The Stylised Facts of Stock Price Movements

Scott THOMPSON*

Abstract

The stylised facts of stock price movements are statistical properties expected to be present in any sufficiently long series of observed stock prices. The below is a review of the current literature on the presence and identification of these stylised facts in observed price series for stocks listed in both developed and developing markets. Frequently identified stylised facts include the heavy tailed distribution of observed stock price returns, the significant autocorrelation of absolute and squared observed returns ("volatility clustering"), the slow decay of the autocorrelation function of absolute observed returns, and the Taylor and Leverage effects.

* thompsscot@myvuw.ac.nz Essay submitted towards FINA411

1 Introduction

More than a century ago Bachelier recognised that the true process determining stock prices was an enigma and proposed that price variations between transactions could be considered independent random variables and could be modelled with a purely random statistical process. This proposal was later regarded as the first complete expression of the Random Walks theory for financial security prices (Fama, 1965a). To allow him to invoke the central limit theorem and derive a model where daily, weekly, monthly, etc returns were independently and identically normally distributed, Bachelier also assumed that these independent random price changes were infinitely frequent and were identically distributed with finite variance. Fifty years later, Bachelier's proposal received widespread attention when Osborne (1959) independently derived Bachelier's model. A vast literature on the Random Walks theory, and on methods of statistically modelling stock price returns in light of it, subsequently emerged (Fama, 1965b).[1]

Over the last half century there has been a proliferation of statistical models for stock returns. While they vary substantially in their complexity, these models share a common purpose: to approximate the behaviour of the unobservable data generating process (DGP) that determines observed stock prices. Assessing the adequacy of this approximation is where the "stylised facts" of stock price movements come in. A stylised fact is a statistical property that we expect to present itself in any series of observed stock prices or returns (the outputs from the

[1] Please see Fama (1965a and b) for a comprehensive explanation of the motivations for the Random Walks theory for security prices, and a review of the origins of the theory.

true DGP) (Taylor, 2005; Cont, 2001). Thus, if a model provides a reasonable approximation of the true DGP for a stock price, the price movements it fits should exhibit these stylised facts. With this motivation in mind, what follows below is a review of the current literature on the presence of and identification of stylised facts.

Following the approach of Taylor (2005), Pagan (1996) and R Cont (2001), calendar effects[2] are not considered to be stylised facts and are excluded from the review. Also, the behaviour of (continuously compounding) stock returns is considered, as opposed to the behaviour of stock prices themselves. This is consistent with the majority of the relevant literature.

The 1 period (potentially a minute, a week, a month, etc) return is defined as:

$$R_t = \ln S_t - \ln S_{t-1}$$

where S_t and S_{t-1} are stock prices (potentially adjusted for dividends) at times t and $t-1$ respectively.

It is also worth mentioning that in this review non-parametric methods for identifying stylised facts are emphasised, as opposed to parametric methods. Parametric methods require the assumption of a specific functional form for the (unknown) DGP. The motivation for focusing on non-parametric methods is to minimise the assumptions the presented results are contingent on. This reduces the risk of our findings simply reflecting the

[2] Calendar effects are anomalous patterns in stock prices and returns that occur regularly with calendar events. For example there is evidence that returns tend to be higher leading up to market holidays (Taylor, 2005)

assumptions made (for example the specific functional form assumed) as opposed to the characteristics of the data itself.

2 Non-normality of returns

The most prevalent stylised fact of stock returns is the peaked and heavy tailed nature, and hence non-normality, of the empirical distribution of daily stock returns. However, non-normality appears to be less pronounced in empirical distributions of monthly returns series.

2.1 Techniques for identifying kurtosis of returns

A distribution is said to have heavy tails and be peaked when it assigns greater probability density to extreme values and values near the mean than would the normal distribution. This implies that extreme values and values close to the mean are more probable than under the normal distribution. It is well accepted that empirical distributions of most daily stock return series tend to exhibit these characteristics (Pagan, 1996; Taylor, 2005; R Cont, 2001).

Summary statistics can be used to identify the presence of heavy tails and "peakedness". Estimated kurtosis is a common example of such a statistic.

The sample Kurtosis of returns can be defined as:

$$k = \frac{1}{n-1} \sum_{t=1}^{n} \frac{(R_t - \overline{R})^4}{s^4}$$

$$s^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left(R_t - \overline{R}\right)^2 \text{ and } \overline{R} = \frac{1}{n}\sum_{t=1}^{n} R_t$$

where n is the number of observations in the sampled returns series.

