Propagation characteristics of differential lines: the odd-mode impedance

Rodolfo Araneo
Department of Electrical Engineering
University of Rome “La Sapienza”
Rome, ITALY
rodolfo.araneo@uniroma1.it

Abstract — **New analytical expressions are proposed for the prediction of the frequency dependence of the characteristic odd-mode impedance of differential interconnects. The classical microwave definition is reviewed as well as analytical formulations available in literature. A two dimensional Finite Difference Frequency Domain method is applied to compare the results and check the accuracy of the proposed expressions.**

Keywords – Odd-mode impedance, frequency-dependence, Finite Difference Frequency Domain, transmission line.

I. INTRODUCTION

Nowadays the technological trends in electronics are driven by the need for a constant increase in the performance of actual high-speed high-populated printed circuit boards (PCBs). Consequently, the increase in operational frequencies, together with the growth of configuration complexity, made arise a large number of signal integrity (SI) and electromagnetic interference (EMI) issues, e.g. propagation delay, signal distortion, crosstalk, radiation, which must be faced at the preliminary design stage.

In this work the attention is focused on analytical formulations for the prediction of the frequency-dependence of the characteristic impedance of differential interconnects, the so called odd-mode impedance. Differential configurations are of practical interest [1-4] since they show better SI and EMI performances than the single-trace transmission lines with current return through the ground plane in all those cases where discontinuities are present on the reference plane, e.g slits.

In particular the very microwave definition of the characteristic impedance has been critically reviewed since it is well-known that in the high frequency range, where the quasi transverse (qTEM) assumption is not fulfilled, the characteristic impedance is not unique [5]. Furthermore analytical formulations available in literature have been revised and further improvements have been proposed. All the expressions have been numerically validated by means of comparison with the results of a compact two dimensional Finite Difference Frequency Domain (FDFD) method in a number of significant configurations and over a wide frequency range.

II. DEFINITION OF CHARACTERISTIC IMPEDANCE

The *proper* definition of the characteristic odd-mode impedance Z_{odd} is strictly related to the study of the full-wave electromagnetic propagation along the differential interconnect depicted in Figure 1, which constitutes a particular example of bidirectional electric waveguide.

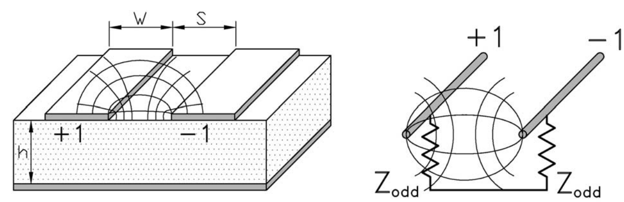

Figure 1. Geometrical sketch of the considered differential interconnect.

It is well-known that the inhomogeneity of the structure with the presence of two distinct dielectric media does not allow the propagation of any TEM mode, except at zero frequency which is a case of no practical interest. Due to the presence of the strips the resulting field can be expressed as a superposition of linked TE and TM modes, named hybrid modes, whose computation is only possible numerically [6,7,15]. At low frequencies the fundamental propagation can be obtained from the lowest order terms of a frequency Taylor expansion of the fields, in order to construct an approximation to the exact solution of the Maxwell equations, the so called quasi-TEM mode [8], which however loses its validity at higher frequencies.

In the past, several candidates have been suggested for a reliable frequency dependent definition, based on two dimensional formulations [10-14] as well as on three dimensional ones [9]. Among them, the definitions on which more attention has been focused, are those in terms of the average propagated power *P* and longitudinal strip current *I* ($Z_{pi} = 2 \cdot P/|I|^2$), the strip center voltage *V* and current *I* ($Z_{vi} = V/I$), power *P* and voltage *V* ($Z_{pv} = 0.5|V|^2/P$). All three definitions of characteristic impedance are identical only if the relation $P = V \cdot I/2$ applies. It has been shown, in the case of a single microstrip line [5], that the trend of the characteristic impedance versus frequency strongly depends on the definition used. Several aspects and arguments [13], especially concerning the definition of voltages [5] which is not unique except in the pure TEM case, have made the power-current formulation prevail through the years in quite all the microwave applications.

