)$$

where the symbol “ m ” refers to the m th reinforcement bar located at (y_m, z_m) ; $E_c = E_c(x, y, z, t)$ and $E_{sm} = E_{sm}(x, t)$ are the generalized moduli of the materials, and $\mathbf{b}(y, z) = [1 \ -y \ z]^T$. It is worth noting that the vectors \mathbf{r} and \mathbf{e} have to be considered as total or incremental quantities depending on the nature of the stiffness matrix \mathbf{H} , which depends on the type of formulation adopted (i.e., secant or tangent) for the generalized moduli of the materials.

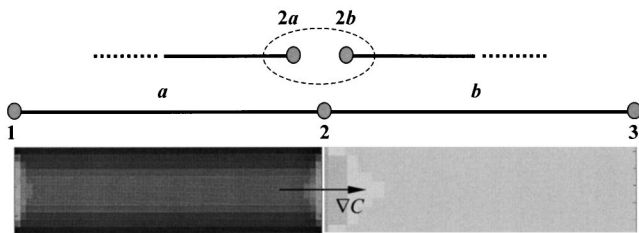


Fig. 8. Continuity of mass transfer at interface between adjacent elements

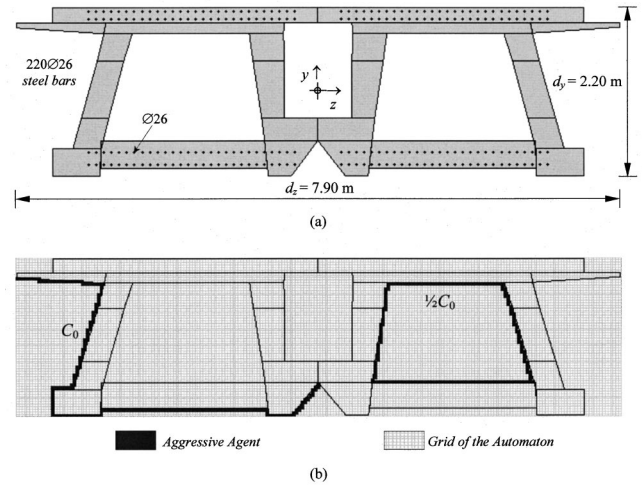


Fig. 9. Box girder cross section undergoing diffusion: (a) geometry and location of reinforcement; (b) grid of cellular automaton and location of aggressive agent

Formulation of a Deteriorating Reinforced Concrete Beam Element

The previous cross-sectional formulation is now extended in order to define the characteristics of a reinforced concrete beam finite element for durability analysis of framed structures (Bon-tempi et al. 1995; Malerba 1998; Biondini 2000).

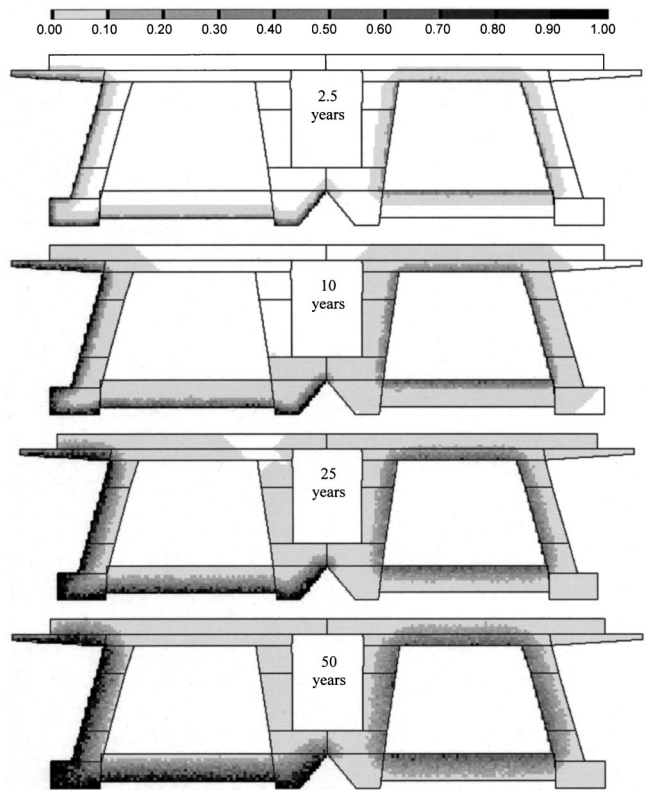


Fig. 10. Box girder cross section undergoing diffusion: distribution of the concentration $C(\mathbf{x}, t)/C_0$ of aggressive agent after 2.5, 10, 25, and 50 years from initial time of diffusion penetration

