Effects of the Dispersive Behavior of Dielectric Substrates on the SPI

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Abstract

Actual printed circuit board configurations require either full-wave simulations or accurate analytical techniques for the analysis and the design of interconnects. This paper focuses on the effects of the dispersive behavior of dielectric substrates and on their simulation by means of new expressions giving a guidance on effective permittivity and per unit length (p.u.l.) parameters at the design stage.

1. Introduction

The dispersive behavior of dielectric substrates used in printed circuit boards is generally identified as one of the most serious factors limiting the speed rate of signal transmission over actual interconnects. Its accurate modeling, without resorting to time-consuming full-wave simulations, as well as the prediction of consequences on the signal propagation over interconnects due to the frequency-dependent properties, is the main goal of this paper. In the past, several quite approximate expressions have been proposed for the so-called effective relative dielectric constant [1-2] ([3,9] for an overview and comparisons), but they are often limited to lossless or zero-thickness configurations and numerical analyses are necessary, when more general situations occur. However, full-wave numerical methods, although extremely accurate, do not give insights into the influence of the various geometrical and physical parameters on the signal propagation and are highly demanding in terms of computer resources. In fact, generally a considerable number of frequency-domain simulations, although sophisticate techniques [4] exist for the reduction of the computer cost, or the expensive direct time domain approach [5], are requested for the accurate simulation of the frequency-dependent interconnect characteristics.

In order to overcome these drawbacks, approximate expressions for the effective permittivity, as well as for the p.u.l. parameters, are useful to account for both the frequency-dependence characteristics of the dielectric substrate and the mode coupling, which is function of the configuration, and allow design choices in a much more efficient way. To this end, very useful are the formulations [e.g. 6] allowing the direct evaluation of the scattering parameters functions of the p.u.l. parameters.

2. Motivations for Improvements of Analytical Formulations for the Effective permittivity

The so-called effective relative dielectric, is defined as

$$\varepsilon_{eff} = \left(\frac{\beta}{k_0}\right)^2 = \left(\frac{\lambda}{\lambda_0}\right)^2 \tag{1}$$

where λ_0 and λ are the wavelengths in air and in the interconnect under analysis, respectively, β represents the propagation constant and k_0 is the wavenumber in air. The frequency-dependence of the effective dielectric constant is observed even in absence of losses, because of the coupling between modes supported by microstrip lines in non-homogeneous media [7].

In the following the attention will be focussed on the so-called odd-mode (or differential-mode) ensuing from a source excitation applied between two traces, as often occurs in high-speed interconnect configurations, instead of between one trace and the ground plane (so-called even-mode).

Losses are generally accounted for by adopting a Debye model, whose parameters may be derived from measured data available in the data-sheets of commercial substrates (e.g. in [8]), as

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau} \tag{2}$$

where ω is the angular frequency, ε_s and ε_∞ are, respectively, the zero-frequency relative permittivity and the relative permittivity at infinite frequency, and τ is the pole relaxation time. In order to demonstrate the need of further investigations, the configuration of two coupled coplanar strips is considered.

0-7803-9821-1/02/$10.00 2002 IEEE

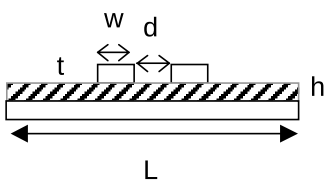


Figure 1 – Coupled microstrip configuration.

In Figure 2 the frequency spectrum of the effective relative permittivity is reported, adopting a full-wave approach based on the Method of Moments (MoM) [9]. The dielectric material is simulated with or without losses, and w = 0.1 mm, d = 0.5 mm, h = 1 mm, L >> h, $t \to 0$ and, in the former case, $\varepsilon_s = 4.5$, $\varepsilon_\infty = 4.19$, $\tau = 0.949$ ns. Inclusion of losses usually implies a variation in the order of 5-10% with respect to the lossless case, much greater than the limit of 1% of accuracy generally considered acceptable for the suitability of analytical expressions. Also strip thickness is recognized as affecting the effective permittivity in the range between 5 and 10 % for usual interconnects dimensions.

