Modelling Monetary Policy Reaction Functions

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"Having looked at monetary policy from both sides now, I can testify that central banking in practice is as much art as science. Nonetheless, while practicing this dark art, I have always found the science quite useful."

– Alan S. Blinder[1]

Abstract

This paper considers how monetary policy is modelled currently, with particular reference to the Reserve Bank of New Zealand's dynamic stochastic general equilibrium model: Kiwi Inflation Targeting Technology (K.I.T.T.). By considering the role and objectives of monetary policy along with its dynamic effects, it is suggested that monetary policy might better be modelled using micro-foundations, where the path of monetary policy is determined by an optimising central bank. In the

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context of the New Zealand economy, the monetary authority's optimisation problem is postulated and developed. To conclude, an alternative monetary policy reaction function is suggested for use in K.I.T.T..

1 Introduction

In the context of a given model, the reaction function is a description of endogenous monetary policy. It takes into consideration the consequences of any economic disturbance and, as such, prompts an automatic response by the model's central bank. The reaction function plays a significant role in the model, particularly via the transmission mechanism of monetary policy which reflects the intrinsic and systemic effects of monetary policy on the dynamic economy it depicts. An effective reaction function provides a reasonable description of the central bank as it responds to economic conditions, appropriately describes the channels through which monetary policy affects behaviour, and ensures cohesive interactions between the central bank and other agents in the model.

This paper seeks to consider the role and importance of the reaction function, particularly in the context of modelling the New Zealand economy. It proceeds as follows: Section 2 considers how monetary policy is modelled, with particular reference to current practice; Section 3 develops an optimising monetary policy rule for use in the Reserve Bank of New Zealand's Kiwi Inflation Targeting Technology model(K.I.T.T.) (the derivations for which can be found in the Appendix); and Section 4 concludes.

2 Modelling monetary policy

This section considers the reaction function and its role in modelling monetary policy; its structure is as follows. Subsection 2.1 introduces the role of the reaction function in communicating monetary policy objectives; Subsection 2.2 introduces the role of the loss function; Subsection 2.3 discusses the transmission mechanism in a modelling context; and Subsection 2.4 describes current practice in modelling monetary policy reaction functions.

2.1   Monetary policy objectives

When considering the reaction function and the role of monetary policy, it is assumed that the monetary authority operates in accordance with a targeting rule rather than an instrument rule. An instrument rule prescribes how the central bank's monetary policy instrument should respond to economic conditions as a specific function of forward-looking economic variables. In practice, no central bank can realistically commit to such a rule. Targeting rules, on the other hand, commit the monetary authority to targeting specific economic variables when conducting monetary policy. Such rules are a closer representation of the true decision framework and practice of inflation-targeting central banks (Svensson, 1999).

It is also assumed that the central bank is inflation-targeting, and that monetary policy is conducted with short-term interest rates as the policy instrument, where interest rates are a key influence on output and inflation. Inflation targeting regimes are

interpreted as having explicit inflation targets and implicit targets for other economic variables, specifically employment (Svensson, 1997). Monetary policy should be modelled in a way which is consistent with these objectives – that is, the monetary policy instrument should be modelled as responding to other target variables in addition to inflation (whether they are implicit or explicit targets), in order for it to be a true depiction of central bank behaviour.

Within a given model, explicit representation of the monetary policy objectives which underpin central bank behaviour is achieved through the inclusion of an appropriate reaction function, which should be consistent with the objectives of the monetary authority. The reaction function therefore describes how the policy rate—that is, the short-term interest rate—is determined within a model. Specifically, the reaction function communicates the underlying goal and means of implementing monetary policy, and creates expectations consistent with the achievement of these goals (Svensson, 1997).

2.2   The role of the loss function

The loss function is a representation of the central bank's preferences with regards to its monetary policy objectives. It is essentially an expression of the central bank's targets and objectives, where the bank is charged with minimising the loss to society arising from instability in the target variables (Svensson, 1997). As assumed previously, the central bank operates according to a targeting rule. This can be interpreted as the assignment of a particular loss function to be minimised by the central bank (Svensson, 1999). In a modelling context, the

loss function therefore plays a key role in communicating the objectives of monetary policy.

Monetary policy in an inflation-targeting framework with an operationally independent central bank works in the following way. Society delegates the role and objectives of monetary policy to the central bank in the form of explicit and implicit targeting rules – that is, society assigns a loss function to the central bank. The monetary authority is then given operational independence to minimise the assigned loss function—this is the optimisation problem of the central bank—where the central bank is held accountable for the minimisation of this loss function (Svensson 1997). The loss function, therefore, provides a description of central bank behaviour based on sound microeconomic foundations.