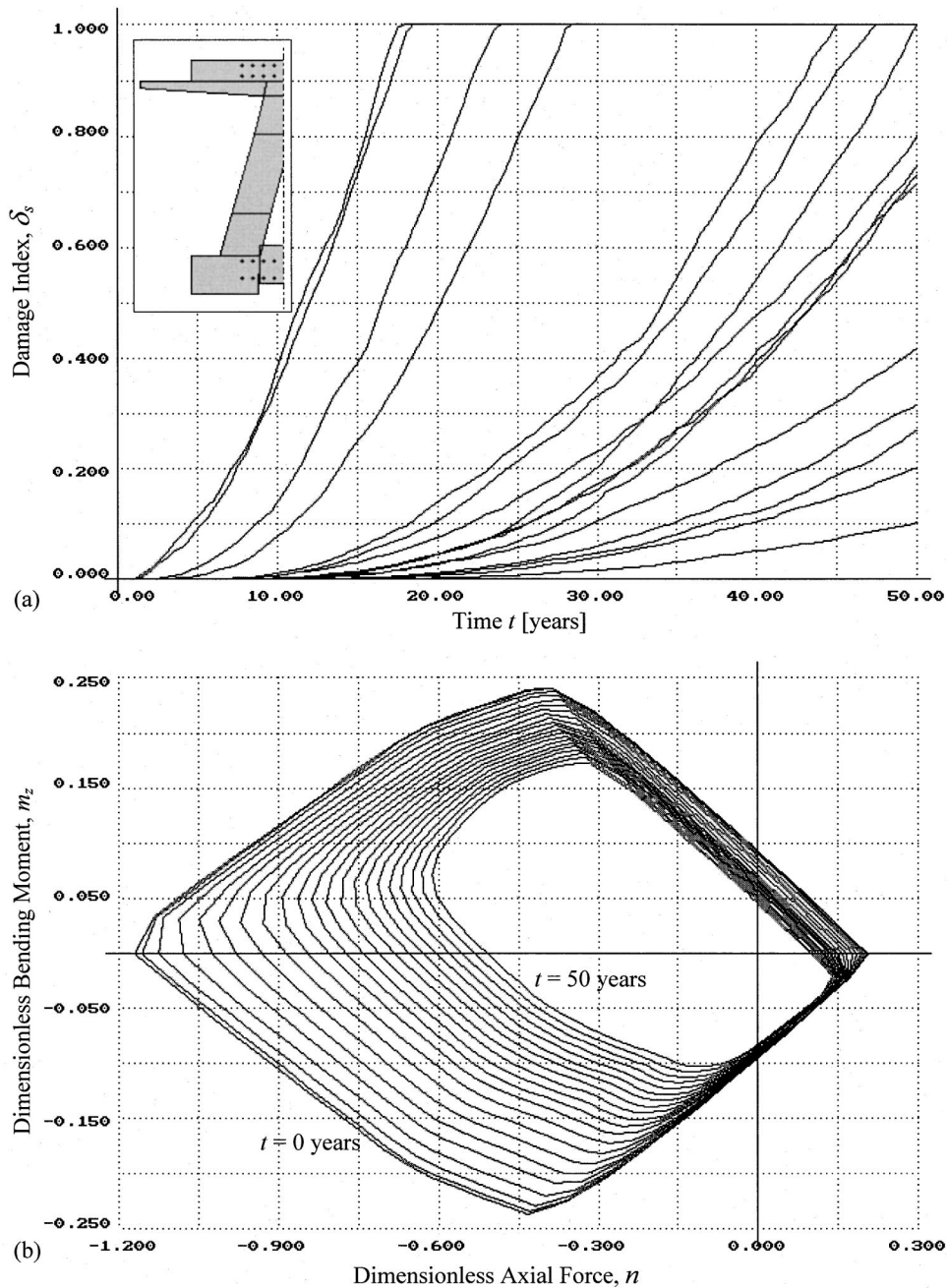


Fig. 11. Structural response during the first 50 years of lifetime: (a) time evolution of damage index δ_s for 16 outermost reinforcement bars on left-hand side; (b) axial force $n=N/(f_c A_{c0})$ versus bending moment $m_z=M_z/(f_c A_{c0} d_y)$ resistance domains with $M_y=0$ ($\Delta t=2.5$ years)

With reference to the finite beam element shown in Fig. 6 and having the cross section shown in Fig. 5, the vector of the displacements $\mathbf{u}=\mathbf{u}(x,t)=[\mathbf{u}_a|\mathbf{u}_b]^T=[u_0|v_0|w_0]^T$ and the vector of the generalized strains $\mathbf{e}=\mathbf{e}(x,t)=[\varepsilon_0|\chi_z|\chi_y]^T$ can be related to the vector of the nodal displacements $\mathbf{s}'=\mathbf{s}'(t)=[\mathbf{s}_a|\mathbf{s}_b]^T=[u_1|u_2|v_1|\varphi_{z1}|w_1|\varphi_{y1}|v_2|\varphi_{z2}|w_2|\varphi_{y2}]^T$ as follows:

$$\mathbf{u}(x,t) = \mathbf{N}(x)\mathbf{s}'(t) \quad (15)$$

$$\mathbf{e}(x,t) = \mathbf{B}(x)\mathbf{s}'(t) \quad (16)$$

where

$$\mathbf{N}(x) = \begin{bmatrix} \mathbf{N}_a(x) & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_b(x) \end{bmatrix} \quad (17)$$

$$\mathbf{B}(x) = \begin{bmatrix} \frac{\partial \mathbf{N}_a(x)}{\partial x} & \mathbf{0} \\ \mathbf{0} & \frac{\partial^2 \mathbf{N}_b(x)}{\partial x^2} \end{bmatrix}$$

In the following, the axial $N_a=N_a(x)$ and bending $N_b=N_b(x)$ displacement functions of a linear elastic beam element having uniform cross-sectional stiffness and loaded only at its ends are adopted (Przemieniecki 1968). Based on this assumption, the element stiffness matrix $\mathbf{K}'=\mathbf{K}'(t)$ and the nodal forces vector $\mathbf{f}'=\mathbf{f}'(t)$, equivalent to the applied loads $\mathbf{f}_0=\mathbf{f}_0(x,t)$, are derived by