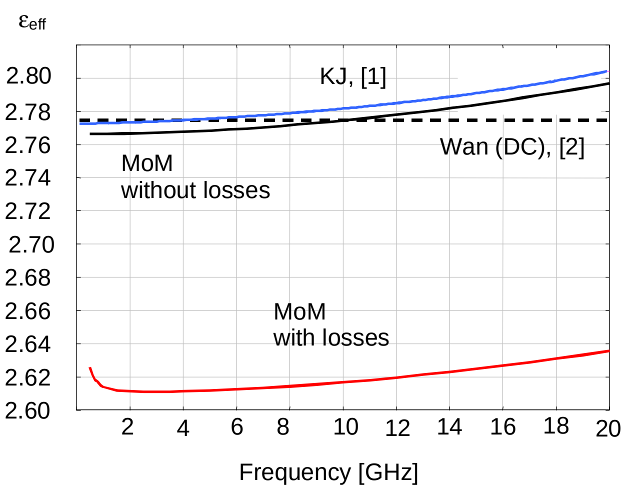


Figure 2 - Frequency-dependence of the effective permittivity evaluated either by means of various analytical expressions or a full - wave MoM - based code.

2.1 Analytical expressions available for the static effective dielectric permittivity

As pointed out in [3, 10], the effective permittivity of a single microstrip line has been investigated by several researchers; the most accurate analytical expressions are those proposed by Kirschning and Jansen [1], valid respectively for single and coupled microstrip lines in the lossless and zero-thickness hypotheses.

However, it is worthy noting that the expressions proposed by Kirschning and Jansen for the zero-frequency case ensue from the numerical approximation of the conformal mapping transformations proposed by Wan [2], as

$$\varepsilon_{re} = 1 + \frac{D_5 + D_6}{K(D_3)}(\varepsilon_r - 1) \tag{3}$$

with

$$D_1 = K(a_3) - F\left(\frac{a_1}{a_3}, a_3\right) \tag{4a}$$

$$D_2 = K(a_3) - F\left(\frac{a_2}{a_3}, a_3\right) \tag{4b}$$

$$D_3 = sn(D_1, a_3) \tag{4c}$$

$$D_4 = sn(D_2, a_3) \tag{4d}$$

$$D_5 = F\left(\frac{D_4}{D_3}, D_3\right) \tag{4e}$$

$$D_6 = F(a_3, D_3) \tag{4f}$$

$$D_7 = \frac{(4-\pi)(4+\pi\varepsilon_r)}{4(4-\pi+2\pi\varepsilon_r)}(D_6 - D_5) \tag{4g}$$

where $K(k)$, $F(z,k)$, $E(k)$ and $E(z,k)$ are respectively complete and incomplete elliptic integral of the first and second kind, where z is the sine of the amplitude and k is the modulus. $Sn(z,k)$, named Jacobian elliptic function *sn*, is the sine of the Jacobi amplitude function *am*, which is defined as the inverse of the trigonometric form of the elliptic integral of the first kind $F(z,k)$, as follows

$$F(z,k) = \int_0^{\arcsin(z)} \frac{1}{\sqrt{1-k^2\sin^2\theta}}\, d\theta \tag{5}$$

$$am(F(z,k),k) = \arcsin(z) \tag{6}$$

$$sn(y,k) = sin(am(y,k)) \quad . \tag{7}$$

The three remaining parameter a_1, a_2 and a_3 can be computed by solving numerically the following system of nonlinear equations:

$$a_2 \cdot a_4 = \sqrt{1 - \frac{E(a_4)}{K(a_4)}} \tag{8a}$$

$$\frac{\pi s}{4h} = K(a_4)E(a_1,a_4) - E(a_4)F(a_1,a_4) \tag{8b}$$

$$\frac{\pi}{2h}\left(\frac{s}{2}+w\right)=K(a_4)E(a_2,a_4)-E(a_4)F(a_2,a_4) \tag{8c}$$

$$\frac{\pi s}{4h}=K(a_4)E(a_3,a_4)-E(a_4)F(a_3,a_4)\ . \tag{8d}$$

It is easy to note that it is possible to solve at first (8a) and (8b) in the unknowns a_1 and a_4, and successively (8c) and (8d) separately to obtain a_2 and a_3. Furthermore it should be noted that equations (8b) and (8d) are of the same form; different real roots have to be chosen in order to obtain a set of roots which satisfies the unequalities $0<a_1<a_2<a_3<1<1/a_4$.

