

# WORKSHOP 2

## EXPANDING HORIZONS

New strategies for multifield fracture problems across scales in heterogeneous systems for energy, health and transport

IMT School for Advanced Studies Lucca  
May 9-12, 2022



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# Preface

These proceedings collect the abstracts presented at the workshop “Expanding Horizons” organized by the NewFrac Training Network at the IMT School for Advanced Studies Lucca, from May 9 to May 12, 2022, as part of the research and training activities of the Marie Skłodowska-Curie project “New strategies for multifield fracture problems across scales in heterogeneous systems for energy, health and transport” (Grant Agreement n. 861061, <https://www.newfrac.eu/>), supported by the European Commission which is gratefully acknowledged. This workshop is the second organized by the NewFracTraining Network, following the previous one organized at the University of Seville in the period October 1-8, 2021.

The workshop, organized in hybrid mode (please use <http://imt.lu/newfracws2> for remote access) to allow the participation of researchers both in person and from remote in response to the rapidly evolving Covid-19 pandemic situation, has been structured over 4 days in a single session with 5 invited lectures from experts in fracture mechanics and mechanics of materials and structures, 8 regular presentations (20 minutes long), and 23 extended presentations (30 minutes long). Noteworthy, 13 of the 36 lectures are delivered by the Early-Stage Researchers of the NewFrac Training Network.

To promote further discussion and exchange of ideas among the NewFrac network members and external researchers, two open events have been organized for that purpose: a round table on computational fracture mechanics and industrial applications on Wednesday afternoon, and open meetings of the committees of the NewFrac project on Thursday afternoon.

The organizers would like to thank Ms. Ninfa Vital, scientific secretary of the event and of the NewFrac project, and the administrative staff of the IMT School for Advanced Studies Lucca (in particular Chiara Quilici, Martina Colozzo, Sibela Salkic and Ivan Chesi) without whose support the event would not have been possible to take place.

The organizers wish you all a successful meeting, full of fruitful professional experiences and exchange of ideas in the diverse fields of fracture mechanics.

Lucca, May 9, 2022.

Prof. Marco Paggi (Chairman)

Prof. Vladislav Mantic (Co-chairman and NewFrac Coordinator)

NEWFRAC Workshop "Expanding Horizons" – IMT School for Advanced Studies Lucca

	MonMay 9	Tue May 10	WedMay 11	ThuMay 12
08:30 h	Registration			
09:00 h	Opening addresses			
09:30 h		Filippo Berto	Stefano Vidoli	Roberta Massabò
10:00 h	Davide Bigoni	Corrado Maurini	Vladislav Mantic	Simone Sangaletti
10:30 h	Zohar Yosibash	Mauro Corrado	AjinkyaDusane	Amir Mohammad Mirzaei
11:00 h	Coffee break	Coffee break	Coffee break	Coffee break
11:30 h	Dominique Leguillon	SindhuBushpalli	Pietro Lenarda	Alessio Gizzi
12:00 h	Maxime Levy	Francesco Vicentini	Thomas Duminy	Marco Lo Cascio (online)
12:30 h	ZengLiu	Camilla Zolesi	Arturo ChaoCorreas	PanayiotisTsokanas
13:00 h	Lunch	Lunch	Lunch	Lunch
		Anatoli Mitrou		

14:00 h	Israel García	Philipp Weissgraeber	<i>Round table on computational fracture mechanics and industrial applications</i>	<i>Meetings of the committees of the NEWFRAC project</i>
14:30 h	Karthik Ambikakumari			
15:00 h	Sara Jimenez	Ángela Fajardo	<i>Cultural visit to the City of Lucca</i>	
15:30 h	<i>Coffee break</i>	<i>Coffee break</i>		
16:00 h	Ángel Valverde	Aurélien Doitrand (online)		
16:30 h	Maria Rosaria Marulli	Maria Herrera (online)		
17:00 h	Juan Manuel Macias	Hyung-Jun Chang		
17:20 h				
18:00 h				
20:00 h				

Caption:

60' invited lecture (45'+15' discussion)
30' lecture (25' + 5' discussion)
20' lecture (15' + 5' discussion)

Location: Scuola IMT Alti Studi Lucca, Piazza San Francesco 19

# INVITED LECTURES

Mon 9, 9:30-10:30: **Davide Bigoni**

Tue 10, 9:00-10:00: **Filippo Berto**

Tue 10, 14:00-15:00: **Philipp Weissgraeber**

Wed 11, 9:00-10:00: **Stefano Vidoli**

Thu 12, 9:00-10:00: **Roberta Massabò**

# Nonconservative forces on structures lead to materials working beyond hyperelasticity

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**Keywords:** *Flutter instability; hypoelasticity.*

**Abstract:** Cauchy-elastic solids include hyper-elasticity and a subset of elastic materials for which the stress does not follow from a scalar strain potential. More in general, hypo-elastic materials are only defined incrementally and comprise Cauchy-elasticity. Infringement of the hyperelastic 'dogma' is so far unattempted and normally believed to be impossible, as it apparently violates thermodynamics, because energy may be produced in closed strain cycles. Contrary to this belief, we show that non-hyper-elastic behavior is possible and we indicate the way to a practical realization of this new concept. In particular, a design paradigm is established for artificial materials where follower forces, so far ignored in homogenization schemes, are introduced as loads prestressing an elastic two-dimensional grid made up of linear elastic rods (reacting to elongation, flexure and shear). A rigorous application of Floquet–Bloch wave asymptotics[1] yields an unsymmetric acoustic tensor governing the incremental dynamics of the effective material. The latter is therefore the incremental response of a hypo-elastic solid, which does not follow from a strain potential and thus does not belong to hyper-elasticity. Through the externally applied follower forces (which could be originated via interaction with a fluid, or a gas, or by application of Coulomb friction, or non-holonomic constraints [2]), the artificial material may adsorb/release energy from/to the environment, and therefore produce energy in a closed strain loop, without violating any rule of thermodynamics. The solid is also shown to display flutter, a material instability corresponding to a Hopf bifurcation, which was advocated as possible in plastic solids, but never experimentally found and so far believed to be impossible in elasticity [3]. The discovery of elastic materials capable of sucking up or delivering energy in closed strain cycles through interaction with the environment paves the way to realizations involving micro and nano technologies and finds definite applications in the field of energy harvesting.

## **Acknowledgements:**

This presentation gathers a series of contributions obtained together with G. Bordiga, A. Cazzolli, L. Cabras, F. Dal Corso, O. Kirillov, D. Misseroni, G. Noselli, and A. Piccolroaz, to whom D.B. wants to express his gratitude. D.B. acknowledges support from the ERC-2022-ADG-BEYOND.

## **References:**

- [1] G. Bordiga, L. Cabras, A. Piccolroaz, D. Bigoni 2021 Dynamics of prestressed elastic lattices: Homogenization, instabilities, and strain localization. *J. Mech. Phys. Solids*, 146, 104198.
- [2] A. Cazzolli, F. Dal Corso, D. Bigoni 2020 Non-holonomic constraints inducing flutter instability in structures under conservative loadings. *J. Mech. Phys. Solids*, 138, 103919.
- [3] G. Bordiga, A. Piccolroaz, D. Bigoni 2022 A way to hypo-elastic artificial materials without a strain potential and displaying flutter instability. *J. Mech. Phys. Solids*, 158, 104665.

## Nature-inspired interlocking structures: why local is so important!

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**Abstract:** Biological structural systems are characterized by complex wavy and re-entrant interlocking features. This allows to mitigate large deformations and deflect or arrest cracks, providing remarkable mechanical performances under static and fatigue loadings, much higher than those of the constituent materials. Nature-inspired engineering interlocking joints has been recently proved to be an effective and novel design strategy. However, currently the design space of interlocking interfaces relies on relatively simple geometries, often built as a composition of symmetric circular or elliptical sutured lines. Data driven methods can be leveraged to enlarge the design space. This work is addressed to give new insights into the study and design of a new generation of advanced and novel interlocked structures through data-driven methods.



# Anticrack — a local collapse that behaves like a crack

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**Keywords:** *Anticrack, crack propagation, crack initiation.*

## **Abstract:**

Anticracks are cracks with crack faces that, instead of separate, move towards each other [1]. While the crack faces of regular cracks would then be in contact with vanishing stress intensity factors, special material mechanisms can occur that avert the interpenetration of the crack faces. Anticracks can then be formulated by means of the classical concepts of mode-I cracks but with negative stress intensity factors.

We discuss several phenomena observed in nature and technical systems that can be described with the concept of anticracks: There are geological phenomena such as pressure dissolution in sedimentary rocks [1], compaction bands in high-porosity rocks or the olivine to spinell phase transformation [2]. Anticracks can also be used to describe the behavior of melt figures in ice. Also collapse in low-density materials such as foams or snow can be studied with this concept.

The collapse of persistent weak layers in stratified snowpacks is studied in detail. Such weak layers can be composed of buried surface hoar, depth hoar or faceted crystals and their failure can ultimately lead to the release of slab avalanches. The understanding of the weak layer behavior and its response to the loading by the weight of snow cover as well as additional loads such as e.g. skier has much improved by the introduction of the concept of anticracks. Whumpf sounds observed in avalanche-prone terrain and also avalanches that were remotely triggered from flat parts of a slope can be explained well by this concept [3].

A snow cover model is presented in detail that allows for analysis of the mechanical response of the stratified snow covers with detailed consideration of the weak layer and its possible collapse [4,6]. The model can be used to compute the energy release rate of anticracks in (compressive) mode I and mode II. A finite fracture mechanics model is established to analyze the conditions of skier-triggered anticrack nucleation [5].

Analogous to the well-known DCB experiment, a fracture experiment for anticracks in snow covers has emerged in the last 15 years in the scientific community of avalanche science: the so-called propagation saw test.

We study the experimental results of current developments of this young experimental technique and the insights into the mechanical behavior of anticracks in snow [7].

## **References:**

- [1] Fletcher, R. C., & Pollard, D. D. (1981). Anticrack model for pressure solution surfaces. *Geology*, 9(9), 419-424.
- [2] Katsman, R., Aharonov, E., & Scher, H. (2006). A numerical study on localized volume reduction in elastic media: some insights on the mechanics of anticracks. *Journal of Geophysical Research: Solid Earth*, 111(B3).
- [3] Heierli, J., Gumbsch, P., & Zaiser, M. (2008). Anticrack nucleation as triggering mechanism for snow slab avalanches. *Science*, 321(5886), 240-243.
- [4] Rosendahl, P. L., & Weißgraeber, P. (2020). Modeling snow slab avalanches caused by weak-layer failure—Part 1: Slabs on compliant and collapsible weak layers. *The Cryosphere*, 14(1), 115-130.
- [5] Rosendahl, P. L., & Weißgraeber, P. (2020). Modeling snow slab avalanches caused by weak-layer failure—Part 2: Coupled mixed-mode criterion for skier-triggered anticracks. *The Cryosphere*, 14(1), 131-145.
- [6] Weißgraeber, P. & Rosendahl, P.L. (2022). A closed-form model for layered snow slabs. *The Cryosphere*, to be submitted.

[7] Bergfeld, B., van Herwijnen, A., Bobillier, G., Rosendahl, P.L., Weißgraeber, P., Adam, V., Dual, J. & Schweizer, J. (2022). Characteristics of anticrack propagation in a weak snowpack layer: conditions of crack arrest and sustained propagation, *Natural Hazards and Earth System Sciences*, *to be submitted*.

## Open problems in the formulation of phase-field damage models describing fatigue

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**Keywords:** *Fatigue, variational formulation, gradient damage models.*

**Abstract:** Engineers currently use the Griffith fracture law to assess the structural limits under monotonically increasing loads, and the Paris law to estimate the structural life under cyclic loads. These two fundamental laws, governing the failing of structures in the short and long run, have often been seen as disconnected in the scientific literature.

The first formal link between the two fields was established in some seminal, but rather unknown, papers by Marigo and coworkers, see for instance [1, 2] and citations therein.

We discuss the detailed derivation of this link and its fundamental ingredients: in the case of a peeling test it is shown how a sequence of cohesive fracturing processes due to a cyclic loading at the micro time scale can accumulate to form a steady-state propagation at the macroscopic time scale. The prefixes micro and macro for the timescales are related to the small parameter  $d/L$  measuring the size of the cohesive zone with respect to the size of the structure.

An implicit law is derived connecting the macroscopic time-rate of the crack length to the ratio  $G/G_c$  between the actual energy release rate  $G$  and the material toughness  $G_c$ .

