Dynamic hyperpaths in transit networks: the stop model with online information

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Abstract

The purpose of this paper is to investigate the possibility of exploiting the hyperpath paradigm to model the route choice in congested metropolitan networks, where both regular and irregular services are available and where passengers are provided at transit stops with information regarding actual waiting times. In this context, we develop different stop models, depending on the layout of the stop and also on the congestion level, for reproducing route choice strategies: how do passengers compare regular and irregular services, how is their choice affected in case of developing queues, which boarding rule is suitable to represent different congestion phenomena. Numerical example are provided showing how congestion and regularity affect the boarding probabilities in case online information is provided.

1. Introduction

The concept of *optimal strategy* was first introduced by Spiess and Florian (1989) to model the travel behaviour of rational passengers in presence of perceived uncertainty in vehicle arrivals when several routes are available to reach the destination from a transit stop, including the relevant case of *common lines* (i.e. partly overlapping). Therefore, it is assumed that, rather than selecting the shortest single routes or itineraries before the beginning of the trip, passengers choose the best strategy, which is defined as "*a set of rules that, when applied allows the traveller to reach his or her destination*". A strategy is chosen before the beginning of the trip and, starting from the origin, it involves the iterative sequence of: walking to a transit stop or to the destination, selecting the attractive lines to board and, for each of them, the stop where the passenger needs to alight.

According to Nguyen and Pallottino (1988), a transit assignment reproducing this strategic behaviour can be modelled by loading a shortest (i.e. with minimal cost) *hyperpath* that connects the origin of the trip to the destination and represents possible diversions at transit stops through waiting *hyperarcs*, each of which identifies a line set. Traditionally, hyperpaths have been exploited to model the frequency-based transit assignment in a static framework, where it is assumed that a passenger, after reaching a stop, waits for the first attractive carrier among a fixed set of lines. It is known (Billi et al., 2004; Noekel, 2007) that this behaviour is rational only when:

- no information is provided at the stop on actual waiting times and on the available capacities of arriving carriers;
- the vehicle arrivals of different lines at the stop are statistically independent, and the same is true for the passenger arrivals with respect to vehicle arrivals (the latter meaning that the service is so irregular and/or so frequent that passengers do not synchronize their arrival at the stop with carriers arrival);
- the headway probability distribution function (p.d.f.) between two successive vehicles of the same line and hence the waiting time for a passenger randomly arriving at the stop are exponential, i.e. memory-less (irregular services).

In contrast with the previous works, we are interested here in extending the *hyperpath* paradigm in order to develop a route choice model in the case where:

- intelligent transport systems provide countdown information at transit stops. This implies that passengers know the actual waiting time before the first arrival of any line serving the stop.
- the urban transit system comprises not only highly irregular services (such as buses), but also regular services (such as underground or LRT) which have deterministic headways. In the latter case the p.d.f. of inter-arrival times and, therefore, passengers' waiting times is uniform and not exponential.
- the transport network is congested, meaning that link travel times and transit frequencies vary during the day, and capacity constraints can prevent passengers from boarding the first attractive line approaching the stop.
- passengers know for each attractive line which is the carrier they will be able to board. We can assume this information is known due to travel experience, or it is dynamically computed off-line by an ITS and communicated to passengers at the stop.

To this aim, the stop model needs to be addressed more specifically in order to describe how the waiting process works in case of congestion and how queues and on-line information affect route choice.

Thus, after a brief overview of assumptions, formulation and notation of the stop model proposed here (Section 2), in Section 3 we recall models developed to evaluate, separately, the effect of online information or congestion on: boarding probabilities, waiting and travel times. Afterwards, our model will be presented focussing at first on a single stop (Section 4) and then on a specific set of attractive lines (Section 5). Numerical examples will be also provided (Section 6) to show the effect of regularity/irregularity and congestion when online information is provided.