2.3   The importance of the transmission mechanism

The literature suggests that, rather than seeking to influence target variables themselves, it is usually more efficient for the monetary policy instrument to respond to determinants of the target variables (Svensson, 1999). How these variables are determined is, therefore, a key aspect of modelling monetary policy and is significant in the derivation of an appropriate monetary policy reaction function. Consequentially it is desirable to express the reaction function of endogenous monetary policy in terms of the variables which influence monetary policy target variables, rather than the target variables themselves.

Not only should a model incorporate the goals of monetary policy in the form of a loss function, it should also include the mechanisms by which these goals are achieved (Coletti *et al*, 1996). An accurate description of the monetary policy transmission mechanism is therefore an important aspect of any model where there is a role for endogenous monetary policy. The transmission mechanism describes the effect of monetary policy in terms of the mechanisms through which the impact on determinant variables is transmitted to its target variables. It is therefore a key determinant of how effective monetary policy is in achieving central bank objectives.

Moreover, the central bank must conduct monetary policy within the context of the model, and subject to the dynamic relationships between economic variables specifically. For example, a tension exists between inflation and employment targeting, at least in the short run. Interaction between the reaction function and an explicit representation of the Phillips curve to model this relationship can provide insight with regards to this issue (Coletti *et al*, 1996). Such relationships between economic variables affect the central bank's ability to conduct monetary policy in an optimal manner. The monetary authority must carry out monetary policy—that is, minimise its loss function—subject to these constraints, where these constraints capture the dynamic effects of monetary policy on the model.

2.4   Current practice

The Forecasting and Policy System (FPS), the Reserve Bank of New Zealand's (RBNZ) previous core macroeconomic model, contains a forward-looking monetary policy reaction function which describes the monetary authority as adjusting short term interest rates when inflation is projected to deviate from target (Black *et al*, 1997). This approach is common in the literature. For example, the Bank of England's Quarterly Model (BEQM) also uses a simple interest rate rule which ensures that inflation is maintained at the target level while assuming some smoothing of interest rates (Harrison *et al*, 2005). The reaction functions in both FPS and BEQM do not describe the policy rate as responding to determinants of its target variables as is desirable. Instead, the policy rate responds to movements in the target variable itself, specifically inflation, as is common in the literature.

The reaction function in FPS is characterised by the distinction it makes between short- and long-term interest rates, where the short-term rate is the policy instrument which determines the term structure of other interest rates, and all rates' impact on agents' decision-making. However, FPS, like most models, does not consider stability of output or other real variables in its description of monetary authority behaviour, nor does it model optimising behaviour on the part of the central bank (Black *et al*, 1997).

The RBNZ's current core economic model, K.I.T.T., is a dynamic stochastic general equilibrium model. K.I.T.T. provides a useful description of the New Zealand economy for

the purpose of forecasting and policymaking. It provides a comprehensive macroeconomic story developed from micro-economic foundations, and is consistent with key features of the New Zealand business cycle (Beneš *et al*, 2009).

Within K.I.T.T. the monetary authority is assumed to set semi-endogenous monetary policy according to a simple interest rate rule like that in BEQM. While the model is otherwise rich in micro-economic foundations, the monetary policy reaction function is not based on optimising behaviour on the part of the central bank. Developing a new policy rule would, therefore, be useful in improving the model currently in use, and is an important avenue of future research (Beneš *et al*, 2009). This idea is further developed in the next section.

3 Developing a new reaction function for use in K.I.T.T

The RBNZ's core macroeconomic model, K.I.T.T, is specifically designed to answer questions about monetary policy. A model designed to answer such questions should contain clear rules for monetary policy along with a reasonable description of the transmission mechanism and dynamic effects on the whole economy (Coletti et al 1996). This section considers how such a rule might be developed for K.I.T.T.. Subsection 3.1 considers the RBNZ's monetary policy objectives; subsection 3.2 proposes a loss function to be minimised by the monetary authority in accordance with these objectives; subsection 3.3 considers the role of the transmission mechanism in determining the model's optimal path of monetary policy; and subsection 3.4 suggests an alternative reaction function for use in determining

the path of endogenous monetary policy within K.I.T.T. Relevant derivations can be found in the Appendix.

3.1   Monetary Policy Objectives

No central bank sets monetary policy with only inflation in mind (Hunt, 2004). This is certainly true of the RBNZ. Clause 4(b) of the Policy Targets Agreement (PTA) states that

> *'In pursuing its price stability objective, the Bank shall implement monetary policy in a sustainable, consistent and transparent manner and shall seek to avoid unnecessary instability in output, interest rates and the exchange rate.' (RBNZ, 2008)*

This implies that the RBNZ does not attempt to control inflation in a rigid manner regardless of the effects on other economic variables, but also attempts to minimise volatility in output, interest rates and the exchange rate, while achieving low and stable inflation. This acknowledges the fact that output, interest rate and exchange rate volatility are likely to be detrimental to growth and economic welfare (Hunt, 2004). Monetary policy may need to respond to other economic variables in achieving its objectives, rather than variability in target variables themselves. This is due to the dynamic and interactive relationships of economic variables (Hunt, 2004). It is therefore desirable to derive a reaction function which captures central bank responsiveness to determinants of its target variables.

