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Effect of Asymmetry in the Modulation Parameters on Self-Focusing of Asymmetric Finite Airy-Gaussian Laser Beam in Collisionless Plasma

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The self-focusing/defocusing of asymmetric finite Airy-Gaussian (AiG) laser beam has been investigated by employing standard Wentzel–Kramers–Brillouin (WKB) and paraxial-ray approximations in a nonrelativistic regime for underdense plasma. The second-order non-linear coupled differential equations have been solved numerically by using the fourth-order Range-Kutta method. The effect of asymmetry in the modulation parameters on the self-focusing/defocusing of the asymmetric finite AiG laser beam in collisionless plasma has been studied. It is observed that the self-focusing/ defocusing of asymmetric finite AiG laser beam is strongly connected with the initial values of the laser and plasma parameters such as modulation parameters and plasma frequency.

Keywords: Airy-Gaussian; Asymmetry; Plasma; Self-focusing

1 Introduction

An interaction of high-intense laser beams with the plasmas gives various non-linear optical effects. The phenomenon of self-focusing is one of the important optical effects because it considerably influences other nonlinear effects. Self-focusing of the laser beam was firstly studied by G. Askaryan in 1962[1]. Its theoretical analysis in nonlinear media was introduced by Akhmanov *et al.*[2] and later extended to plasmas by Sodha *et al.*[3]. Self-focusing of laser beams in plasmas has many applications like high harmonic generation[4], laser-driven plasma accelerators[5], laser-driven inertial confinement fusion[6], X-ray lasers[7], *etc*. A review of the literature highlights the fact that most of the work on the self-focusing of laser beams in plasmas has been done for Gaussian beams due to their specific characteristics. In the last six decades, most of the work has been done on the self-focusing of non-Gaussian laser beams such as Hermite-Gaussian beams[8], cosh-Gaussian beams[9], Hermite-cosh-Gaussian beams[10], elliptical-Gaussian beams[11], Bessel-Gaussian beams[12], *etc*. Apart from these beams, Airy-Gaussian (AiG) beams have their specific properties such as high penetration[13], Airy beams retain their shape[14], invariant under small-angle approximation. Hence, it is very suitable to study the finite AiG laser beam under paraxial approximations. Due to these silent features, AiG beams are useful in various applications such as plasma guidance[15], optically clearing particles[16], trapping and guiding microparticles[17], vacuum electron acceleration[18], *etc*.

Belafhal *et al.*[19,20] presented the self-focusing of the finite Airy-Gaussian (AiG) beam in collisionless plasma. They have reported that the self-focusing of the finite AiG beam can be controlled by varying the modulation parameter. Recently, Pawar *et al.*[21,22] explored the domains of modulation parameters in the interaction of finite AiG laser beams in plasmas. They found that the extent of self-focusing of AiG beams depends on a range of modulation parameters.

In the present paper, asymmetry in the modulation parameter of finite AiG beams has been introduced. Such asymmetry in modulation parameters is due to the two transverse dimensions of the beam. In Section 2, coupled differential equations governing beam width parameters have been established. In Section 3 results are presented graphically and discussed. Some brief conclusions are added in Section 4.

2 Basic theoretical formulation

The expression for the electric field distribution of the asymmetric finite AiG laser beam is given as[19],

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$$E(x,y,z) = \sqrt{\frac{{E_0}^2}{f_1 f_2}} Ai\left(\frac{x}{r_0 f_1}\right) exp\left(\frac{a\,x}{r_0 f_1}\right) exp\left(\frac{-x^2}{r_0^2 f_1^2}\right) Ai\left(\frac{y}{r_0 f_2}\right) exp\left(\frac{b\,y}{r_0 f_2}\right) exp\left(\frac{-y^2}{r_0^2 f_2^2}\right) \quad \ldots(1)$$

Where, E_0 is constant amplitude of electric field and the direction of propagation is along the z-axis, $Ai(\cdot)$ is the Airy function of the first kind, r_0 is the radius of the finite AiG beam and the modulation parameters as a along x and b along y-direction respectively.