Our stop models will allow us to evaluate the expected waiting time of the corresponding hyperarc, as well as the probability to board each attractive line, addressing the case of exponential or uniform p.d.f. for inter-arrival times and availability of on-line information at stops. In this situation, even if there is no queue, it is not always convenient to board the first attractive carrier approaching the stop, but the best strategy is to keep waiting (Gentile *et al.*, 2005). Hence the boarding probability and *under saturation delay* (or waiting time) cannot be computed according to the formula given by Spiess and Florian (1989) for the classic case of exponential headways and no information.

Moreover, if the flow willing to board is over the available capacity on the approaching line vehicle, passengers have to queue until the service becomes actually accessible to them, thus suffering an additional over saturation delay. Therefore, the model has to be adjusted to represent the dynamic queue, which depends also on the layout of the stop.

2. Hyperpaths: formulation and notation

The transit network is formally represented here by an oriented *hypergraph* $G = (N, A)$, where N is the set of nodes and A is set of arcs and hyperarcs.

As usual, the generic arc $a \subseteq A$ is identified by an ordered pair of nodes, referred to respectively as the *tail*, denoted by $TL(a) \in N$, and the *head*, denoted by $HD(a) \in N$; that is $a = (TL(a), HD(a))$. While for the generic hyperarc the head can be a set of nodes, i.e. $HD(a) \subseteq N$. We distinguish the following different types of arcs and hyperarcs:

- *AP pedestrian arcs*, used by passengers to walk from the origin to a boarding stop, from a stop to another stop, and from the alighting stop to the destination;
- *AH waiting hyperarcs*, used to model the under saturation delay due to the discontinuity of the service. Passengers at the stop do not know which attractive carrier will arrive first, therefore they associate a probability to each head of the hyperarc that represents the boarding on a particular line;

- *AQ queuing arcs*, used to model the over saturation delay due to passenger flow exceeding the available capacity of the line at the stop;
- *AA alighting arcs*, used to model the alighting process;
- *AL line arcs*, connecting two subsequent stops of a same line;

Therefore we have: $A = AP \cup AH \cup AQ \cup AA \cup AL$.

Note that the assumption of representing first the waiting process and then the queuing process as in Figure 1 is questionable from a phenomenal point of view, since exactly the opposite occurs in reality; however this proves to be a valid choice from a modeling point of view.

First of all, doing so allows us to develop a model with separate queues. On the contrary, if queues were not separate, the model should also represent overtaking among passengers wanting to board different attractive lines and therefore FIFO rule would not hold true.

Secondly, both the expected waiting time and the expected travel time once boarded affect passenger choices as part of the generalized travel costs. Instead, representing the waiting process after the queuing process impedes to include the queuing time in the computation of the optimal strategy.

Finally, adopting the approach proposed by Meschini *et al.* (2007), transit frequencies are conceived here as a continuous flow of line carriers. This allows representing explicit capacity constraints on line vehicles and reproducing "over saturation" queuing times of passengers at transit stops. On the other hand, we have to force into the model the simulation of the "under saturation" waiting time of passengers at transit stops, due to the intrinsic discontinuity of the service. Under this consideration, we can add this term wherever it is more convenient from a modelling point of view, in this case associating it to hyperarcs before the queuing time, as far as all components of generalized costs are correctly taken into account.

A *support hyperarc a* is associated with all the lines serving the stop. However, a passenger directed to a given destination d will consider only a subset of services. Therefore, we associate a specific hyperarc b to each possible attractive line set, i.e. $HD(b) \subseteq HD(a)$, as shown in Figure 2.

A hyperpath h is an acyclic sub-graph on G connecting a single origin o to a single destination d, where there is one single arc or hyperarc exiting from each node (except for the destination) and all nodes are connected to d; that is, diversions occur only at waiting hyperarcs.

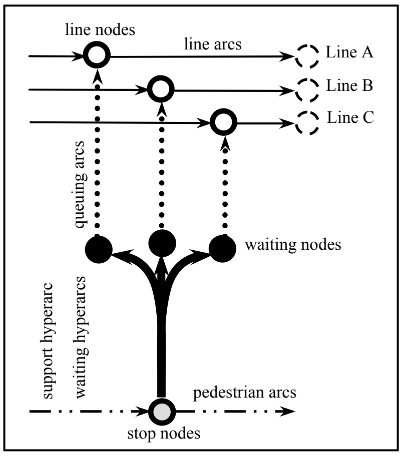

FIGURE 1 – Representation of a stop in the hypergraph.