The very same law reduces to Paris fatigue law when  $G$  is much smaller than  $G_c$ , whilst it tends to the Griffith law when  $G$  approaches  $G_c$  from below.

From these results a phase-field model is extrapolated having this dual capability of describing both sudden and cumulated fracture phenomena. Despite some promising numerical results, there are still some open problems of technical and theoretical nature.

### References:

[1] Jaubert A, Marigo JJ. 2006. Justification of Paris-type fatigue laws from cohesive forces model via a variational approach. *Continuum Mech. Thermodyn.* 18: 23–45

[2] Abdelmoula R, Marigo JJ, Weller T. 2010. Construction and justification of Paris-like fatigue laws from Dugdale-type cohesive models. *Annals of Solid and Structural Mechanics* 1 (3): 139–158

## Mechanics of Multilayers: Delamination Fracture and Root Displacements

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**Keywords:** *Bimaterial interfaces; delamination; elasticity; sandwich composites; coatings.*

**Abstract:** Multilayer systems are used in many technological applications and often involve materials with extreme elastic properties or elastic mismatch between the layers. Examples are: layered systems used in micro- and flexible electronics (metals on elastomers), sandwich composites for aeronautical, naval and wind energy applications (metals/fiber-reinforced polymers on metal/polymeric foams) and protective coatings (polymer/TBCs on metals). In the presentation we will focus on the mechanics of multilayers and review recent results on the problem of interfacial delamination and debonding in orthotropic strips with extreme orthotropy ratios, thin films on substrates and sandwich composites [1-3]. We will discuss the modeling of multilayer systems and fracture specimens using structural theories and present a recently formulated technique to define boundary conditions for the detached layers which accurately account for the local deformations at the crack tip [4]; the technique extends to 2-D elasticity the original ideas formulated in [5] and uses the exact fields far from the crack tip, the reciprocity theorem and the J-integral to derive compliance coefficients which describe root rotations and displacements. Finally, we will present applications of these concepts into the design of novel testing procedures for the characterization of the interfacial toughness of sandwich composites and thermal barrier coatings [6,7].

**Acknowledgement:** RM is supported by the U.S. Office of Naval Research, ONR Global, contract # N62909-21-1-2048.

### References:

- [1] K.B., Ustinov, R. Massabò, D. Lisovento, (2020), Orthotropic strip with central semi-infinite crack under arbitrary loads applied far apart from the crack tip. Analytical solution. Eng. Failure Analysis, 110, 104410, <https://doi.org/10.1016/j.engfailanal.2020.104410>
- [2] L. Barbieri, R. Massabò, C. Berggreen(2018), The effects of shear and near tip deformations on interface fracture of symmetric sandwich beams. Eng.Fract. Mech., 201:298–321.<https://doi.org/10.1016/j.engfracmech.2018.06.039>
- [3] R. Massabò, K.B., Ustinov, L. Barbieri, C., Berggreen, (2019) Fracture Mechanics Solutions for Interfacial Cracks between Compressible Thin Layers and Substrates. Coatings 2019, 9, 152.<https://doi.org/10.3390/coatings9030152>
- [4] K.B., Ustinov, Massabò, R., (2022), On elastic clamping boundary conditions in plate models describing detaching bilayers, Int. J. Solids Struct., in press, <https://doi.org/10.1016/j.ijsolstr.2022.111600>
- [5] M.G., Andrews, R., Massabò, (2007) The effects of shear and near tip deformations on energy release rate and mode mixity of edge-cracked orthotropic layers. Eng.Fract. Mech., 74, 2700-2720, <https://doi.org/10.1016/j.engfracmech.2007.01.013>
- [6] I. Monetto, R., Massabò, R., (2021) An analytical solution for the inverted four-point bending test in orthotropic specimens. Eng.Fract. Mech., 245, art. no. 107521, <https://doi.org/10.1016/j.engfracmech>.

[7] R., Massabò, (2021), Upper and lower bounds for the parameters of one-dimensional theories for sandwich fracture specimens. J. Appl. Mech., Trans. ASME, 88 (3), art. no. 031014, <https://doi.org/10.1115/1.4049141>

# MONDAY, MAY 9

## MORNING SESSION

10:30-11:00: **Zohar Yosibash**

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11:30-12:00: **Dominique Leguillon**

12:00-12:30: **Maxime Levy**

12:30-13:00: **Zeng Liu**



# Can the finite fracture mechanics coupled criterion be applied to V-notch tips of a quasi-brittle steel alloy?

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**Keywords:** *Coupled Criterion, Finite Fracture Mechanics, V-notch, Metallic*

**Abstract:** Structures made of steel alloys with V-notches may fracture at the V-notch tip at which a small plastic zone usually evolves. Failure criteria for predicting fracture loads for such quasi-brittle alloys, as a function of the V-notch opening angle are very scarce and have not been validated, to the best of our knowledge, by a set of experimental observations.

Herein we present a set of experiments performed on AISI 4340 steel alloy specimens with three different V-notch opening angles and three different tempering temperatures, loaded in mode I.

The experimental observations are used to investigate to which extent the finite fracture mechanics coupled criterion (FFMCC) for brittle materials [1] may predict the failure load of quasi-brittle steel alloys, depending on the plastic zone size. Finite element analyses were used to compute the parameters required by the FFMCC. We compared the predicted versus the experimental fracture load for the different V-notch opening angles and tempering temperatures.

For small opening angles ( $30^\circ$ ) for which the plastic area is very small, the FFMCC under-predicts the fracture load by about 20% which is within the experimental error range. The under-prediction of the failure load constantly increases to  $\sim 50\%$  as the V-notch angle increases to  $90^\circ$  and plastic zone area increases to  $\sim 0.5 \text{ mm}^2$  (for a V-notch depth of 5 mm). The under-estimated fracture load at the  $30^\circ$  may be attributed to an under-estimation of the fracture toughness.

The detailed experimental database is suggested to be used for validation purposes for various failure criteria developed for quasi-brittle materials. The recently developed quasi-brittle FFMCC in [2] will be investigated (in a follow-up work) whether it can better predict failure loads for such steels with V-notches having a V-notch angle larger than  $30^\circ$ .

## References:

[1] D Leguillon. Strength or toughness? A criterion for crack onset at a notch. *Eur. Jour. Mech. A - Solids*, **21**(1):61–72, 2002.

[2] D. Leguillon and Z. Yosibash. Failure initiation at V-notch tips in quasi-brittle materials. *Int. J. Solids and Structures*, **122–123**:1–13, 2017.

# Considering small scale yielding in predicting failure initiation of quasi-brittle materials

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**Keywords:** *Plasticity, quasi-brittle materials, coupled criterion*

**Abstract:** Within the framework of Finite Fracture Mechanics, the Coupled Criterion (CC) has shown to be effective in predicting failure initiation in brittle materials like ceramics, some polymers, rocks... [1]. On the other hand, ductile fracture models can be used to describe the failure of fully plastic materials like copper, aluminum, some steel grades... Between these two behaviors, there is a wide range of materials that develop a damaged or a plastic zone but which remains small and confined to the vicinity of stress concentration points. They are so-called quasi-brittle materials, there are some polymers, some other steel grades...

In a theoretical analysis performed in [2] we tried to analyze the influence of a small damaged zone around the root of a V-notch where Young's modulus, toughness and tensile strength evolve under the effect of an increasing load. The crack nucleation is not a direct consequence of damage (i.e. a vanishing damage parameter), but it obeys the conditions imposed by the CC and is strongly influenced by the surrounding degraded material properties.

Following a series of experiments on 4340 steel specimens with V-notches of 30°, 60° and 90° [3] we try to extend the CC to cases where a small-scale yielding plasticity is evident at the V-notch tips. The plastic zone size was measured using DIC and the failure load was recorded. Having available this information and using the Ramberg-Osgood power law for the plastic zone, the stresses still exhibit a singular behavior when approaching the notch root like the Rice and Rosengren model. The main difference in our approach compared to the damage model lies in the elastic unloading phase due to the crack jump at onset (a damage model unloads along a slope of the damaged material whereas in our present analysis it follows the elastic material slope). At the crack onset the plastic zone is assumed frozen. The next step, now ongoing, is to split the measured fracture energy into a part due to plasticity and the other one to crack onset (somehow Griffith's term  $2\gamma$ ).

We shall present a preliminary analysis in an effort to enhance the CC to cases where a small-scale yielding zone develops prior to crack initiation at the V-notch tip.

## References:

[1] Weissgraeber P., Leguillon D., Becker W., A review of Finite Fracture Mechanics: Crack initiation at singular and non-singular stress-raisers. Arch. Appl. Mech. 86, 2016, 375-401.

[2] Leguillon D., Yosibash Z., Failure initiation at V-notch tips in quasi-brittle materials. Int. J. Solids Structures 122-123, 2017, 1-13.

[3] Yosibash Z., Mendelovich V., Gilad I., Bussiba A., Can the finite fracture mechanics coupled criterion be applied to the V-notch tip of a quasi-brittle steel alloy. Submitted.

# Preliminary phase field implementation for bone fracture considering heterogeneous elastic and fracture properties

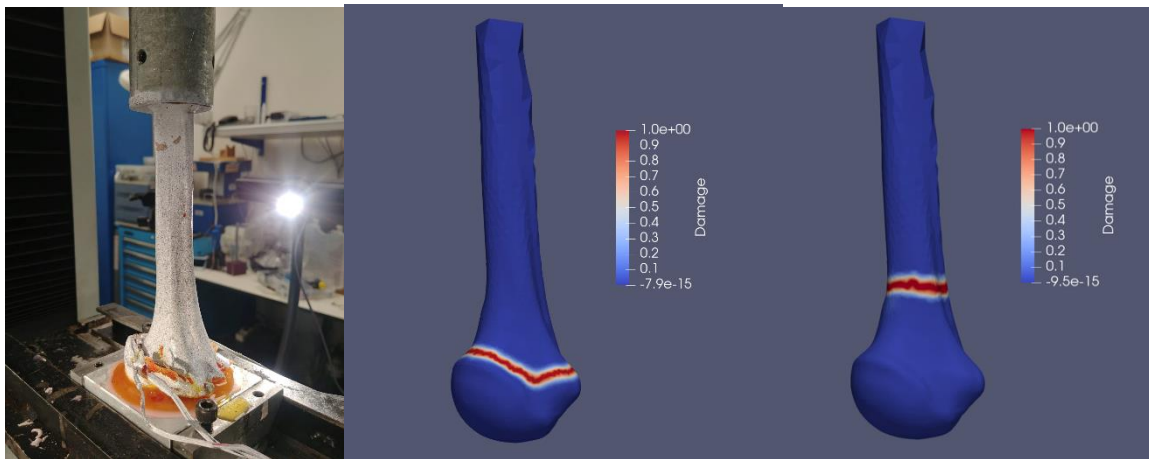
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**Keywords:** Human long bone fracture, phase field method, heterogeneous Young modulus, heterogeneous fracture toughness, digital image correlation

**Abstract:** Bone fracture prediction by CT-based Finite Element Analysis has been studied in [1] based on maximum principal strain criterion. This simplified criterion can predict fracture initiation load within 20% error (conservative prediction). It cannot predict the crack path which may also be of interest for clinical application. In an attempt to improve bone fracture prediction, FEA and Phase Field Models (PFMs) are investigated.

CT-based mesh and heterogeneous Young modulus  $E(x)$  are imported into FEniCs for linear elastic FEA and PFM implementation for a humeral bone under compressive loading condition. Comparison with experimental results similar to those presented in [2] indicates PFM does not capture fracture location by only considering density-based heterogeneous  $E(x)$ . Introducing an assumed density-based heterogeneous fracture toughness  $G_c(x)$  leads to good prediction of fracture location ([3] and Fig. 1). Density-based correlation of bone fracture toughness is investigated through three-point bending fracture testing on cortical bone specimens and estimation of critical stress intensity factor  $K_{Ic}$  and energy release rate  $G_c$  using Digital Image Correlation (DIC).



**Fig. 1:** Phase field implementation for predicting fracture location in humeral bone under compressive loading applied on top boundary (distal) and fully clamped on bottom boundary (proximal). From left-to-right: experimental observation, PFM considering heterogeneous  $E(x)$  and  $G_c(x)$ , PFM only considering  $E(x)$ .