The PTA makes it clear that only unnecessary volatility in output, interest rates and exchange rates need be avoided. Some

degree of economic volatility is necessary for the economy to adjust to changing economic conditions. This is especially true of the exchange rate. Exchange rate movements reduce the need for adjustment in other areas of the economy in response to economic shocks (Hunt, 2004).

Furthermore, exchange rate stability is likely to be associated with volatility in other key economic variables, particularly inflation, output and interest rates. As a consequence, the targeting of exchange rate volatility may conflict with other monetary policy objectives.

It is also difficult for the monetary authority to respond to determinants of exchange rates, as the rate fluctuates more than interest rate differentials alone can justify, making exchange rate targeting difficult to model (Hargreaves, 2002). When deriving a reaction function for use in modelling the New Zealand economy, we do not consider exchange rate stabilisation as a monetary policy objective, though it is acknowledged that this is an important aspect of the RBNZ's role in stabilising the New Zealand economy.

3.2   The loss function

The following quadratic loss function is proposed for the monetary authority in K.I.T.T, as is standard in the literature.

$$L_t = \frac{1}{2}\{(\pi_t - 2)^2 + \lambda(Y_t - Y)^2 + \mu(r_t - r_{t-1})^2\}$$

where L_t is the total loss to the central bank in period t; 2% is the middle of the inflation target band; π_t is CPI inflation in period t; Y_t is actual output in period t; Y is potential output; $Y_t -$

Y measures the output gap; $\lambda \geq 0$ is the weight the RBNZ places on stabilising the output gap relative to inflation; r_t is the policy rate in period t; $\mu \geq 0$ is the relative weight of interest rate smoothing.

This describes the RBNZ as being concerned with price, interest-rate and output stability in accordance with its monetary policy objectives, as outlined in the PTA. Its preferences in achieving output and interest rate stability relative to its goal of price stability are captured by parameters λ and μ respectively.

Substituting in the definition of CPI found in K.I.T.T. yields,

$$L_t = \frac{1}{2}\left\{\left(\left(1 - v_c - v_\tau - v_f\right)\pi_t^n + v_c\pi_t^c + v_\tau\pi_t^\tau + v_f\pi_t^f - 2\right)^2 \right.$$
$$\left. + \lambda(Y_t - Y)^2 + \mu(r_t - r_{t-1})^2\right\}$$

where v_c, v_τ, v_f are the weights of construction-cost, tradables inflation and petrol-price inflation in CPI respectively; π_t^n is non-tradables inflation; π_t^c is construction-cost inflation; π_t^τ is tradables inflation; and π_t^f is petrol-price inflation.

3.3   The transmission mechanism

Modelling the behaviour of the monetary authority requires effective representation of the transmission mechanism of monetary policy. In K.I.T.T., there are four major channels through which monetary policy operates. These are via the exchange rate, consumption-demand, investment-demand, and expectations formation.

The exchange rate channel is described by the uncovered interest parity (UIP) condition and describes how the nominal exchange rates respond to interest-rate differentials caused by changes in monetary policy (Beneš *et al*, 2009). The RBNZ minimises the loss function subject to the uncovered interest parity condition. This relationship is represented by expressing movements in the exchange rate in terms of interest rate differentials and, thus, as a function of the policy rate. This is substituted into the loss function where appropriate.

The consumption-demand, investment-demand and expectations channels are captured in the Phillips curve relationship. The consumption-demand channel describes the effect that monetary policy has on agents who substitute between leisure and consumption as the price of borrowing changes. This, in turn, affects both tradables and non-tradables inflation. The investment-demand channel captures the effect of monetary policy on the expected return of business and residential investment. This has subsequent effects on demand for the intermediate good and construction-cost inflation. The expectations channel describes the systemic effect of changes in monetary policy on expectations formation, where agents incorporate rational expectations into their optimising behaviour (Beneš *et al*, 2009).

The effects on expectations formation, investment demand and consumption demand feed through to the Phillips curve which, in turn, influences output and inflation. With regards to deriving a reaction function for K.I.T.T., the effects of changing policy rates on expectations, investment and consumption demand are taken into account by minimising the loss function subject to the Phillips curve relationship. This relationship is

expressed by restating the Phillips curve for the respective components of CPI inflation in terms of policy rates and substituting these into the loss function. This is effective in capturing the dynamic effects of changes in the policy rate on output, inflation and, ultimately, the endogenous path of monetary policy in the model.