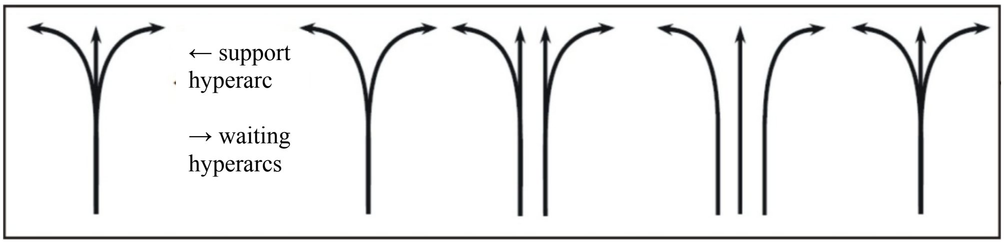

FIGURE 2 – Waiting hyperarcs defining the attractive set associated with the support hyperarc of all available lines

3. The new Stop Model

Adopting the approach of Nguyen and Pallottino (1988), the shortest hyperpath search represents the choice of the best travel strategy, where, at every stop, each attractive line is associated with a probability to board. Probability to board line $l(\pi_l)$ depends, in general, on: its inter-arrival times, its line travel time, which is the expected travel time from the stop to destination, once boarded line l ; inter-arrival times and line times of other attractive lines.

In the following section we highlight how the stop model for hyperpath search should be changed in order to compute probabilities to board (or *diversion probabilities*) and expected waiting times in a number of alternative scenarios.

3.1 Stop model for multimodal transit networks with online information at stops: the uncongested case

Dynamic passengers' information systems are nowadays able to give simultaneously the next departure time for all lines serving a stop. Consequently, it is no longer possible to rely on a stop model based on the assumption that the only information available is the observation of the next line to be served, as in the original works by Spiess and Florian (1989) and Nguyen and Pallottino (1988).

The basic assumptions of the new model are therefore (Gentile *et al.* 2005):

- Passengers arrive randomly at the stops;
- Transit line waiting times are statistically independent with continuous distributions, namely exponential for irregular and uniform for regular services;
- Passengers can retrieve at the stop the actual line waiting times w_l ;
- For each attractive line l serving the stop, passengers evaluate with sufficient accuracy the expected travel time from the stop to destination, s_l .

In this context, the rational user would choose the line l associated with the minimum total travel time:

$$w_l + s_l = \min\{w_h + s_h : h \in L\} \tag{3.1}$$

Any value w_l showed by the count-down system is a stochastic variable of which the p.d.f. $f_l(w)$ is known.

Therefore, when a carrier of line l arrives at time w , the boarding condition not only requires that line h has not arrived before w , but, if h is a faster line, it must not even arrive before $w-(s_l - s_h)$. Given the independency of headways of different lines (and, consequently, of waiting times) the joint probability that the total time of line l is shorter than or equal to any other line, when the actual waiting time displayed for the first service of l is, w is:

$$\prod\nolimits_{h \in L_n^i \setminus \{l\}} \Pr(w_h + s_h \geq w + s_l) \tag{3.2}$$

Where $L_n^i = \{1,2,...,n\}$ is the set of lines serving the generic stop i. Probability to board line l is, hence, computed as:

$$\pi_l = \int_0^{+\infty} \left\{ f_l(w) \cdot \prod\nolimits_{h \in L_n^i \setminus \{l\}} \Pr(w_h + s_h \geq w + s_l) \right\} dw \tag{3.3}$$

If $f_l(w) \cdot \prod\nolimits_{h \in L_n^i \setminus \{l\}} \Pr ob(w_h + s_h \geq w + s_l)$ is interpreted as the p.d.f. of the waiting time at the stop, conditional to boarding line l, the expected waiting time conditional to board the same line is:

$$EW_l = \int_0^{+\infty} w \cdot \left\{ f_l(w) \cdot \prod\nolimits_{h \in L_n^i \setminus \{l\}} \Pr(w_h + s_h \geq w + s_l) \right\} dw \tag{3.4}$$

