SIMULATING CAR-PEDESTRIAN INTERACTIONS DURING MASS EVENTS WITH DTA MODELS: THE CASE OF VANCOUVER WINTER OLYMPIC GAMES

Lorenzo Meschini, Guido Gentile
Università di Roma “La Sapienza”, Dipartimento di Idraulica Trasporti e Strade,
Via Eudossiana, 18 - 00184 Roma (RM), Italy
lorenzo.meschini@uniroma1.it, guido.gentile@uniroma1.it
Kean Lew
PTV America Inc.

This paper is dedicated to the memory of our friend and colleague Kean Lew.

1. INTRODUCTION

This paper presents the application of a within-day Dynamic Traffic Assignment (DTA) Model to simulate ordinary, evacuation and emergency scenarios for downtown Vancouver during the forthcoming Winter Olympic Games.
Within this context, the main problem was to reproduce different kinds of pedestrians and vehicles-pedestrians interactions; these congestion phenomena can occur in presence of the unusual demand produced by important public events, such as sport games, music concerts and political rallies, when significant levels of pedestrian and vehicle flows are concentrated in space and time, i.e. converge to or diverge from one point/area in a relatively short time interval.
Additionally to the above, other phenomena that needed to be addressed where:

- *pedestrian route choice*, since several routes are available to reach and leave event locations;
- *special event temporal demand*, with time peaks concentrated around begin and end of events;
- *pedestrian capacity constraints*, due to the limited capacity of road and sidewalks;
- *short term closure of streets*, due to Olympic security measures.

In order to address the above modelling needs, different approaches where considered, from classical static assignment, to meso simulation (Di Gangi and Velonà, 2007), to micro simulation applied to pedestrians (Cepolina, 2005; Cepolina *et al.*, 2008) and vehicles (Vitetta *et al.*, 2007). Finally, a Macroscopic Dynamic Assignment model calculating Dynamic User Equilibrium was adopted, suitably extended in order to represent pedestrian flows and

vehicle-pedestrian interactions.
Aim of this paper, beside briefly presenting the Dynamic Assignment Model adopted, is thus to outline the modelling solutions devised to represent pedestrian and vehicle-pedestrian congestion.
The rest of the paper is structured as follows: section 2 is devoted to a brief recall of the dynamic traffic assignment model adopted within this paper; section 3, after introducing the formalization of the network performance model, illustrates all the model devised to properly represent pedestrian and vehicle-pedestrian interaction phenomena; section 4 finally presents dynamic equilibrium formulations based on the supply models previously introduced.

2. DYNAMIC USER EQUILIRBIUM MODEL

The adopted Dynamic Traffic assignment (DTA) model for road networks, named DUE (Dynamic User Equilibrium) and presented in Gentile *et al.* (2007), is based on a macroscopic representation of time-continuous flows, where the DTA is regarded as a User Equilibrium and is expressed as a fixed point problem in terms of maneuver flows at nodes. The fixed point problem, represented in Figure 1, is indeed formalized by combining the Network Loading Map, yielding the maneuver flow temporal profiles corresponding to given arc travel time and cost temporal profiles, and the Arc Performance Function, yielding the arc travel time and cost temporal profiles corresponding to given maneuver flow temporal profiles. On this basis, it is possible to devise efficient assignment algorithms, whose complexity is equal to the one resulting in the static case multiplied by the number of time intervals introduced.

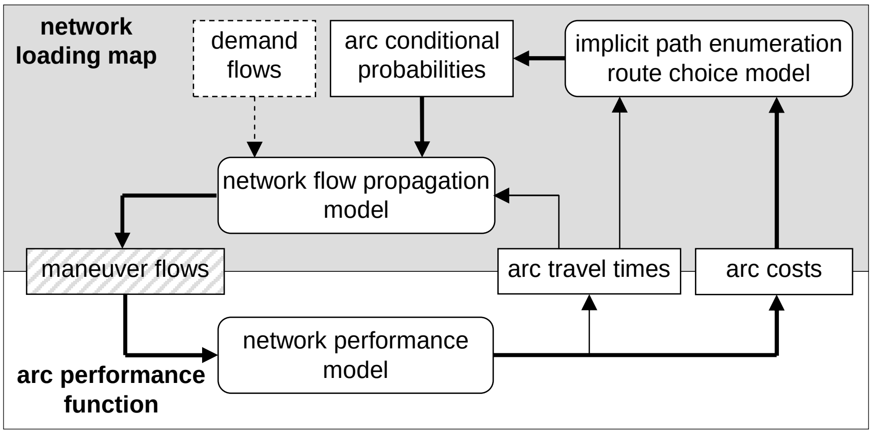

Figure 1. Scheme of the fixed point formulation for the DTA with spillback congestion.