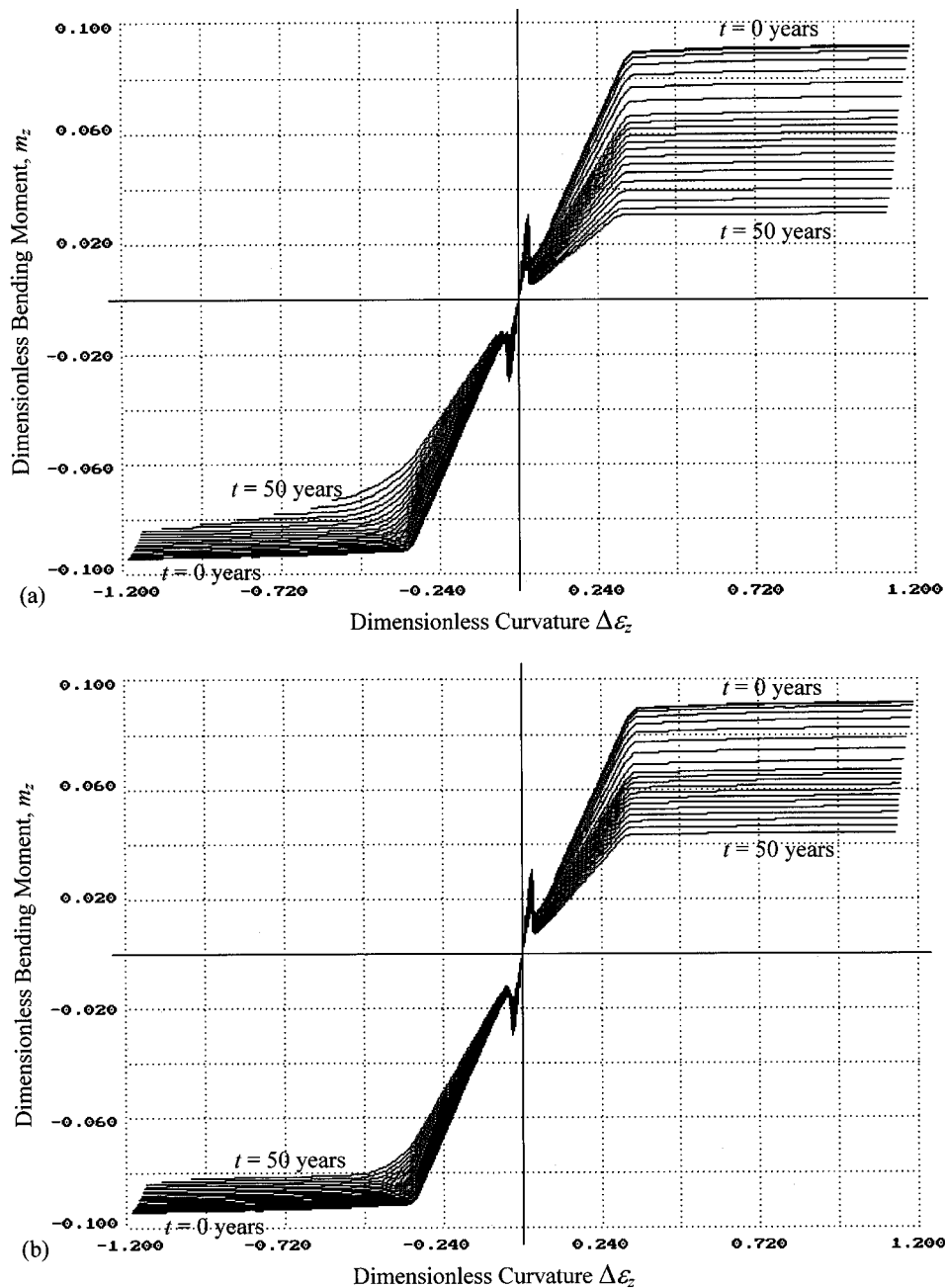


Fig. 12. Structural response during the first 50 years of lifetime ($\Delta t = 2.5$ years): bending moment $m_z = M_z / (|f_c| A_c d_y)$ versus curvature $\Delta \varepsilon_z = \chi_z d_y$ diagrams for $N = M_y = 0$ with (a) $\psi_c^* = 1$ and (b) $\psi_c^* = 0$

applying the principle of the virtual displacements and then evaluated at each time instant t by integration over the length l of the beam

$$\mathbf{K}'(t) = \int_0^l \mathbf{B}(x)^T \mathbf{H}(x, t) \mathbf{B}(x) dx \quad (18)$$

$$\mathbf{f}'(t) = \int_0^l \mathbf{N}(x)^T \mathbf{f}_0(x, t) dx \quad (19)$$

Finally, by assembling the stiffness matrix $\mathbf{K} = \mathbf{K}(t)$ and the vectors of the nodal forces $\mathbf{f} = \mathbf{f}(t)$ in a global reference system, the equilibrium of the whole structure can be expressed as follows:

$$\mathbf{K}(t) \mathbf{s}(t) = \mathbf{f}(t) \quad (20)$$

where $\mathbf{s} = \mathbf{s}(t)$ = global vector of the nodal displacements. It is worth noting that, as it has been already pointed out at the cross sectional level, the vectors \mathbf{f} and \mathbf{s} have to be considered as total or incremental quantities depending on the nature of the global stiffness matrix \mathbf{K} , or if a secant or a tangent formulation is adopted for the sectional stiffness matrix \mathbf{H} . Finally, it is outlined that the diffusive parameters defining the distribution function $f_\Psi(\psi)$ in cracked concrete (i.e. ψ_a , ψ_b , and ψ_c in Fig. 3) must be properly set according to the proposed finite element formulation. This formulation assumes smeared cracks along the beams and computes the equilibrium conditions in terms of average stresses over the cracks spacing.