Therefore, the expected waiting time at the considered stop i is given by the summation of EW_h over all the attractive lines:

$$EW = \sum_{h \in L_n^i} EW_h \tag{3.5}$$

While the expected travel time once boarded is:

$$ES = \sum_{h \in L_n^i} \pi_h \cdot s_h \tag{3.1}$$

As pointed out by Nökel and Wekeck (2009), in this context, given that a generic stop i is served by L lines, the attractive set L^* consists of all lines which are "*optimal at least in the extreme case that a vehicle of the line arrives after zero wait time, while the wait time for any other line amounts to its full headway*". To determine L^* we can therefore apply the approach proposed by Gentile *et al.* (2005), computing in turn ET^{L_j}, for $j = 1,...,n$ and then choose $L^* = \arg\min \{ET^{L_j} : j = 1,...,n\}$.

3.2 Stop model for multimodal transit networks when congestion occurs

The previous sub-section presented a stop model for multimodal transit networks (regular and irregular services), where online information is provided at the stops.

A second important issue to address is the formation and dispersion of queues, in case not all the passengers can board the first approaching carrier because of capacity constraints.

In order to properly reproduce this phenomenon, a dynamic model has to be built where passengers' demand, line frequency and travel times depend on the time of the day (Trozzi *et al.* 2009).

Therefore, in the following we introduce the variables utilized to describe the stop model.

$\lambda_l(t)$	Frequency of line l departing from the terminal at time t
$T_l^i(t)$	Instant when the carrier of line l departed from the terminal at time t reaches stop i
$\varphi_l^i(t)$	Frequency of line l at stop i at time t; it is the inverse of the headway expected value
$Q_l^i(t)$	Available capacity on line l at stop i at time t

$N_l^i(t)$	Number of passengers waiting in a queue to access line l at stop i at time t (namely, the number of passengers exceeding the available capacity of the approaching carrier)
$M_l^i(t)$	Number of passengers waiting for service at stop i at time t that are able to board the next carrier of line l
$W_L^i(t)$	Expected waiting time of the hyperarc identified by the set of attractive lines L serving stop i at time t
$\pi_{l\in L^i}(t)$	Probability of boarding line l among the attractive set L at stop i at time t ; equal to the internal coefficients of the corresponding hyperarc
$e_l^i(t)$	Flow of passengers on line l approaching stop i at time t
Φ_l	Vehicle capacity of line l
$P_l^i(t)$	Probability to be able of boarding the next vehicle of line l approaching stop i at time t
$\psi_l^i(t)$	Effective frequency of line l at stop i at time t

The temporal profile $\varphi_l^i(t)$ of the frequency at a given stop can be determined on the basis of the temporal profile $\lambda_l(t)$ of the frequency departing from the terminal and of the travel times on the network by applying a basic dynamic formula:

$$\varphi_l^i(T_l^i(t)) = \lambda_l(t)/(dT_l^i(t)/dt) \tag{3.7}$$

The available capacity is then given by:

$$Q_l^i(t) = \Phi_l \cdot \varphi_l^i(t) - e_l^i(t) \tag{3.8}$$

Therefore, we have:

$$M_l^i(t) = Q_l^i(t)/\varphi_l^i(t) \tag{3.9}$$

Based on the above equations, all variables are time-varying, including the main characteristic of the headway distribution, which is the frequency. However, it is very difficult to consider this feature in the computation of expected waiting time and line probabilities, which require integration over time. We will henceforth refer to the values of all the variables at the instant when the passenger reaches the stop and consider them to be constant during the wait.

Now that the basic formulation is introduced, we will focus, for the sake of clarity, on a single line stop. In this case the waiting hyperarc collapses into a normal arc and probability to board the only attractive line is equal to one. Consequently, we are only interested in describing the variation of waiting times due to temporary oversaturation.