**Acknowledgement:** The authors thank Prof. CorradoMaurini (Sorbonne Université) for assistance and insights.

## References:

- [1] Z. Yosibash, R. Plitman Mayo, G. Dahan, N. Trabelsi, G. Amir, C. Milgrom. 2014. Predicting the stiffness and strength of human femurs with real metastatic tumors. Bone, volume 69: pages 180-190.
- [2] G. Dahan, N. Trabelsi, O.Safran, Z.Yosibash. 2019. Finite element analyses for predicting anatomical neck fracture in the proximal humerus. Clinical Biomechanics, volume 68: pages 114-121



[3] R. Shen, H. Waisman, Z. Yosibash, G. Dahan. 2019. International Journal for Numerical Methods in Biomedical Engineering, volume 35.

## **A computational framework for the modeling of coupled hygro-thermo-mechanical problems in the thin-walled photovoltaic laminates**

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**Keywords:** *Hygro-thermo-mechanical coupling, Thermo-visco-elastic behavior, Moisture diffusion, Finite element method*

**Abstract:** A three-dimensional hygro-thermo-mechanical computational framework for the photovoltaic (PV) laminates as well as its numerical implementation is proposed in this work. Aiming at the efficient thermo-mechanical modeling of the thin-walled structures with polymeric interfaces [1, 2], the solid shell element, which incorporates the enhanced assumed strain (EAS) method and assumed natural strain (ANS) method for the alleviation of locking pathologies, and interface element with thermo-visco-elastic cohesive zone model using the fractional calculus method is formulated. Besides, the finite element (FE) implementation of moisture diffusion in the 3D setting along the polymeric interfaces [3] is also derived with the consideration of spatial and temporal variation of diffusivity due to its temperature and gap dependency. Given the difference between the time scales of moisture diffusion and thermo-mechanical problems, a staggered scheme is proposed to solve the coupled hygro-thermo-mechanical governing equations. Specifically, the relative displacement and temperature fields are firstly solved from the thermo-mechanical analysis, and then projected to the FE model of moisture diffusion to determine the diffusion coefficient for its subsequent analysis. The proposed method is applied to the simulation of three international standard tests of PV modules, namely the damp heat test, the humidity freeze test, and the thermal cycling test, and numerical predictions are compared with analytical solution for the damp heat case with the constant temperature boundary condition, as well as experimental electroluminescence (EL) images obtained from the thermal cycling test with the cyclic temperature boundary condition, in which good consistency demonstrates the effectiveness and reliability of this framework.

### **References:**

[1] Paggi, M., Kajari-Schröder, S. and Eitner, U., 2011. Thermomechanical deformations in photovoltaic laminates. *The Journal of Strain Analysis for Engineering Design*, 46(8), pp.772-782.

[2] Paggi, M. and Sapora, A., 2015. An accurate thermoviscoelastic rheological model for ethylene vinyl acetate based on fractional calculus. *International Journal of Photoenergy*, 2015.

[3] Kempe, M.D., 2006. Modeling of rates of moisture ingress into photovoltaic modules. *Solar Energy Materials and Solar Cells*, 90(16), pp.2720-2738.

# MONDAY, MAY 9

## AFTERNOON SESSIONS

14:00-14:30: **Israel Garcia**

14:30-15:00: **KarthikAmbikakumari**

15:00-15:30: **Sara Jimenez**

-

16:00-16:30: **Angel Valverde**

16:30-17:00: **Maria Rosaria Marulli**

17:00-17:20: **Juan Manuel Macias**

## On the effect of the interface radius on the ability to arrest cracks of curved weak interfaces

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**Keywords:** Curved interfaces, Crack arrester, Crack deflection, Finite Fracture Mechanics, Phase Field.

**Abstract:** Curved weak interfaces present promising advantages to be implemented as crack arrestors in structures designed under the tolerant-design principles. Among other advantages, they neither add extra weight nor affect significantly to the global stiffness of the structural element.

To be employed as crack arrestor, it is key that the interface is able to deviate the crack. If the crack penetrates across the interface, the effect of the weak interface as crack arrestor is canceled. In view of this, this work studies how to set the interface parameters to promote the crack deviation along the interface.

In particular, following the dimensional analysis of the problem, the effect of three significant dimensionless parameters are studied: interface to bulk fracture toughness, interface to bulk tensile strength and the interface curvature radius normalized with the material characteristic length.

The study is carried out using three approaches widely used currently for this type of problems: Linear Elastic Fracture Mechanics, Finite Fracture Mechanics, and a combination of Phase field and Cohesive Zone Model.

The results presents a clear effect of some parameters, as the ratio of interface to bulk fracture toughness, for which the three approaches agree. However, the results are moderately diverse in which correspond to the effect of the ratio of interface to bulk tensile strength and quite divergent in what respect to the effect of the radius. The results are interpreted and explained as a consequence of the main assumptions behind the approaches studied.

## Finite Element implementation of the Coupled Criterion based on the Principle of Minimum Total Energy subjected to a Stress Condition to predict crack onset and growth

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**Keywords:** *Crack onset and growth, Finite Fracture Mechanics (FFM), FEM, PMTE-SC*

**Abstract:** The aim of this work is to develop a numerical procedure predicting crack onset and growth in brittle materials. This procedure is based on the Coupled Criterion of Finite Fracture Mechanics (CCFFM) proposed by Leguillon [1], assuming crack advances by finite steps, and requiring that both stress and energy conditions are fulfilled. The prospective crack advance is modelled by a discontinuous representation of crack bridged by a continuous distribution of stiff springs with a linear elastic behaviour up to its breakage, which is a similar approach as used previously in the Linear Elastic-(perfectly) Brittle Interface Model (LEBIM) [2] combined with CCFFM [3]. The load stepping algorithm applies the Principle of Minimum Total Energy subjected to a Stress Condition (PMTE-SC), where the energy criterion is imposed by minimizing the total energy change due to a crack advance allowed by the stress criterion. This new formulation of the CCFFM was introduced by Mantič [4], see also [5]. The PMTE-SC appears to be more versatile than the classical formulation of CCFFM using stress and energy criterion curves. Thus, an implementation of PMTE-SC in FEM could provide a computational tool capable of solving complex practical problems.

A most simple implementation PMTE-SC in FEM seems to consider cracks geometrically modelled as topological discontinuities in the mesh, with cracks introduced explicitly during the discretization of the domain, thus matching the crack faces with the element edges. In this work, we develop the PMTE-SC implementation within the FEM software ABAQUS version 6.4 using subroutines such as UINTER, USDFLD, etc. The contact routine in ABAQUS was adapted to account for the bridging stresses. The user subroutine UINTER was programmed to specify the stress-opening relations. Several numerical examples are presented to show the capabilities of the developed procedure.

### References:

- [1] D. Leguillon. 2002. Strength or toughness? A criterion for crack onset at a notch. *European Journal of Mechanics A/Solids*, 21, 61–72.
- [2] V. Mantič, L. Távara, A. Blázquez, E. Graciani, F. París. 2015. A linear elastic-brittle interface model: application for the onset and propagation of a fibre-matrix interface crack under biaxial transverse loads. *International Journal of Fracture* 195, 15-38.
- [3] M. Muñoz-Reja, L. Távara, V. Mantič, P. Cornetti. 2016. Crack onset and propagation at fibre-matrix elastic interfaces under biaxial loading using finite fracture mechanics. *Composites Part A*, 82, 267–278.
- [4] V. Mantič, 2014. Prediction of initiation and growth of cracks in composites. Coupled stress and energy criterion of the finite fracture mechanics (Keynote Lecture). In: *Proceedings of the 16th European Conference on Composite Materials (ECCM16)*, F. París (Ed.), <http://www.escm.eu.org/eccm16/assets/1252.pdf>.
- [5] M. Muñoz-Reja, V. Mantič, L. Távara. 2022. Comparative analytical study of the coupled criterion and the principle of minimum total energy with stress condition applied to linear elastic interfaces. *Theoretical and Applied Fracture Mechanics*, 103274.

# Toughening mechanisms in platelets reinforced glass. Numerical implementation of the Coupled Criterion: matched asymptotic approach

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**Keywords:** Finite Fracture mechanics – Coupled Criterion - Micro-scale – Toughening mechanisms

**Abstract:** According to [2], the fracture toughness of the glass can be increased introducing a reinforcing second element, i.e., a secondary constituent with high modulus, strength and/or ductility, implemented in the form of fibers, whiskers, platelets, or particulates. In such composites different toughening mechanisms appear. An example is the borosilicate glass/ $\text{Al}_2\text{O}_3$  platelet composite, proposed by [1], widely used in the industry for being environmentally friendly and low-cost. Previous experimental studies about its fracture properties were made in [2], in which the influence of the reinforcement volume fraction and the mechanical properties were investigated.

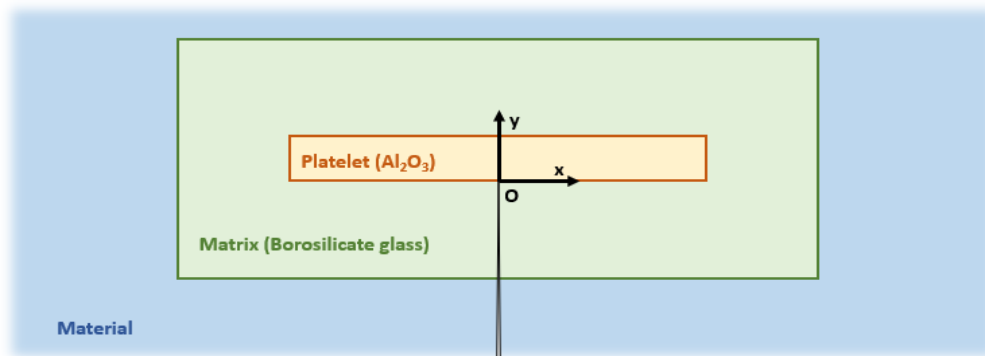


Figure 1: 2D model considered in the matched asymptotic approach of the CC

The aim of this work is to analyze the answer brought by the Coupled Criterion (CC) [3] considering this kind of materials. In the framework of Finite Fracture Mechanics, the CC joins two conditions for a crack to be nucleated: an energy and a stress condition, depending on the critical energy release rate and the strength of the material. For this study a Matched Asymptotic approach (Figure 1) is applied, the platelet being small compared to the specimen. The interface between the matrix and the platelet is confirmed to be strong, see [2]. Moreover, a preexisting crack is assumed to impinge the platelet.

We study the influence of different parameters, such as the size of the platelet, various inclinations and positions of the preexisting crack. Our objective is to determine the predominant failure mechanism among a wide range of possibilities, such as a debonding of the interface between the platelet and the matrix at the end of the platelet, a deviation of the initial preexisting crack along the interface, a penetration of the crack in the platelet, or a crack jump into the matrix, with no penetration in the platelet. The toughening influence of the platelet is then derived.

## References:

- [1] Boccaccini, A. R., & Trusty, P. A. (1996). Toughening and strengthening of glass by  $\text{Al}_2\text{O}_3$  platelets. *Journal of materials science letters*, 15(1), 60-63.
- [2] Kotoul, M., Pokluda, J., Šandera, P., Dlouhý, I., Chlup, Z., & Boccaccini, A. R. (2008). Toughening effects quantification in glass matrix composite reinforced by alumina platelets. *Acta materialia*, 56(12), 2908-2918.

[3] Leguillon, D. (2002). Strength or toughness? A criterion for crack onset at a notch. *European Journal of Mechanics-A/Solids*, 21(1), 61-72.

## Computational modelling of hydrogen embrittlement in polycrystalline materials

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**Keywords:** *Phase-field, Hydrogen Embrittlement, Cohesive Zone Model, Elastoplastic Fracture, FEM*

**Abstract:** Hydrogen Embrittlement (HE), which usually occurs in metallic structures exposed to corrosive environments with Hydrogen (H), is one of the main obstacles for boosting and upscaling green solutions based on H. This phenomenon consists of a dangerous and unpredictable degradation of ductility and strength caused by the presence of gaseous H in the local environment. From a physical perspective, the micro-mechanism governing HE is not yet completely understood. Proof of that is the absence of a database on the mechanical response of steels affected by HE. Consequently, there is a strong need to understand, quantify, and model how H affects the mechanical performance of metals, inducing fracture, and reduces its durability. The effect of HE is observed, among others, in polycrystalline structures, where H plays an essential role in the crack pattern and, moreover, in the mechanical performance. Specifically, the entrance on H within these structures is not only associated to a premature failure, but also on the onset of a competition in the crack pattern: trans-granular (crack propagating through the grains, ductile) vs. inter-granular (crack propagating through the grain boundaries, fragile) fracture. In the field of HE, this competition happens in steels, aluminum alloys and specially, in nickel superalloys, which exhibit fracture in the grain boundaries (interface delamination) under highly corrosive environments.