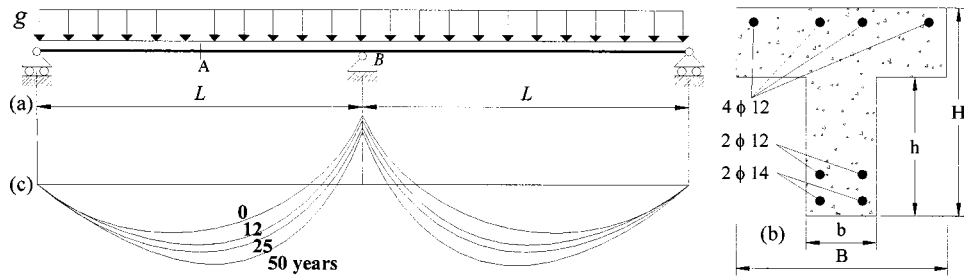


Fig. 13. Reinforced concrete continuous beam: (a) structural model and load condition; (b) geometry of cross section and location of reinforcement; and (c) time evolution of bending moment diagram

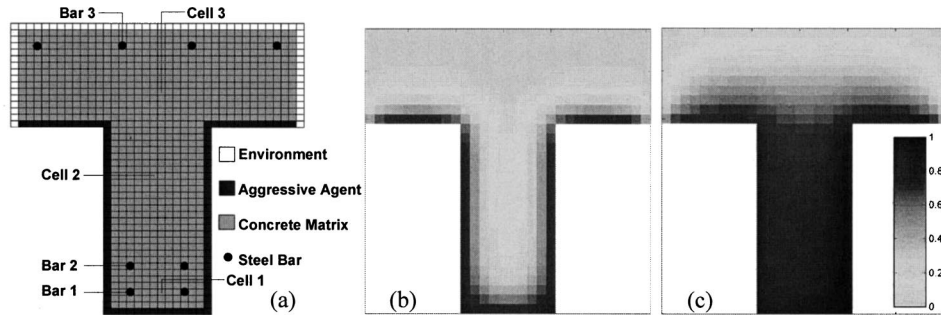


Fig. 14. (a) Grid of cellular automaton and location of aggressive agent along beam; maps of concentration $C(x,t)/C_0$ of agent in cross section B after (b) 4 years and (c) 30 years

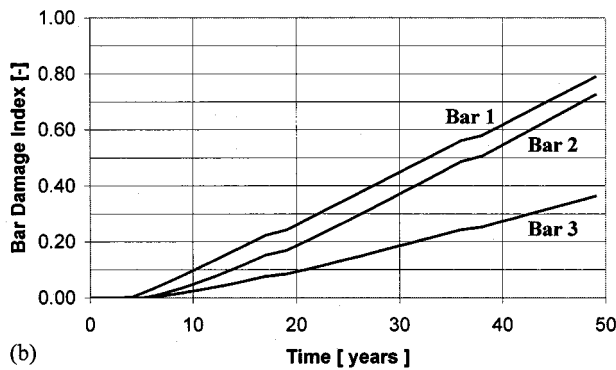
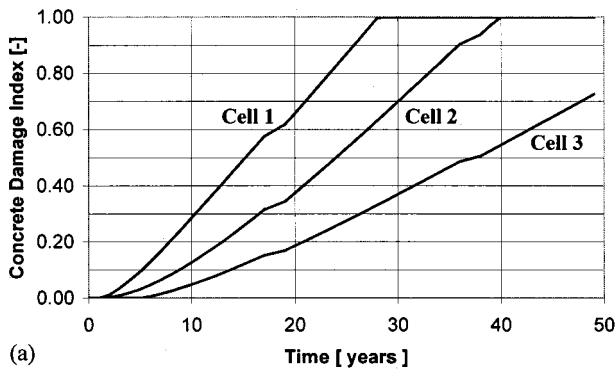


Fig. 15. Time history of damage indices of (a) three concrete cells and (b) three steel bars belonging to cross section B [see Figs. 13(a) and 14(a)]

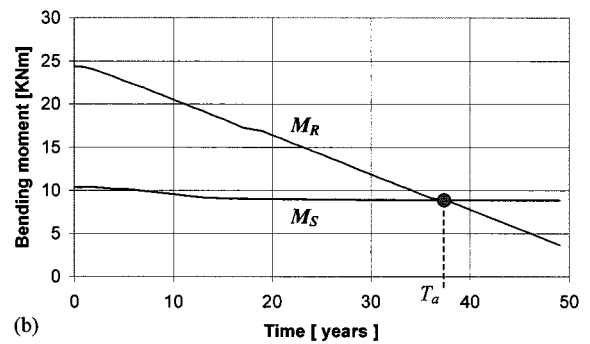
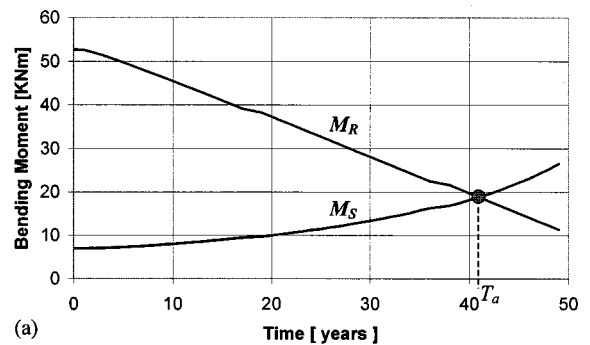


Fig. 16. Comparison between time evolution of both acting M_S and resistant M_R bending moment in (a) cross section A and (b) cross section B [see Fig. 13(a)]