4. The stop model for a single line

4.1 Mingling queue

If the stop is designed as a platform (namely, in the underground case), passengers *mingle* on it and cannot respect any boarding order, as they do not know exactly where the carrier is going to stop. Thus, if a passenger stands just in front of the point where the doors will open, then he will probably board on the next carrier approaching the stop. But if he stands far from that point, he may have to wait for a subsequent arrival. Therefore, the waiting time does not decrease only because a passenger has already missed one or two runs due to congestion. These are the same assumptions made in Schmoecker *et al.* (2008). On such basis, a passenger has the same probability of boarding at each carrier arrival, that can be evaluated as:

$$P_l^i(t) = M_l^i(t)/(M_l^i(t) + N_l^i(t)) \tag{4.1}$$

Therefore, passengers perceive a service with the following *effective frequency*:

$$\psi_l^i(t) = \varphi_l^i(t) \cdot P_l^i(t) \tag{4.2}$$

Given the hypothesis of exponential arrivals with rate $\varphi_l^i(t)$ it can be proved by simulation that the p.d.f. of the waiting time for the generic line l is still exponential with a rate equal to the effective frequency:

$$f_{l,IRR}^i(w,t) = \begin{cases} \psi_l^i(t) \cdot e^{-\psi_l^i(t) * w}, & \text{if w} \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{4.3}$$

Hence the expected waiting time is:

$$W_{l,IRR}^i(t) = \frac{1}{\psi_l^i(t)} = \frac{1}{\varphi_l^i(t)} + \frac{N_l^i(t)}{Q_l^i(t)} \tag{4.4}$$

If we assume that the same rule holds true also in the deterministic case, the p.d.f. of the waiting time for the generic line l is still uniform with a rate equal to the effective frequency:

$$f_{l,REG}^i(w,t) = \begin{cases} \psi_l^i(t), & \text{if } 0 \leq w \leq \dfrac{1}{\psi_l^i(t)} \\ 0, & \text{otherwise} \end{cases} \tag{4.5}$$

$$W_{l,REG}^i(t) = \frac{1}{2 \cdot \psi_l^i(t)} = \frac{1}{2 \cdot \varphi_l^i(t)} + \frac{N_l^i(t)}{2 \cdot Q_l^i(t)} \tag{4.6}$$

This is consistent with the intuition that calls for scaling the frequency by the probability to be able of boarding. Moreover the waiting time can be decomposed into a *service time* and a *queuing time*.

Clearly, when the congestion level is extremely high the queue may spill back from the platform and can assume FIFO behaviour; this phenomenon can be modelled by a link to access the platform with a final bottleneck.

4.2 FIFO queue

If the stop is designed so that passengers have to respect a FIFO service order, then the passenger who arrives first at the stop is the first to board. Hence, the $N_l^i(t)$-th queuing passenger will have to wait for the $k_l^i(t)$-th arrival, when the service will be truly available to him:

$$k_l^i(t) = 1 + INT[N_l^i(t)/M_l^i(t)] \tag{4.7}$$

where *INT[x]* is the first integer not smaller than *x*.

Headways are independently and equally distributed according to an exponential distribution of parameter $\varphi_l^i(t)$. Therefore, the waiting time before the $N_l^i(t)$-th arrival is distributed according to a Gamma (α, β), where parameter $\alpha = n$ and $\beta = 1/\varphi_l^i(t)$.

$$f_{l,IRR}^i(w,t) = \begin{cases} \varphi_l^i(t)^{k_l^i(t)} \cdot e^{-\varphi_l^i(t) \cdot w} \cdot \dfrac{w \cdot [k_l^i(t) - 1]}{[k_l^i(t) - 1]!}, & \text{if } w \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{4.8}$$

Hence the expected waiting time is:

$$EW_{l,IRR}^{i}(t) = \frac{k_l^i(t)}{\varphi_l^i(t)} = \frac{1}{\psi_l^i(t)} = \frac{1}{\varphi_l^i(t)} + \frac{N_l^i(t)}{Q_l^i(t)} \tag{4.9}$$

Consequently, the expected waiting time at the stop is the same as in the mingling case, given by the service time and a queuing time. The variance of the Gamma function is instead lower than the variance of the Exponential function for the same expected value.