For a random sample from a normal distribution the population value of kurtosis is 3, and the above sample estimator has a standard error of:

$$se(k) = \sqrt{\frac{24}{n}}$$

Under the null that the distribution randomly sampled from is normal, the test statistic:

$$\frac{\sqrt{n}(k-3)}{\sqrt{24}} = \frac{k-3}{se(k)}$$

is asymptotically distributed as standard normal. If a distribution has population kurtosis of more than 3 it exhibits positive excess kurtosis, such a distribution is called “leptokurtic”. A leptokurtic distribution exhibits peakedness and heavy tails.

Graphs, as opposed to summary statistics, can also provide a simple way to identify leptokurtosis. For many daily returns series peakedness and heavy tails are readily apparent in a simple relative frequency histogram overlaid with the normal density that corresponds to the estimated mean and variance of the returns series. Alternatively, non-parametric kernel based estimators can be used to estimate the density of standardised returns, and these density estimates can be compared to the density of the standard normal distribution to identify the presence of heavy tails and peakedness. It is common to use a standard normal distribution kernel (Pagan, 1996; Taylor, 2005), leading to the density estimator:

$$f(R) = \frac{1}{n \times h} \sum_{t=1}^{n} \phi\left(\frac{R - R_t}{h} \right)$$

where $\phi(.)$ is the density of the standard normal distribution, n is the number of observations in the sampled returns series, and the smoothing parameter h is often chosen to be:

$$0.9 \times \hat{\sigma}_R \times n^{-0.2} \text{ , where } \hat{\sigma}_R = \frac{1}{n} \sum_{t=1}^{n} \left(R_t - \overline{R} \right)$$

So-called "semi-non-parametric methods" have also been proposed to estimate the density of returns (see examples in Pagan, 1996 and R Cont, 2001). However, these require (admittedly limited) assumptions on the form of the density of returns. Attempts to classify the distribution of tail events for stock returns more precisely have also been made using extreme value theory. However, many of the relevant theorems require the questionable (see below) assumption that returns are independent across time (R Cont, 2001).

2.2 The Kurtosis of Daily Returns

As early as the 1960s the leptokurtic nature of daily stock returns had been identified. Fama (1965a) utilised a data set consisting of the daily returns between 1957 and 1962 of the 30 stocks then included in the DJIA (Dow Jones Industrial Average) and identified leptokurtosis in the observed distribution of returns for every stock. Fama (also citing earlier work) claimed that the presence of leptokurtosis in the empirical distribution of daily stock returns was "indisputable" (Fama, 1965a). This claim has remained well accepted (Pagan, 1996; R Cont, 2001). Taylor (2005), for example, cites leptokurtosis as featuring in almost any series of daily stock returns. To illustrate this Taylor utilises a data set of daily returns between the 1980s and 1990s for the S&P500, FT 100 and Nikkei 225 and a selection of frequently traded NYSE and LSE listed stocks.[3] He shows that the sample kurtosis for all of these data sets is at least 10 standard errors (based on the above definition) greater than 3.

Similar results are found when longer time series are used. For example Ding, et al (1993) utilise a series of daily returns for the S&P500 spanning the period from 1928 to 1991 to estimate kurtosis of 25.42. Ding and Granger (1996) present similar results for the Nikkei index between 1970 and 1992. Such excess kurtosis is also apparent in studies that utilise more recent returns series. For example, Karoglou (2010) identifies significant excess kurtosis in the empirical distribution of daily returns of market indexes in 27 OECD countries between 1994 and 2006. Doan, et al (2010) present similar findings of excess kurtosis for the daily returns of 25 US and 25 Australian stock portfolios constructed by size for a period spanning from 1992 to 2007.

The findings described above are far from unique and it has become expected that almost any series of daily stock returns will exhibit leptokurtosis and hence be non-normal (Taylor; Andersen et al, 2001; R Cont, 2001). New Zealand index returns are not exempt; Yu (2002) identifies significant leptokurtosis of

[3] Taylor removes the week of the 87 crash from these series.

daily NZSE40 returns between 1980 and 1998. Nor are daily index returns for emerging markets in Latin America, Asia (Aggarwal et al, 1999) and Africa (Youwei et al, 2010). The evidence of leptokurtosis appears even more extreme for intra-daily returns (see for example Areal and Taylor, 2002; Taylor, 2005; R Cont, 2001).