2.2 *Influence of dielectric losses and frequency-dependence*

Various formulae have been proposed to correct the values of the static effective permittivity in order to account for the dispersion, but to-date, no formulation is available for the simultaneous effects of mode conversion and dielectric losses. The first phenomenon may be accounted for by means of the correction proposed by Kirschning and Jansen or that by Kobayashi. The difference between the two formulations relies on the correction introduced at each frequency, f;

Kirschning and Jansen proposed:

$$\varepsilon_{eff}(f)=\varepsilon_r-\frac{\varepsilon_r-\varepsilon_{eff}(0)}{1+P(f)} \tag{9}$$

$$P(f)=P_1P_2\left[(0.1844+P_3P_4)\cdot f_n\cdot P_{15}\right]^{1.5763} \tag{10}$$

where f_n is a normalized frequency, obtained considering the product of f (in GHz) by the height of traces (in mm), and the expressions for the coefficients P_k may be found in [1] and are not reported here for the sake of conciseness.

While Kobayashi's expression [3] reads:

$$\varepsilon_{eff}(f)=\varepsilon_r-\frac{\varepsilon_r-\varepsilon_{eff}(0)}{1+\left(\frac{f}{f_{50}}\right)^m}\ . \tag{11}$$

However, the analytical expressions for m and f_{50} may be found in [3] are valid for a single microstrip and have to be modified for coupled strips. The new expressions are:

$$f_{50}=\frac{f_{TE1}}{0.75+\left(0.75-\frac{1}{3\varepsilon_r^{\sqrt{3}}}\right)\cdot u} \tag{12}$$

where $m=m_0m_c$ as in [3], $u=w/h$, but now

$$m_0=1+\frac{1}{1+\sqrt{u}}+\frac{1}{3}\left(\frac{1}{1+\sqrt{u}}\right)^3+\frac{1}{5}\left(\frac{1}{1+\sqrt{u}}\right)^5\ . \tag{13}$$

It should be noted that original expressions give raise to large differences with respect to numerical results. Some improvements in the expression of m_c are still required.

The effect of losses is well incorporated by entering, for example into the first formulation, with the values obtained by a Debye formulation, as per (2). In Figure 3 the same configuration considered previously is analyzed and a comparison of the results obtained by means of MoM, (9) or (11) are reported.

Even though other expressions [e.g. 10 - 11], revealed to be accurate enough, they yields to large errors in some configurations and seem unsuitable for reliable predictions.

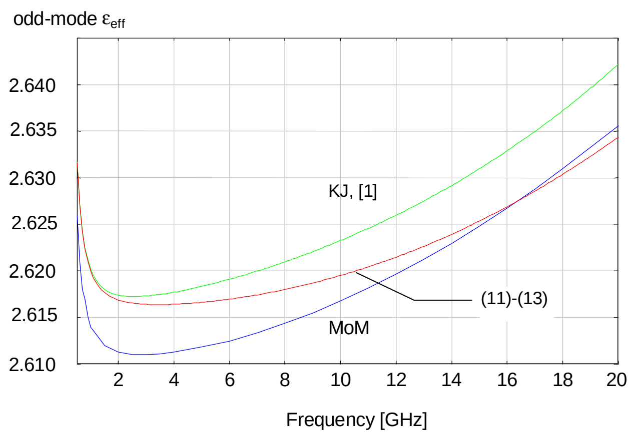


Figure 3 - Frequency-dependence of the real part of the effective permittivity including dielectric losses.