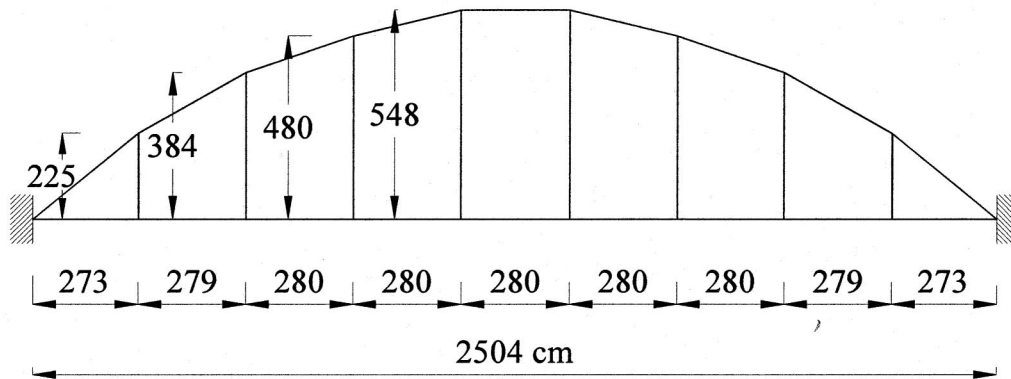
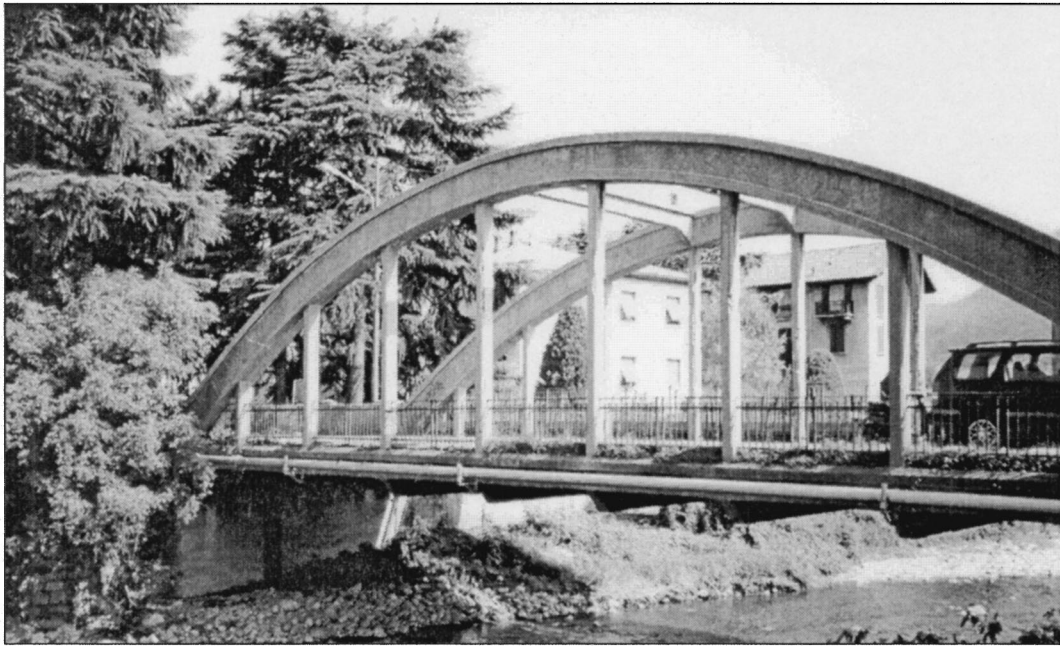


Fig. 17. Arch bridge over Breggia river (Como, Italy): pictorial view and structural model

Numerical Integration and Boundary Conditions

In order to couple the diffusion process to the structural response, each finite element is linked to a local three-dimensional grid oriented according to the beam axis. The assemblage of such grids forms the geometrical support of the cellular automaton which describes the diffusion process. In this regard, two aspects need to be discussed further.

Firstly, the characteristics of the deteriorating reinforced concrete finite beam element must be obtained at each time instant from the integrals previously introduced. To this aim, a numerical integration is clearly required and, without loss of generality and for sake of simplicity, it can be effectively based on the three-dimensional grid of the cellular automaton adopted to describe the diffusion process, as shown for example in Fig. 7 for a beam element having rectangular box cross section. In this way, the integrals Eqs. (12), (18), and (19) can be reduced to the following algebraic forms:

$$\mathbf{H}_c(x, t) = \Delta x^2 \sum_{i \in A_c(x)} E_{ci}(x, t) \mathbf{b}_i^T \mathbf{b}_i [1 - \delta_{ci}(x, t)] \quad (21)$$

$$\mathbf{K}'(t) = \Delta x \sum_{i \in l} \mathbf{B}_i^T \mathbf{H}_i(t) \mathbf{B}_i \quad (22)$$

$$\mathbf{f}'(t) = \Delta x \sum_{i \in l} \mathbf{N}_i^T \mathbf{f}_{0i}(t) \quad (23)$$

where the symbol “ i ” denotes the cell i whose center is located at point $\mathbf{x}_i = (x_i, y_i, z_i)$, $\Delta x = \Delta y = \Delta z =$ grid dimension, and the sums are extended only to the cells belonging to the actual structural volume of the beam element.

The second aspect deals with a consistent application of the evolutionary rule of the cellular automaton to this ensemble of grids. In fact, the grids associated with adjacent beam elements must be interconnected in order to assure the continuity of the diffusion flow at their boundaries, as indicated in Fig. 8. To this aim, the hypothesis of absorbing boundaries previously introduced is neglected in these regions and the neighborhoods of the cells located at the common ends of linked elements are formed by including the cells belonging to both the corresponding adjacent grids within the radius specified by the evolutionary rule.