On the other hand, if headways are deterministic, the waiting time before $k_l^i(t)$ -th vehicle can be computed as the waiting time before the first arrival (uniformly distributed) plus the (deterministic) waiting time from the second up to the $k_l^i(t)$ -th vehicle:

$$f_{l,REG}^{i}(w,t) = \begin{cases} \varphi_l^i(t), & \text{if } \dfrac{[k_l^i(t)-1]}{\varphi_l^i(t)} \leq w \leq \dfrac{k_l^i(t)}{\varphi_l^i(t)} \\ 0, & \text{otherwise} \end{cases} \tag{4.10}$$

Therefore the expected waiting time is:

$$EW_{l,REG}^{i}(t) = \frac{1}{2 \cdot \varphi_l^i(t)} + \frac{[k_l^i(t)-1]}{\varphi_l^i(t)} \tag{4.11}$$

5. The stop model for multiple lines

When passengers wait at a stop served by several attractive lines, they do have the possibility to choose the best one and, therefore, their travel behaviour can be regarded as strategic.

In this case, not only we are interested in understanding which is the exact boarding rule characterizing different dynamic phenomena of congestion. In fact, we also want to represent comparisons between regular and irregular services and the way online information at stops affects users' route choice. In the following, different models are proposed depending on the layout of the stop and on the congestion level.

5.1 Mingling queue

Let us firstly consider a stop where passengers *mingle* while waiting to board the first attractive line. When the available capacity of approaching carriers is lower than the number of passengers at the stop willing to board a line, the waiting time has a continuous distribution with rate equal to the effective frequency computed by equation (4.2).

The second assumption we now have to take into account is that passengers are provided at the stop with online information regarding the actual waiting time before the first arrival of all attractive lines.

On this basis it is possible to compute as in Gentile *et al.* (2005) the probability of line l to be the chosen one among all the attractive lines, given the capacity constraints due to transit congestion.

Therefore, the internal coefficient of the corresponding *hyperarc* is equal to the probability:

$$\pi_l^i(t) = \int_0^{+\infty} \left\{ f_l^{\,i}(w,t) \cdot \prod_{h \in L_n^{REG} \setminus \{l\}} \overline{F}_h^{REG} \cdot \prod_{h \in L_n^{IRR} \setminus \{l\}} \overline{F}_h^{IRR} \right\} dw \tag{5.1}$$

While the expected waiting time is:

$$EW_l^i(t) = \int_0^{+\infty} w \cdot \left\{ f_l^i(w,t) \cdot \prod_{h \in L_n^{REG} \setminus \{l\}} \overline{F}_h^{REG} \cdot \prod_{h \in L_n^{IRR} \setminus \{l\}} \overline{F}_h^{IRR} \right\} dw \tag{5.2}$$

In the previous two equations, if line l is irregular, $f_l^i(w,t)$ is given by equation (4.3), while, if it is a regular one, then $f_l^i(w,t)$ is given by equation (4.5).

$\overline{F}_h^{REG}$ and $\overline{F}_h^{IRR}$ rrespectively represent the complement of the cumulative distribution function (c.d.f) of the waiting time before being able to board a regular or an irregular line. It is possible to compute them in the following way:

$$\overline{F}_h^{REG} = \Pr\{w_h + s_h(t) \geq w + s_l(t)\} = \overline{F}_h^{REG}(w + s_l(t) - s_h(t))$$

$$= \begin{cases} 1, & \text{if } \ w < s_h(t) - s_l(t) \\ 0, & \text{if } \ w > \dfrac{1}{\psi_h^i(t)} + s_h(t) - s_l(t) \\ 1 - \psi_l^i(t) \cdot [w + s_l(t) - s_h(t)], & \text{otherwise} \end{cases} \tag{5.3}$$

$$\overline{F}_h^{IRR} = \Pr\{w_h + s_h(t) \geq w + s_l(t)\} = \overline{F}_h^{IRREG}(w + s_l(t) - s_h(t))$$

$$= \begin{cases} e^{-\psi_h^i(t) \cdot [w + s_l(t) - s_h(t)]} & \text{if } \ w > s_h(t) - s_l(t) \\ 0, & \text{otherwise} \end{cases} \tag{5.4}$$

5.2 FIFO queue

A different case is the one of transit stops, served by a bunch of attractive lines, where passengers queue until the chosen service becomes truly available to them.