2.3 The Kurtosis of Monthly Returns

However, monthly returns generally appear to exhibit weaker leptokurtosis than do daily or intra-daily returns (See for example Richardson and Smith, 1993; Affleck-Graves and McDonald, 1989; Taylor, 2005; R Cont, 2001). Recent evidence of this is provided by Schrimpf (2010), who identifies excess kurtosis of monthly index returns in the US, UK, Japan, German and France between 1973 and 2007 that is below that typically observed for daily returns.

2.4 Flaws of Conventional Tests for Leptokurtosis and Proposed Solutions

It is worth noting that Pagan (1996) identifies an issue with relying on the above conventional test statistics to identify leptokurtosis (and hence non-normality) of returns. Such test statistics are appropriate to test the null that individual stock returns (R_t) are independently (across time) normally distributed. Accordingly, tests for leptokurtosis/non-normality based on such statistics are not robust to dependence over time between returns (R_t). The assumption that stock returns are independent across time is questionable given the significant autocorrelation of nonlinear transformations of returns present in most empirical stock return series (see below). However, applying alternative tests for the presence of leptokurtosis which are robust to such return dependence to data sets of monthly and daily NYSE returns between 1928 and 1989 Pagan (1996) still rejects the null hypothesis of normality.

More recently, Kim and White (2004) provided a different criticism; they argued that conventional sample kurtosis statistics are extremely sensitive to "outlier values" as such statistics raise these outliers to the 4^{th} power. They provide a survey of outlier robust quantile based kurtosis statistics and apply these to a data set of daily S&P500 returns between 1982 and 2001. Their statistics identify much milder leptokurtosis than traditional sample kurtosis statistics. They argue that traditional sample kurtosis statistics should be viewed with scepticism and that the "stylised facts [that stock returns exhibit severe excess kurtosis] might have been accepted too readily" (Kim and White, 2004, p.56).

While it is clear that extreme values will have a large impact on conventional kurtosis statistics, their conclusion is questionable. Taylor (2005) for example omitted the week of the 87 stock market crash from his data sets (which cover a similar period to that covered by Kim and White's data set) and still identified extremely significant leptokurtosis in numerous market indices (including the S&P500) and stocks. Also, whether extreme return values (not caused by recording errors) should even be considered "outliers" is questionable. They are still outputs from the true DGP for stock returns, and their presence should be relevant to the modelling choice. As a result of these concerns, there does not appear to be evidence that Kim and White's above conclusion has received wide acceptance.

3 Autocorrelation of Returns

3.1 Absence of Autocorrelation of Daily Returns

Another widely accepted stylised feature of daily returns for liquid stocks is the absence of significant (linear) autocorrelation of returns for all lags (Pagan, 1996; Taylor, 2005; Ding et al, 1993; R Cont, 2001). The sample autocorrelation at lag, τ, can be defined as:

$$\rho_{\tau,R} = \frac{\sum_{t=1}^{n-\tau}\left(R_t - \overline{R}\right)\left(R_{t-\tau} - \overline{R}\right)}{\sum_{t=1}^{n}\left(R_t - \overline{R}\right)^2}$$

Taylor, utilising this definition calculates autocorrelations for lags 1 to 30 using the data set of indices and individual stocks described above. He finds more than 90% of the autocorrelation estimates lie between -0.05 and 0.05. Ding et al (1993) also find only negligible autocorrelation of daily S&P 500 returns for lags between 1 and 100 days when utilising the above described data set.

3.2 Autocorrelation of Returns in Illiquid Markets

Significant first order autocorrelation of returns does seem to appear when indices for smaller markets outside the US, UK and Japan are considered. Bailey and Chung (1995) report significant first order correlation of returns for a 44 security index on the Mexican stock exchange between 1986 and 1994 also, Aggarwal et al (1999) cite the appearance of autocorrelation for some markets in Latin America and Asia between 1985 and 1995. For New Zealand, Yu (2002) reports a "not negligible" first order autocorrelation of 0.281 for daily NZSE40 returns between 1980 and 1998. These results at first appear inconsistent with those found by Taylor etc for larger markets. However, the presence of illiquid stocks with stale prices in these indices may explain such findings.[4] As Taylor (2005) cites, as early as the 1970s it had been identified that the presence of illiquid stocks in a portfolio can cause that portfolio's daily returns to exhibit positive first order autocorrelation. Illiquid stock prices tend to reflect common information later than liquid stock prices (as there is typically a larger gap between the release of information and a subsequent trade). Hence, common information may be incorporated into the prices of the component stocks in the portfolio on different days, leading the positive autocorrelation.