The dynamic arc performance function adopts link-based macroscopic flow models (Gentile *et al.*, 2005) and is capable to represent spillback congestion, that is propagation of congestion among adjacent arcs, achieved through the introduction of time-varying exit and entry capacities that limit the inflow on downstream arcs in such a way that their storage capacities are never exceeded. Specifically, the Network Performance Model (NPM), depicted in Figure 2, is specified as a circular chain of three models, which can be formulated and solved as a system through a fixed point problem to determine the exit capacity temporal profiles, and thus the arc travel times and costs, for given maneuver flows.

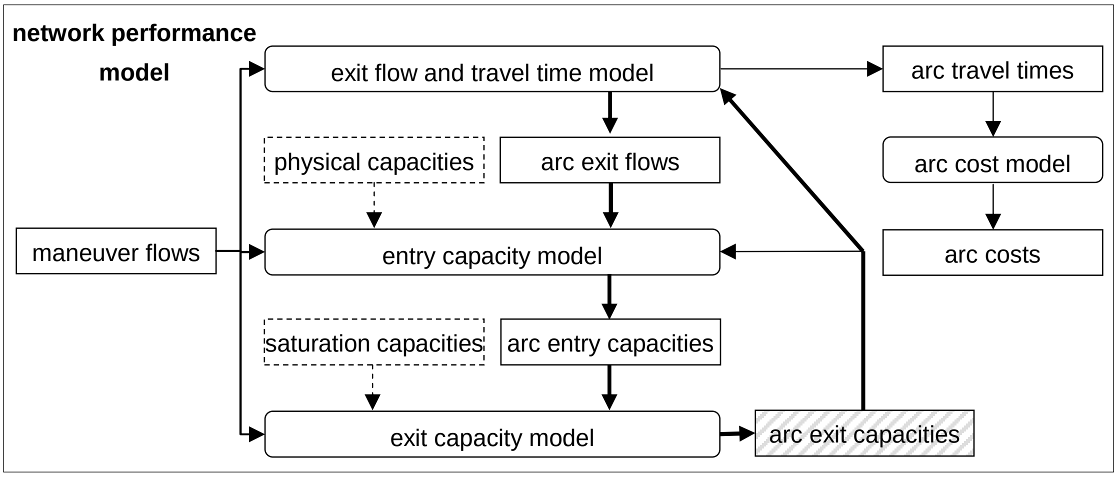

Figure 2. Scheme of the fixed point formulation for the NPM.

While the DUE model has a time continuous formulation, its numerical solution requires, as usual, the discretization of time in intervals. However, a key feature of the above approach is that no upper bound is set on the interval length by the solution method itself; in fact, this approach is intended to work with time intervals of several minutes, and allows the modeller to choose the time discretization based on the best trade-off between results accuracy and calculation times.

3. VEHICLE-PEDESTRIAN CONGESTIONS MODELS

3.1 Formalization of the Network Performance Model

In this section we want to achieve a representation of the multimodal network such that the relations between road and pedestrian elements, which are necessary to formalize non-separable cost functions modeling intra and inter modal congestion, can be correctly and univocally identified. In the following, we will represent two travelling modes: the *road mode R* and *the pedestrian mode P*, and we will refer to the generic mode $m \in M = \{R, P\}$.

The road and pedestrian networks are modeled through *directed graphs* $GR = (N, AR)$ and $GP = (N, AP)$, where N is the set of the *nodes*, each representing an intersection or a “centroid” (i.e. a trip terminal, origin or destination), and AR , AP are respectively the sets of *road* and *pedestrian arcs*, each representing a road or pedestrian link or a “connector” between a centroid and the road or pedestrian network.

The generic arc a is univocally identified by its *tail* $TL(a) \in N$ and *head* $HD(a) \in N$, that is $a=(TL(a), HD(a)) \in AR \cup AP$, and consists of a homogeneous channel of *length* L_a and *physical capacity* Q_a with a final bottleneck of *saturation capacity* $S_a \leq Q_a$.