Considering the inherent heterogeneous character of this micro-mechanism, a Finite Element micro-scale approach employing the phase-field (PF) technique is proposed. Specifically, the current approach is based on the work proposed in [1], being applied on the level of grain-size structures. With the aim of providing a further insight into this problem, the previous HE bulk-fracture methodology is combined with a Cohesive Zone Model (CZM) approach in surfaces to model grain boundaries, which will follow a traction-separation cut-off law developed in [2]. The proposed framework is applied to simulate fracture in poly-crystalline samples. It has shown very appealing predictive capabilities leading to a robust correlation with several experimental cases regarding nickel superalloys [3,4], being the first of its kind to model HE-induced failure in grain frontiers within the context of the PF approach.

**Acknowledgements:** AVG and MP would like to acknowledge support from the Italian Ministry of University and Research to the Project of National Interest (PRIN2017) 'X-FAST-SIMS: Extra fast and accurate simulation of complex structural systems'. EMP acknowledges financial support from EPSRC funding under grant No. EP/R010161/1 and from support from the UKCRIC Coordination Node, EPSRC grant number EP/R017727/1, which funds UKCRIC's ongoing coordination. JR is grateful to the support of the Ministry of Science, Innovation and Universities (Project PGC2018-099197-B-I00) and Consejería de Economía y Conocimiento, Junta de Andalucía, and European Regional Development Fund (Projects US-1265577 and P20-00595).

### References:

- [1] E. Martínez-Pañeda, A. Golahmar, C. F. Niordson. 2018. A phase field formulation for hydrogen assisted cracking. *CMAME* 342: 742–761.
- [2] M. Paggi, J. Reinoso. 2017. Revisiting the problem of a crack impinging on an interface: a modeling framework for the interaction between the phase field approach for brittle fracture and the interface cohesive zone model. *CMAME* 321: 145 – 172
- [3] E. Martínez-Pañeda, Z. D. Harris, S. Fuentes-Alonso, J. R. Scully, J. T. Burns. 2020. On the suitability of slow strain rate tensile testing for assessing hydrogen embrittlement susceptibility. *Corrosion Science* 163 108291.

[4] Z. D. Harris, S. K. Lawrence, D. L. Medlin, G. Guetard, J. T. Burns, B. P. Somerday. 2018. Elucidating the contribution of mobile hydrogen-deformation interactions to hydrogen-induced intergranular cracking in polycrystalline nickel. *Acta Materialia* 158: 180–192.

### **Failure analysis of thin layer–flexible substrate systems through a combined phase-field and cohesive zone model approach**

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**Keywords:** *Phase-field method, Cohesive Zone Model, thin layer-substrate systems, hyperelastic materials, delamination vs. crack propagation in the bulk.*

**Abstract:** Many cutting-edge technologies integrate materials with a notable mismatch between the mechanical and fracture properties of the components: new-generation of flexible-electronic devices, bio-medical sensors, adhesive-bonded joints, and solar-cell wafers, where polymeric substrates are usually combined with stiffer layers. These structural components can experience different failure mechanisms: crack penetration through the layers, cohesive delamination at the interfaces, and mixed-mode mechanisms integrating the previous two damage modes.

In this work, a phase-field approach has been formulated in the framework of finite elasticity and used to study the effect of material properties mismatch on the crack propagation in thin layer–flexible substrate systems. The nonlinear material response of the constituents has been implemented following the constitutive model proposed in [1]. The framework has been further enriched by considering the existence of imperfect interfaces between the joint components, modeled using the Cohesive Zone Model (CZM) and a polynomial-based Tvergaard traction-separation law [2, 3]. This constitutive law provides a gradual degradation with a larger extent of the so-called cohesive process zone compared to the cut-off interface law implemented in [4].

The computational framework has been applied in various analyses studying the dependence of the damage pattern on the material (ceramic, polymer, metal) of the thin layer and the competition between crack propagation in the bulk and delamination at the interface. Although the phase-field approach already captures the crack propagation at the interface in some configurations, the coupling with CZM gives a better insight into the different failure mechanisms and the accuracy of the simulations for Mixed Mode failure providing a plausible tool of interest in industrial uses.

**Acknowledgements:** The authors would like to acknowledge support from the Italian Ministry of University and Research to the Project of National Interest (PRIN2017) 'X-FAST-SIMS: Extra fast and accurate simulation of complex structural system' (Grant agreement no. 20173C478N).

#### **References:**

[1] C. Miehe and L. M. Schänzel, 2014, Phase field modeling of fracture in rubbery polymers. Part I: Finite elasticity coupled with brittle failure, *Journal of the Mechanics and Physics of Solids*, vol. 65 (2): 93–113.

[2] V. Tvergaard, 1995, Fibre debonding and breakage in a whisker-reinforced metal, *Materials Science and Engineering: A*, vol. 190 (1–2): 215–222.

[3] H. D. Espinosa and P. D. Zavattieri, 2003, A grain level model for the study of failure initiation and evolution in polycrystalline brittle materials. Part I: Theory and numerical implementation, *Mechanics of Materials*, vol. 35 (3-6): 333–364.

[4] M. Paggi and J. Reinoso, 2017, Revisiting the problem of a crack impinging on an interface: A modeling framework for the interaction between the phase field approach for brittle fracture and the interface cohesive zone model, *Computer Methods in Applied Mechanics and Engineering*, vol. 321: 145–172.

## Micromechanical analysis of composite materials using Phase-Field fracture coupled with plasticity

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**Keywords:** *Finite element method, Phase-Field, Micromechanics, Matrix cracking.*

**Abstract:** Failure in composite materials when studied at the component or micro-scale is a collection of several interacting damaging phenomena such as fiber and matrix fracture, interface debonding and even plasticity effects in the matrix. Most of these events take place simultaneously and depend on a great number of material parameters. These parameters can potentially be related to the scale such as the fracture toughness and strength; but also, to the degree of volumetric constraint, such as the plastic response and fracture initiation. As a consequence, realistic material parameters are very difficult to measure and approximation of bulk properties may be inaccurate if used to approximate micro-scale behavior. In this work, the Phase-Field (PF) method coupled with pressure dependent and pressure independent plasticity models have been used to investigate the influence of material behaviour in the process of failure initiation and progression of composite materials at the micro-scale. Matrix brittle fracture using standard PF formulations is compared with PF coupled with Von Mises (pressure independent) and a pressure dependent plasticity model for thermoset matrices. This work studies the influence of the implementation (in the form of a User Material Model -UMAT- or a User defined Element -UEL- in Abaqus) and different solution schemes in the running time and convergence of the simulations. It is found that coupling Phase-Field fracture with plasticity requires special measures to overcome convergence problems, and the implementation of the UEL turns out to be the most interesting solution to overcome those problems.



# TUESDAY, MAY 10

## MORNING SESSIONS

10:00-10:30: **CorradoMaurini**

10:30-11:00: **Mauro Corrado**

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11:30-12:00: **SindhuBushpalli**

12:00-12:20: **Francesco Vicentini**

12:20-12:40: **Camilla Zolesi**

12:40-13:00: **Anatoli Mitrou**



# Numerical Bifurcation and Stability Analysis of Gradient Damage Models

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**Keywords:** *damage, fracture, quasi-static evolution, stability.*

**Abstract:** Modern approaches to fracture mechanics cast the irreversible evolution problem of a brittle solid as a series of constrained incremental energy-minimisation problems where time plays the role of an independent parameter. Their underlying mechanical model is that of an elastic-damageable material which, in the context of irreversible dissipative systems, has a variational (energetic) link with classical theories of fracture [1, 2]. Such models are characterised by material softening and instabilities, are nonconvex and strongly nonlinear, and their associated evolution problems may exhibit multiple solutions, or none. In this context, and even in simple (2dof) toy models, typical counter-intuitive features of such non-smooth systems manifest continuous families of bifurcations, the conceptual distinction between bifurcation and stability thresholds, and that between the stability of a state vs. that of an evolution. For large systems of applicative interest, a fundamental problem is to select among all possible evolution trajectories those which are observable in physical experiments. We present [3] novel numerical techniques for the bifurcation and stability analysis of quasi-static evolution paths as well as practical tools to select stable evolutions based on the solution of an eigenvalue problem under irreversibility constraints. As we illustrate with several numerical examples, this approach allows us to filter unstable solutions provided by standard (first-order) minimisation algorithms and to effectively compute stable evolution paths. We demonstrate our purpose on a fragmentation problem featuring the emergence of complex fracture patterns with robust morphological properties.

## References:

- [1] B. Bourdin, G. A. Francfort, and J.-J. Marigo. The variational approach to fracture. *Journal of elasticity*, 91(1-3):5–148, 2008.
- [2] K. Pham, H. Amor, J.-J. Marigo, and C. Maurini. Gradient damage models and their use to approximate brittle fracture. *International Journal of Damage Mechanics*, 20(4):618–652, 2011.
- [3] A. A. León Baldelli and C. Maurini. Numerical bifurcation and stability analysis of variational gradient-damage models for phase-field fracture. *Journal of the Mechanics and Physics of Solids*, 152:104424, 2021.

## A probabilistic FEM approach for structural integrity assessment of glass components

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**Keywords:** *Glass, Probability of failure, XFEM, Monte Carlo, Flaws size distribution*

**Abstract:** Stress-based failure criteria are usually adopted for the design of glass components [1], although they rely on a parameter, namely the glass strength, that is not a real material property since it depends on the fracture toughness, the surface flaws size distribution, the test setup and the specimen geometry [2]. Consequently, large safety factors are introduced, to ensure a safe design. Furthermore, interaction between cracks and the effect of stress concentrations are usually neglected. In order to overcome the limitations of such approaches, a new computational methodology is herein proposed, adopting a stress-intensity factor based failure criterion, which is by far a more suitable criterion for glass, a material highly sensitive to cracks.

The newly developed methodology consists in: (i) modeling the structural element through the finite element method, (ii) randomly applying to the FE model a population of surface flaws extracted from a pre-defined statistical distribution function, (iii) computing the related stress-intensity factors, (iv) evaluating the load carrying capacity by equating the maximum stress-intensity factor to the fracture toughness. Since a single simulation is meaningless because of the stochastic character of the problem, the procedure is repeated several times by varying the population of surface flaws in order to obtain the probability density function of the failure load. Finally, the critical load referred to a chosen probability of failure is derived.

The extended Finite Element method is used because of its inherent capability to deal with multiple cracks of any position and length without adapting the mesh topology [3], and the enrichment functions proposed by Liu et al. [4] are implemented since they enable a direct computation of the stress-intensity factors at the tip of the cracks. The current version of the numerical methodology is limited to plane stress/strain models, although its extension to 3D problems is quite straightforward.

Different case studies are presented, to prove the reliability of the method for the structural integrity assessment of glass components.

### References:

- [1] EN 16612:2019 (2019) Glass in building - Determination of the lateral load resistance of glass panes by calculation, European Committee for Standardization, Brussels.
- [2] L. Afferrante, M. Ciavarella, E. Valenza (2006) Is Weibull's modulus really a material constant? Example case with interacting collinear cracks, *International Journal of Solids and Structures* 43, 5147–5157.
- [3] T. Belytschko, R. Gracie, G. Ventura (2009) A review of extended/generalized finite element methods for material modeling, *Modelling and Simulation in Materials Science and Engineering* 17, 043001.
- [4] X. Y. Liu, Q. Z. Xiao, B. L. Karihaloo (2004) XFEM for direct evaluation of mixed mode SIFS in homogeneous and bi-materials, *International Journal for Numerical Methods in Engineering* 59, 1103–1118.

## Study of unfolding failure in multidirectional CFRP laminates using phase field fracture method

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**Keywords:** *Unfolding, Delamination, Intralaminar cracks, Interlaminar failure, Phase Field, Finite Fracture Mechanics*

**Abstract:** Delamination, which is one of the most critical failure modes in laminated composites, requires meticulous evaluation to ensure the structural integrity of the component. Such interlaminar failure, which typically occurs in curved composite laminates when they are loaded under a bending moment trying to open the curvature, is termed unfolding failure. This failure is characterized by the interlaminar tensile strength (ILTS) and is generally obtained by the four-point bending test when applied to a unidirectional laminate with plies oriented at 0° [1].

The present study focuses on investigating the delamination behavior in the curved part of L-shaped specimens. To achieve this, several options are thoroughly examined.

