Sensitivity of Equivalent Circuits on the Extraction Procedure

Rodolfo Araneo [1], Sami Barmada [2], and Antonino Musolino [2]

[1] Department of Electrical Engineering, University of Roma La Sapienza, Rome, Italy
rodolfo.araneo@uniroma1.it

[2] Department of Electrical Systems and Automation, University of Pisa, Pisa, Italy
sami.barmada@dsea.unipi.it

Abstract: The present paper proposes an approach to evaluate the “quality” of equivalent circuits of complex devices obtained by a novel technique based on constitutive sub circuits and an Particle Swarm Optimization. In particular the robustness of the obtained circuits is evaluated by a sensitivity analysis, which leads to the identification of the range of variation of its frequency response, since different runs of the global extraction procedure lead to slightly different equivalent circuits (yet topologically coincident). The analysis of the numerical results give an insight on the robustness of as single case and at the same time attest the efficiency of the extraction technique.

Keywords: Equivalent circuit, Finite Integration Technique , Particle Swarm Optimization, wavelets, sensitivity.

1. Introduction

With the increase of system performance requirements, it has become important to provide the designer with accurate models for passive components and interconnects based on data obtained by measurements or electromagnetic fields solvers, typically, in the form of frequency-domain network parameters (S, Y, or Z parameters). Traditionally, these data calls for the extraction of an electrical equivalent circuit model to be inserted inside a time-domain circuit simulator, such as Spice, which is the preferred simulation environment for mixed-signal designers. The construction of physically based models, i.e. a model where each element represents a physically meaningful electrical characteristic of the structure under test, is very difficult and time-consuming, becoming a prohibitive task when the frequencies and the complexity of the object increase. To overcome this problem, several macromodeling techniques have been proposed in the recent years, to systematically extract equivalent structures, based on a pure mathematical fitting approach, which allow the direct insertion of the measured/computed behavior of the structure inside the Spice environment. In these approaches, a number of constraints must be satisfied by the final model, such as casuality, stability and passivity (the most difficult to be guaranteed). In this framework, a novel technique has been recently proposed [1] for an efficient systematic extraction of passive equivalent circuits and it is here revised, through a Particle Swarm Optimization, in order to improve its reliability and efficiency.

Moreover, a sensitivity analysis is performed on the extracted model, to assess both the robustness of the extracted circuit and the reliability of the proposed extraction technique.

To perform such analysis the authors use a technique based on a hybrid wavelet – scattering parameters approach which has been previously formulated to analyze the effects of the parameters and topology uncertainties in power grids when used as a Powerline Communication channel [2].

In particular the computation of a network’s sensitivity (described by the scattering parameters in the wavelet domain), with respect to any component, is performed adopting an adjoint technique. Once the sensitivities are known, an estimate of response's bounds is obtained both in the time and frequency domain.

The sensitivity is calculated by the use of an adjoint technique since it requires a reduced computational overhead while providing equivalent results ([3] – [5]). The wavelet transform is a well known and efficient tool for the transient simulation of linear and non linear networks, while the usefulness of the scattering matrix representation in describing network performances is of common knowledge in the electrical engineering area [6] – [8]. The good interpolating properties of the wavelet functions and the sparsity of the obtained scattering matrices make this approach computationally efficient and accurate.

The result of the analysis gives an insight on the accuracy of the extracted equivalent circuit and on the robustness of the method.

The proposed study is applied here to a spiral inductor as test case.

2. Extraction Technique

To extract the equivalent circuit of the spiral inductor, a novel improved version of the technique described in [1] is applied. The procedure is aimed at synthesizing an equivalent circuit which is not only passive on the whole, but with all passive lumped components, which is more physically meaningful since the structure under test is passive. The technique requires several subsequent steps.

At first a full-wave analysis is conducted to accurately describe the electromagnetic behavior of the structure and, eventually, a deembedding procedure is applied in order to isolate the discontinuity from the remaining part of the structure, as in [1]. The commercial code CST Microwave Studio [9] based on the Finite Integration Technique has been used for the computation of the two-port scattering matrix $\mathbf{S}(\omega)$ of the spiral inductor. Then a check on the accuracy of the computed scattering parameters has been performed to assess the validity of the simulation: in fact, since some structures may present very low losses, the computed scattering matrix may not represent a passive system due to the errors brought into analysis by the numerical approximation, and some corrections could be necessary [1].