Moreover, we define the sidewalk network as an *undirected graph* $GS = (N, AS)$ where the generic undirected sidewalk arc $s = \{TL(s), HD(s)\} \in AS$, $TL(s) \in N$ and $HD(s) \in N$, is a longitudinal element where pedestrians may walk in two opposite directions, characterized by *length* L_s and *total capacity* Q_s , where the latter is shared by the two possible directions.
We then identify the following relation among the different elements introduced above:

- each pedestrian arc $a \in AP$ is univocally associated to a sidewalk arc, that is: $PS(a) = \{s \in AS : \{TL(s), HD(s)\} = \{TL(a), HD(a)\}\} \subseteq AS$;
- each sidewalk arc $s \in S$ is associated to the set of the corresponding pedestrian arcs (which can be at the most 2), that is: $SP(s) = \{a \in AP : \{TL(a), HD(a)\} = \{TL(s), HD(s)\}\} \subseteq AP$;
- each pedestrian arc $a \in AP$ may be associated to the opposite pedestrian arc sharing the same sidewalk, that is: $OP(a) = \{b \in AP : \{TL(b), HD(b)\} = \{HD(a), TL(a)\}; PS(b) = PS(a)\} \subset AP \cup \varnothing$;
- each road link $a \in AR$ is associated the set of pedestrian links sharing the same final node, that is: $RPH(a) = \{b \in AP : HD(b)\} = HD(a)\} \subset AP \cup \varnothing$;
- each road link $a \in AR$ may be associated to pedestrian links corresponding to sidewalks adjacent to the road, that is: $RPTH(a) = \{b \in AP : \{TL(b), HD(b)\} = \{TL(a), HD(a)\}\} \subset AP \cup \varnothing$;

Finally, the sets of origins and destinations of pedestrian and car trips, referred to as *origins and destinations*, are two subsets $ORIG \subseteq N$ and $DEST \subseteq N$ of the nodes, with $ORIG \cap DEST = \varnothing$. Each *origin* $o \in ORIG$ has no entering arcs except for one dummy arc, and its exiting connectors have an infinite physical capacity; each *destination* $d \in DEST$ has no exiting arcs except for one dummy arc, and its entering connectors have an infinite saturation capacity; dummy arcs, which have infinite saturation capacity and zero length, allow defining inflows and outflows of connectors in terms of the maneuver flows at centroids. Let $\varphi_{ab}(\tau)$ be the *maneuver flow* at time τ from arc $a \in AR[AP]$ to arc $b \in AR[AP]$ at node $HD(a) = TL(b)$. More aggregated and familiar flow variables can be easily derived as follows:

$$f_a(\tau) = \sum_{b \in BS(TL(a))} \varphi_{ba}(\tau) , \tag{1}$$

$$F_a(\tau) = \int_{-\infty}^{\tau} f_a(\sigma) \cdot d\sigma , \tag{2}$$

$$\phi_a(\tau) = \sum_{b \in FS(HD(a))} \varphi_{ab}(\tau) , \tag{3}$$

$$\Phi_a(\tau) = \int_{-\infty}^{\tau} \phi_a(\sigma) \cdot d\sigma , \tag{4}$$

where $f_a(\tau)$ is the *inflow*, $F_a(\tau)$ is the *cumulative inflow*, $\phi_a(\tau)$ is the *outflow* and $\Phi_a(\tau)$ is the *cumulative outflow* of arc $a \in AR[AP]$ at time τ , while $BS(i)$ =

$\{a \in AR[AP]: HD(a) = i\}$ and $FS(i) = \{a \in AR[AP]: TL(a) = i\}$ are, respectively, the *forward* and the *backward star* of node $i \in N$. Note that the dummy arcs relative to each centroid are introduced specifically to let (1) and (3) hold also for such nodes.

3.2 Pedestrian Concordant interaction model

Accordingly with Gentile *et al.* (2007), flow states are determined accordingly with the Simplified Theory of Kinematic Waves (STKW) assuming the parabolic/trapezoidal fundamental diagram depicted in Figure 3, where $KQ_a > 0$ is the *critical density*, $KJ_a > KQ_a$ is the *jam density*, $V_a \geq Q_a / KQ_a$ is the *free flow speed*, and w_a is the absolute value of the *kinematic wave speed* corresponding to hypercritical conditions, that is for $k \in [KQ_a , KJ_a]$ (see Gentile *et al.,* 2005).