In Fig. 1, (a), (c) and (e), shows the 3D initial intensity distribution (I/I_0) and Fig. (b), (d) and (f) show its density plots of initial normalized intensity distribution of asymmetric finite AiG laser beam

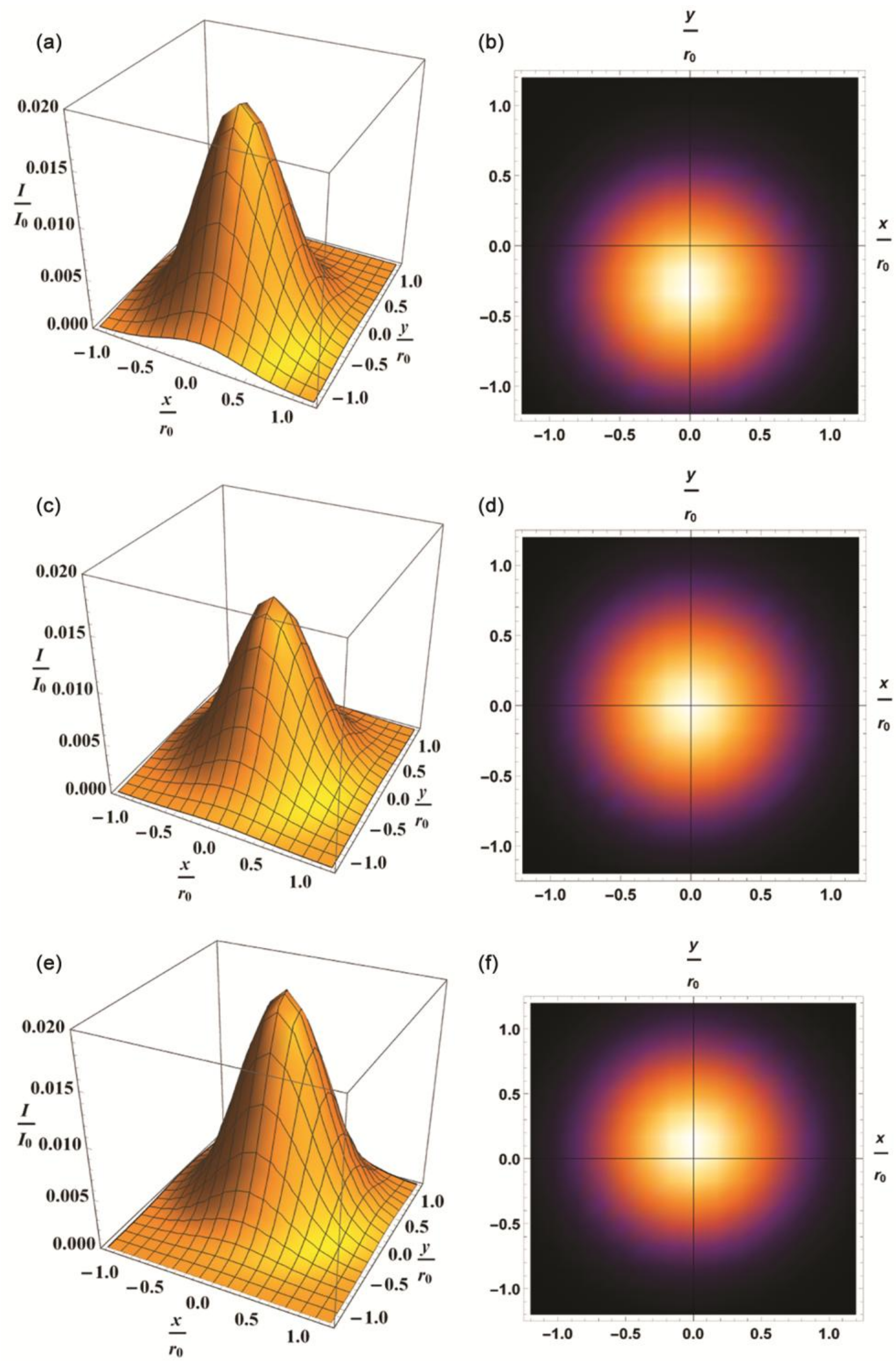

Fig 1 — 3D intensity profile of finite AiG beams with $a = 0.70$ for (a) $b = 0.00$, (c) $b = 0.70$ and (e) $b = 1.50$. The transverse view of the intensity profile with $a = 0.70$ for (b) $b = 0.00$, (d) $b = 0.70$ and (f) $b = 1.50$.

under initial conditions $(f_1 = f_2 = 1)$ for the asymmetric variation of the modulation parameters as; $(a = a_{cr}, b = 0.00)$, $(a = a_{cr}, b = b_{cr})$ and $(a = a_{cr}, b = 1.50)$ respectively. From the normalized intensity distributions it is clear that, peak of the intensity distribution shifts towards the axis of the beam by the variation of one modulation parameter (b) when other modulation parameter (a) is fixed and vice versa. This shifting of intensity peak towards the center of the beam is occurred till $(b = b_{cr})$. The peak of the intensity distribution shifts away from the center of the beam along the y-axis as b increases beyond the b_{cr} by keeping a fixed and vice versa. In Fig. 1, (c) and (d) clearly shows that the peak of the intensity distribution exact coincide with axis of the beam at $a = a_{cr}$ and $b = b_{cr}$. The variation in any one of the modulation parameter (a or b) of the finite asymmetric AiG laser beam can affect the overall intensity distribution of the beam.