Nevertheless, a more thorough investigation of the matter should start from a whole analogy between the physical coupled microstrip electric waveguide, on which several fundamental and higher order field modes propagate, and the model of the coupled transmission lines, with a set of modes expressed in terms of currents and voltages. The analogy should be aimed at the construction of a transmission line representation of the modes which propagates [8], outside the context of the quasi-TEM assumption. Clearly uniqueness of the representation is lost and some latitude appears in the definition of which is the best possible model. In the derivation of a *correct* model, several criteria should be satisfied:

- the TL model should reduce to the quasi-TEM transmission line for the fundamental mode at low frequencies;
- the conservation of dispersion should be enforced imposing the equality between the propagation eigenvalues of the respective eigenmodes γ_i;
- the conservation of reciprocity should be imposed in the TL model;
- the same power transmitted in the waveguide structure should be transmitted in the TL model.

Defining in the multiconductor TL model the modal voltage matrix [**V**] and the modal current matrix [**I**], where the element ij corresponds respectively to the voltage or current on the ith conductor due to the jth mode, it is possible to define mainly a reciprocity-based and a power-based TL model [8].

In the reciprocity-based model, ensured the conservation of reciprocity by enforcing the orthogonality of the modes $[\mathbf{V}]^T[\mathbf{I}] = 2\,[\mathbf{U}]$, where [**U**] is the identity matrix, the characteristic impedance matrix can be computed as

$$[\mathbf{Z}_{\text{char}}] = [\mathbf{V}][\mathbf{I}]^{-1} = 2\left([\mathbf{I}]^{T}\right)^{-1}[\mathbf{I}]^{-1}, \tag{1}$$

where the longitudinal current matrix can be computed by the integration of the magnetic field of each eigenmode along the boundary of each perfect conductive strip, in the case of a TL model for only the fundamental propagation modes. The odd mode impedance can be extracted straightforwardly from the characteristic impedance matrix, simply applying a differential signal, as depicted in Figure 1. In general the conservation of the propagated power is not guaranteed between the model and the original microstrip configuration.

In the power-based model the conservation of power is enforced $[\mathbf{V}]^T[\mathbf{I}]^* = 2\,[\mathbf{P}]$, where [**P**] is the power matrix [8], and the characteristic impedance matrix can be computed as

$$[\mathbf{Z}_{\text{char}}] = [\mathbf{V}][\mathbf{I}]^{-1} = 2\left([\mathbf{I}]^{T*}\right)^{-1}[\mathbf{P}]^{T}[\mathbf{I}]^{-1}. \tag{2}$$

Anyway, the characteristic matrix is not generally symmetric, which means that the derived TL model is not reciprocal in contrast to the original waveguide itself. Thus the odd mode impedance loses its uniqueness since it changes from line to line. Only in the lossless case, considered in the paper, since the complex power can be decomposed into independent contributions of the different eigenmodes, the enforcement of the conservation of power imposes the reciprocity. The current matrix is real and the characteristic matrix is symmetric.

III. MODELING

A. Full-wave modeling

To compute the propagation characteristics of the microstrip differential line and check the agreement with the proposed analytical formulation, a novel compact two dimensional full-wave Finite Difference Frequency Domain method [15] has been used. The method benefits from several favorable features which allow an efficient computation of both the propagation constant β for a given frequency and the transverse field pattern, necessary to compute the characteristic impedance. The geometry of the differential interconnect is depicted in Figure 2: it is assumed that its structure is uniform along the *z* axis and that the traveling wave propagates along the positive *z* direction. The two dimensional transversal section of the interconnect is discretized as shown in Figure 2. The Yee's grid is reduced to a compact 2-D grid and all the spatial derivatives with respect to the variable *z* are substituted by the frequency operator $-j\beta$.