Using equations found in K.I.T.T., derivations of general functional forms for the components of the loss function in terms of the policy rate are found in the appendix. These capture the transmission mechanism of monetary policy described by UIP and Phillips curve relationships found in K.I.T.T.. Substituting these into the loss function yields the general functional form of the loss function subject to the policy rate,

$$L_t$$

$$= \frac{1}{2}\Bigg\{\Bigg((1 - v_c - v_\tau - v_f)(f_1(r, r_t, r_{t-1} \dots)$$

$$- g_2(r_t, r_{t-1}, \dots), h_1(r, r_t, r_{t-1} \dots), E_t(\pi_t^n - \pi_{t-1}^n), \dots)$$

$$+ v_c g\Bigg(f_1(r, r_t, r_{t-1} \dots)$$

$$- g_2(r_t, r_{t-1}, \dots), \ln\frac{g_0(r_t, r_{t-1}, \dots)}{I^h}, E_t(\pi_t^c - \pi_{t-1}^c), \dots\Bigg)$$

$$+ v_\tau f\Bigg\{\ln\frac{f_2(r, r_t, r_{t-1}, E_t S_{t+1})}{Pf}$$

$$- f_3(r_t, r, \dots), f_1(r, r_t r_{t-1}, \dots), \ln\frac{f_0(r, r_t, r_{t-1} E_t S_{t+1}, \dots)}{P^q}$$

$$- f_3(r_t, r, \dots), E_t(\pi_{t+1}^\tau - \pi_t^\tau), \dots\Bigg\} + v_f j(r, r_t, r_{t-1}, E_t S_{t+1}, \dots)$$

$$- 2\Bigg)^2 + \lambda(k(r_t, E_t r_{t+1}, \dots) - Y)^2 + \mu(r_t - r_{t-1})^2\Bigg\}$$

where $E_t S_{t+1}$ is the nominal exchange rate expressed as units of foreign currency per New Zealand dollar in period $t+1$ as expected in period t.

3.4   The reaction function

The optimisation problem of the monetary authority is described as follows. The monetary authority chooses the optimal path of monetary policy in period t so as to minimise the loss function (above) over the medium term. This is subject to the Phillips curve relationship and UIP condition within the

context of the model, where these constraints capture the role and dynamic effects of monetary policy on the model.

According to the PTA, the central bank is responsible for achieving its desired objectives over the medium term, where we assume that the medium term is a twelve quarter time horizon. Hence, in period t, the RBNZ chooses $(r_t, r_{t+1}, \ldots, r_{t+12})$ to minimise,

$$V_t = \sum_{i=t}^{t+12} L_i$$

where V_t is the loss to the central bank over a 12 period horizon—the intertemporal loss function—and $(r_t, r_{t+1}, \ldots, r_{t+12})$ describes the path of monetary policy. This implies that to achieve its monetary policy objectives, the central bank chooses the optimal path of monetary policy at time t such that this intertemporal loss function is minimised, and adjusts the policy rate accordingly.

The optimisation problem of the RBNZ is therefore given by,

$$\min_{r_t, r_{t+1}, \ldots, r_{t+12}} V_t$$

First order conditions are found by computing,

$$\frac{\partial V_t}{\partial r_t} = 0; \ \frac{\partial V_t}{\partial r_{t+1}} = 0; \ \ldots; \ \frac{\partial V_t}{\partial r_{t+12}} = 0$$

Solving these first order conditions simultaneously yields a reaction function of the following general form,

$$r_t = q_0(\pi_{t-1}, E_t\pi_t, E_t\pi_{t+1}, \dots)$$
$$+ q_1(\dots, r_{t-2}, r_{t-1}, E_t r_{t+1}, E_t r_{t+2}, \dots)$$
$$+ q_2(Y, \dots)$$

This is the endogenous reaction function of the loss-minimising central bank. To allow for exogenous monetary policy shocks we introduce a shock term, ε_t^m, where the following semi-endogenous interest rate rule is postulated for use in K.I.T.T.

$$r_t = q_0(\pi_{t-1}, E_t\pi_t, E_t\pi_{t+1}, \dots)$$
$$+ q_1(\dots, r_{t-2}, r_{t-1}, E_t r_{t+1}, E_t r_{t+2}, \dots)$$
$$+ q_2(Y, \dots) + \varepsilon_t^m$$

As can be seen from the proposed reaction function, the optimal policy rate at time *t* is specifically related to, among other variables, the path of monetary policy which minimises the inter-temporal loss function over the medium term. The choice of policy rate in period *t*, as determined by the reaction function, is consistent with the smoothing of interest rates while stabilising output and inflation. This rule arises as a result of optimising behaviour on the part of the model's central bank, where this is consistent with the objectives of the RBNZ according to the PTA.

In comparison, the reaction function currently in use has no loss-minimising behavioural underpinnings. Rather, it is an ad hoc rule where the policy rate in the current period responds to the expected inflation gap one period in advance subject to interest rate smoothing. This is not consistent with the objectives of the PTA, nor is it determined by microeconomic behavioural underpinnings. As such, an alternative policy rule like the general form reaction function derived here would provide a

more detailed description of the RBNZ's policy setting behaviour consistent RBNZ objectives and with the proposed approach described by the central bank modelling literature.