3.3 Autocorrelation of Intra-Daily Returns

Intra-daily returns on liquid stocks for periods of more than 20 minutes typically do not exhibit autocorrelation (Taylor,

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[4] For the NZSE40 there appears to be evidence of the presence of stale prices over the period of Yu's study (see Cao et al, 2009).

2005; R Cont, 2001). However, for shorter periods, distortions introduced by market microstructure can lead to negative estimates of autocorrelation (see below). For example, using S&P100 tick by tick data for the month of February 2002 Bandi and Russell (2006) find negative first order autocorrelation of returns for almost all stocks, most of these autocorrelation estimates are highly statistically significant. Andersen et al (2001) find similar results using 5 minute returns for all firms included in the DJIA30 between 1993 and 1998. Such negative first order autocorrelation estimates are typically attributed to "bid-ask bounce" or other sources of "microstructure noise" (Bandi and Russell, 2006; Taylor, 2005; Andersen et al, 2001; R Cont, 2001).

Bid-ask bounce occurs when bid and ask prices do not change and as buy and sell market orders are placed the last price "bounces" between the bid and the ask price, leading to negative first order correlation of short period returns (calculated using last prices). Another related issue is the presence of errors in intraday price data sets. Errors are more likely in intra-daily price data sets as opposed to daily price data sets simply because information is being collected more frequently. Also, given that the sample variance of returns will be smaller for intra-daily returns, outlier errors (e.g. those that arise when a decimal is misplaced) will have a greater impact on estimated autocorrelation (and kurtosis) statistics (Taylor, 2005). Accordingly, our "stylised facts" of negative first order autocorrelation of returns and extreme leptokurtosis of returns for very short periods may simply reflect market microstructure noise and recording errors. A developing area of the literature focuses on identifying the component of observed variance of intra-daily returns that can be attributed to such microstructure noise (see Bandi and Russel, 2006).

4 Autocorrelation of Squared and Absolute Returns

Although stock returns themselves typically do not exhibit autocorrelation, they certainly do not appear independent. The autocorrelation of absolute and squared returns are significant for many lags for most daily stock return series.[5] It is this tendency for periods of extreme and slight absolute returns or squared returns to persist that is commonly referred to as "volatility clustering".

Ding et al (1993) cite that under the null of *iid* returns the asymptotic standard error of the autocorrelation of absolute and squared returns (assuming returns exhibit finite variance and fourth moments respectively[6]) is:

$$\frac{1}{\sqrt{n}}$$

Using this asymptotic standard error and a data set of daily returns on the S&P500 between 1928 and 1991, they present significant (at at least the 5% level) autocorrelation estimates for

[5] Independence requires $E\left(g(R_t)h(R_{t-k})\right)=E\left(g(R_t)\right)\times E\left(h(R_{t-k})\right)$ which implies

$$Cov\left(g(R_t),h(R_{t-k})\right)=E\left(g(R_t)h(R_{t-k})\right)-E\left(g(R_t)\right)\times E\left(h(R_{t-k})\right)$$
$$=E\left(g(R_t)\right)\times E\left(h(R_{t-k})\right)-E\left(g(R_t)\right)\times E\left(h(R_{t-k})\right)=0$$

for any measureable functions *g(.)* and *h(.)*. Hence, the presence of correlation between absolute and squared returns implies returns cannot be independent.

[6] Note that the presence of finite variance and fourth moments may be questionable for many returns series (R Cont, 2001).

absolute returns and squared returns for lags between 1 and 100. For absolute returns, they find positive autocorrelation estimates for all lags between 1 and 2705. They find positive autocorrelation of squared returns for all lags between 1 and 2598. Also evident in Ding et al (1993)'s results is the slow decay of the autocorrelations of squared and absolute returns over time. For example, for the first order autocorrelation of absolute returns is found to be 0.318, but after 100 lags only fell to 0.162.