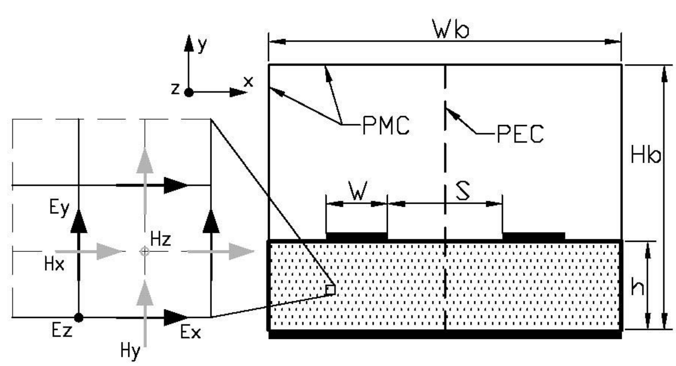

Figure 2. Geometry of the considered differential interconnect with its 2-D finite difference discretization.

After normalizing the field components with the square root of the free-space wave impedance, $\mathbf{H}' = \mathbf{H}\sqrt{\eta_0}$ and $\mathbf{E}' = \mathbf{E}/\sqrt{\eta_0}$, starting from the Maxwell's curl equations and eliminating the longitudinal components E_z and H_z as in [15], it is possible to obtain the following four finite difference equations in the frequency domain:

$$\frac{\beta}{k_0}E_x^{i,j}=\frac{1}{k_0^2\delta_x\delta_y}\left(\frac{H_x^{i,j}}{\varepsilon_z^{i,j}}+\frac{H_x^{i+1,j-1}}{\varepsilon_z^{i+1,j}}-\frac{H_x^{i,j-1}}{\varepsilon_z^{i,j}}-\frac{H_x^{i+1,j}}{\varepsilon_z^{i+1,j}}\right)+ \\ +\frac{1}{k_0^2\varepsilon_z^{i,j}\delta_x^2}H_y^{i-1,j}+\left(1-\frac{\varepsilon_z^{i,j}+\varepsilon_z^{i+1,j}}{k_0^2\varepsilon_z^{i,j}\varepsilon_z^{i+1,j}\delta_x^2}\right)H_y^{i,j}+\frac{1}{k_0^2\varepsilon_z^{i+1,j}\delta_x^2}H_y^{i+1,j} \tag{3a}$$

$$\frac{\beta}{k_0}E_y^{i,j}=\frac{1}{k_0^2\delta_x\delta_y}\left(\frac{H_y^{i-1,j}}{\varepsilon_z^{i,j}}+\frac{H_y^{i,j+1}}{\varepsilon_z^{i,j+1}}-\frac{H_y^{i,j}}{\varepsilon_z^{i,j}}-\frac{H_y^{i-1,j+1}}{\varepsilon_z^{i,j+1}}\right)+ \\ -\frac{1}{k_0^2\varepsilon_z^{i,j}\delta_y^2}H_x^{i,j-1}-\left(1-\frac{\varepsilon_z^{i,j}+\varepsilon_z^{i,j+1}}{k_0^2\varepsilon_z^{i,j}\varepsilon_z^{i,j+1}\delta_y^2}\right)H_x^{i,j}-\frac{1}{k_0^2\varepsilon_z^{i,j+1}\delta_y^2}H_x^{i,j+1} \tag{3b}$$

$$\frac{\beta}{k_0}H_x^{i,j}=\frac{1}{k_0^2\delta_x\delta_y}\left(E_x^{i-1,j}+E_x^{i,j+1}-E_x^{i,j}-E_x^{i-1,j+1}\right)+ \\ -\frac{1}{k_0^2\delta_x^2}E_y^{i-1,j}-\left(\varepsilon_y^{i,j}-\frac{2}{k_0^2\delta_x^2}\right)E_y^{i,j}-\frac{1}{k_0^2\delta_x^2}E_y^{i+1,j} \tag{3c}$$