4 Conclusion

Monetary policy reaction functions are a useful tool in modelling central bank behaviour. Effective modelling of monetary policy requires an accurate depiction of central bank objectives and the transmission mechanism through which monetary policy operates. The development of a reaction function, such as the one developed here for use in K.I.T.T., would be advantageous for any model where there is a role for endogenous monetary policy. This suggested improvement on current practice is premised on the idea that reaction functions should be founded on micro-economic relationships, consistent with the decision framework and practice of monetary policy, and expressed in terms of variables which influence the achievement of these objectives.

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Appendix: Deriving a monetary policy reaction function for K.I.T.T

Equation references are to K.I.T.T: Kiwi Inflation Targeting Technology (Beneš *et al*, 2009). The numbers of original equations are indicated in the text below, which may have been simplified for clarity; all other equations are derived. The loss function and central bank optimisation problem are imposed.

A1 The loss function

A1.1   General form

I propose a quadratic loss function as is standard in the literature. This is given by,

$$L_t = \frac{1}{2}\{(\pi_t - 2)^2 + \lambda(Y_t - Y)^2 + \mu(r_t - r_{t-1})^2\}$$

where 2% is the middle of the inflation target band; π_t is CPI inflation in period t; Y_t is actual output in period t; Y is potential output; $Y_t - Y$ measures the output gap; $\lambda \geq 0$ is the weight of stabilising the output gap relative to inflation; r_t is the policy rate in period t; $\mu \geq 0$ is the relative weight of interest rate smoothing.

A1.2   Defining CPI and the resulting loss function for K.I.T.T.

From equation 2.69, the CPI inflation rate in K.I.T.T. is defined as,

$$\pi_t = \left(1 - v_c - v_\tau - v_f\right)\pi_t^n + v_c\pi_t^c + v_\tau\pi_t^\tau + v_f\pi_t^f$$

where v_c, v_τ, v_f are the weights of construction-cost, tradables inflation and petrol-price inflation in CPI respectively. π_t^n is non-tradables inflation; π_t^c is construction-cost inflation; π_t^τ is tradables inflation; π_t^f is petrol-price inflation.

Hence, we can substitute this into the loss function,

$$L_t = \frac{1}{2}\Big\{ \big((1 - v_c - v_\tau - v_f)\pi_t^n + v_c\pi_t^c$$
$$+ v_\tau\pi_t^\tau + v_f\pi_t^f - 2 \big)^2 + \lambda(Y_t - Y)^2$$
$$+ \mu(r_t - r_{t-1})^2 \Big\}$$

A2 Deriving functional forms for components of the loss function

A2.1 Tradables inflation:

From the following simplified version of K.I.T.T. equation 2.91, the Phillips curve for the tradable sector, we know that,

$$\pi_t^\tau = f(p^f, p^z, p^q, E_t(\pi_{t+1}^\tau - \pi_t^\tau), \dots)$$

where $p^f = ln\frac{P_t^f \psi_t^f}{P^f} - ln\frac{P_t^\tau}{P^\tau}$; $p^z = ln\frac{P_t^z}{P^z} - ln\frac{P_t^\tau}{P^\tau}$; $p^q = ln\frac{P_t^q}{P^q} - ln\frac{P_t^\tau}{P^\tau}$

Now, P_t^q is the price of non-oil imports in domestic currency and S_t is the nominal exchange rate expressed in terms of units of foreign currency per New Zealand dollar. Variables without time subscripts denote steady state values. Therefore,

$$P_t^q = \frac{P_t^{\$q}}{S_t},$$

where $P_t^{\$q}$ is the price of non-oil imports determined by an exogenous process. Call this f_0,

$$f_0(S_t, \dots) = \frac{P_t^{\$q}}{S_t}$$

UIP is described in K.I.T.T. equation 2.138,

$$
\begin{aligned}
(r_t - r) - \left(r_t^f - r^f\right) + E_t\left(ln\frac{S_{t+1}}{S}\right) - ln\frac{S_t}{S} \\
= \theta\left((r_{t-1} - r) - \left(r_{t-1}^f - r^f\right)\right. \\
\left. + ln\frac{S_t}{S} - ln\frac{S_{t-1}}{S}\right) + \varepsilon_t^u
\end{aligned}
$$

where r_t^f is the foreign interest rate at time t. Hence, we can generalise this as

$$S_t = s_0(r, r_t r_{t-1} E_t S_{t+1}, \dots)$$

Therefore,

$$f_0(r, r_t, r_{t-1} E_t S_{t+1}, \dots) = \frac{P_t^{\$q}}{s_0(r, r_t r_{t-1} E_t S_{t+1}, \dots)}$$

P_t^z is the price of the intermediate good. Now, from equation 2.118,

$$ln\frac{P_t^z}{P^z} = r_t - r + ln\frac{K_{t-1}}{K} - ln\frac{Z_t}{Z}$$

From K.I.T.T. equations 2.26 and 2.29, we can see that the production of the intermediate good is positively correlated with the capital stock. Further, from equation 2.114, we can see that the capital stock in the current period is determined by the

capital stock in the previous period and business investment. We generalise this relationship by the function f_z.