Ding and Granger (1996), Taylor (2005), R Cont (2001), Pagan (1996) and more recently McMillan and Ruiz (2009), among many others, present similar results regarding the significance and slow decay of the autocorrelation of absolute returns and squared returns, using returns series for indices and stocks listed on the NYSE, LSE and Nikkei. There is also evidence that such persistence of squared returns appears for index returns in emerging markets (Aggarwal et al, 1999).

The slowly decaying nature of the autocorrelation of squared and absolute daily stock returns is well accepted; however what this implies about the process stock returns follow is a matter of contention (see for example Diebold and Inoue, 2001; Granger and Hyung, 2004; Banerjee and Urga, 2005; Stărică and Granger, 2005). Slow decay of the autocorrelation of absolute returns could be consistent with stock returns following a stationary process that exhibits long memory (i.e. events that occurred many periods ago actually are relevant to the present dynamics of returns).

Alternatively, nonstationarities in the process for stock returns (for example structural breaks or the unconditional mean level of returns shifting over time) could also lead to a slowly decaying autocorrelation function of absolute returns. The latter explanation appears to be receiving more support in recent literature. It seems improbable for example that the true mean daily return of the S&P500 was constant during the entirety of Ding and Granger's (1993) 1928 to 1991 data set (Granger and Hyung, 2004).

5 The Taylor Effect

Another feature highlighted by Granger and Ding (1993) that has been generally accepted as a stylised fact (R Cont, 2001; Taylor, 2005), is the "Taylor effect".[7] The Taylor effect typically refers to the result that for observed returns series the autocorrelation of absolute returns tends to be greater than the autocorrelation of squared returns, implying that absolute returns are more predictable than squared returns. However, the term Taylor effect is also often used to refer to the more general result that the first order autocorrelation of $|R_t|^d$ is maximised when d=1.

5.1 Evidence of the Taylor Effect

Utilising their aforementioned sample of daily S&P500 returns Granger and Ding (1993) estimated the sample autocorrelation of $|R_t|^d$ for values of *d* between 0.125 and 3 and lags between 1 and 100 days. For all lags, the autocorrelation of $|R_t|^d$ was maximised by a value for *d* between 0.75 and 1.25.

[7] This result was first identified by Taylor in 1986, hence why Granger and Ding (1995) coined the name "Taylor effect".

Their results for the first order sample autocorrelation of $|R_t|^d$ are reproduced below:

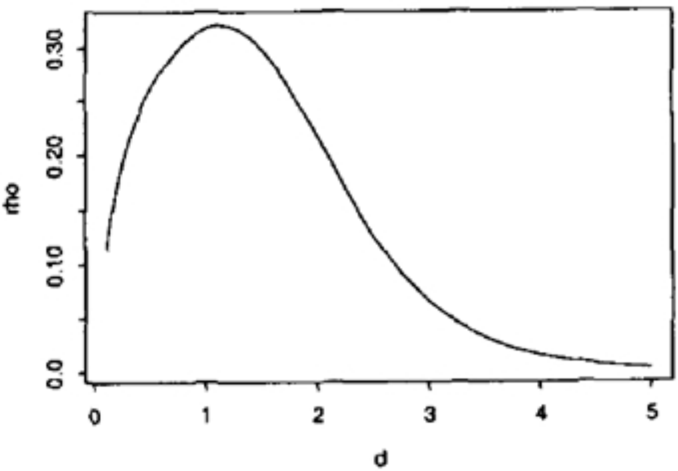

Figure 1. Autocorrelation of |r|**d at lag 1 (Granger and Ding, 1993, p. 88)

Ding and Granger (1996) later identified the Taylor effect in a number of other returns series, including a series of daily returns on the Japanese Nikkei between 1970 and 1992, daily individual stock returns for Chevron between 1962 and 1991 and minute by minute returns for Japanese food company Ajinomoto between April 3, 1989 and April 30, 1992. Interestingly, the effect was most pronounced in the minute by minute Ajinomoto returns where the autocorrelation of absolute returns was between 10 and 20 times larger than that of squared returns for all lags between considered (between 1 and 500).

5.2 Does the Finite Sample Bias of the Estimators Used Account for the Taylor Effect?

More recently, evidence from simulation based studies has led to speculation that observations of the Taylor effect in observed returns series may simply be due to the relative finite sample biases of sample autocorrelation estimators for squared and absolute returns (see for example Dalla, 2008). For example, if the sample autocorrelation of absolute returns is biased upwards whereas the sample autocorrelation estimator for squared returns is biased downwards this may cause the sample autocorrelation of absolute returns to exceed that of squared returns, even if the population autocorrelation of absolute returns is identical to that of squared returns.