$$\frac{\beta}{k_0}H_y^{i,j}=\frac{1}{k_0^2\delta_x\delta_y}\left(E_y^{i,j}+E_y^{i+1,j-1}-E_y^{i,j-1}-E_y^{i+1,j}\right)+ \\ +\frac{1}{k_0^2\delta_y^2}E_x^{i,j-1}+\left(\varepsilon_x^{i,j}-\frac{2}{k_0^2\delta_y^2}\right)E_x^{i,j}+\frac{1}{k_0^2\delta_y^2}E_x^{i,j+1}\quad , \tag{3d}$$

where the classical notation of the finite difference method has been used to name the field components, δ_x and δ_y are the mesh sizes in the x and y direction, respectively, k_0 is the free-space wavenumber at the given frequency and ε_x, ε_y and ε_z are the diagonal components of the relative dielectric permittivity tensor $[\boldsymbol{\varepsilon}]$.

The novelty of equations (3), unlike [15], lies in the fact that a better discretization of the dielectric has been introduced since the permittivity tensor has been defined as a constant on each component of the electric field, rather than on each cell of the electric grid. It is worth to note that only the transverse components of the electric and magnetic field appear in the equations, thus speeding up the computational efficiency with respect to classical six components finite difference or finite elements formulations. Moreover the ratio between the propagation constant β and the wavenumber k_0 is directly computed as the eigenvalues of the eigenproblem, allowing a direct accurate computation of the effective dielectric constant.

Since the longitudinal components do not appear in the final equations, the boundary conditions on the surface of perfect electric (PEC) and magnetic (PMC) conductors must be imposed indirectly as explained in [15]. Furthermore the continuity condition of the electric field across the interface of the dielectric substrate is ensured by setting the relative permittivity components ε_x and ε_z which lie on the boundary, as the average of the dielectric constant of the substrate and the vacuum. In order to enforce all the necessary boundary conditions from a numerical point of view, it is more suitable to solve a generalized eigenvalue problem

$$[\mathbf{A}]\mathbf{x}=\lambda[\mathbf{B}]\mathbf{x}\quad , \tag{4}$$

where [**A**] is the right-hand sparse coefficient matrix which is computed from (3), **x** is the column eigenvector with the transverse field components $\{E_x, E_y, H_x, H_y\}^T$, $\lambda = \beta/k_0$ is the eigenvalue and [**B**] is a unit matrix. In order to enforce to zero the i^{th} component of **x**, it is necessary [15] to multiply the diagonal matrix elements a_{ii} and b_{ii} by a very high number, set to 10^5 in the following computations, setting to zero the i^{th} row and column of the matrix [**A**].

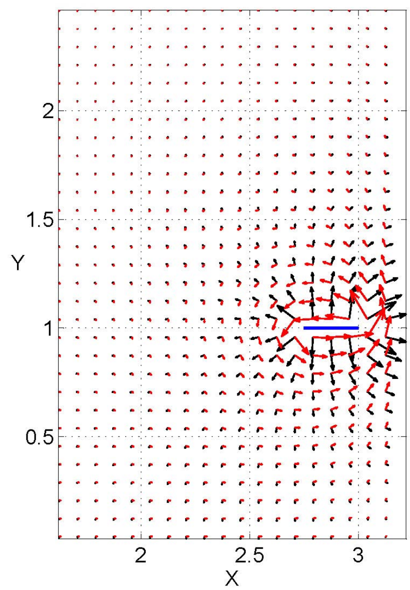

Figure 3. Fundamental differential quasi-TEM mode computed at the frequency of 1 GHz for the configuration #1(units in mm).

Since the attention is focused on the odd mode, it is possible to reduce the computational effort by a factor of two subdividing the transversal section with a perfect electric wall placed between the two microstriplines as shown in Figure 1. Furthermore, since the geometry must be closed, perfect magnetic walls are placed on the outer boundaries to simulate free space radiation. Anyway, since a magnetic symmetry is really imposed, which brings to the study of a periodic structure, the dimensions of the transversal 2-D domain must be carefully chosen because the size must be properly electrically small in order to reduce the dimension of the eigenproblem and, at the same time, properly large to avoid any distortion of the field pattern. In the following computations, the height H_b of the domain has been set to 4h, where h is the height of the dielectric substrate, and the width W_b to 7(2W+G), where W is the width of the single line and G is the spacing between the lines. These dimensions which must be considered only as guidelines, proved to give accurate results maintaining the dimension of the problem reasonable.