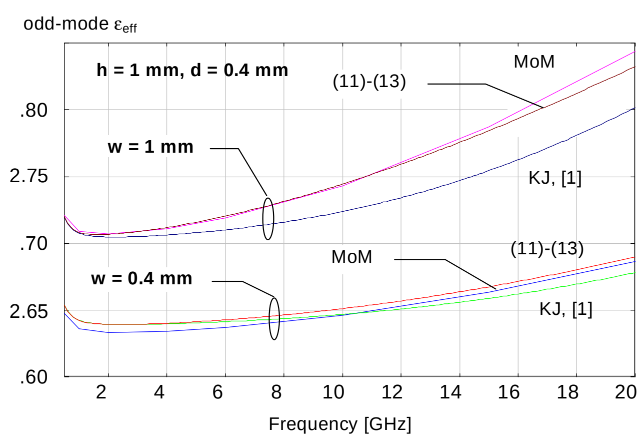
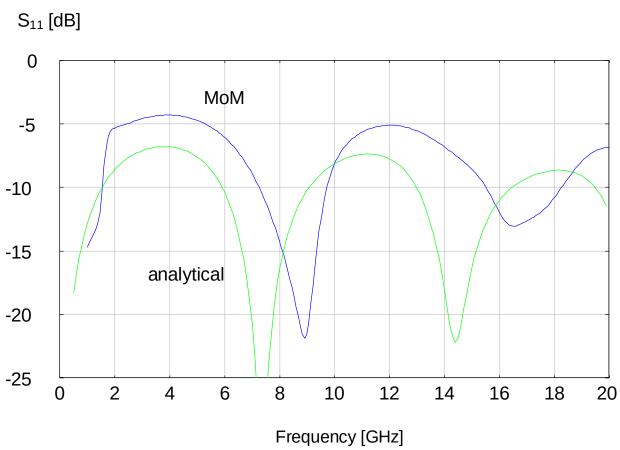


Figure 4 - Comparisons of effective permittivity values, including dielectric losses, in different configurations.

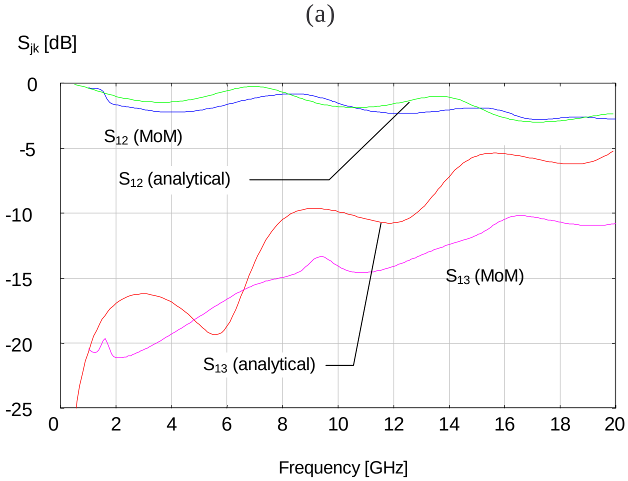
3. Scattering Parameters

Scattering parameters are widely considered for the input-output characterization of any kind of devices and very useful is the possibility of using reliable analytical predictions at a preliminary design stage without resorting to resource-demanding numerical simulations.

The configurations analyzed have the following dimensions: w=0.4 mm, d=0.4 mm, h=1 mm, length=1 cm, permittivity as in Sections 2-3, and, for the resistance matrix data from [12] have been used. As reference, a 50- ohm impedance been considered. Standard expressions in [13] have been applied for the evaluation of the p.u.l. parameters from the even- and odd-mode quantities. In Figures 5 the results obtained by means of previous expressions have been compared with MoM-based numerical data. A reduction in the computer runtime in the order of 10^3-10^4 has been observed. The accuracy of the analytical predictions, although satisfactory for preliminary evaluations, puts in evidence the need of further improvements.



(a)



(b)

Figure 4 - Comparisons between scattering parameters for a typical differential line configuration evaluated analytically or numerically by MoM: S_{11} (a) andS_{12} and S_{13} (b).

4. Conclusions

An analytical formulation and new expressions for the prediction of the effective permittivity and the scattering parameters has been presented. Results can be considered satisfactory for preliminary design evaluations.

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