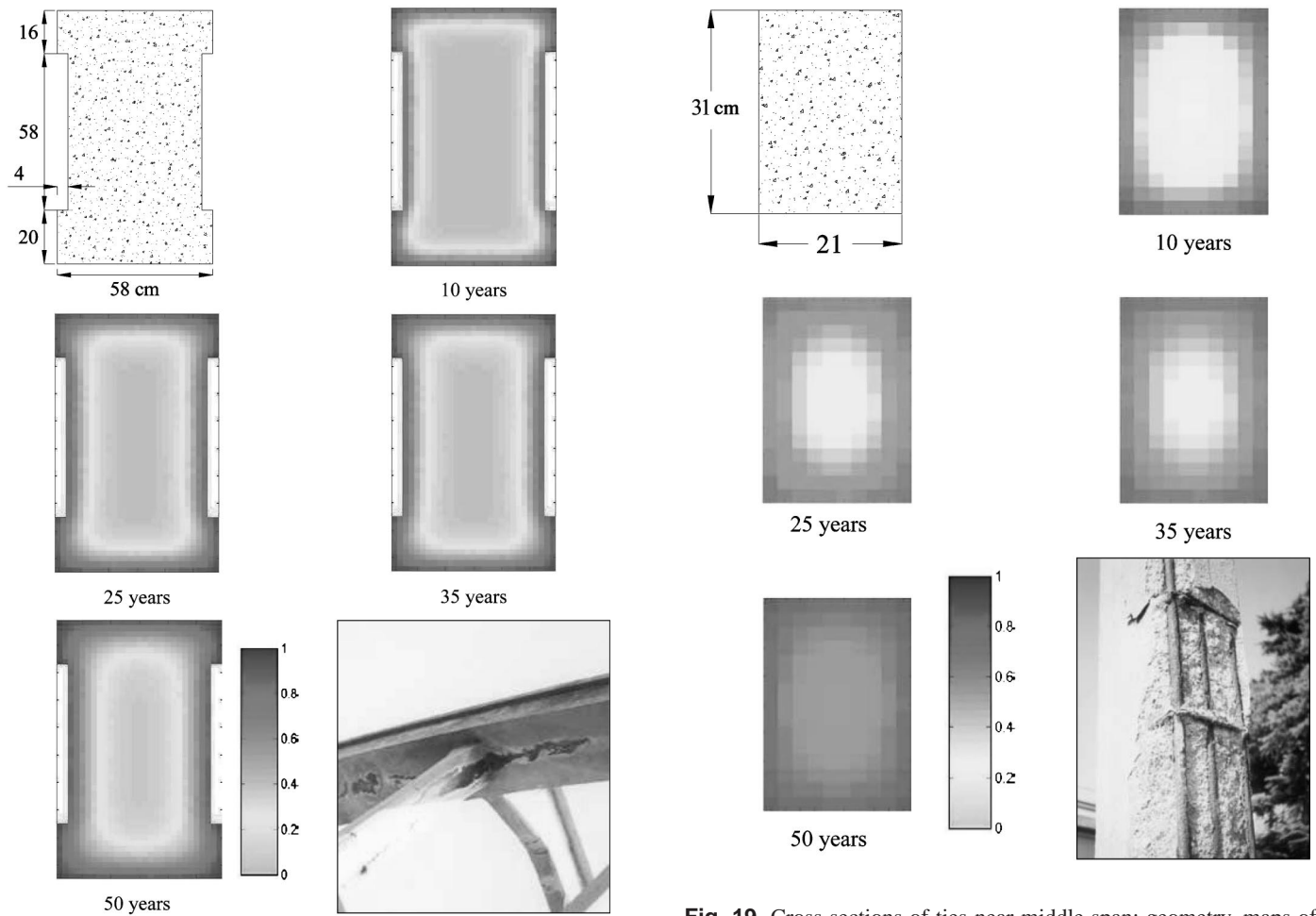


Fig. 18. Cross sections of arches at middle span: geometry, maps of concentration $C(\mathbf{x}, t)/C_0$ of agent during time, and comparison with actual damaged state

Fig. 19. Cross sections of ties near middle span: geometry, maps of concentration $C(\mathbf{x}, t)/C_0$ of agent during time, and comparison with actual damaged state

Applications

In the following, the effectiveness of the proposed methodology is demonstrated through several applications. A box girder cross section, a continuous T-beam, and an existing arch bridge are all analyzed in order to predict their structural lifetime performance under the attack of external aggressive agents. Since the applicability of the procedure does not depend on the type of chemicals which diffuse inside the structure, the basic diffusion and damage parameters are only specified and no details are given on the nature of the aggressive agents.

Durability Analysis of a Reinforced Concrete Box Girder Cross Section

The durability analysis of a reinforced concrete box girder cross section is performed. The cross section has the geometry shown in Fig. 9(a), with main dimensions $d_y=2.20$ m and $d_z=7.90$ m, and is reinforced with 220 bars having diameter $\varnothing=26$ mm. For concrete, the stress-strain diagram is described by the Saenz's law in compression and by an elastic perfectly plastic model in tension, with the following parameters: compression strength $f_c=-40$ MPa; tension strength $f_{ct}=0.25|f_c|^{2/3}$; initial modulus $E_{c0}=0.25|f_c|^{1/3}$; peak strain in compression $\varepsilon_{c0}=-0.20\%$; strain limit

in compression $\varepsilon_{cu}=-0.35\%$; strain limit in tension $\varepsilon_{ctu}=2f_{ct}/E_{c0}$. For steel, the stress-strain diagram is described by an elastic-perfectly plastic model in both tension and compression, with the following parameters: yielding strength $f_{sy}=500$ MPa; elastic modulus $E_s=206$ GPa; strain limit $\varepsilon_{su}=1.00\%$. With reference to a diffusivity coefficient $D=10^{-11}$ m²/s, the cellular automaton is defined by a grid dimension $\Delta x=35.5$ mm and a time step $\Delta t=0.5$ years. Fig. 9(b) shows the location of the aggressive agent, with concentration $C(t)=C_0$ along the bottom side on the left and $C(t)=\frac{1}{2}C_0$ within the cell on the right. Damage rates are defined by the values $C_{cr}=0$, $C_c=C_s=C_0$, $\Delta t_c=10$ years and $\Delta t_s=15$ years. Stochastic effects are accounted for by assuming $\psi_b=-\psi_a=1$, with $\psi_c=0$ and $\psi_c^*=1$ for uncracked and cracked concrete, respectively.