The generalized eigenproblem has been solved with the built-in Matlab function *eigs*, which makes optimum use of the

Arpack library together with the matrix sparse form of Matlab. Only one eigenvalue, corresponding to the fundamental differential mode, has been computed. In Figure 3 it is reported the pattern of the differential mode for the configuration #1 of Table I at the frequency of 5 GHz.

B. Analytical Modeling

The new analytical expression for the characteristic impedance of the differential interconnect, based on the power formulation (2), reads as

$$Z_{odd}(f) = Z_{odd}(0)\sqrt{\frac{\varepsilon_{eff}(0)}{\varepsilon_{eff}(f)}}\frac{1}{1+50D} \qquad (5)$$

where $Z_{odd}(0)$ and $\varepsilon_{eff}(0)$ are respectively the odd-mode impedance and the odd-mode effective dielectric constant at zero frequency which can be computed with high accuracy with the conformal mapping formulation proposed in [16]. The coefficient D can be computed as

$$D = \frac{\alpha(f)[\varepsilon_{eff}(0)-\varepsilon_r(f)]f}{2\varepsilon_{eff}(f)}\left[m_2(f) + f\log\left(\frac{f}{f_{50}}\right)m_p(f)\right] \qquad (6a)$$

$$\alpha(f) = \left(\frac{f}{f_{50}}\right)^{m_2(f)}\frac{1}{f}\left[1+\left(\frac{f}{f_{50}}\right)^{m_2(f)}\right]^{-2} \qquad (6b)$$

$$m_p(f) = \frac{0.15\exp\left(-0.45\frac{f}{f_{50}}\right)}{f_{50}(1+u)} \qquad (6c)$$

$$m_2(f) = 2.2\cdot m(f) \qquad (6d)$$

where $u = W / h$ and f is the frequency. To develop this model, the analytical expressions for the frequency dependent effective dielectric constant $\varepsilon_{eff}(f)$ reported in [4] have been used, with the following changes:

$$f_{50}(f) = \frac{2\cdot f_{TE1}(f)}{0.75+\left(0.75-\frac{1}{3\varepsilon_r(f)^{\sqrt{3}}}\right)\cdot u} \qquad (7a)$$

$$f_{TE1}(f) = c_0\frac{\left(\frac{\pi}{2}+\arctan\left(\varepsilon_r(f)\sqrt{\frac{\varepsilon_{eff}(0)-1}{\varepsilon_r(f)-\varepsilon_{eff}(0)}}\right)\right)}{2\pi h\sqrt{\varepsilon_r(f)-\varepsilon_{eff}(0)}} \qquad (7b)$$

$$m(f) = m_0\, m_c(f) \qquad (7c)$$

$$m_0 = 1+\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{1}{1+\sqrt{u}}\right)^n \qquad (7d)$$

$$m_c = \begin{cases} 1+\dfrac{\sqrt{2}}{1+u}\left[0.15-0.235\,e^{-0.45\frac{f}{f_{50}}}\right] & \text{for } u \le \sqrt{2}/2 \\ 1 & \text{for } u > \sqrt{2}/2 \end{cases}, \qquad (7e)$$

where h is the height of the substrate as reported in Figure 2 and $\varepsilon_r(f)$ is the relative dielectric constant of the substrate which generally depends upon frequency due to the dielectric dispersion.

IV. RESULTS

The accuracy of the proposed formulation has been compared with the analytical expressions of Kirschning and Jansen reported in [11-14] and with the full-wave data obtained by means of the FDFD code.