At first, the delamination at a 0°//0° interface in unidirectional curved laminate is modelled using cohesive elements and the behavior is analyzed. Since delamination propagation is very unstable, a control algorithm explained in [2,3] is used to capture these instabilities and displacement jumps. While the ILTS problem is subjected to load control, the algorithm uses special auxiliary nodes to load the model under displacement control with the crack tip opening displacements.

In quasi-isotropic and cross-ply layup configurations, failure is primarily initiated by transverse matrix cracks, which further propagate as delaminations in the presence of high interlaminar normal stresses (INS). The phase field fracture method in terms of the UMAT/HETVAL subroutine [4] is exploited to demonstrate these matrix-dominated intralaminar damages in multiangle configurations.

In the first stage, the onset of interlaminar or intralaminar damage is analyzed by means of FEM models in which only the desired fracture mechanism is considered. In a subsequent stage, a detailed study is carried out to determine the interactions between (a) multiple delaminations and (b) intralaminar damage with the respective delaminations.

To this end, more complicated FEM models are employed, combining phase field methodologies and interfacial cohesive damage methodologies.

### References:

- [1] J.M. González-Cantero, E. Graciani, B. López-Romano, and F. París. 2018. Competing mechanisms in the unfolding failure in composite laminates. *Compos Sci Technol*, 156:223–230.
- [2] E. Martínez-Pañeda, S. del Busto and C. Betegón. 2017. Non-local plasticity effects on notch fracture mechanics. *Theoretical and Applied Fracture Mechanics*, 92:276-287.
- [3] J. Segurado and J. Llorca. 2004. A new three-dimensional interface finite element to simulate fracture in composites. *International Journal of Solids and Structures* 41:2977-2993.
- [4] Y. Navidtehrani, C. Betegón, E. Martínez-Pañeda. 2021. A Unified Abaqus Implementation of the Phase Field Fracture Method Using Only a User Material Subroutine. *Materials*, 14(8):1913.

# Phase-field modeling of brittle fracture in heterogeneous materials: a one-dimensional study

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**Keywords:** *Heterogeneity, Phase-field, 1D, Effective fracture toughness, Effective strength.*

**Abstract:** Phase-field modeling of brittle fracture, first proposed by Bourdin et al. [1] as regularization of Francfort and Marigo's variational approach to fracture mechanics [2] and later re-interpreted as a special family of gradient damage models [3], provides a remarkably flexible variational framework to describe the nucleation and propagation of cracks with arbitrarily complex geometries and topologies in two and three dimensions.

The approach is based on the assumption that the brittle material exhibits homogeneous elastic and fracture properties (fracture toughness). On the other hand, many brittle materials are characterized by strongly heterogeneous properties, one important category being biological tissues such as bones. Phase-field modeling of fracture in these tissues is particularly attractive due to their typically complex crack patterns, yet it requires the extension of the approach to the case of heterogeneous mechanical properties.

Previous studies addressing fracture in heterogeneous materials have adopted a pragmatic approach, by simply substituting the constant fracture toughness of the original model with a fracture toughness depending on the material point [4, 5, 6, 7]. However, the implications of heterogeneous material properties on the key predictions of the phase-field model have not been thoroughly investigated yet.

In this contribution, we perform such investigation with the one-dimensional case. We revisit the fundamental mathematical analysis in [3] by assuming that the material properties are heterogeneous with different possible profile shapes. Our main goal is to quantitatively assess how the heterogeneity in material properties influences the fracture toughness and the tensile strength of a one-dimensional bar.

## References:

- [1] G. A. Francfort, J. J. Marigo. 1998. Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids*, 46(8), pp.1319-1342.
- [2] B. Bourdin, G.A. Francfort, J.J. Marigo. 2000. Numerical experiments in revisited brittle fracture. *Journal of the Mechanics and Physics of Solids*, 48(4), pp.797-826.
- [3] K. Pham, H. Amor, J. J. Marigo, C. Maurini. 2011. Gradient damage model and their use to approximate brittle fracture. *International Journal of Damage Mechanics*, 20(4), pp.618-652.
- [4] P. A. V. Kumar, A. Dean, J. Reinoso, P. Lenarda, M. Paggi. 2021. Phase field modeling of fracture in Functionally Graded Materials:  $\Gamma$ -convergence and mechanical insight on the effect of grading. *Thin-Walled Structures*, 159, p.107234.
- [5] S. Natarajan, R. K. Annabattula, E. Martínez-Pañeda. 2019. Phase field modelling of crack propagation in functionally graded materials. *Composites Part B: Engineering*, 169, pp.239-248.
- [6] M. Z. Hossain, C. J. Hsueh, B. Bourdin, K. Bhattacharya. 2014. Effective toughness of heterogeneous media. *Journal of the Mechanics and Physics of Solids*, 71, pp.15-32.
- [7] R. Shen, H. Waisman, Z. Yosibash, G. Dahan. 2019. A novel phase field method for modeling the fracture of long bones. *International journal for numerical methods in biomedical engineering*, 35(8), p.e3211.

## Nucleation and propagation of cracks under multiaxial loading in phase-field modelling

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**Keywords:** *damage, fracture, phase field, variational methods, finite elements.*

**Abstract:** The phase-field approach to brittle fracture is based on the following minimization problem

$$\min_{\mathbf{u}, \alpha} \mathcal{E}_{\text{tot}}(\mathbf{u}, \alpha) = \int_{\Omega} [\varphi(\boldsymbol{\varepsilon}(\mathbf{u}), \alpha) + \psi_{\text{diss}}(\alpha, \nabla \alpha)] d\Omega$$

where  $\mathcal{E}_{\text{tot}}$  is the total energy functional,  $\mathbf{u}, \alpha, \boldsymbol{\varepsilon}(\mathbf{u})$  are respectively the displacement and damage fields and the strain tensor, and  $\varphi$  and  $\psi_{\text{diss}}$  are respectively the elastic and the dissipated energy density. The minimization problem is solved under the irreversibility constraint for the damage field. Recent results have proven that phase-field approaches can quantitatively predict crack nucleation for mode-I loading [1] applied to compressible materials. However, the prediction of crack nucleation under multiaxial loading and for materials whose bulk moduli tends to infinity is crucially influenced by the elastic energy density expression.

Under multiaxial loading, the elastic energy density is split into a damageable part ( $\varphi_D$ ) and a residual part ( $\varphi_R$ ), which serves the double purpose of avoiding interpenetration of the crack surfaces under compression and reflecting the physical asymmetry of fracture behaviour between tension and compression. Currently available decompositions (e.g., those in [2,3]) for compressible materials are insufficiently flexible [4] as they do not allow to reproduce the experimentally measured tensile and compressive (or shear) strengths. To solve the issue, a novel decomposition was proposed in [4], giving a parametric strength surface à la Drucker-Prager to be adjusted based on the experimentally measured tensile and compressive strength.

In this contribution, the new decomposition is implemented numerically to verify the analytical nucleation curves. Moreover, the issue affecting nucleation and propagation in materials close to the compressibility limit is discussed. An example of a basic code of phase-field modelling for different elastic energy densities is shown using FEniCSx. In addition, the analysis is extended beyond nucleation to analyse the localization modes under multiaxial loading and to study the propagation behaviour. This implies dealing with the non-linearities introduced by the energy decompositions and bad conditioning and locking issues related to the corresponding linear system.

### References:

- [1] E. Tanné, T. Li, B. Bourdin, J.-J. Marigo, C. Maurini. 2018. Crack nucleation in variational phase-field models of brittle fracture. *J. Mech. Phys. Solids*, 110: 80-99.
- [2] H. Amor, J.-J. Marigo, C. Maurini. 2009. Regularized formulation of the variation brittle fracture with unilater contact: numerical experiments. *J. Mech. Phys. Solids*, 57: 1209–1229.
- [3] F. Freddi, G. Royer-Carfagni. 2010: Regularized variational theories of fracture: a unified approach. *J. Mech. Phys. Solids*, 58: 1154–1174.

[4] L. De Lorenzis, C. Maurini. 2021. Nucleation under multi-axial loading in variational phase-field models of brittle fracture. *Int. J. Frac.*

## Extending the PF method to the fracture of generally layered thin-ply laminates

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**Keywords:** *Phase Field, Anisotropy, Thin-ply composites, Fracture*

**Abstract:** Thin-ply carbon fiber reinforced polymer (CFRP) composites are gaining significant appeal in the aerospace world as they can lead to significant weight reductions without any compromise in plain strength. Their thin nature though can render modeling them in ways typical up to now for composites inefficient. But since they present a quasi-brittle type of failure with a significant reduction in the complexity of the damage mechanisms, contrary to standard CFRP laminates. So, as mentioned by Arteiro et al. [1], and showcased in their early work with Reinoso et al. [2] for the open-hole tension (OHT) of quasi-isotropic thin-ply laminates, treating fracture in this type of CFRPs might be best done at the macroscale, by means of an equivalent single layer (ESL) representation approach, utilizing theories and methods for brittle fracture. In line with that, the Phase Field (PF) method for fracture, is considered here. In this case, full consideration of elastic and fracture anisotropy is included in the modeling methodology, based on the approach of Camanho and Catalanotti [3], that provides an analytical way of obtaining a laminate's toughness requiring as inputs, only the intralaminar toughness of a 0-degree ply, the lay-up of the laminate and the laminas's properties itself. The anisotropic PF model is implemented in a combination of a UMAT along with a UMATHT extending the approach of Navidtehrani et al. [4] to keep the application of the model at a rather simple level. Numerical off-axis OHT results to show the methods capabilities and the influence of the various parameters included in the anisotropic PF model are presented.

### References:

[1] Arteiro, Albertino, Giuseppe Catalanotti, José Reinoso, Peter Linde, and Pedro P. Camanho. "Simulation of the mechanical response of thin-ply composites: from computational micro-mechanics to structural analysis." *Archives of Computational Methods in Engineering* 26, no. 5 (2019): 1445-1487.

[2] Reinoso, José, Albertino Arteiro, Marco Paggi, and Pedro P. Camanho. "Strength prediction of notched thin ply laminates using finite fracture mechanics and the phase field approach." *Composites Science and Technology* 150 (2017): 205-216.

[3] Camanho, Pedro P., and Giuseppe Catalanotti. "On the relation between the mode I fracture toughness of a composite laminate and that of a 0 ply: Analytical model and experimental validation." *Engineering Fracture Mechanics* 78, no. 13 (2011): 2535-2546.

[4] Navidtehrani, Yousef, Covadonga Betegón, and Emilio Martínez-Pañeda. "A simple and robust Abaqus implementation of the phase field fracture method." *Applications in Engineering Science* 6 (2021): 100050.



# TUESDAY, MAY 10

## AFTERNOON SESSIONS

15:00-15:30: **Ángela Fajardo**

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16:00-16:30: **Aurélien Doitrand**

16:30-17:00: **Maria Herrera**

17:00-17:20: **Hyung-Jun Chang**

## Multiscale phase-field modeling of fracture in short glass fiber reinforced polymers

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**Keywords:** *Short glass fiber reinforced polymers, phase field modeling of fracture, multiscale modeling.*

**Abstract:** Short glass fiber reinforced polymers (SFRPs) are of increasing interest in industrial applications. To ensure a safe use of SFRP components subjected to complex operational loads and harsh environments, the knowledge of their operating limits is of great importance. Therefore, we are interested in modeling and simulating fracture mechanical processes in SFRP components subjected to monotonic loads. Typically, such components are manufactured by injection molding, which results in locally varying fiber orientations [1]. This in turn significantly influences the macroscopic mechanical behavior of the composite material.

We approach the modeling task by a multiscale database approach. To resolve the relationship between the microstructure and the macroscopic behavior we extend the established, isotropic phase field models of fracture [2-5] towards the anisotropic case making use of the fiber orientation interpolation concept [6]. The developed model is implemented in the commercial finite element package Abaqus with the algorithmic approach in [7]. To feed the database, the anisotropic elastic coefficients are obtained from previously executed micromechanical simulations on realistic microstructures [1] using the efficient microscopic solver FeelMath [8]. The performance of the proposed approach is demonstrated by means of several numerical investigations.

### References:

[1] Schneider, M. [2017]: *The sequential addition and migration method to generate representative volume elements for the homogenization of short fiber reinforced plastics*. Computational Mechanics, **59**: 247 - 263.

[2] Bourdin, B.; Francfort, G. A.; Marigo, J.-J. [2000]: *Numerical experiments in revisited brittle fracture*. Journal of the Mechanics and Physics of Solids, **48** (4): 797-826.