Hence, define the following function, f_1,

$$f_1(r, r_t, \dots) = r_t - r + ln\frac{K_{t-1}}{K} - ln\frac{f_z(K_{t-1}, I_t^k, \dots)}{Z}$$

where r_t is the policy rate in period t, r is the natural interest rate.

From K.I.T.T. equation 2.25, we can see that investment in capital, I_t^k, is positively correlated with the household's discounted future stream of (imputed) earnings from business investment and negatively correlated with the cost of tradable goods (where the price of tradable goods is negatively correlated with the effective interest rate – see equations 2.105, 2.103 and 2.24).

Equation 2.14 shows that the household's discounted future stream of (imputed) earnings from business investment is negatively correlated with the effective interest rate r_t^h over all periods. We can express this as,

$$\Phi_t^k = f_K(r_t^h, r_{t-1}^h, \dots)$$

where the household's effective interest rate is given by equation 2.24 below,

$$r_t^h = r_t + \zeta\left(\frac{B_t}{\Phi_t^h H_t} - \lambda\right)$$

Therefore,

$$f_1(r, r_t r_{t-1}, \ldots)$$

$$= r_t - r + + ln\frac{K_{t-1}}{K}$$

$$- ln\frac{f_z(K_{t-1}, r_t, r_{t-1}, \ldots)}{Z}$$

P_t^f is the petrol price in domestic currency and S_t is the nominal exchange rate expressed in terms of units of foreign currency per New Zealand dollar. Therefore,

$$P_t^f = \frac{P_t^{\$f}}{S_t},$$

where $P_t^{\$f}$, is the price of petrol in foreign currency and determined by an exogenous process. Call this f_2,

$$f_2(S_t, \ldots) = \frac{P_t^{\$f}}{S_t}$$

As derived previously,

$$S_t = s_0(r, r_t, r_{t-1}, E_t S_{t+1}, \ldots)$$

Therefore

$$f_2(r, r_t, r_{t-1}, E_t S_{t+1}, \ldots) = \frac{P_t^{\$f}}{s_0(r, r_t, r_{t-1}, E_t S_{t+1}, \ldots)}$$

Now, by K.I.T.T. equations 2.105, 2.103 and 2.24,

$$ln\frac{P_t^\tau}{P^\tau} = \frac{\chi}{1-\chi}ln\frac{C_{t-1}^\tau}{C^\tau} - \frac{1}{1-\chi}ln\frac{C_t^\tau}{C^\tau} - ln\frac{\Lambda_t}{\Lambda}$$

$$ln\frac{\Lambda_t}{\Lambda} = r_t^h - r + ln\frac{E_t \Lambda_{t+1}}{\Lambda} - \varepsilon_t^c$$

$$r_t^h = r_t + \zeta\left(\frac{B_t}{\Phi_t^h H_t} - \lambda\right)$$

Combining these,

$$\ln \frac{P_t^\tau}{P^\tau}$$

$$= \frac{\chi}{1-\chi} \ln \frac{C_{t-1}^\tau}{C^\tau} - \frac{1}{1-\chi} \ln \frac{C_t^\tau}{C^\tau} - r_t$$
$$- \zeta \left(\frac{B_t}{\Phi_t^h H_t} - \lambda \right) + r - \ln \frac{E_t \Lambda_{t+1}}{\Lambda}$$

Call this f_3,

$$f_3(r_t, r, \dots)$$

$$= \frac{\chi}{1-\chi} \ln \frac{C_{t-1}^\tau}{C^\tau} - \frac{1}{1-\chi} \ln \frac{C_t^\tau}{C^\tau} - r_t$$
$$- \zeta \left(\frac{B_t}{\Phi_t^h H_t} - \lambda \right) + r - \ln \frac{E_t \Lambda_{t+1}}{\Lambda}$$

Hence, we can express tradables inflation in the following way,

$$\pi_t^\tau$$
$$= f \left\{ \ln \frac{f_2(r, r_t, r_{t-1}, E_t S_{t+1})}{P^f} \right.$$
$$- f_3(r_t, r, \dots), f_1(r, r_t r_{t-1}, \dots), \ln \frac{f_0(r, r_t, r_{t-1} E_t S_{t+1}, \dots)}{P^q}$$
$$\left. - f_3(r_t, r, \dots), E_t(\pi_{t+1}^\tau - \pi_t^\tau), \dots \right\}$$