In this case, the presence of online information, not only affects diversion probabilities and the expected waiting time, but thoroughly changes the way of modelling passengers' behaviour. The following example helps to clarify.

If the transit system is highly crowded, the stops shared by several lines can be designed to have physically separate queues for the different lines. In this case, if no information is provided, each line where $N_l^i(t) > 0$ cannot be considered for a strategic behaviour, since the passenger has to join the corresponding queue as soon as he/she reaches the stop and then it may be difficult for him/her to change row. This case thus reduces to that of a FIFO queue for a single line that we already have examined in section 4.2. By contrast, if at a multiple line stop passengers wait together in a single queue their behavior can be regarded as strategic (Trozzi *et al.* 2009).

However, if passengers were provided with information at the stop regarding the arrival times of carriers and the available capacity on-board (or the passenger has sufficient experience to guess it), whichever is the stop layout (separate or mixed queues) we can always model passengers' behaviour through hyperpaths. Indeed, the information anticipates the event of vehicle's arrival to the moment when the passenger reaches the stop; hence, his optimal travel strategy comes true at this instant, when he/she actually chooses which line to board taking into account the number of passengers waiting for the different lines.

For the sake of simplicity, we will refer in the following only to separate queues, as the model for mixed queues can easily be developed on this basis.

As in the mingling case, the stop model in case of FIFO separate queues can be developed by applying the general formulas due to Gentile *et al.* (2005).

Case 1 is when l is an irregular line.

In this case, when computing boarding probabilities (equation 5.1) and expected waiting times (equation 5.2), the complements of c.d.f. assume the following expressions:

$$\overline{F}_h^{REG} = \Pr\left\{ w_h \geq w + s_l(t) - \frac{k_h^i(t)-1}{\varphi_h^i(t)} - s_h(t) \right\}$$

$$\overline{F}_h^{REG} = \begin{cases} 1, & \text{if } \alpha_{hl} < {k_h^i(t)-1}\big/{\varphi_h^i(t)} \\ 0, & \text{if } \alpha_{hl} > {k_h^i(t)}\big/{\varphi_h^i(t)} \\ 1-\psi_l^i(t)\cdot\left[\alpha_{hl} - {k_h^i(t)-1}\big/{\varphi_h^i(t)}\right], & \text{otherwise} \end{cases} \tag{5.5}$$

where we recall that ϑ is a function such that:

$$\alpha_{hl} = w + s_l(t) - s_h(t) - \frac{k_h^i(t)}{\varphi_h^i(t)}$$

$$\overline{F}_h^{IRR} = \Pr\{ w_h \geq w + s_l(t) - s_h(t) \} = \begin{cases} \displaystyle\sum_{j=0}^{k_h^i(t)-1} \frac{\left[\varphi_h^i(t)\cdot\beta_{hl}\right]^j \cdot e^{\left[\varphi_h^i(t)\cdot\beta_{hl}\right]}}{j!}, & \text{if } \beta_{hl} \geq 0 \\ 1, & \text{otherwise} \end{cases} \tag{5.6}$$

where $\beta_{hl} = w + s_l(t) - s_h(t)$.

Case 2 is when l is a regular line.