The time evolution of the diffusion process is described by the maps of Fig. 10, which show the distribution of concentration $C(\mathbf{x}, t)/C_0$ of the aggressive agent after 2.5, 10, 25, and 50 years from the initial time of diffusion penetration. These maps highlight how cellular automata are able to provide to the whole system some emergent properties (the diffusive patterns) self-induced only by its local dynamics (the evolutionary rules). The mechanical damage induced by diffusion can be appreciated from the diagrams of Fig. 11, which show the time evolution of both the damage index δ_s for the 16 outermost reinforcement bars on the left side and the dimensionless resistance domains of the axial force N versus bending moment M_z with $M_y=0$. Finally, Fig.

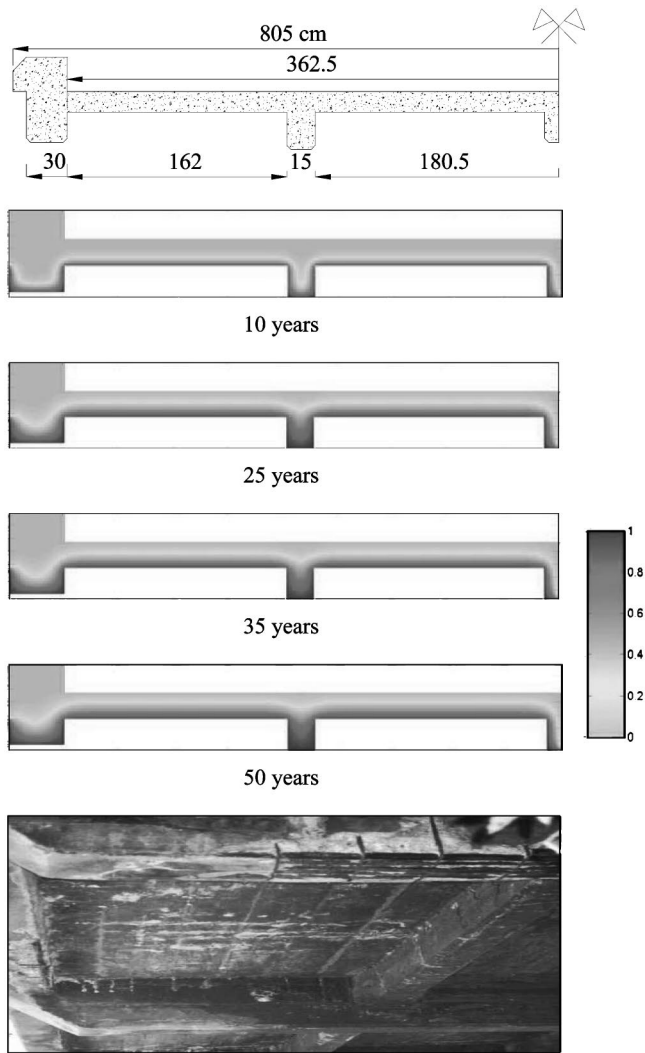


Fig. 20. Cross section of deck at middle span: geometry, maps of concentration $C(\mathbf{x},t)/C_0$ of agent during time, and comparison with actual damaged state

12(a) shows the corresponding time evolution of the structural response in terms of dimensionless diagrams of the bending moment M_z versus curvature χ_z with $N=M_y=0$. In addition, Fig. 12(b) shows the same curves computed by neglecting the directionality effects in the stochastic model, or by adopting a symmetric density function $f_\psi(\psi)$ for both cracked and uncracked concrete ($\psi_c=\psi_c^*=0$). In this way, the direct comparison of the diagrams in Figs. 12(a) and (b) allows the quantification of the negative role played by cracking.

Lifetime Assessment of a Continuous Reinforced Concrete T-Beam

The continuous reinforced concrete T-beam shown in Fig. 13(a), with span lengths $L=3.00$ m and load $g=10$ kN/m, is now studied. The cross section has the geometry shown in Fig. 13(b), with main dimensions $H=0.40$ m, $h=0.25$ m, $B=0.40$ m, and $b=0.15$ m. The behavior of concrete is linear elastic with modulus $E_c=35$ GPa in compression and with no strength in tension. For steel, a linear elastic behavior with modulus $E_s=206$ GPa is assumed. With reference to a diffusivity coefficient $D=1.37 \times 10^{-11}$ m²/s, the cellular automaton is defined by a grid dimen-

sion $\Delta x=10$ mm and a time step $\Delta t=0.02$ years. Fig. 14(a) shows the grid of the automaton and the location of the aggressive agent with constant concentration $C(t)=C_0$. The damage rates are defined by the values $C_{cr}=0.30C_0$, $C_c=C_s=C_0$, $\Delta t_c=30$ years and $\Delta t_s=40$ years. The stochastic effects are accounted for by assuming $\psi_b=-\psi_a=1$, with $\psi_c=0$ and $\psi_c^*=1$ for uncracked and cracked concrete, respectively.