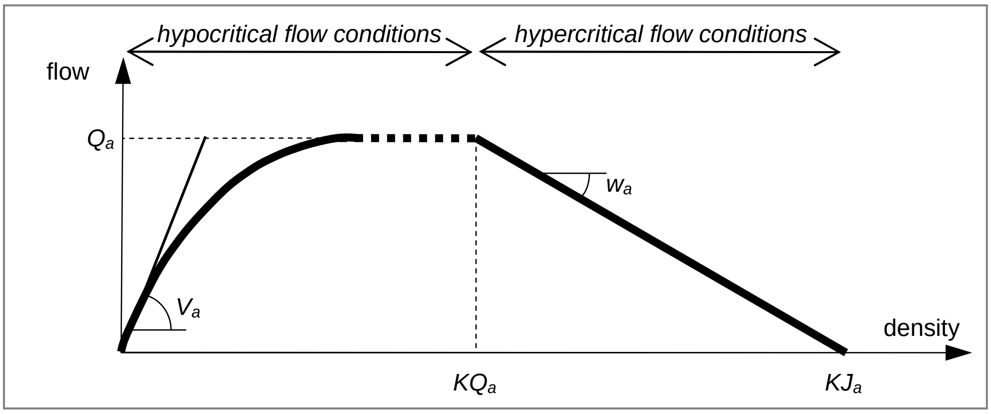

Figure 3. The Fundamental diagram adopted within the DUE model.

In this section the above macroscopic flow model is suitably extended to represent the interactions between pedestrians in presence of explicit capacity constraints on sidewalks. To this end, the model proposed by Daamen and Hoogendoorn (2003) was utilized, expressing the following empirical relation between the longitudinal space used by pedestrians and their speed over the generic pedestrian arc *a*:

$$A(v_a) = AJ_a - 0.52 \ln\left(1 - \frac{v_a}{V_a}\right) \tag{5}$$

where v_a is the walking speed, V_a is the average free walking speed ($V_a \approx 1.34$ m/s), A denotes the required area, and AJ_a is the largest area for which walking is impossible ($JA_a \approx 0.19$ m^2, as depicted on left side of Figure 4).

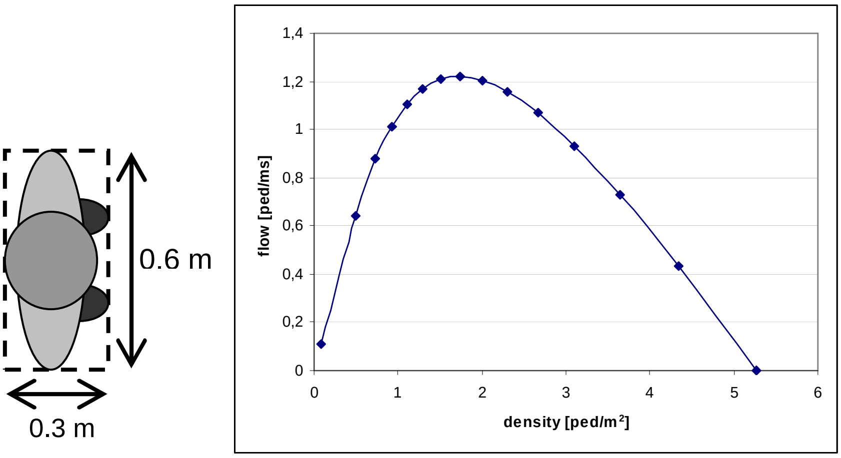

Figure 4 - (left side) minimal area for a single pedestrian; (right side) pedestrian fundamental diagram for a 1 m wide sidewalk

The above relation and values yield the fundamental diagram depicted on right side of Figure 4, which is valid for a 1 m wide sidewalk, where the capacity Q_a is about 1.2 ped/s, that is 4,320 ped/h, and the pedestrian jam density KJ_a is , of course, 1 / 0.19 ≈ 5.26 ped/m.