It should be noted that both analytical expressions do not match exactly low frequency values given by the numerical code. The accuracy of the proposed expressions is generally better than in [11]. Through extensive numerical tests it have been found that u must be less than 10 as in [11] while no particular limit has been found on the spacing, g = S/h.

In Figure 4 the configurations of Table I are analyzed by means of the proposed expressions and the results for the odd mode impedance are reported. In this initial study, the dielectric substrate has been modeled with a constant dielectric permittivity of 3.89.

Configuration	H [mm]	w [mm]	s [mm]
# 1	1	0.25	0.25
# 2	1	0.5	0.5
# 3	1	0.5	0.25

Table I – Interconnect configurations

V. CONCLUSIONS

The prediction of the frequency dependence of the odd-mode impedance of differential interconnect has been carried out by means of analytical and numerical formulations. The definition of the characteristic impedance has been reviewed and new expressions have been proposed for the odd-mode impedance in presence of dispersion starting from an improved power-based model. The proposed expressions are validated through comparisons with a novel two dimensional full-wave FDFD code. They proved to be more accurate than existing available formulations, while remaining of immediate use.

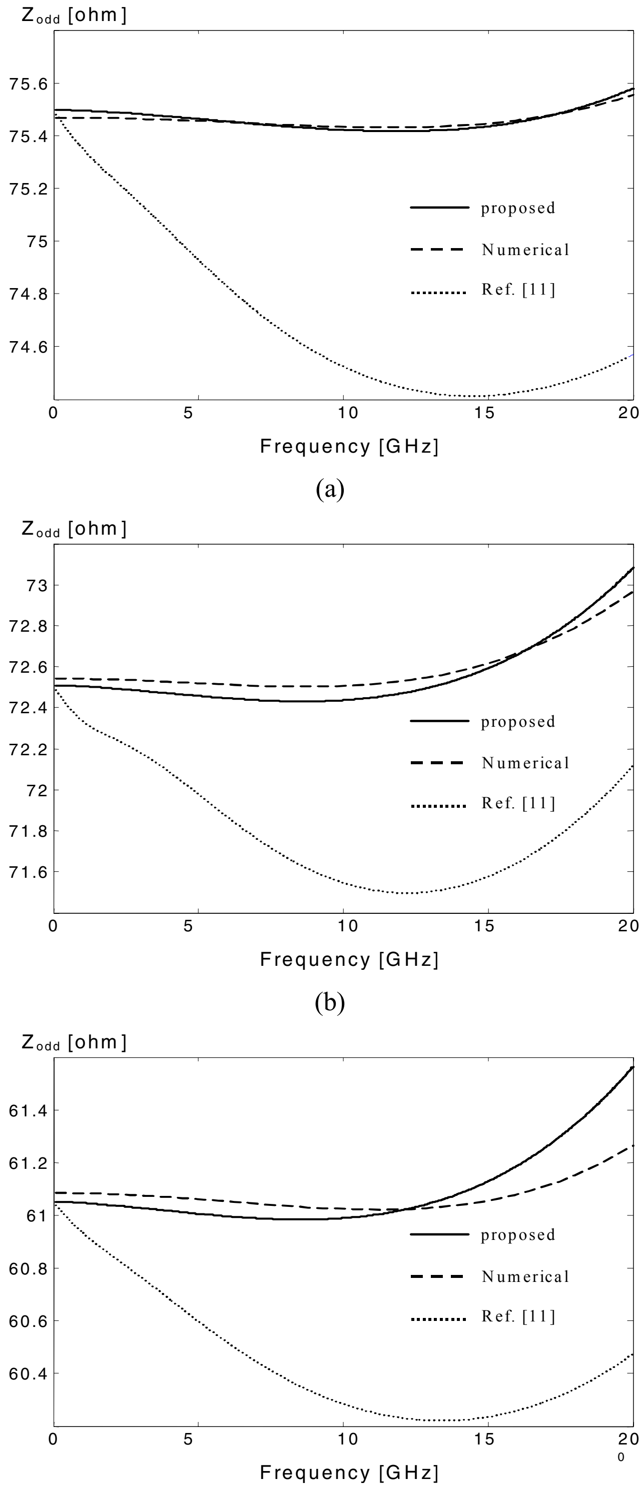

Figure 4. Frequency dependent odd-mode impedance in the configurations of Table I, in the same order.