A2.2 Construction-cost inflation:

From K.I.T.T. equation 2.93, the Phillips curve for the construction sector, we know that,

$$\pi_t^c = g(p^z, y^c, E_t(\pi_t^c - \pi_{t-1}^c), \dots)$$

where

$$y^c = \ln \frac{Y_t^c}{Y^c}$$

As shown previously from equation 2.118,

$$p^z = \ln \frac{P_t^z}{P^z} = r_t - r + \ln \frac{K_{t-1}}{K} - \ln \frac{Z_t}{Z}$$

Further, as we derived in the previous example,

$$p^z = f_1(r, r_t, \dots)$$
$$= r_t - r + \ln \frac{K_{t-1}}{K}$$
$$- \ln \frac{f_z(K_{t-1}, I_t^k, \dots)}{Z}$$

Now, we are told that $Y_t^c = I_t^h$, where I_t^h is aggregate housing investment demand. Therefore,

$$y^c = \ln \frac{I_t^h}{I^h}$$

The household's first order condition for residential investment, K.I.T.T. equation 2.16, is given by,

$$\frac{\gamma_h \Phi_t^h (I_t^h)^{\gamma_h - 1}}{P_t^c} = 1 + \iota_h (\ln I_t^h - \ln I_{t-1}^h - \varepsilon_t^{ih})$$

From this expression we can see that housing investment demand is positively correlated with the household's discounted future stream of (imputed) earnings from housing services and negatively correlated with the cost of residential investment. We generalise this relationship by function g_0.

$$I_t^h = g_0(\Phi_t^h, P_t^c, \dots)$$

Equations 2.11 and 2.13 also show that the household's future stream of (imputed) earnings from housing services is negatively correlated with the effective interest rate over all periods. We can generalise this relationship by function g_1.

$$\Phi_t^h = g_1\left(r_t^h, r_{t-1}^h, \ldots\right)$$

where

$$r_t^h = r_t + \zeta\left(\frac{B_t}{\Phi_t^h H_t} - \lambda\right)$$

Hence, housing investment is a decreasing function of the policy rate.

This implies,

$$I_t^h = g_0(P_t^c, r_t, r_{t-1}, \ldots)$$

The cost of residential investment is given by K.I.T.T. equation 2.112,

$$
\ln\frac{P_t^c}{P^c}
$$
$$
= \ln\frac{\Phi_t^h}{\Phi^h} + (\gamma_h - 1)\ln\frac{I_t^h}{I^h}
$$
$$
- \iota_h\left(\ln\frac{I_t^h}{I^h} - \ln\frac{I_{t-1}^h}{I^h} - \varepsilon_t^{ih}\right)
$$

We can express this relationship by the function g_2,

$$\ln\frac{P_t^c}{P^c} = g_2\left(\Phi_t^h, I_t^h, \ldots\right)$$

where from g_1 and g_2,

$$\ln\frac{P_t^c}{P^c} = g_2(r_t, r_{t-1}, \ldots)$$

Hence, we can express construction-cost inflation in the following way,

$$\pi_t^c$$

$$= g\left(f_1(r, r_t, r_{t-1} \dots) \right.$$

$$- g_2(r_t, r_{t-1}, \dots), ln\frac{g_0(r_t, r_{t-1}, \dots)}{I^h}, E_t(\pi_t^c$$

$$\left. - \pi_{t-1}^c), \dots \right)$$

A2.3 Non-tradables inflation:

From equation 2.92, the Phillips curve for the non-tradable sector, we know that,

$$\pi_t^n = h(p^z, y^n, E_t(\pi_t^n - \pi_{t-1}^n), \dots)$$

where

$$y^n = ln\frac{Y_t^n}{Y^n}$$

As shown previously from equation 2.118,

$$p^z = ln\frac{P_t^z}{P^z} = r_t - r + ln\frac{K_{t-1}}{K} - ln\frac{Z_t}{Z}$$

and, as with the previous two examples,

$$p^z = f_1(r, r_t, \dots)$$

$$= r_t - r + ln\frac{K_{t-1}}{K}$$

$$- ln\frac{f_z(K_{t-1}, I_t^k, \dots)}{Z}$$

Furthermore, from the previous example we know that,

$$ln \frac{P_t^c}{P^c} = g_2(r_t, r_{t-1}, \dots)$$

Now, from equation 2.121 we know,

$$ln \frac{Y_t^n}{Y^n} = ln \frac{P_t^z}{P^z} + ln \frac{Z_t^n}{Z^n} - ln \frac{\Phi_t^n}{\Phi^n}$$

Therefore,

$$ln \frac{Y_t^n}{Y^n} = f_1(r, r_t, r_{t-1} \dots) + ln \frac{Z_t^n}{Z^n} - ln \frac{\Phi_t^n}{\Phi^n}$$