3.3 Pedestrian discordant interaction

This section addresses the problem of correctly allocating to each direction the capacity of a sidewalk used by opposite pedestrian flows. This phenomenon is clearly dependent on directional pedestrian flows conflicting with each other, which means that the pedestrian capacity for each direction of a sidewalk is an endogenous variable resulting from the assignment itself.
Then, two entities are to be determined: the entry capacity $Q_a(\tau)$ of arc $a \in AP$ at each time τ; and the jam density $KJ_a(\tau)$ of arc a at each time τ.

The entry capacities $Q_a(\tau)$ and $Q_{OP(a)}(\tau)$ of the two directional arcs belonging to the same sidewalk are determined as follows:

$$Q_a(\tau) = \frac{\min\{f_a(\tau);\varepsilon\}}{\min\{f_a(\tau);\varepsilon\} + \min\{f_{OP(a)}(\tau);\varepsilon\}} \cdot Q_{s(a)} \tag{6}$$

$$Q_{o(a)}(\tau) = Q_{s(a)} - Q_a(\tau) = \frac{\min\{f_{s(a)}(\tau);\varepsilon\}}{\min\{f_a(\tau);\varepsilon\} + \min\{f_{s(a)}(\tau);\varepsilon\}} \cdot Q_{s(a)} \tag{7}$$

Where: $f_a(\tau)$ and $f_{OP(a)}(\tau)$ are the two directional inflows of the sidewalk; $Q_{s(a)}$ is the total sidewalk capacity; and ε is a number greater than zero and much smaller than the total sidewalk capacity, that is:

$$0 < \varepsilon << Q_{s(a)} \tag{8}$$

The directional capacity model defined by (6) and (7), which is non-separable, shares the total capacity between the two directions proportionally to the directional inflows, in such a way that the sum of the directional capacities is always equal to the total sidewalk capacity. Note that when the sum of the directional inflows at time τ is smaller than the total sidewalk capacity, meaning that flows are not constrained, (6) and (7) yield $Q_a(\tau) > f_a(\tau)$ and $Q_{OP(a)}(\tau) > f_{OP(a)}(\tau)$, thus not constraining directional flows, as expected; in particular, if both directional flows are zero, it results from the above model:

$$Q_a(\tau) = Q_{OP(a)}(\tau) = 0.5 \cdot Q_{s(a)} > 0 \tag{9}$$

As a matter of fact, ε is introduced within the model specifically to guarantee the correctness of the model also in presence of null directional flows; its effect is to reserve anyhow to each direction a minimum capacity, even if it is not completely used (as it wouldn't be in reality, if the constraint is active on the opposite direction). However, ε can be set small enough so that the possible wasted capacity would be negligible in practice.
Combining relations (6) and (7) with (1) the *sidewalk capacity allocation model* can be expressed in the following compact form for all the arcs at once:

$$\boldsymbol{Q} = \mathrm{Q}(\boldsymbol{\varphi}) \tag{10}$$

where bold symbols denote temporal profiles of vector variables.

The jam densities $KJ_a(\tau)$ and $KJ_{OP(a)}(\tau)$ of the two directional arcs corresponding to the same sidewalk are determined similarly with the entry capacities, that is:

$$KJ_a(\tau) = \frac{\min\{k_a(\tau);\varepsilon\}}{\min\{k_a(\tau);\varepsilon\} + \min\{k_{OP(a)}(\tau);\varepsilon\}} \cdot KJ_{s(a)} \tag{11}$$

$$KJ_{OP(a)}(\tau) = KJ_{s(a)} - KJ_a(\tau) = \frac{\min\{k_{s(a)}(\tau);\varepsilon\}}{\min\{k_a(\tau);\varepsilon\} + \min\{k_{OP(a)}(\tau);\varepsilon\}} \cdot KJ_{s(a)} \tag{12}$$

Where: $KJ_{s(a)}$ is the maximum density of the sidewalk; ε is defined as above; $k_a(\tau)$ and $k_{OP(a)}(\tau)$ are the two directional average densities of the sidewalk at time τ, defined as:

$$k_a(\tau) = \frac{1}{L_{SP(a)}}\left(F_a(\tau) - \Phi_a(\tau)\right) \tag{13}$$

$$k_{OP(a)}(\tau) = \frac{1}{L_{SP(a)}}\left(F_{OP(a)}(\tau) - \Phi_{OP(a)}(\tau)\right) \tag{14}$$

Combining relations (11)÷(14) with relations (1)÷(4) the *sidewalk jam density allocation model* can be expressed in the following compact form for all the arcs at once:

$$\boldsymbol{KJ} = \mathrm{KJ}(\boldsymbol{\varphi}) \tag{15}$$

where bold symbols denote temporal profiles of vector variables.