[3] Pham, K.; Amor, H.; Marigo, J.-J.; Maurini, C. [2011]: *Gradient damage models and their use to approximate brittle fracture*. International Journal of Damage Mechanics **20** (4): 618-652.

[4] Miehe, C.; Welschinger, F.; Hofacker, M. [2010]: *Thermodynamically consistent phase field models of fracture: variational principles and multi-field FE implementations*. International Journal for Numerical Methods in Engineering, **83**: 1273 – 1311.

[5] Miehe, C.; Hofacker, M.; Welschinger, F. [2010]: *A phase field model for rate-independent crack propagation: robust algorithmic implementation based on operator splits*. Computer Methods in Applied Mechanics and Engineering, **199**: 2765 – 2778.

[6] Köbler, J.; Schneider, M.; Ospald, F.; Andrä, H.; Müller, R. [2018]: *Fiber orientation interpolation for the multiscale analysis of short fiber reinforced composite parts*. Computational Mechanics, **61**: 729 - 750.

[7] Seles, K.; Lesicar, T.; Tonkovic, Z.; Soric, J. [2019]: *A residual control staggered solution scheme for the phase-field modeling of brittle fracture*. Engineering Fracture Mechanics, **205**: 370 – 386.

[8] Information on <http://www.itwm.fraunhofer.de/feelmath>.

## Influence of debonding and substrate plasticity on thin film multi-cracking

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**Keywords:** *Coupled criterion, thin film, multicracking.*

**Abstract:** This work focuses on multicracking of porous ceramic thin films in microelectronic assemblies under bending. We study the initiation and of a periodic array of cracks using the coupled criterion (CC) [1] and its subsequent divisions following the approach proposed by Leguillon *et al.* [2]. The crack spacing at initiation is determined as the smallest crack spacing corresponding to the minimum imposed bending stress. Crack density increase with increasing loading is then predicted based on the stress and energy conditions of the CC. The proposed model allows determining the crack density as a function of imposed loading, which is compared to experimental measurements obtained by in-situ bending tests. The influence of debonding of the thin layer and plasticity of the substrate on crack density variation as a function of imposed loading is studied. A nonlinear implementation of the CC is adopted to consider the plastic strain energy variation due to crack initiation. The final crack spacing is mainly influenced by the debonding length ahead of the thin film crack. Considering the substrate plasticity results in crack initiation at a smaller imposed loading level and subsequent divisions occurring at larger loading levels compared to the linear elastic case. However, the saturation spacing is not much influenced by the substrate plasticity. An inverse identification of the thin film fracture toughness and tensile strength is finally established.

### References:

- [1] Leguillon D. 2002. Strength or toughness? A criterion for crack onset at a notch *European Journal of Mechanics A/solids*, vol. 21(#1), pp 61-72.
- [2] Leguillon D, Li J, Martin E. 2017. Multi-cracking in brittle thin layers and coatings using a FFM model. *European Journal of Mechanics A/Solids*, 63:14-21.

## Finite element analysis of singular elastic solutions in multi-material corners with frictional sliding contact

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**Keywords:** *finite element method, frictional contact, anisotropic elasticity, multi-material corner*

**Abstract:** A semi-analytic computational code based on the matrix formalism developed in [1,2,3] to compute the singularity exponent and the singular stress and displacement fields in anisotropic multi-material corners under generalized plane strain was presented in [4,5]. The considered homogeneous boundary conditions considered are stress free, clamped faces, faces with some restricted or allowed displacement direction and frictionless or frictional sliding contact. The interface conditions between two consecutive materials considered are perfectly bonded and frictionless or frictional sliding interfaces. The code was successfully verified by comparing the computed result with the results obtained by closed-form expressions of singularity exponents or eigenequations presented previously by many authors for relevant special cases. In several cases including frictional sliding contact, for both isotropic and monoclinic materials, beside those singular solutions found in the literature, various additional singular solutions, not found by other authors, were obtained by our semi-analytic computational code. In this work, some of these new solutions are checked by using a finite element software, to achieve a full verification of the matrix formalism proposed and the computational code developed for corner singularity analysis. Once these singular solutions are verified, the generalized stress intensity factors for the most singular term will be determined for some practical cases of multi-material corners including frictional sliding contact.

### References:

- [1] V. Mantič, F. París, J. Cañas (1997) Stress singularities in 2D orthotropic corners. *International Journal of Fracture*. 83: 67-90.
- [2] A Barroso, V Mantič, F París (2003) Singularity analysis of anisotropic multimaterial corners. *International Journal of Fracture*. 119, 1-23..
- [3] V. Mantič, A. Barroso, F. París (2014) Singular elastic solutions in anisotropic multimaterial corners. Applications to composites. In: *Mathematical Methods and Models in Composites*, V.Mantič (Ed.), 425-495. Imperial College Press.
- [4] M.A. Herrera-Garrido, V. Mantič, A. Barroso (2022) A powerful matrix formalism for stress singularities in anisotropic multi-material corners. Homogeneous (orthogonal) boundary and interface conditions. *Theoretical and Applied Fracture Mechanics* (under review).
- [5] M.A. Herrera-Garrido, V. Mantič, A. Barroso (2022) A powerful matrix formalism for stress singularities in anisotropic multimaterial corners for frictional boundary and interface conditions. *Theoretical and Applied Fracture Mechanics* (to be submitted).

## Crack propagation modeling at SAFRAN – AN OVERVIEW

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**Keywords:** *Crack modeling, industrial application.*

### **Abstract:**

We will be discussing the importance of crack propagation prediction at Safran. Safran manufactures components for airplanes and helicopters and these components are required to meet stringent safety requirements. Part of these safety requirements is determining the path a crack will follow once initiation occurs. For this, we use several tools, both 2D and 3D to predict the propagation of cracks. These tools have been validated using both elementary tests and industrial applications, such as bladed disks, engine disks and turbine blades.

We will give a brief overview of these tools and test cases and then close with future perspectives for our studies and the proposed application for next year's secondment.

# WEDNESDAY, MAY 11

## MORNING SESSIONS

10:00-10:30: **Vladislav Mantic**

10:30-11:00: **Ajinkya Dusane**

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11:30-12:00: **Pietro Lenarda**

12:00-12:30: **Thomas Duminy**

12:30-13:00: **Arturo ChaoCorreas**

# Numerical Implementation of the Coupled Criterion Applied to Spring Interfaces by means the Principle of Minimum Total Energy Subjected to a Stress Condition

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**Keywords:** CC-FFM, PMTE-SC, LEBIM, load stepping scheme, ABAQUS

**Abstract:** The Coupled Criterion of Finite Fracture Mechanics (CC-FFM) has shown to be a powerful analytical tool for prediction of crack onset in brittle materials. CC-FFM has been applied to many structural configurations to compute failure load, usually achieving a very good agreement with the experimental results. Application of CC-FFM to the Winkler (spring) interfaces was initiated in [1], with the aim to develop more realistic models, in comparison with the so-called Linear Elastic-(perfectly) Brittle Interface Model (LEBIM), for prediction of crack onset in thin adhesive layers. With the aim of developing a general computational tool for the prediction of fracture initiation and propagation, a new methodology based on the principle of minimum total energy subjected to a stress condition (PMTE-SC), with discontinuous representation of cracks, was introduced in [2]. PMTE-SC is essentially equivalent to the original formulation of the CC-FFM. This new methodology was applied for the first time to a crack growing in a Winkler (spring) interface in [3], using analytical solutions of the Double Cantilever Beam (DCB) test, and demonstrating the equivalence between the original proposal of the CC-FFM and the new PMTE-SC methodology. In this work, a quite general numerical implementation of the PMTE-SC is presented employing a load stepping procedure in the commercial FEM code ABAQUS®. The proposed tool can predict onset and propagation of multiple cracks in several Winkler (spring) interfaces.

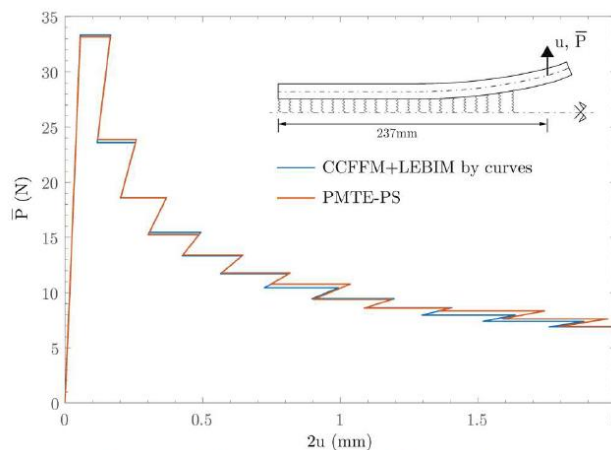


Fig. 1. Load-displacement curves for the DCB test under load control using the original formulation of the CCFFM and the new PMTE-SC.

## References:

- [1] P. Cornetti, V. Mantič, A. Carpinteri. 2012. Finite fracture mechanics at elastic interfaces. *International Journal of Solids and Structures*, 49, 1022-1032.
- [2] V. Mantič. 2014. Prediction of initiation and growth of cracks in composites. Coupled stress and energy criterion of the Finite Fracture Mechanics (Keynote Lecture), In: *Proceedings of the 16th European Conference on Composite Materials (ECCM16)*, F. París (Ed.) <http://www.escm.eu.org/eccm16/assets/1252.pdf>.

[3] M. Muñoz-Reja, V. Mantič, L. Távara. 2022. Comparative analytical study of the coupled criterion and the principle of minimum total energy with stress condition applied to linear elastic interfaces. *Theoretical and Applied Fracture Mechanics*. <https://doi.org/10.1016/j.tafmec.2022.103274>.

## Phase field modeling of fracture in Viscoelastic Materials

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**Keywords:** *Viscoelastic materials; Fractional calculus; Prony series model; Phase-field method*

**Abstract:** Fracture of viscoelastic solids plays an essential role in many areas of science and technology, including biological attachments, adhesives, polymers, and soft robotics [1]. Nearly all properties of these materials are time-dependent and may tremendously affect their failure behavior. Since time appears in the stress-strain relations, viscoelastic problems differ from the corresponding elastic problems. Compared with elastic or elastic-plastic materials, viscoelastic fracture received far less attention in the literature, and it is not yet well understood. This work focuses on the Phase-field (PF) method for modeling the fracture of viscoelastic materials. Due to the ability to model complex crack propagation, including branching, PF has significant relevance in fracture studies [2]. Various mathematical models have been developed and used to represent the viscoelastic material functions. The most versatile model is obtained by connecting the number of Maxwell's arms in the Generalized Maxwell model (GMM) [3] through standard Prony series type expansion. The material properties are determined from fitting Prony series in the real data. Alternatively, Fractional calculus has proved to be very effective in modeling the power-law time dependency of relaxation behavior of polymers. It offers the easiest way to estimate the model parameters compared to other models [4, 5]. This work proposes a phase-field formulation for crack propagation in viscoelastic materials using a material model using GMM and fractional calculus. A comparative study is performed with several benchmark problems to determine the effectiveness of models. Further, the proposed model is used to numerically solve some test cases related to fracture tests conducted on samples of polymer Ethylene Vinyl Acetate (EVA), a major encapsulant of solar cells in photovoltaics.

### References:

- [1] S.P. Marques, G.J. Creus. 2012. *Computational viscoelasticity*. Springer Science Business Media.
- [2] Christian Miehe, Martina Hofacker, Fabian Welschinger. 2010. A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits. *Computer Methods in Applied Mechanics and Engineering*. 199(45-48):2765–2778.
- [3] Rilin Shen, Haim Waisman, and Licheng Guo. 2019. Fracture of viscoelastic solids modeled with a modified phase field method. *Computer Methods in Applied Mechanics and Engineering*. 346:862–890.
- [4] Marco Paggi, Alberto Sapora. 2015. An Accurate Thermoviscoelastic Rheological Model for Ethylene Vinyl Acetate Based on Fractional Calculus. *International Journal of Photoenergy*.
- [5] P. Lenarda, M. Paggi. 2022. A computational framework for rheologically complex thermo-visco-elastic materials. *International Journal of Solids and Structures*. 236-237.



## Failure through crack propagation in components with holes and notches: an experimental assessment of the phase field model

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**Keywords:** *Phase field approach to fracture; Photoelasticity; PMMA; Circular holes and V-notches; Numerical-experimental comparison.*

**Abstract:** Fracture growth in a material is strongly influenced by the presence of inhomogeneities, which deviate crack trajectories from rectilinearity and deeply affect failure. Increasing crack tortuosity is connected to enhancement of fracture toughness. Often a crack may even be stopped when it impinges a void, which releases the stress concentration. Therefore, the determination of crack trajectories is important in the design against failure of materials and mechanical pieces [1].