However, whether such a difference in relative finite sample bias of the two estimators is actually present and can account for observations of the Taylor effect is not clear. The true finite sample bias of the sample autocorrelation estimators depends on the (unknown) underlying data generating process (DGP) for stock returns. Hence, a finding by a study that the relative finite sample biases of autocorrelation estimators may account for the Taylor effect is contingent upon the model (and the specification and parameters of that model) used by the study to approximate the true DGP.

Conflicting results between different simulation studies illustrate this caveat. Dalla (2008) presents findings from his own and others earlier studies that use various specifications of a Long Memory Stochastic Volatility model that show the sample autocorrelation of squared returns to suffer from a greater negative bias than that of squared returns. These results imply that the relative bias of the autocorrelation estimators could account, at least in part, for the Taylor effect. Conversely, a study by Perez and Ruiz (2003) suggests the opposite. They also use a Long Memory Stochastic Volatility model; however their specification differs from those considered by Dalla. Using various parameter choices Perez and Ruiz find only negligible differences between the bias of the sample autocorrelation of absolute returns and the bias of the sample autocorrelation of squared returns. In fact, for some parameter combinations Perez and Ruiz considered the sample autocorrelation of absolute returns to suffer from a slightly greater negative bias than that of squared returns.

6 The Leverage Effect

The leverage effect, which was first identified by Black (1976), refers to the tendency for most measures of the volatility of returns (for example the sample variance of returns over a given period, or the size of squared returns or absolute returns) to increase as the price of a stock decreases (R Cont, 2001). This suggests positive and negative returns have an asymmetric effect on most measures of volatility (Taylor, 2005); negative returns correspond to reductions in stock prices and hence tend to correlate with increases in volatility measures, whereas positive returns correspond to increases in stock prices and hence tend to correlate with reductions in volatility measures. A number of stock return models have been developed that incorporate this asymmetric volatility effect (see Taylor, 2005 chapters 8 and 10).

6.1 Techniques Used to Identify the Leverage Effect and Evidence of the Leverage Effect

The technique used to identify the leverage effect is dependent on the measure of the volatility of returns used. For example, Christie (1982) considered the standard deviation of the rate of return to be volatility, and accordingly ran the regression:

$$\ln\left(\frac{\hat{\sigma}_t}{\hat{\sigma}_{t-1}}\right) = \beta_0 + \theta_t\left[\ln\left(\frac{S_t}{S_{t-1}}\right)\right] + u_t$$

where each time period t corresponded to a quarter of a year, and $\hat{\sigma}^2{}_t$ was the sum of squared daily returns over quarter t, implying that:

$$\frac{\hat{\sigma}_t}{\hat{\sigma}_{t-1}}$$

was approximately equal to the ratio of the sample standard deviation of daily returns from quarter t to the sample standard deviation of daily returns from quarter *t-1*.[8] In this regression θ_t approximates the elasticity of the standard deviation of the firm's returns to the firm's stock price S_t. Christie (1982) ran this regression for each of 379 NYSE listed firms using stock price data spanning the period 1962 to 1978. His θ_t estimates were negative for 85% of the 379 firms and yielded t-stats that were significant at the 5% level for 25% of the firms.

[8] This is only approximate as the number of trading days varied between quarters.

More recently, the size of absolute or squared returns have become a more common measure of volatility than sample standard deviations. Accordingly, more recent studies have used other methods to identify the leverage effect. For example Pagan (1996) utilises the fact that a negative cross correlation between lagged returns and absolute (or squared) returns implies the leverage effect; greater volatility (as defined as larger absolute or squared returns) tends to be observed following negative returns. Accordingly, Pagan estimates the sample cross correlation between squared returns and lagged returns, for lag lengths between 1 and 12 periods. Utilising daily stock returns for the S&P500 composite index between 1928 and 1987 he finds significant negative cross correlations for all 12 lags.

