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Materials with memory: free energies & solution exponential decay. (English summary)

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The main goal of the paper is to prove an exponential decay of the energy of a rigid heat conductor with memory. The material fills a domain Ω of $\mathbb{R}^3$. The author uses the thermodynamic model already introduced in [M. Fabrizio, G. Gentili and D. W. Reynolds, Internat. J. Engrg. Sci. **36** (1998), no. 7-8, 765–782; MR1629806 (99i:80006)]. The problem is first written as $\dot{u}(\mathbf{x},t) = \nabla \cdot \int_0^\infty k'(\tau)\overline{g}^t(\mathbf{x},\tau)d\tau$; $\overline{g}^t(\mathbf{x},\tau) = \nabla u(\mathbf{x},t) - \nabla u(\mathbf{x},t-\tau)$, with $\overline{g}^t(\mathbf{x},\tau) = \int_{t-\tau}^t g(\mathbf{x},s)ds$. Here $k' \in L^1(\mathbb{R}^+) \cap L^2(\mathbb{R}^+)$. The initial conditions $u(\mathbf{x},0) = u_0(\mathbf{x})$; $\overline{g}^t(\mathbf{x},0) = \int_{-\tau}^0 \nabla u(\mathbf{x},s)ds$ are added. This problem can be put in a semigroup framework as $\dot{\chi} = A\chi$ for the operator A corresponding to the above system. A being proved to be maximal dissipative on a functional space $\mathcal{H}$, it is known that $\chi(t) = S(t)\chi_0$ involving the semigroup S associated to A and the initial conditions issued from that of the above system. The main result of the paper proves that the energy $\mathcal{E}(t) = \langle S(t)\chi_0, (t)\chi_0\rangle /2$ is such that $\mathcal{E}(t) \leq Me^{\gamma t}$ for some positive constants M and γ assuming that $0 < -k'(t) \leq \delta k''(t)$, for some positive constant δ. The proof is a quite immediate consequence of the further properties of the operator A which ensure those of the corresponding semigroup.

Reviewed by *Alain Brillard*

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