Successively the scattering matrix is transformed into the admittance matrix $\mathbf{Y}(\omega)$ on which the Foster canonical representation in terms of a partial fraction expansion can be computed (the method works on the impedance matrix $\mathbf{Z}(\omega)$ representation as well as explained in [1]) as:

$$\mathbf{Y}(s) = \mathbf{A}_0 + s \cdot \mathbf{A}_\infty + \sum_{k=1}^{N_{cp}} \left(\frac{c_k}{s - a_k} + \frac{c_k^*}{s - a_k^*} \right) \mathbf{A}_k + \sum_{k=1}^{N_{rp}} \frac{c_k}{s - a_k} \mathbf{A}_k \tag{1}$$

where a_k are the poles, c_k the residues, N_{cp} and N_{rp} the number of complex conjugate and real-single poles, respectively, $\mathbf{A}_{k,0,\infty}$ are real matrices and s is the Laplace variable. The poles, residues and the matrices must fulfill several requirements [1] to ensure that the admittance matrix is positive real. Through the Vector Fitting methodology [10, 1], the poles, residues and the matrices can be computed, given a prescribed number of complex and real poles: it is observed in [1] that, even if these numbers must be calibrated and customized to the application in hand, the vector fitting is a robust technique which helps somehow to their optimum choice through few attempts.

The synthesis is carried out by means of the constitutive elementary sub-circuits [1] which are connected in parallel in order to reproduce the admittance parameters in the frequency domain. Each pole of the partial fraction expansion is reproduced by means of two sub-circuits with different component values.

After the synthesis of the equivalent circuit, a final optimization was performed on the values of its components [1] through a nonlinear programming problem and a Genetic Algorithm, in the most complex cases, both to correct those components with negative values and to improve the whole accuracy of the circuit response in comparison with the full wave data. In the novel approach, here proposed, this final step is carried out through a Particle Swarm Optimizer, which presents several advantages over the GA and non-liner deterministic techniques.

Inspired by a model of social behavior of bees seeking for food, the Particle Swarm Optimization (PSO) [11,12] is a robust stochastic evolutionary technique which utilizes swarm intelligence to achieve the goal of optimization. Assuming a N-dimensional problem and a swarm composed of M particles, the position vector $\mathbf{x}_t^m(t) = \left[x_{1,t}^m, x_{2,t}^m, \ldots x_{N,t}^m\right]^T$, with $m = 1,2\ldots M$, represent the evolution of a particle of the swarm (a possible solution of the problem) in the solution space at time *t*. Each particle moves inside the solution space with a velocity vector $\mathbf{v}_t^m(t) = \left[v_{1,t}^m, v_{2,t}^m, \ldots v_{N,t}^m\right]^T$, searching for food (the best solution). The core idea of the PSO algorithm is the exchange of information about the global and local best values, that can be done determining the position and the velocity of the particle at the next iteration through the following two updating equations:

$$\begin{aligned} \mathbf{v}_{t+1}^m &= w\mathbf{v}_t^m + c_1\,\mathbf{\Phi}_1\,(\mathbf{p}_L^m - \mathbf{x}_t^m) + c_2\,\mathbf{\Phi}_2\,(\mathbf{p}_\mathbf{G} - \mathbf{x}_t^m) \\ \mathbf{x}_{t+1}^m &= \mathbf{x}_t^m + \mathbf{v}_{t+1}^n \end{aligned} \tag{2}$$

The essence of the entire optimization is the manipulation of the particles' velocities, whose equations must be fully understood: w is the inertia factor, which keeps the particle in its current trajectory, while the last two terms injects deviation according to the distances to the personal $\mathbf{p}_L^m$ and global $\mathbf{p}_\mathbf{G}$ best location, through the cognitive factor c_1 and the social factor c_2, respectively (also called acceleration coefficients). The two diagonal matrices $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ are a set of statistically independent random numbers uniformly distributed in the range [0,1], to inject the unpredictability of the particles' movement. The convergence of the algorithm depends on the proper tuning of the acceleration coefficients [13], and on the boundary conditions used to prevent the explosion of the particles [14]. PSO shares many features with GA but has several advantages: at each iteration, the GA considers three genetic operators (selection, crossover and mutation) while the PSO considers only one simple operator (the velocity updating), thus requiring an easier tuning of the control parameters (whose number is the same between the two algorithms, i.e. four). While in the GA, the stagnation can occur when the individuals assume a genetic code close to that of the fittest chromosome of the population, in the PSO, a proper control of the inertial weight and of the acceleration coefficients allows to find new fittest locations in the solution space. In the present problem, the inertia and acceleration coefficients have been chosen according to [13,14] and the damping boundary conditions [14] have been used to relocate the particles that fly outside the allowable solution space.