ACKNOWLEDGMENTS

This work has been partially supported by the Italian Ministry of University (MIUR) under a Program for the Development of Research of National Interest (PRIN grant # 2002093437).

REFERENCES

[1] R. Araneo, S. Celozzi, F. Maradei, "Effects of the Dispersive Behavior of Dielectric Substrates on the SPI", *IEEE 6th Workshop on Signal Propagation on Interconnects*, Lucca, Italy, May 2002, pp. 105-108.

[2] R. Araneo, S. Celozzi, F. Maradei, "Digital signal transmission thru differential interconnects: full-wave vs. spice modeling", *IEEE 2003 Int. Symp. on EMC*, Boston, 2003, pp. 855-858.

[3] R. Araneo, S. Celozzi, F. Maradei, "Transient Analysis of Differential Interconnects by Spice-Like Circuits Accounting for Frequency-Dependent Losses", *2004 Int. Symp. On EMC*, Sendai, 2004, pp. *.*

[4] R. Araneo, S. Celozzi, F. Maradei, G. Guida, "Prediction and Validation of Propagation Characteristics of Differential Lines in PCB Configurations", *EMC Europe 2004*, Eindhoven, 2004, pp. 438-443.

[5] B. Bianco, L. Panini, M. Parodi, S. Ridella, "Some Considerations About the Frequency Dependence of the Characteristic Impedance of Uniform Microstrips", *IEEE Trans. On MTT*, 1978, vol. 26, pp. 182-185.

[6] T. Ito, R. Mittra, "Spectral-Domain Approach for Calculating the Dispersion Characteristic of Microstrip Lines", *IEEE Trans. On MTT*, 1973, pp. 496-499.

[7] K. C. Gupta, R. Garg, I. Bahl, P. Bhartia, *Microstrip Lines and Slotlines*, Artech House, Inc., Norwood, 1996.

[8] F. Olyslager, *Electromagnetic Waveguides and Transmission Lines*, Clarendon Press, Oxford, 1999.

[9] J. C. Rautio, "A New Definition of Characteristic Impedance", *IEEE 1991 MTT –S Int. Symp.*, 1991, pp. 791-764.

[10] W. J. Getsinger, "Measurement and Modeling of the Apparent Characteristic Impedance of Microstrip", *IEEE Trans. on MTT*, 1983, pp. 624-632.

[11] M. Kirschning, R. H. Jansen, "Accurate Wide-Range Design Equations for the Frequency- Dependent Characteristic of Parallel Coupled Microstrip Lines", *IEEE Trans. on MTT*, 1984, pp. 83-90.

[12] E. Hammerstad, O. Jensen, "Accurate models for microstrip computer-aided design", *IEEE MTT 1980 Int. Symp.*, Washington DC, 1980, pp.407-409.

[13] R. H. Jansen, M. Kirschning, "Arguments and an Accurate Model for the Power-Current Formulation of Microstrip Characteristic Impedance", *Arch. Elek, Ubertragung (AEÜ)*, 1983, vol. 37, pp. 108-112.

[14] M. Kirschning, R. H. Jansen, "Accurate Model for Effective Dielectric Constant of Microstrip with Validity up to Millimeter-wave frequencies", *Electron Letters*, 1982, vol. 18, pp. 272-273.

[15] Y.J. Zhao, K. L. Wu, K. K. Cheng, "A Compact 2-D Full-Wave Finite-Difference Frequency-Domain Method for General Guided Wave Structures", *IEEE Trans. Microwave Theory Tech.*, 2002, vol. 50, pp. 1844-1848.

[16] Changhua Wan, "Analytically and Accurately Determined Quasi-Static Parameters of Coupled Microstrip", *IEEE Trans. on MTT*, 1996, vol. 44, pp. 75-80.