In this case, when computing equations (5.1) and (5.2), the complements of c.d.f. are:

$$\overline{F}_h^{REG} = Pr\left\{ w_h \geq \frac{k_l^i(t)-1}{\varphi_l^i(t)} + w + s_l(t) - \frac{k_h^i(t)-1}{\varphi_h^i(t)} - s_h(t) \right\}$$

$$\overline{F}_h^{REG} = \begin{cases} 1, & \text{if } \gamma_{hl} < {k_h^i(t)-1}\big/{\varphi_h^i(t)} \\ 0, & \text{if } \gamma_{hl} > {k_h^i(t)}\big/{\varphi_h^i(t)} \\ 1-\psi_l^i(t)\cdot\left[\gamma_{hl} - {k_h^i(t)-1}\big/{\varphi_h^i(t)}\right], & \text{otherwise} \end{cases} \tag{5.7}$$

where $\gamma_{hl} = \dfrac{k_l^i(t)-1}{\varphi_l^i(t)} + w + s_l(t) - \dfrac{k_h^i(t)-1}{\varphi_h^i(t)} - s_h(t)$.

$$\overline{F}_h^{IRR} = \Pr\left\{ w_h \geq w + \frac{k_l^i(t)-1}{\varphi_l^i(t)} + s_l(t) - s_h(t) \right\} = \begin{cases} \displaystyle\sum_{j=0}^{k_h^i(t)-1} \frac{\left[\varphi_h^i(t)\cdot\delta_{hl}\right]^j \cdot e^{-\varphi_h^i(t)\cdot\delta_{hl}}}{j!}, & \text{if } \delta_{hl} \geq 0 \\ 1, & \text{otherwise} \end{cases} \tag{5.8}$$

where $\delta_{hl} = w + \dfrac{k_l^i(t)-1}{\varphi_l^i(t)} + s_l(t) - s_h(t)$.

6. Numerical Example and Conclusions

A numerical example is provided here in order to demonstrate the stop models proposed in this paper. We only consider three cases with separate queues where the FIFO rule applies. In the first case we assume no congestion; therefore the first carrier is always available. In the second case, there is congestion and passengers need to wait for the second carrier, while in the last case only the third carrier is available. We assume three separate lines one of which is regular and the other two irregular. The details of the lines are shown in Table 1 below.

Line i	Line frequency φ_i	Regularity	Travel time s_i
1	1/20 min $^{-1}$	regular	30 min
2	1/15 min $^{-1}$	irregular	40 min
3	1/10 min $^{-1}$	irregular	45 min

TABLE 1. Line attributes

The probability to board the first available carrier for each line, together with the expected waiting time, line travel time and total travel time are calculated using the formulas developed in section 5.2. The results of the computation are shown in the following table.

	π_1	π_2	π_3	*EW*	*ES*	*ET*
$k = 1$[1]	0.834	0.131	0.035	9.049	31.831	39.470
$k = 2$	0.044	0.433	0.523	14.480	42.179	56.659
$k = 3$	0.002	0.389	0.61	24.48	43.023	67.508

TABLE 2. Boarding probabilities and expected waiting times

Line 1, which is the service with the longest average headway, is the most attractive only in the case without congestion. By contrast, if passengers have to wait for the second or third carrier, line 3, which is the one with the shortest average headway, becomes the most attractive line.

The results obtained in this study, where lines with different level of regularity are considered together, are being compared to results of the paper by Gentile *et al* (2005), where for the case without congestion, only lines with the same level of regularity were considered together. The results are compared in table 3.

Line regularity	π_1	π_2	π_3	*EW*	*ES*	*ET*
Irr-Irr-Irr	0.587	0.257	0.156	6.81	34.92	41.73
Re-Irr-Irr	0.834	0.131	0.035	7.63	31.83	39.47
Re-Re-Re	0.805	0.160	0.035	7.27	32.12	39.39

TABLE 3. Results comparison in the case of no congestion

In the absence of congestion, it can be seen that there is an improvement in the expected total travelling time (i.e. *ET*) of 2.26 minutes in respect to the case where all lines are irregular. However, there seems to be not a great improvement when all the lines are regular

[1] This is the case where there is no congestion and the first carrier is always available

compared to the case considered here. This can be due to the fact that the only regular line has the lowest travelling time.

In this study, we have shown how regularity and the congestion level affect route choice.

The next steps of our research will address the possibility to include in the same route choice model also timetable-based services and, then, embedding this model in an assignment procedure.

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