3. Sensitivity and Response Bounds Analysis

For the sake of conciseness we report here only the final results of the technique used for the evaluation of the sensitivity and time the time and frequency domain of a general circuit: let $v(t,\gamma)$ be the time domain dynamic of the output voltage as a function of time and of the general uncertain parameter γ; by $\bar{\gamma}$ is indicated the "nominal" value of the parameter γ.

According to eq. (3) the voltage response is given by $v(t,\bar{\gamma})$ plus a sum of m terms $\Delta v_n(t)$, where m is the number of parameters of the equivalent circuits.

$$\begin{aligned} v(t,\gamma) &\cong v(t,\bar{\gamma}) + \sum_{n=1}^{m} \Delta v_n(t) \\ \Delta v_n(t) &= (\gamma_n - \bar{\gamma}_n) \sum_{l=1}^{M}\sum_{i=1}^{M} \frac{\partial \hat{w}_{l,i}}{\partial \gamma_n} \hat{e}_i w_l(t) \end{aligned} \tag{3}$$

In eq. (3) the terms $\hat{w}_{l,i}$ are the elements of matrix $\hat{\boldsymbol{W}}$, the wavelet representation of the voltage transfer function, while the terms $\hat{\boldsymbol{e}}_i$ are the applied voltages provided by the generators. Eq. (3) can be used to estimate the bounds of the response by using the worst case condition. We evaluate the $\Delta v_n(t)$ corresponding to the maximum allowed tolerance of γ_n obtaining:

$$v(t,\bar{\gamma}) - \sum_{n=1}^{m}\left|\max_{\gamma_n}\{\Delta v_n(t)\}\right| \le v(t,\gamma) \le v(t,\bar{\gamma}) + \sum_{n=1}^{m}\left|\max_{\gamma_n}\{\Delta v_n(t)\}\right| \quad (4)$$

By the use of the Fourier transform we obtain:

$$\boldsymbol{V}(j\omega,\gamma) = \sum_{l=1}^{M}\hat{v}_l(\gamma)\boldsymbol{W}_l(j\omega) \cong \boldsymbol{V}(j\omega,\bar{\gamma}) + \sum_{n=1}^{m}(\gamma_n - \bar{\gamma}_n)\sum_{l=1}^{M}\sum_{i=1}^{M}\frac{\partial \hat{w}_{l,i}}{\partial \gamma_n}\hat{e}_i\boldsymbol{W}_l(j\omega) \quad (5)$$

where $\boldsymbol{W}_l(j\omega)$ indicates the Fourier transform of the l^{th} function $w_l(t)$ of the chosen wavelet basis.

The bounds of the amplitude response of the circuit in the frequency domain are calculated by using the worst case condition as for the bounds in the time domain:

$$\left|\boldsymbol{V}(j\omega,\bar{\gamma})\right| - \sum_{n=1}^{m}\max_{\gamma_n}\left\{\left|\Delta\boldsymbol{V}_n(j\omega)\right|\right\} \le \left|\boldsymbol{V}(j\omega,\gamma)\right| \le \left|\boldsymbol{V}(j\omega,\bar{\gamma})\right| + \sum_{n=1}^{m}\max_{\gamma_n}\left\{\left|\Delta\boldsymbol{V}_n(j\omega)\right|\right\} \quad (6)$$

where the terms $\left|\Delta\boldsymbol{V}_n(j\omega)\right|$ are calculated as a function of the sensitivities in the wavelet domain by exploiting the chain rule.

4. Analysis of a Spiral Inductor

The test case presented here is a spiral inductor whose geometry and electrical parameters are shown in Figure 1, while Figure 2 shows the full wave model of the inductor implemented on CST Studio.