We can express this using the function h_1,

$$h_1(r, r_t, r_{t-1} \dots) = f_1(r, r_t, r_{t-1} \dots) + ln \frac{Z_t^n}{Z^n} - ln \frac{\Phi_t^n}{\Phi^n}$$

Hence, we can express non-tradable inflation in the following way,

$$\pi_t^n = h(f_1(r, r_t, r_{t-1} \dots) - g_2(r_t, r_{t-1}, \dots), h_1(r, r_t, r_{t-1} \dots), E_t(\pi_t^n - \pi_{t-1}^n), \dots)$$

A2.4 Petrol-price inflation

Petrol-price inflation follows an exogenous process given by equation 2.150. Petrol-price inflation is a function of the nominal exchange rate.

$$\pi_t^f = j(\Delta ln S_t, \dots)$$

From the UIP condition let,

$$\Delta ln S_t = s_1(r, r_t, r_{t-1}, E_t S_{t+1}, \dots)$$

Hence, we can express petrol price inflation as,

$$\pi_t^f = j(r, r_t, r_{t-1}, E_t S_{t+1}, \dots)$$

A2.5 Output

Actual output—that is, real GDP at time t—to be Y_t, is given by,

$$Y_t = C_t^h + C_t^n + C_t^\tau + C_t^f + I_t^k + I_t^h + G_t + X_t^v + X_t^d \\ - M_t^q - M_t^o$$

It is beyond the scope of this paper to derive the IS equation implied by the model. However, we know that this would imply that output is negatively correlated with current and expected future policy rates and we can generalise this relationship in the following way,

$$Y_t = k(r_t, E_t r_{t+1}, \dots)$$

A3 The optimisation problem:

As described previously, the period t loss function is given by,

$$L_t = \frac{1}{2}\Big\{\big((1 - v_c - v_\tau - v_f)\pi_t^n + v_c \pi_t^c \\ + v_\tau \pi_t^\tau + v_f \pi_t^f - 2\big)^2 + \lambda(Y_t - Y)^2 \\ + \mu(r_t - r_{t-1})^2\Big\}$$

Substituting in the derived equations for output and the components of CPI inflation gives,

$$L_t$$

$$= \frac{1}{2}\Biggl\{ \Biggl((1 - v_c - v_\tau - v_f)(f_1(r, r_t, r_{t-1} \dots)$$

$$- g_2(r_t, r_{t-1}, \dots), h_1(r, r_t, r_{t-1} \dots), E_t(\pi_t^n$$
$$- \pi_{t-1}^n), \dots)$$

$$+ v_c g \Biggl(f_1(r, r_t, r_{t-1} \dots)$$

$$- g_2(r_t, r_{t-1}, \dots), ln\frac{g_0(r_t, r_{t-1}, \dots)}{I^h}, E_t(\pi_t^c$$

$$- \pi_{t-1}^c), \dots \Biggr)$$

$$+ v_\tau f \Biggl\{ ln\frac{f_2(r, r_t, r_{t-1}, E_t S_{t+1})}{P^f}$$

$$- f_3(r_t, r, \dots), f_1(r, r_t r_{t-1}, \dots), ln\frac{f_0(r, r_t, r_{t-1} E_t S_{t+1}, \dots)}{P^q}$$

$$- f_3(r_t, r, \dots), E_t(\pi_{t+1}^\tau$$

$$- \pi_t^\tau), \dots \Biggr\} + v_f j(r, r_t, r_{t-1}, E_t S_{t+1}, \dots) - 2 \Biggr)^2$$

$$+ \lambda(k(r_t, E_t r_{t+1}, \dots) - Y)^2 + \mu(r_t - r_{t-1})^2 \Biggr\}$$

Now in accordance with the PTA, in period t the RBNZ wishes to choose the optimal path of monetary policy such that the intertemporal loss function is minimised over the medium term. We assume that the medium term is a 12 quarter time horizon.

Hence, in period t, the RBNZ chooses $(r_t, r_{t+1}, \dots, r_{t+12})$ to minimise,

$$V_t = \sum_{i=t}^{t+12} L_i$$

Hence, the optimisation problem of the RBNZ is given by,

$$\min_{r_t, r_{t+1}, \dots, r_{t+12}} V_t$$

First order conditions are found by computing,

$$\frac{\partial V_t}{\partial r_t} = 0; \ \frac{\partial V_t}{\partial r_{t+1}} = 0; \ \dots; \ \frac{\partial V_t}{\partial r_{t+12}} = 0$$

Solving these first order conditions simultaneously yields the following reaction function,

$$\begin{aligned}
r_t = q_0(&\pi_{t-1}, E_t\pi_t, E_t\pi_{t+1}, \dots) \\
+ \ q_1(&\dots, r_{t-2}, r_{t-1}, E_t r_{t+1}, E_t r_{t+2}, \dots) \\
+ \ q_2(&Y, \dots)
\end{aligned}$$