The recently developed phase-field approach (AT1 and AT2 models), based on a variational approach to damage localization, is believed to be particularly suited to describe complex crack trajectories [2,3]. This belief is examined through a comparison between simulations and photoelastic experiments on PMMA plates designed in a new way, to highlight the effects of notches and circular holes on fracture propagation. The latter is shown to initiate from a notch and to be strongly attracted by voids. When a void is hit, fracture is arrested, unless the void contains a notch on its internal surface, from which a new crack nucleates and propagates.

Several different mechanical models are tested where fracture initiates and grows under Mode I compact tension, four-point bending and tension indirectly generated during compression of samples containing a circular hole. The experiments show that the fracture propagation may be designed to develop in different tortuous paths, involving multiple arrests and secondary nucleation.

Simulations performed with an *ad hoc* implemented version of the AT1 and AT2 phase field methods equipped with spectral decomposition, in which a crack is simulated as a highly localized zone of damage accumulation, are shown to be in close agreement with experiments and therefore confirm the validity of the approach and its potentialities for mechanical design.

**Acknowledgements:** D.B. and R.C. gratefully acknowledge funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 955944-REFRACTURE2. M.P. and P.L. gratefully acknowledge funding from the Italian Ministry of University and Research (MIUR) to the project of national interest (PRIN 2017) XFAST-SIMS: Extra fast and accurate simulation of complex structural systems (grant agreement No. 20173C478N). D.M. gratefully acknowledges funding from H2020-MSCA-ITN-2020-LIGHTEN-956547.

### References:

- [1] D. Misseroni, A.B. Movchan, N.V. Movchan, D. Bigoni. 2015. Experimental and analytical insights on fracture trajectories in brittle materials with voids. *International Journal of Solids and Structures*, 63:219–225.
- [2] G.A. Francfort, J.J. Marigo. 1998. Revisiting brittle fracture as an energy minimization problem, *J. Mech. Phys. Solids*, 46:1319–1342.
- [3] C. Miehe, M. Hofacker, F. Welschinger. 2010. A phase field model for rate independent crack propagation: robust algorithmic implementation based on operator splits. *Comput. Methods Appl. Mech. Engng.* 199:2765–2778.

# Material fracture parameter identification using digital image correlation and the coupled criterion

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**Keywords:** Fracture parameter determination, digital image correlation, coupled criterion

**Abstract:** The coupled criterion [1] relies on the knowledge of two material parameters, *i.e.* the strength ( $\sigma_c$ ) and the critical energy release rate ( $G_c$ ). Many methods exist to determine them. In digital image correlation (DIC) the displacement in a material is computed by maximizing the correlation between pixel intensity on images taken during a mechanical test.

DIC can be coupled with finite elements (FE) to determine *a priori* unknown material properties in an iterative process. Each step, material properties are modified and the displacement [2] and/or force [3] difference between the FE model and the experiment is calculated. The material parameters are those who minimized this difference.

However, this method induces uncertainty from the experimental setup and the application of the boundary condition in FE model. They cannot be neglected when dealing with small size samples, *i.e.* all dimensions inferior to 10  $\mu\text{m}$ .

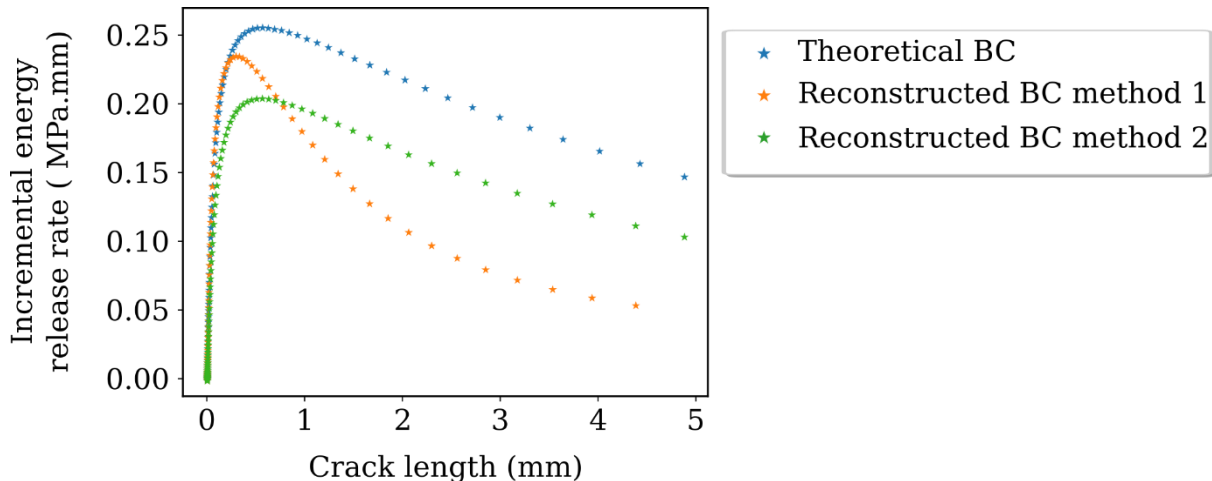


Figure 1 - Incremental energy release rate for different boundary condition application methods. For large crack, the incremental energy release rate, used in  $G_c$  determination, fails to be computed.

Wedge splitting tests coupled with DIC have been carried out on small size samples from isotropic (PMMA and gypsum) and anisotropic (nacre-like alumina) materials. Strategies to identify and minimize uncertainty on the determination of  $\sigma_c$  and  $G_c$  were identified, and fracture parameters were determined.

## References:

[1] D. Leguillon, 2002, Strength or toughness? A criterion for crack onset at a notch, European Journal of Mechanics - A/Solids, 61-72

[2] R. Vargas, J. Neggers, R.B. Canto, J.A. Rdorigues, F. Hild, 2019. Analysis of a castable refractory using the wedge splitting test and cohesive zone model. *Journal of the European Ceramic Society* 39, 3903–3914.

[3] C. Melo, M. Furlan, F. Hild, N. Schmitt, R. Canto, 2020. Uniaxial compression test on ceramic green compact with bending consideration using digital image correlation. *Powder Technology*.

### **Crack onset from circular holes under biaxial loading conditions**

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**Keywords:** *Finite Fracture Mechanics, Cohesive Zone Model, Phase Field, Safety domains, Energy decompositions.*

**Abstract:** The phenomenon of crack onset stemming from a circular hole present in a biaxially loaded infinite plate is herein investigated. A thorough analysis on the dependence of both the stress field and the stress intensity factor upon the degree of biaxiality revealed a wide range of scenarios in what concerns their sign and trend spatial distributions, thus rendering this a powerful case study for comparing different failure criteria. Subsequently, three different approaches were used to determine the biaxial safety domains: Finite Fracture Mechanics, Cohesive Zone Model and Phase Field. In particular, the original formulation of Finite Fracture Mechanics (FFM) proved to be consistently more optimistic than its averaged-stress variant (FFM-avg); likewise, both turned out to agree in predicting the existence of a region in the loading space where failure is governed only by the energy condition. Besides, Dugdale's Cohesive Zone Model (CZM) predictions proved to be fairly close to FFM's, while the differences between CZM and FFM-avg resulted more noticeable. Lastly, the numerical implementation of the Phase Field method was performed in FeniCS-X [1]. Special attention was paid to the choice of the energy decomposition, being herein implemented two options: No-Decomposition and No-Tension decomposition (see [2]). In particular, the latter showcased reasonable agreement with FFM (and CZM), thus rendering it a solid contender for its use in subsequent implementations of the Phase Field in which combined tension-compression stress states appear but damage is mostly expected to affect tensile-like states, such as in the dynamic failure of brittle materials.

#### **References:**

[1] M. S. Alnaes, J. Blechta, J. Hake, A. Johansson, B. Kehlet, A. Logg, C. Richardson, J. Ring, M. E. Rognes and G. N. Wells. 2015. The FEniCS Project Version 1.5. *Archive of Numerical Software* 3. <https://doi.org/10.11588/ans.2015.100.20553>

[2] L. De Lorenzis, C. Maurini. 2021. Nucleation under multi-axial loading in variational phase-field models of brittle fracture. *International Journal of Fracture*. <https://doi.org/10.1007/s10704-021-00555-6>

# THURSDAY, MAY 12

## MORNING SESSIONS

10:00-10:20: **Simone Sangaletti**

10:20-10:40: **Amir Mohammad Mirzaei**

10:40-11:00: **Hugo Girard**

-

11:30-12:00: **Alessio Gizzi**

12:00-12:30: **Marco Lo Cascio**

12:30-13:00: **PanayotisTsokanas**

## Phase field fracture modelling for 3D printed materials: a preliminary study

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**Keywords:** *3D printing, Carbon Fibre, Phase Field*

**Abstract:**In recent years, Additive Layer Manufacturing (ALM) has captured more and more interest among both industrial and academic players due to the incredible opportunities that this technology offers. In this field, one particular branch seems to be promising for the possibilities offered from the design point of view: the continuous carbon fibre deposition. The possibility of depositing continuous Carbon Fibre (CF) along with common polymeric material allows the designer to give particular attention to regions of the component which could be subjected to higher stresses for whichever reason (notches, discontinuities, joints). The reinforcement of such areas with CF allows a better resistance to catastrophic phenomena like fracture which could lead to the failure of the component. Therefore, this deposition technique is of particular interest and object of study in this work. In the first part, a problem studied experimentally by Akasheh et al. [1] is analyzed numerically by means of the Phase Field fracture modelling, demonstrating the optimal capabilities of this approach to capture not only the experimental crack path but also the Stress-Strain response of the specimens tested. Validation of the material parameters is of particular importance especially for novel materials like the ones used in the experiments performed by the authors. Once both the capabilities of Phase Field and the material properties have been verified by means of the numerical simulations described above, these are now exploited to study a well know problem in the field of mechanics and aeronautics: structural joints. The aim is to use Phase Field as a tool able to predict failure load and failure mode of a component subjected to operating loading conditions, for instance. This could allow the designer to exploit the capabilities of continuous carbon fibre deposition to generate fracture resistance components, by means of accurate reinforcement of the most critical regions, where failure is most likely to occur. The Open Hole Tension test is simulated numerically by means of the same tools described above, taking into consideration different geometries for the reinforcement around the hole. The specimens with different reinforcements show different mechanical responses and crack patterns, thus highlighting the influence of the continuous carbon fibre reinforcement on the fracture scenario.

### References:

- [1] Akasheh, F. and H. Aglan (2019). Fracture toughness enhancement of carbon fiber–reinforced polymer com-posites utilizing additive manufacturing fabrication. *Journal of Elastomers and Plastics* 51 (7-8), 698–711

## A phase field model for fatigue fracture of brittle materials

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**Keywords:** *Phase-field; Fatigue; numerical modeling; Variational methods.*

**Abstract:** Phase field modeling of fracture has attracted a lot of interest because of its advantages in numerical simulations, especially for complex crack problems[1]. So far, mostly, the approach has been utilized to investigate quasi-static and dynamic fracture in brittle and ductile materials having isotropic as well as anisotropic properties. However, fatigue fracture which can be considered as the most common failure mechanism of real engineering structures has most recently also been addressed with the phase field approach. One of the first ideas for phase field modeling of fatigue was to introduce a fatigue degradation function that reduces the fracture toughness using a proper history variable [2,3]. Simulations based on the explicit resolution of each loading cycle are very computationally intensive, due to the need for a fine discretization inherent to the phase-field approach combined with the expensive time resolution of a large number of cycles. In this investigation, we highlight possibilities to accelerate these simulations and comparatively test a few of them with appropriate numerical examples.

### References:

- [1] B. Bourdin, G.A. Francfort, J.-J. Marigo, Numerical experiments in revisited brittle fracture, *J. Mech. Phys. Solids.* 48 (2000) 797–826.
- [2] P. Carrara, M. Ambati, R. Alessi, L. De Lorenzis, A framework to model the fatigue behavior of brittle materials based on a variational phase-field approach, *Comput. Methods Appl. Mech. Eng.* 361 (2020) 112731.
- [3] R. Alessi, S. Vidoli, L. De Lorenzis, A phenomenological approach to fatigue with a variational phase-field model: The one-dimensional case, *Eng. Fract. Mech.* 190 (2018) 53–73.