Figs. 14(b) and (c) show the maps of concentration $C(\mathbf{x},t)/C_0$ of the agent in the cross section B of Fig. 13(a) after 4 and 30 years, respectively. Moreover, Figs. 15(a) and (b) show the corresponding evolution of the damage indices δ_c and δ_s in the same cross section for three concrete cells and three steel bars, respectively. Since the coupling effects between the diffusion process and the cracking state lead to a non uniform distribution of damage along the beam, the diagram of the bending moment evolves during time as shown in Fig. 13(c). In particular, Figs. 16(a) and (b) make a comparison between the evolution of the acting bending moments M_S and the corresponding resistant bending moment M_R , computed by accounting for the actual nonlinear material constitutive laws, in cross sections A and B, respectively. Such diagrams highlight the effectiveness of the proposed methodology, which allows the assessment of the service life of the structure and the necessity of a possible rehabilitation.

Identification of Critical Members of an Existing Arch Bridge

The arch bridge over the Breggia River in Como (Italy) shown in Fig. 17 is finally considered. The geometry of the cross sections of the two arches, ties, and deck is shown at the top left-hand side of Figs. 18–20, respectively. In particular, the height of the arch cross section varies from 84 cm at the ends to 76 cm in the middle. The structure is modeled with homogeneous beam elements having linear elastic behavior with elastic modulus $E_c=35$ GPa. A uniform load $g=50$ kN/m acting on the deck is considered. With reference to a diffusivity constant $D=1.37 \times 10^{-11}$ m²/s, the cellular automaton is defined by a grid dimension $\Delta x=20$ mm and a time step $\Delta t=0.08$ years. The aggressive agent is located around the whole external surface of both the arches and the ties, as well as under the deck, with constant concentration $C(t)=C_0$. The damage rates are defined by the values $C_{cr}=0.30C_0$, $C_c=C_0$ and $\Delta t_c=20$ years. The stochastic effects are accounted for by assuming $\psi_b=-\psi_a=1$ and $\psi_c=0$.

The time evolution of damage can be obtained from the inspection of Figs. 18–20 which show the maps of the concentration $C(\mathbf{x},t)/C_0$ of the agent in the cross sections at the middle span of the bridge. These figures make also a comparison between the concentration maps and the actual damaged state of the structure, which is characterized by a relevant deterioration with spalling of the concrete cover and corrosion of the reinforcement bars. Such a comparison leads to confirm the effectiveness of the proposed methodology, which, through a reliable assessment of the damage distribution and the identification of the most critical members (the ties), allows to address the lifetime performance and maintenance planning processes.

Conclusions

1. Durability analysis of concrete structures under attack of aggressive agents is generally investigated through the study of the local deterioration of the materials, dealing for example with carbonation of concrete and corrosion of reinforcement,

and limited attention is devoted to the global effects of these local phenomena on the overall performance of the structure. The main reason for this approach is that analytical solutions of the diffusion problem exist only in a limited number of classical cases, characterized by simple and well defined boundary conditions. However, these solutions are able to accurately reproduce the diffusion process only in very limited parts of the structure such as the region around the concrete cover. General numerical approaches (e.g., finite differences or finite elements) provide better and more general solutions, but in these cases complexities arise from the necessity of an additional model to simulate the structural response of the damaged structure. In fact, due to the particular nature of the problem, these two models are characterized by different discretizations in space and time. Thus, it appears very costly to couple these models in order to account for the interactive effects of diffusion, damage, and structural behavior. In addition, finite difference and finite element models are based on the assumption that the global phenomenon of diffusion is governed by given mathematical relationships. This assumption introduces an implicit and additional source of approximation.

2. The approach presented in this paper aims to overcome the previous drawbacks. The principal novelty of this approach is the use of a special class of evolutionary algorithms, called cellular automata, to investigate the deterioration processes induced by diffusion. The main advantages of these algorithms in the context of durability analysis and lifetime assessment of deteriorating structures are as follows: (1) They provide a general approach to modeling the diffusion processes and lead to very accurate results when compared with the solution of the governing differential equations. (2) Complex geometrical and mechanical boundary conditions, that usually hold even for simple structural problems, can be easily handled. (3) The grid of the cellular automaton which simulates the diffusion process can be efficiently associated with the finite element model of the structure in order to describe the mechanical material damage and to perform the numerical integration required for the time-variant non linear structural analyses. (4) The link between diffusion and mechanical models leads to a cellular automata formulation of deteriorating reinforced concrete finite elements able to account for both the stochastic diffusion process of the aggressive agent inside the structure and the corresponding quantification of material damage, as well as the interactive effects between diffusion, damage, and structural behavior. (5) Models based on cellular automata must be considered as an alternative to the differential equations of diffusion. In fact, they show a complex behavior analogous to that associated with complex differential equations. However, in this case complexity emerges from the interaction of simple entities following simple rules. For this reason, the knowledge of the global dynamics of the system is not required and a more general and comprehensive modeling of diffusion may be attempted in order to account for the uncertainties involved in the assessment process.
3. The effectiveness of the proposed methodology has been demonstrated through several applications. A box girder cross section, a continuous T-beam, and an existing arch bridge have been all analyzed by this methodology in order to predict their structural lifetime performance. The accuracy of the results clearly depends on the values of the material parameters which define both the diffusive and damage pro-

cesses, like the diffusivity coefficient D , the critical threshold of concentration C_{cr} , the damage rates q_c and q_s , and the stochastic factors ψ_a , ψ_b , and ψ_c . Of course, further research aimed to develop both experimental tests on structural prototypes and monitoring activities on existing structures is required in order to achieve a proper calibration of such parameters. However, despite the necessity of this calibration, the results of the considered applications prove that the proposed approach represents a powerful engineering tool in both lifetime design and assessment of reinforced concrete structures in aggressive environments.

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