We conclude this section pointing out that within the above models, directional entry capacities and jam densities are determined assuming, at each instant and for each direction, a unique flow state along the whole sidewalk, which in general is not true in reality.

3.4 Vehicle-pedestrian transversal interaction

The following model represents the effect of vehicle pedestrian transversal interaction, that is the delay suffered by vehicular flows due to pedestrian crossings, assuming that pedestrians would have priority over car when crossing roads.
In order to represent this phenomenon, we define the following *saturation capacity reduction model*, reducing the final capacity of road link $a \in AR$ as a function of flows on pedestrian links sharing the same head node, identified by set $RPH(a)$ defined previously.

$$S_a = \frac{SM_a}{1 + \alpha \cdot (\sum_{b \in RPH(a)} f_b)^{\beta}} \ , \tag{16}$$

where SM_a is the maximum saturation capacity.
The above function, combined with (1), can be expressed in the following compact form for all the road arcs at once:

$$\boldsymbol{S}_R = \mathrm{S}(\boldsymbol{\varphi}_P) \tag{17}$$

where bold symbols denote temporal profiles of vector variables, and subscript R and P denote respectively road and pedestrian elements.

3.5 Vehicle-pedestrian longitudinal interaction

Longitudinal interaction occurs in presence of highly crowded sidewalks, where pedestrian may randomly and discontinuously occupy part of the car lane near to the sidewalk, since faster pedestrians “hop-off” and “hop-on” it in order to pass slower people. This partially chaotic pedestrian behaviour leads both to a loss of road capacity and to a reduction of the road free-flow speed, since drivers have to reduce their speed for safety reasons.
Then, this phenomenon is represented by the following *physical capacity*, *jam density* and *free-flow speed reduction models*, were the physical capacity, the jam density and the free-flow speed of the generic road arc $a \in AR$ are reduced by the pedestrian flows on sidewalks adjacent to the road:

$$Q_a = \frac{QM_a}{1 + \gamma \cdot (\sum_{b \in RPTH(a)} f_b)^{\delta}} \ , \tag{18}$$

$$KJ_a = \frac{KJM_a}{1 + \gamma \cdot (\sum_{b \in RPTH(a)} f_b)^{\delta}} \ , \tag{19}$$

$$V_a = \frac{VM_a}{1 + \eta \cdot (\sum_{b \in RPTH(a)} f_b)^{\kappa}} \ , \qquad (20)$$

where QM_a, KJM_a and VM_a are respectively the maximum physical capacity, the maximum jam density and the maximum free-flow speed on arc *a*.

The above functions, combined with (1), can be expressed in the following compact form for all the road arcs at once:

$$\boldsymbol{Q}_R = \mathrm{Q}(\boldsymbol{\varphi}_P) \qquad (21)$$

$$\boldsymbol{KJ}_R = \mathrm{KJ}(\boldsymbol{\varphi}_P) \qquad (22)$$

$$\boldsymbol{V}_R = \mathrm{V}(\boldsymbol{\varphi}_P) \qquad (23)$$

where bold symbols denote temporal profiles of vector variables, and subscripts *R* and *P* denote respectively road and pedestrian elements.

4. MULTI MODAL DYNAMIC USER EQUILIBRIUM MODELS UNDER DIFFERENT VEHICLE-PEDESTRIAN INTERACTIONS

In this chapter, after recalling briefly the formalization of the dynamic user equilibrium model as a fixed point problem, five different types of possible vehicle-pedestrian dynamic assignment are defined, each one characterized by a different vehicle-pedestrian interaction, namely *Normal*, *Controlled*, *Random*, *Chaotic concordant*, *Chaotic conflicting*.