## Numerical simulation of debonding in thermoplastic composites: impact of adjacent fibers

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**Keywords:** *Fiber-matrix debonding, Finite Fracture Mechanics, Cohesive Zone Model, adjacent fibers, composites.*

**Abstract:** Thermoplastic composites are nowadays increasingly employed, in particular for both aeronautic and aerospace usages. This widening application requires a thorough understanding of thermoplastic composites behavior. According to several authors [1-2], the fiber-matrix interface is a key aspect of global composite mechanical properties. We aim to investigate the interaction between two adjacent fibers and its impact on stress fields at the vicinity of interfaces. A simulation study is carried out in order to attest numerical model implemented with the help of experimental results from the literature [3].

Multiple numerical models were developed to describe debonding process. Two major approaches can be identified based on Finite Fracture Mechanics and Cohesive Zone Model. These two were largely detailed for different configurations, 2D and 3D, for instance in [4-5]. Based on these works and Livingston experimental results [3], FFM and CZM models are applied for debonding simulations in a two fibers composite and compared to experimental results.

The influence of the interaction between two fibers is considered by studying several configurations with various inter-fibers angle ( $\alpha$ ) and distance ( $d$ ) (see Figure 1). All these observations open up new possibilities, such as the integration of non-linearity and the prediction of transverse crack initiation.

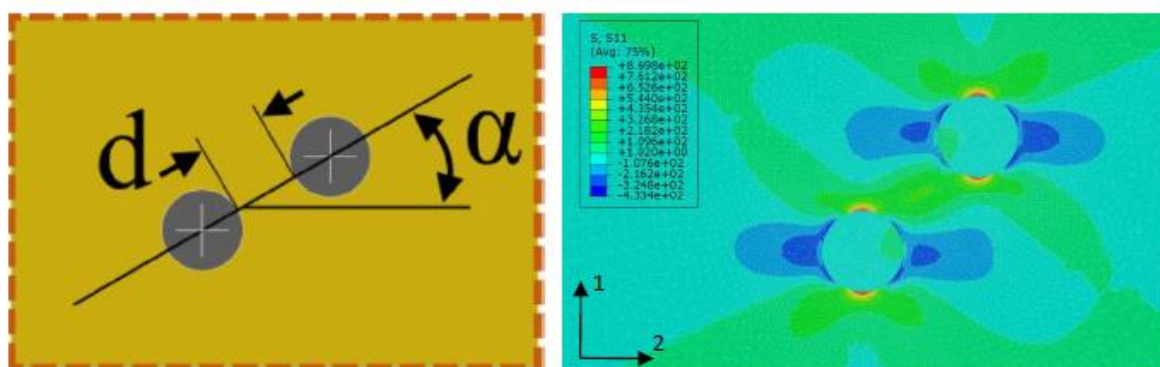


Figure 1 : Double fibers sample configurations studied by Livingston [3] (left), and corresponding finite element simulation from Abaqus with tensile test along direction 1 (right)

### References:

- [1] Shengdao W., Yanchao Y., Yongfeng M., Jiyao S., Xin C., Jiashuang L., Guibin W. (2021). Synergy of electrochemical grafting and crosslinkable crystalline sizing agent to enhance the interfacial strength of carbon fiber/PEEK composites. *Composites Science and Technology*. Volume 203. 108562.
- [2] Junlin C., Kai W., Yan Z. (2018). Enhanced interfacial interactions of carbon fiber reinforced PEEK composites by regulating PEI and graphene oxide complex sizing at the interface. *Composites Science and Technology*. Volume 154. 175-186.
- [3] Livingston R. (2021). In situ characterization of fiber-matrix interface debonding via full-field measurements. Rowan University.
- [4] Doitrand A., Leguillon D. (2018). 3D application of the coupled criterion to crack initiation prediction in epoxy/aluminum specimens under four point bending. *International Journal of Solids and Structures*. Volume 143. 175-182.
- [5] Koyanagi J., Shah P.D., Kimura S., Ha S.K., Kawada H. (2009). Mixed-mode interfacial debonding simulation in single-fiber composite under a transverse load. *Journal of Solid Mechanics and Materials Engineering*. Volume 3. 796-806.

# Computational Fracture Analysis of Metastatic Human Bones in the presence of Fixation Screws and accounting for Bone-Metastasis Interaction

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**Keywords:** *Constitutive modeling, Finite element modeling, Bone fracture, Metal screws.*

**Abstract:** Bone metastases significantly increase the risk of fracture. Screw fixation improves the stability of a diseased bone, reducing fracture occurrence. However, a comprehensive understanding of the bone mechanical behavior in the presence of metastases and after screw fixation is still lacking for optimal surgical treatment planning. In such a framework, finite element modeling is widely recognized as the main computational tool to study the mechanical response of patient-specific human bones under pathological conditions. The present contribution investigates metastatic human bones in the presence of screw fixation by developing a novel biological rationale for bone-metastasis interaction. We parametrize the effect of metastasis size, location, and material model with respect to fracture load and fracture patterns under physiological loads. Two selected case studies are proposed, a patient-specific femur and vertebra geometry was derived by segmenting the CT images. The CAD models of two pedicle screws were virtually inserted in the vertebra considering the most critical insertion angle [1] and including ellipsoidal-shaped metastasis [2]. Bone was modeled as a linearly elastic, isotropic material with CT-based heterogeneous properties. For the vertebra, a custom algorithm to accurately identify the thin cortical layer was developed and applied, reducing partial volume effects. The screws' behavior was assumed homogeneous, isotropic, and linear elastic. The main novelty is based on a specific constitutive description for the metastasis, accounting for bone-metastasis interaction, assuming that the metastasis induces a structural change in the bone region close to the lesion. Lateral left bending and extension loading modes were simulated for the vertebra, whereas a single leg stance was chosen as a loading condition for the femur. The screws-vertebra and femur models were discretized using ten-nodes tetrahedra. The bone's local progressive damage was simulated using quasi-static force (for vertebra) and displacement (for femur) -driven incremental approaches, implementing and comparing both stress and strain-based failure criteria. An extended parametric analysis combined failure criteria, type of metastasis, lesion size and location, and loading mode. Computational findings showed that: i) size, location, and type of metastasis significantly impact the bone mechanical response and fracture pattern; ii) the combination of loading mode and metastasis type is a critical modeling parameter in determining the fracture risk; iii) the incorporation of a specific constitutive description for the metastasis allows to capture stress/strain localization mechanisms within the metastatic tissue, revealing the model capability in describing possible strain-induced mechano-biological stimuli driving onset and evolution of the lesion. The proposed computational approach serves as a basis for an integrated computational tool of potential support in clinical practice.

## References:

- [1] L. Molinari, C. Falcinelli, A. Gizzi, A. Di Martino. 2021. Effect of pedicle screw angles on the fracture risk of the human vertebra: A patient-specific computational model. *Journal of the Mechanical Behavior of Biomedical Materials*, 116:104359.
- [2] C. Falcinelli, A. Di Martino, A. Gizzi, G. Vairo, V. Denaro. 2019 Mechanical behavior of metastatic femurs through patient-specific computational models accounting for bone-metastasis interaction. *Journal of the Mechanical Behavior of Biomedical Materials*, 93:9-22



# Virtual Element Method for Micro Damage and Cracking in Fibre-Reinforced Composites

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**Keywords:** *Virtual Element Method, Damage, Matrix Cracking, Composites.*

**Abstract:** Prediction of micro-damage initiation and evolution is a critical challenge in designing fibre-reinforced composites. The widely employed standard Finite Element Method (FEM) models crack propagation by sequentially adapting the problem's mesh. This approach is computationally intensive as the continuous mesh refinement process generally affects the entire analysis domain due to preserving a mesh-conforming transition between the refined target region and the surrounding areas [1].

The Virtual Element Method (VEM) [2] is a recent generalization of FEM that can use polygonal elements of arbitrary shapes. This unique feature allows avoiding mesh dependency of the crack propagation direction, as any crack path can be represented as computed by modifying the topology of the virtual element over which the crack propagation occurs, including the crack edges as new element edges without the need for further remeshing. This approach can be employed [3] either when the computed crack length increment is large enough to split an element entirely or when it splits this element only partially. In order to improve numerical accuracy near the crack tip, a local mesh refinement can be introduced by subdividing one or more local elements in any number of arbitrary shape elements without affecting large portions of the analyzed domain. This contribution presents a recently developed VEM-based framework for crack path tracking that is readily applicable to matrix cracking modelling in composite materials.

## References:

- [1] M. Kuna. 2013. Finite elements in fracture mechanics. Springer, Dordrecht.
- [2] L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, and A. Russo. 2013. Basic principles of virtual element methods. *Mathematical Models and Methods in Applied Sciences*, 23: 199–214.
- [3] M. Lo Cascio, I. Benedetti, V. Mantič. 2020. Micro Damage and Cracking in Fibre Reinforced Composites by a Novel Hybrid Numerical Technique. *AIP Conference Proceedings*, 2309:020001.

# Interfacial fracture analysis of layered beams with elastic couplings and hygrothermal stresses using an elastic-interface model

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**Keywords:** *layered beam, elastic-interface model, energy release rate, mode partitioning, bending-extension coupling, residual hygrothermal stresses,  $\mathcal{J}$ -integral.*

**Abstract:** This work presents our progress in studying the problem of an interfacial crack in a layered beam with bending-extension coupling (BEC) and residual hygrothermal stresses (RHTS). These effects have occupied us in our recent work [1–3]. Here, we consider a beam element that is an assemblage of two sublaminates connected by a linearly elastic (until fracture) interface of negligible thickness. Both sublaminates may have arbitrary stacking sequences (which introduces BEC) and are modeled as shear-deformable laminated beams. In addition, we assume that the beam element is affected by RHTS. The mathematical problem is formulated and then reduced to two differential equations in the interfacial stresses. To formulate and solve this mathematical problem, we rely heavily on our recent work in [3], which, in turn, extended the so-called enhanced beam theory model proposed in [4]. In simpler words, the present work updates the model of [3] to account for RHTS, while it also discusses some issues not mentioned there (*e.g.*, on the boundary conditions). The aim of the work is twofold. First, we intend to explore the mechanical behavior of the beam element. Thus, we derive explicit analytical expressions for various mechanical quantities: internal forces and moments, strain measures, and generalized displacements in both sublaminates. The effect of the RHTS on these expressions is highlighted through comparison with the respective solutions in [3] that ignore this effect. The second aim of the work is to determine the energy release rate (ERR) and its mode I and mode II contributions. For this purpose, we adopt the  $\mathcal{J}$ -integral method, also using the so-called interface potential energy. For the beam element under consideration that is affected by residual stresses,  $\mathcal{J}$ -integral ceases to be path independent [5]. We address this issue using a recently proposed approach [5], which, in a nutshell, splits the loading into two steps. Thus, we compose a valid  $\mathcal{J}$ -integral solution that allows computing the ERR. Mode partitioning is achieved by assuming, as in [3], that the compressive normal stresses at the crack tip do not promote the crack opening.

Ongoing work on validating the proposed analytical solutions using finite element analysis will be presented in a subsequent publication. Lastly, future extensions for the calculations of other essential mechanical quantities (*e.g.*, compliance) are possible.

## References:

- [1] P. Tsokanas, T. Loutas. 2019. Hygrothermal effect on the strain energy release rates and mode mixity of asymmetric delaminations in generally layered beams. *Engineering Fracture Mechanics*, volume 214: 390–409.
- [2] P. Tsokanas, T. Loutas, G. Kotsinis, W. M. van den Brink, P. Nijhuis. 2021. Strain energy release rate and mode partitioning of moment-loaded elastically coupled laminated beams with hygrothermal stresses. *Composite Structures*, volume 259: 113237.
- [3] S. Bennati, P. Fiscaro, L. Tagliallegne, P. S. Valvo. 2019. An elastic interface model for the delamination of bending-extension coupled laminates. *Applied Sciences*, volume 9 (no. 17): 3560.
- [4] S. Bennati, M. Colleluori, D. Corigliano, P. S. Valvo. 2009. An enhanced beam-theory model of the asymmetric double cantilever beam (ADCB) test for composite laminates. *Composites Science and Technology*, volume 69 (no. 11–12): 1735–1745.
- [5] H. L. Toftegaard, B. F. Sørensen. 2019. General J integral solution for specimens loaded by moments, axial forces and residual stresses – A unifying stiffness formulation. *Engineering Fracture Mechanics*, volume 217: 106500.