

# A Measurement-based Model of High-speed Electro-Optic Mach-Zehnder Modulators

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## Abstract

An empirical model of Mach-Zehnder modulators, which allows evaluating the output optical field as a function of the modulating electrical field and is suitable of straightforward implementation within commercially available microwave CAD tools, is proposed. The model of a Mach-Zehnder modulator for optical transmitters designed and fabricated at Corning-OTI labs, has been determined and implemented in Agilent ADS.

**Keywords:** Mach-Zehnder, Electrooptic modulators, Coplanar waveguides.

## 1. INTRODUCTION

Mach-Zehnder (MZ) electrooptic amplitude modulators are key components for high-speed optical communication links. Their design and optimization is increasingly more critical, as the operating bit-rate approaches 40 Gb/s. Accurate models able to take into account of the electrooptic interaction and evaluate both the output optical intensity and the frequency chirping are required for analysis and design purpose. Several model based on geometrical and technological parameters have been developed in recent years: the evaluation of the optical response is achieved starting from the physical and geometrical parameters by means of finite-elements approaches for quasi-static analysis [1]-[2], and full-wave analysis [3] of coplanar waveguides. Empirical equivalent models have been also developed in recent years in order to provide independence from technological parameters, straightforward implementation within commercially available CAD tools, and fast simulation. An empirical lumped-element equivalent circuit was proposed to evaluate the frequency response of both an electro-absorption intensity modulator [4] and a MZ modulator [5]. In [6] an empirical model was proposed to determine the instantaneous voltage at each section of the modulator by means of a frequency transfer function. A pseudo-electrical non-linear model was used in [7] to simulate the optical field propagation within microwave CAD environment. Here, a non-linear dispersive model of the MZ modulator is proposed in which the effects of input and output tapers on electric field propagation are taken into account, and separated from the propagation in the active region, i.e. the coplanar wave line, where the electrooptic interaction comes. The pulse response of each section of the modulator, evaluated from measured S-parameters as proposed in [8], allow to simulate the propagation of the electric field within the modulator and the electrooptic interaction in both frequency-domain and time-domain. The extraction and CAD implementation of a MZ modulator fabricated at Corning-OTI laboratories are presented and discussed.

## 2. THE MODULATOR MODEL

The electrical linear model is divided into three blocks as shown in Fig. 1: an input taper which matches the 50  $\Omega$  electrical coplanar input to the lower impedance active region, the active L-length coplanar region where the electrooptic interaction comes, and the output taper with load termination. An empirical measurements-based model has been used to describe the propagation of the electric field  $E(t,y)$  within the three sections of the modulator: the time-domain voltage  $v_{1,2}(t,y)$  at position  $y$  of each of the two branches at each section is computed by means of the pulse response which is evaluated from measured S-parameters. If a variation  $\delta\tau_{1,2}$  due to electrooptic interaction is found for the phase delay in each of the two branches of the modulator, the expression for the optical field  $E_{out}(t)$  and for the optical intensity  $I_{out}(t)$  at the output of the modulator are the following:

$$E_{out}(t) = E_{in}(t - \tau) \cdot \frac{e^{-j\omega_0 \cdot \delta\tau_1} + e^{-j\omega_0 \cdot \delta\tau_2}}{2}, \quad (1)$$

$$I_{out}(t) = |E_{out}|^2 = |E_{in}(t - \tau)|^2 \cdot \frac{1 + \cos[\omega_0(\delta\tau_1 - \delta\tau_2)]}{2}, \quad (2)$$