4.1 Road equilibrium model

We recall preliminarily from Gentile *et al.* (2007) that the Network Loading Map, yielding vector $\boldsymbol{\varphi}$ of node manoeuvre flow temporal profiles for given vectors $\boldsymbol{c}$ and $\boldsymbol{t}$ of arc cost and travel time temporal profiles and given demand vector $\boldsymbol{D}$ of origin destination flow temporal profiles, can be formalized as the following functional:

$$\boldsymbol{\varphi} = \omega^*(\boldsymbol{c}, \boldsymbol{t} \,; \boldsymbol{D}) \qquad (24)$$

where bold symbols denote temporal profiles of vector variables.
Analogously, the Arc Performance Function, yielding arc cost and travel time temporal profiles for given manoeuvre flow temporal profiles, can be formalized as the following functionals:

$$\boldsymbol{t} = \mathrm{t}^*(\boldsymbol{\varphi}) \qquad (25)$$

$$\boldsymbol{c} = \mathrm{c}^*(\boldsymbol{\varphi}) \ . \qquad (26)$$

In their turn, c* and t* results from the solution of the fixed point problem defining the Netwok Performance Model, as described in details in Gentile *et al.* (2007).
On this basis, the DTA is formalized as a fixed-point problem in terms of maneuver flow temporal profiles by substituting into the Network Loading Map

(24) the Arc Performance Function (25)-(26):

$$\varphi = \omega^*(c^*(\varphi), t^*(\varphi); \boldsymbol{D}) \ . \qquad (27)$$

4.2 Road and Pedestrian equilibrium model with normal interaction

Normal interaction occurs in ordinary condition scenarios, where pedestrians use only sidewalks and cross roads at signals or at crossing points; within this situation, vehicle-pedestrian interaction can be neglected or taken into account through *a priori* turn delays for cars, while we need to represent pedestrian discordant interaction effects.

However, equilibrium formulation (27) is not well suited to represent pedestrian assignment. In fact, the formulation of the network performance model expressed by (25) and (26) is not explicitly dependent from supply parameters (namely maximum capacity, jam density and speed), since they where considered exogenous variables; however, in the previous sections we showed how they may become endogenous variables in presence pedestrian and vehicle-pedestrian interactions. Then, in order to formalize the pedestrian equilibrium, we modify (25)and (26) explicitly introducing the dependency of the supply models from vectors of physical capacities $\boldsymbol{Q}$ (arc physical capacities), $\boldsymbol{S}$ (arc saturation capacities), $\boldsymbol{V}$ (arc free-flow speeds) and $\boldsymbol{KJ}$ (arc jam densities):

$$\boldsymbol{t} = t^*(\varphi, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{V}, \boldsymbol{KJ}) \qquad (28)$$

$$\boldsymbol{c} = c^*(\varphi, \boldsymbol{Q}, \boldsymbol{S}, \boldsymbol{V}, \boldsymbol{KJ}) \qquad (29)$$

Then, combining the Network Loading Map (24), the Arc Performance Function (28)-(29) and the sidewalk capacity allocation models (10)-(15), we obtain the following formulation of pedestrian DTA with discordant interaction:

$$\varphi_P = \omega^*(c^*(\varphi_P, Q(\varphi_P), \boldsymbol{S}_P, \boldsymbol{V}_P, KJ(\varphi_P)), t^*(\varphi_P, Q(\varphi_P), \boldsymbol{S}_P, \boldsymbol{V}_P, KJ(\varphi_P)); \mathbf{D}_P) \ . \qquad (30)$$

Finally, road-pedestrian DUE with normal interaction is formalized as two separate fixed point problem: (30) for pedestrian assignment, and the following for road assignment, obtained combining (24), (28) and (29):

$$\varphi_R = \omega^*(c^*(\varphi_R, \boldsymbol{Q}_R, \boldsymbol{S}_R, \boldsymbol{V}_R, \boldsymbol{KJ}_R), t^*(\varphi_R, \boldsymbol{Q}_R, \boldsymbol{S}_R, \boldsymbol{V}_R, \boldsymbol{KJ}_R); \mathbf{D}_R) \ , \qquad (31)$$

where subscripts *R* and *P* denote respectively road and pedestrian elements.
To be noted that, in this case, the two assignment are independent from each other.

4.3 Road and Pedestrian equilibrium model with controlled interaction

Controlled interaction occurs during special events, where we can plan to assign some or road lanes to pedestrians, which however can't spread on lanes reserved to vehicles. While we can still reasonably assume that no longitudinal interaction occurs, transversal interaction may not be negligible, due to the high number of pedestrians involved.