Bouchaud et al. (2001) follow a similar approach to Pagan, however they utilise a more recent and much more comprehensive data set. Their data consists of daily prices between January 1990 and May 2000 for 437 individual U.S. stocks that were included in the S&P500, along with daily levels for the S&P 500, NASDAQ, CAC 40, FTSE, DAX, Nikkei and Hang Seng indices between January 1990 and October 2000. Bouchaud et al. find that on average both the individual stocks and the indices display significant negative correlation between squared and lagged returns (for a number of lag lengths). Interestingly, for small lags the correlation appears to be greater for indices compared to individual stocks. However, the effect appears to decay to zero as the lag length increases much faster for indices than for individual stocks. Bouchaud et al. also report another interesting result. The estimated correlation between *lagged* squared returns and *current* returns was on average insignificant for the individual stocks for all positive lag lengths.

It was also insignificant for the indices, except for a positive correlation for the first four lags. That is, price falls appear to be followed by increases in volatility, not vice versa.

6.2 Economic Explanations for the Leverage Effect

The economic explanation for the leverage effect is a matter of contention. Black argued that when a stock's price falls, this represents a decline in the value of the firm's equity that tends to be greater than the decline in the market value of the firm's debt, leading to an increase in the firm's leverage (as measured by the market value of its debt divided by the market value of its equity), which increases the volatility of the value of the firms equity (its stock price) (Taylor, 2005). Indeed, it is from this explanation that the effect received its name. Early evidence also appeared to support it. Christie (1982) for example finds a significant positive correlation between stock return volatility and financial leverage, and that the leverage effect he identified tended to be greater for firms with higher debt/equity ratios.

However, later studies have questioned this explanation. Taylor (2005) claims that the small daily changes in debt/equity ratios that occur cannot account for the magnitude of the leverage effect that is observed. Duffee (1995) also questions Black's explanation. Using a data set of daily returns between 1977 and 1991 for 2,494 firms listed on the NYSE, Duffee (1995) finds no evidence that the leverage effect is greater for firms with higher debt/equity ratios. He claims Christie's result may have been unique to the smaller sample of larger S&P500 firms Christie considered.

It has been suggested instead that “time-varying risk premia” may instead explain the leverage effect (See review in Duffee, 1995). This explanation asserts that an increase in return volatility increases the expected future returns investors require from a stock, leading to an immediate fall in its price. Taylor (2005) and Bouchaud et al (2001) also cite other proposed explanations for the leverage effect, including that the “herding” behaviour of uninformed traders selling in response to price falls contributes to increased volatility.

7 Conclusion

Many “stylised facts” of stock price movements are now widely accepted. The non-exhaustive review above has identified a number of examples: the kurtosis of the empirical distribution of daily returns, the absence of linear autocorrelation of returns (at least for liquid US, UK and Japanese markets), the presence of significant and slowly decaying autocorrelations of absolute and squared returns, and the so called Taylor and leverage effects. These facts can be interpreted as conditions a model should be consistent with if it is to be considered a reasonable approximation of the true DGP for stock prices.

Unlike calendar effects and other anomalies, the stylised facts identified above are consistent with the efficient markets hypothesis.

The absence of autocorrelation between daily returns is precisely what would be expected if all publically available information is immediately and fully incorporated into stock prices (Fama, 1965a). If prices responded slowly, positive autocorrelation of returns would occur, whereas if they tended to over respond then correct, negative autocorrelation would.

The significant autocorrelation of absolute and squared returns does suggest that the Random Walk theory in its strictest sense (that returns are statistically independent) is invalid. However it does not challenge the efficient market hypothesis requirement that abnormal profits cannot be achieved by utilising publically available past price information. Although large absolute returns may tend to follow large absolute returns, this result does not allow the direction of price changes to be predicted. Similarly, the leptokurtosis of returns, and Taylor and leverage effects do not suggest the existence of opportunities to generate abnormal returns based on publically available information.

Another interesting point to note from the above is that methods to identify stylised facts of stock return series can be relatively unsophisticated. Simple autocorrelation and kurtosis estimates in many cases appear to have been satisfactory.

While the evidence for the presence of the above stylised facts is vast for US, UK and Japanese indices and individual stocks, studies of smaller markets (including New Zealand and many emerging markets) appear to focus primarily on indices. Studies of the above stylised facts that utilise returns series for individual stocks, especially illiquid stocks, listed on such smaller markets appear to be rare. Literature making use of returns series that include the recent (2008-2010) period of decline and rebound in most equity markets has also been difficult to locate. Such studies are presumably forthcoming. It will be interesting to see whether recent price movements have strengthened the evidence for the presence of some of these stylised facts (for example, we may expect there to now be even more evidence of leptokurtosis), or weakened it for others.

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