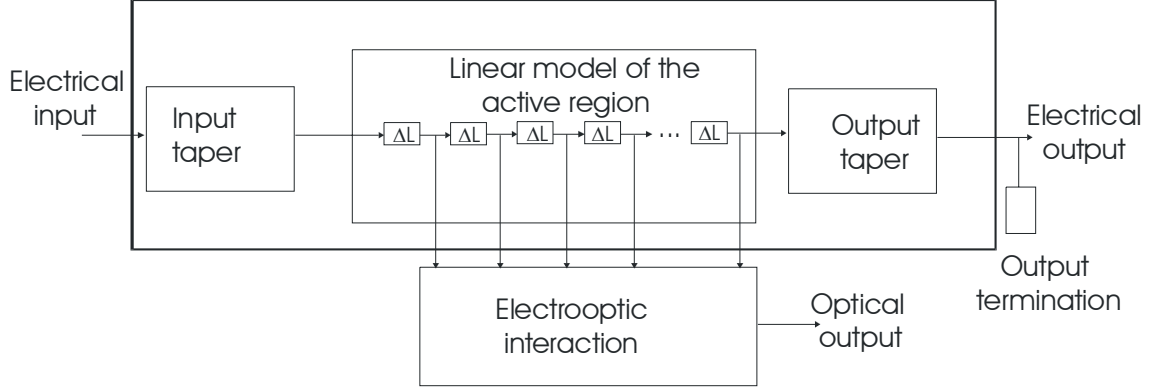


Fig. 1 Block scheme of the Mach-Zehnder modulator model

where  $\tau$  is the group delay and ideal 3 dB power splitting/combining is considered at the input and the output of the modulator, and

$$E_{in}(t) = A_{in}(t) e^{j[\omega_0 t + \varphi_{in}(t)]} \quad (3)$$

is the optical field at the input of the modulator, for which the bandwidth of both the amplitude function  $A_{in}(t)$  and the phase function  $\varphi_{in}(t)$  are much lower than  $\omega_0$ .

The phase shift  $\delta\tau_{1,2}$  in a  $\Delta L$ -length coplanar section is evaluated as a function of the voltage  $v_{1,2}(t,y)$  of the electrodes 1, 2, as:

$$\delta\tau_{1,2}(t) = -\frac{\pi}{LV\pi} \int_0^L v_{1,2} \left( t - \frac{L-y}{c} n_e, dy \right) dy, \quad (4)$$

where the voltage  $v_{1,2}(t,y)$  is evaluated from the corresponding electric field  $E(t,y)$ ,  $V_\pi$  is the half-wave voltage at DC, and  $n_e$  is the effective index of the guided mode. In this work the integral in (4) has been approximated with a summation, under the hypothesis that the voltage  $v_{1,2}(t,y)$  can be considered as a constant value within the  $\Delta L$ -length coplanar section:

$$\delta\tau_{1,2}(t) = -\frac{\pi}{LV\pi} \sum_{m=1}^{L/\Delta L} v_{1,2} \left( t - \frac{L-m\Delta L}{c} n_e, m\Delta L \right) \Delta L. \quad (5)$$

Therefore, in the model the coplanar active part has been subdivided in  $\Delta L$ -length coplanar sections, where the hypothesis holds that  $\Delta L$  is much lower than the minimal wavelength propagated. The S-parameters of the input and output tapered lines, and of a  $\Delta L$ -length coplanar line, that are needed to complete the electrooptic model, are evaluated by using the procedure proposed in [8].

### 3. MODEL EXTRACTION AND CAD IMPLEMENTATION

The model of a 29 mm z-cut Mach-Zehnder modulator, designed and fabricated at Corning-OTI laboratories, has been extracted and implemented in Agilent-ADS CAD tool. The set of test-structures described in [8] has been designed and measured together with the modulator in order to extract the model: two test patterns composed of coplanar microstrip lines with 5 and 15 mm length, respectively, connected by means of tapered transitions with  $80\mu\text{m}$  pads and  $80\mu\text{m}$  gap, have been used to extract the propagation constants  $\alpha$  and  $\beta$ ; for both the input and the output taper a THRU structure, a LINE structure (comprising a 5mm-length coplanar line), and a HYBRID structure comprising the taper under test and the reference taper (i.e. the tapered transitions with  $80\mu\text{m}$  pads and  $80\mu\text{m}$  gap used to evaluate  $\alpha$  and  $\beta$ ) have allowed the evaluation of input/output tapers S-parameters. A 5 V  $V_\pi$  has been measured for the modulator. Moreover, the ANSOFT HFSS software tool has been used to provide simulation of the reference taper and of the characteristic impedance  $Z_c$  of the coplanar line as in [8].

The CAD implementation of the model has been carried out in order to allow accurate evaluation of the optical output for a 10 Gb/s input data pattern: in particular, as the impulsive response is calculated from frequency-domain measurements, at least measured S-parameters up to 35 GHz are required to use the 7<sup>th</sup> harmonic of the 5 GHz fundamental tone for impulsive response evaluation. Moreover, as the approximation in (5) has been made (i.e. the electric field has to be considered as a constant value within the  $\Delta L$ -length coplanar section),  $\Delta L$  has been chosen to be lower than the minimal propagated wavelength:

$$\Delta L \cong \frac{l}{10} \cdot \frac{c}{f_{Max} n_e} \cong 0.5 \text{ mm} , \quad (6)$$

where  $f_{Max} = 35$  GHz.