This case is formalized as two related DTA, (pedestrian and car), where pedestrian assignment is expressed again by (30), while road assignment is formalized combining the fixed point formulation (31) with the road saturation capacity reduction model (17); that is:

$$\varphi_R = \omega^*(c^*(\varphi_R, \boldsymbol{Q}_R, S(\varphi_R), \boldsymbol{V}_R, \boldsymbol{KJ}_R), t^*(\varphi_R, \boldsymbol{Q}_R, S(\varphi_R), \boldsymbol{V}_R, \boldsymbol{KJ}_R); \mathbf{D}_R) \quad , \qquad (32)$$

where subscripts *R* and *P* denotes respectively road and pedestrian elements. Contrary to the previous case, here the road equilibrium depends on the solution of the pedestrian equilibrium.

4.4 Road and Pedestrian equilibrium model with random interaction

Random interaction occurs during special event and/or evacuation scenario, in which, although pedestrians should stay on sidewalks (or assigned lanes), they may randomly and discontinuously occupy part of the car lane near to the sidewalk, causing the vehicle-pedestrian longitudinal interaction described in section 3.5.

Also this case is formalized as two related DTA, (pedestrian and car), where pedestrian assignment is expressed again by (30), while road assignment is formalized combining the fixed point formulation (32) (thus assuming that controlled and random interaction occur jointly) with the road capacity and speed reduction models (21)-(23); that is:

$$\varphi_R = \omega^*(c^*(\varphi_R, Q(\varphi_p), S(\varphi_P), V(\varphi_P), KJ(\varphi_P)), t^*(\varphi_R, Q(\varphi_P), S(\varphi_P), V(\varphi_P), KJ(\varphi_P)); \mathbf{D}_R) \quad (33)$$

where subscripts *R* and *P* denotes respectively road and pedestrian elements. Also in this case, the road equilibrium depends on the solution of the pedestrian equilibrium.

4.5 Road and Pedestrian equilibrium model with Chaotic concordant interaction

Chaotic concordant interaction occurs in presence of evacuation scenarios where it is not possible to separate pedestrian and car flows, which are then completely mixed together; however, on each road, pedestrian and cars will flow in the same direction (this assumption may be reasonable if evacuation/collecting point are identified and known both by cars and pedestrians, so that everybody is basically trying to reach the same point).

In this case, the sidewalk capacity is assigned to pedestrians, while the entire road capacity is assigned to both car and pedestrians, where cars are thus forced to travel at pedestrian speed.

This scenario is formalized through a unique pedestrian DTA, where cars are considered as "pedestrian equivalents"; that is:

$$\varphi_P = \omega^*(c^*(\varphi_P, \boldsymbol{Q}_{RP}, \boldsymbol{S}_{RP}, \boldsymbol{V}_P, \boldsymbol{KJ}_{RP}), t^*(\varphi_P, \boldsymbol{Q}_{RP}, \boldsymbol{S}_{RP}, \boldsymbol{V}_P, \boldsymbol{KJ}_{RP}); \mathbf{D}_{RP}) \quad , \qquad (34)$$

where the generic component of the generic vector $\boldsymbol{X}_{RP}$ is given by a linear combination of the corresponding pedestrian and road variables; for example, the generic demand component $d_{od} \in \mathbf{D}_{RP}$ from origin $o \in ORIG$ to destination $d \in DEST$ at time τ is defined as:

$$d_{od}(\tau) = d_{od}^P(\tau) + \zeta \cdot d_{od}^R(\tau) \quad .$$

4.6 Road and Pedestrian equilibrium model with Chaotic conflicting interaction

Chaotic conflicting interaction is the worst case, occurring in presence of evacuation scenarios where it is not possible to separate pedestrian and car flows, which are then completely mixed together, and where pedestrians do not respect car directions. In this case, cars are basically stuck on the road, while pedestrians flow around them. We state that the best (and more conservative) way to model this case is to assign to pedestrians sidewalk capacities, plus a small part of the road capacities, due to the presence of blocked cars that will occupy the rest of it; then, the evacuation time is the time needed to evacuate pedestrians, plus the time needed to evacuate cars, both resulting from two separate dynamic traffic assignments; then, in terms of equilibrium formalization, this case is analogous to the normal interaction case.

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