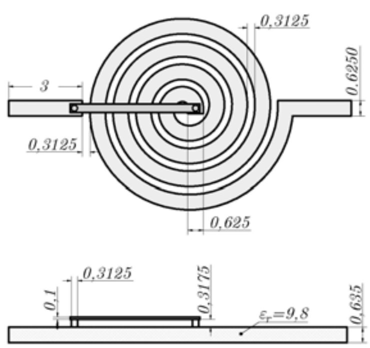

Fig. 1. Geometry of the spiral inductor.

The equivalent circuit according to the procedure summarized in I is composed by a set of parallel branches and transformers as shown in Figure 2.

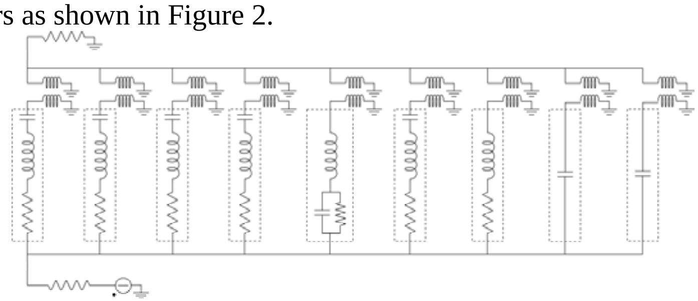

Fig. 2. Topology of the equivalent circuit of the spiral inductor.

Different runs of the extraction procedure will lead to equivalent circuits of the same topology but with different values of the electrical parameters; based on the authors' experience this difference can be quantified with a 2.5% of uncertainty on each electrical parameter, which has been assumed as uncertainty for this test case.

The results of the analysis are shown in Figure 3 and Figure 4, respectively being the time domain (impulse) and frequency domain bounds of the voltage response of the circuit. In the graphs the bold lines represent the calculated bounds, while the grey cloud has been obtained by running 1000 simulations with a random variation of the parameters value within the range previously defined. The cloud has been added only to show the accuracy of the bounds calculation technique.

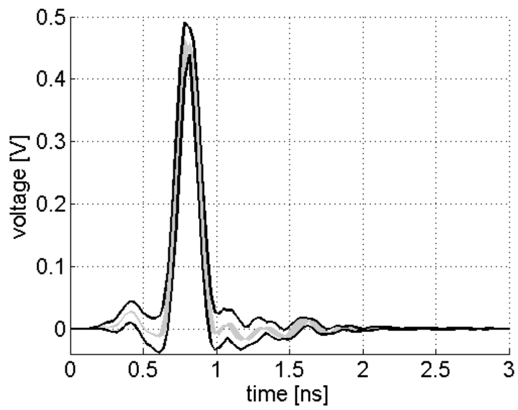

Fig. 3. Time domain bounds of the impulse response

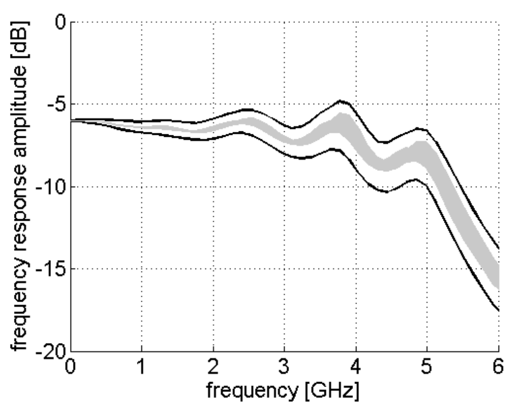

Fig. 4. Frequency domain bounds

A close look at Figures 4 and 5 show both the robustness of the extraction technique (hence of the obtained equivalent circuit) and the accuracy of the sensitivity analysis performed to evaluate the above mentioned characteristics. In particular the frequency response amplitude shows a variation of at most 3dB, which is extremely satisfying.

5. Conclusion

An improved fitting-based procedure for the systematic extraction of equivalent circuits, with a new final Particle Swarm Optimization, has been presented. Through a hybrid wavelet-scattering parameters approach, a sensitivity analysis has been conducted on the resulting circuit to asses its robustness, since different runs of the extraction procedure may lead to slightly different circuits. The preliminary results confirm the effectiveness of the proposed methodology for the extraction of reliable circuits.

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