The S-parameters of a 0.5 mm CPW, and of the input and output tapers of the modulator have been extracted by means of the proposed procedure up to 40 GHz. The e.m. simulation of the reference taper has been carried out up to 40 GHz but sufficient accuracy has been found up to 30 GHz only. In both the input and the output taper models some spurious peaks have been found at 13 GHz and 27 GHz, and anomalous behavior is shown at frequencies greater than 30 GHz due to the lower accuracy of e.m. simulations at such frequencies. For what concerns the spurious peaks, the hypothesis has been made that it is caused by the steep transition between the taper under test and the reference taper in the HYBRID structure: a more gradual transition could allow the higher-order mode to vanish at the ports. In the redesign of the test structures, a 5 mm line will be inserted between the test and the reference taper. Analogous results have been obtained for the output taper. Finally, the linear model of the 29 mm modulator has been built by cascading the two port S-parameter matrices of the input taper, the 58 lines with 0.5 mm length, and the output taper. The S-parameters of the modulator linear model have been compared to the measured S-parameters (dot line), as shown in Fig. 2, denoting good accuracy up to 30 GHz. Then, in each of the 58 lines, the phase shift related to the electrooptic interaction has been evaluated, and the total phase variation is calculated by using (5), in which the measured value of  $V_\pi$  is used. Finally, the output optical field has been calculated, according to (1), by means of time-domain or Harmonic Balance simulations. In Fig. 3 the output optical intensity  $I_{out}(t)$  for a 10 Gb/s input stream with amplitude equal to  $V_\pi$  and  $V_\pi \pm 1$  V is shown.

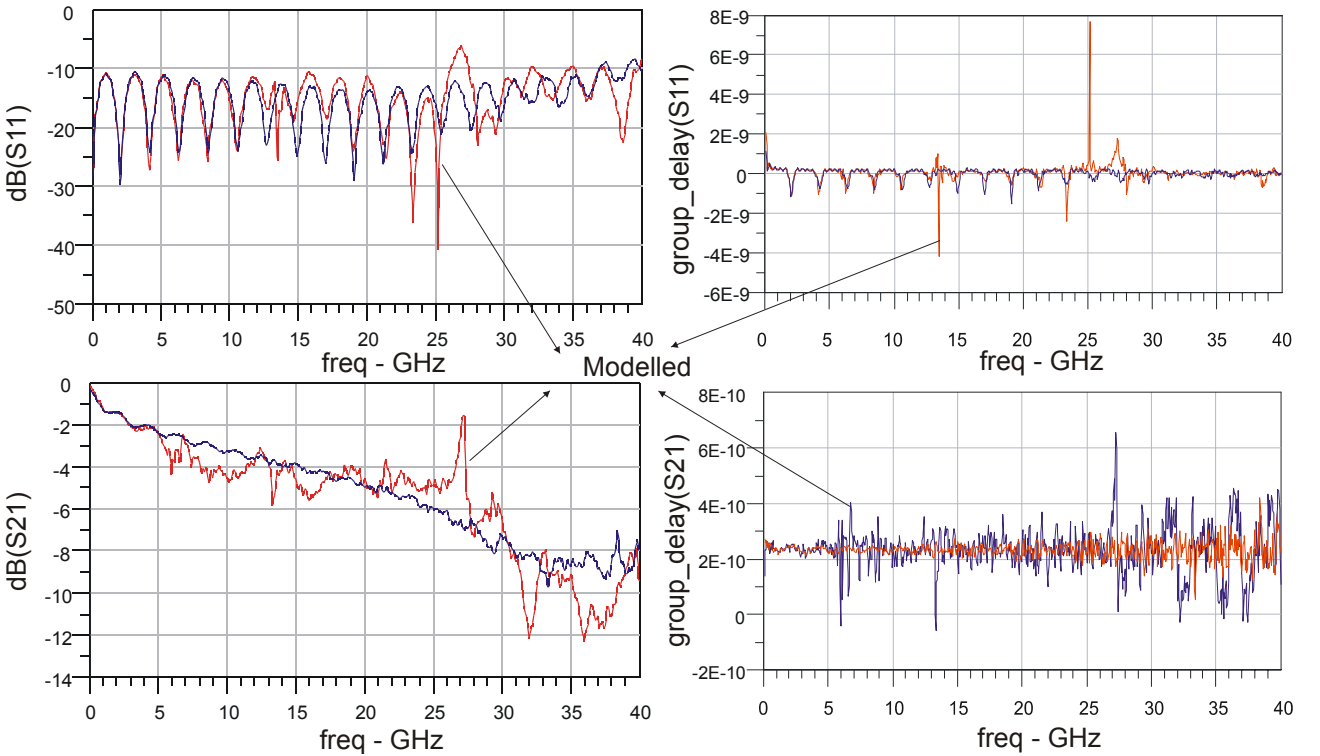


Fig. 2 Comparison between simulated and measured S-parameters of the fabricated modulator

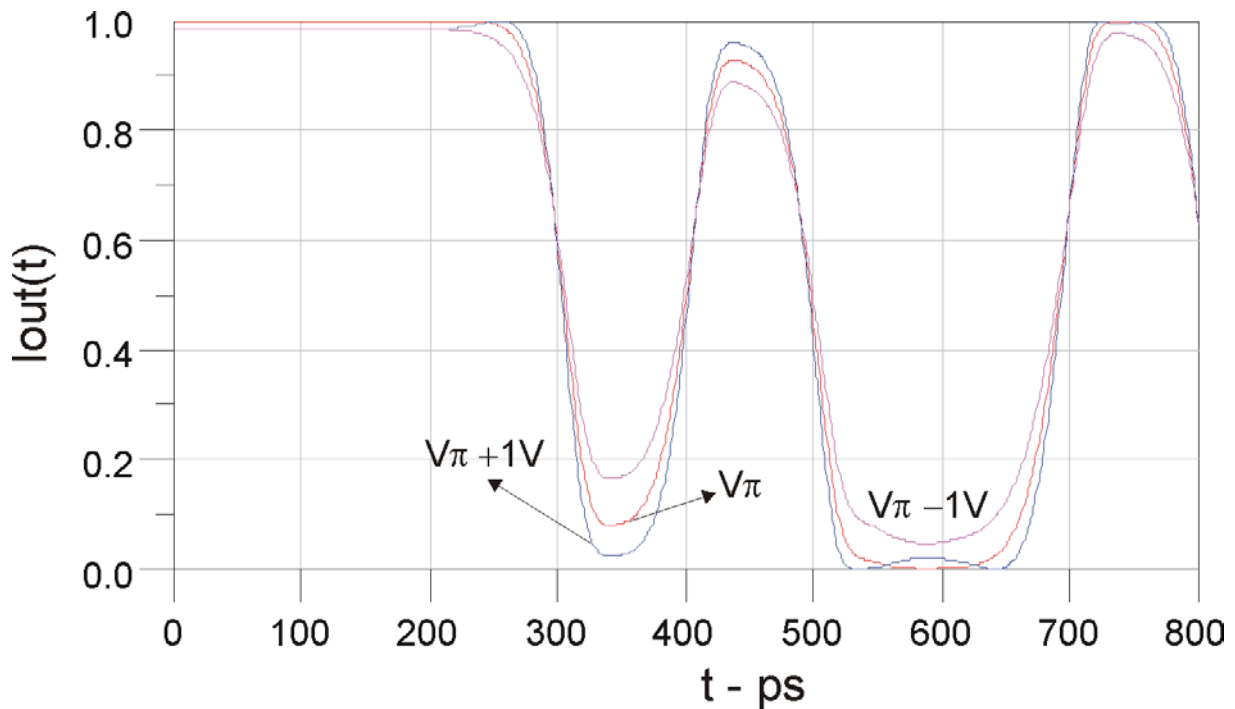


Fig. 3 Output optical intensity for an input voltage of  $V_\pi$  and  $V_\pi \pm 1V$

#### 4. CONCLUSIONS

A large-signal model of Mach-Zehnder electrooptic modulators has been proposed which allows to perform both time-domain and non-linear frequency-domain simulations within commercial microwave CAD tools. The extraction procedure, based on both S-parameters measurements and e.m. simulations, has been presented and validated on a device designed and fabricated for optical systems beyond 10 Gb/s.

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