The intensity dependence dielectric constant in collisionless plasma can be given as; $\varepsilon = \varepsilon_0 + \varepsilon_2 \Phi(EE^*)$. Where, $\varepsilon_0 = 1 - \omega_p^2/\omega_o{}^2$ and $\varepsilon_2\Phi(EE^*)$ are the linear and nonlinear part of the intensity dependent dielectric constant of the plasma. Non-linear measure of intensity dependent dielectric constant for the collisionless plasma can be given as[20],

$$\Phi\left(EE^*\right) = \frac{\omega_p^2}{\omega_0^2}\left[1\text{-}exp\left(\frac{-3m\alpha EE^*}{4M}\right)\right] \quad \text{...(2)}$$

Here, $\omega_p = \sqrt{4\pi ne^2/m}$ is the plasma frequency with n, e and m are the equilibrium electron density of plasma, charge of the electron, rest mass of the electron, ω_0 is angular frequency of the asymmetric finite AiG laser beams and $\alpha = e^2/8\,k_B T\omega_0{}^2 m$, where k_B and T are the Boltzmann constant and temperature of the plasma.

By employing Akhmanov *et al.*[2] and its later modification by Sodha *et al.*[3],we have following wave equation as;

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial x^2} + \frac{\partial^2 A_0}{\partial y^2}\right) + \frac{\Phi\left(A_0 A_0^*\right)}{\varepsilon_0} \quad \text{...(3)}$$

By using an expression for eikonal of the beam, we can obtain the general solutions of parabolic equation as,

$$S = \beta_1(z)\frac{x^2}{2} + \beta_2(z)\frac{y^2}{2} + \emptyset(z), \quad \text{...(4)}$$

and

$$A_0^2 = \frac{E_0{}^2}{f_1 f_2}\left(Ai\left(\frac{x}{r_0 f_1}\right)\mathrm{Ai}\left(\frac{y}{r_0 f_2}\right)\right)^2 exp\left(\frac{2ax}{r_0 f_1} + \frac{2by}{r_0 f_2} - \frac{2x^2}{r_0^2 f_1^2} - \frac{2y^2}{r_0^2 f_2^2}\right), \quad \text{...(5)}$$

Where $A_0(x, y, z)$ and $S(x, y, z)$ are the real functions of x, y and z co-ordinates. Here f_1 and f_2 are the dimensionless beam width parameters along x and y-axis direction respectively. By considering $\beta_1(z) = \frac{1}{f_1(z)}\frac{df_1(z)}{dz}$ and $\beta_2(z) = \frac{1}{f_2(z)}\frac{df_2(z)}{dz}$ as the curvatures of the beam along x and y directions. From Eq. (3), we set up the following beam width parameter differential equations as;

$$\frac{d^2 f_1}{d\xi^2} = \frac{A(a)}{f_1^3} + \frac{B(a)D\rho^2}{f_1^2 f_2}\, exp\left(\frac{-Dg_2^4}{f_1 f_2}\right) \quad \text{... (6)}$$

and

$$\frac{d^2 f_2}{d\xi^2} = \frac{A(b)}{f_2^3} + \frac{B(b)\, D\rho^2}{f_2^2 f_1} exp\left(\frac{-Dg_2^4}{f_1 f_2}\right) \quad \text{... (7)}$$

Where, $D = 3m\alpha E_0{}^2/4M$, $\rho = \rho'\Omega$ here, $\rho' = r_0\omega_0/c$, $\Omega = \omega_p/\omega_0$ and $\xi = z/R_d$, with ρ' is initial beam radius, ξ is the dimensionless normalized distance of propagation, z is the distance propagated by the laser beam in plasma and R_d is the Rayleigh length. $A(u)$ and $B(u)$ are functions of the modulation parameters as: $A(u) = 4 + u - \frac{2ug_1^3}{g_2^3} + \frac{4g_1^2}{g_2^2}$, $B(u) = (g_1^2 g_2^2 - 4ug_1 g_2^3 - 2g_2^4 + 2u^2 g_2^4)$,

where u is the modulation parameter of asymmetric finite AiG beams, $g_1 = 1/3^{\frac{1}{3}}\,\Gamma(1/3)$ and $g_2 = 1/3^{\frac{2}{3}}\,\Gamma(2/3)$ with Γ is the "Gamma function". The nonlinear coupled ordinary differential Eqs. (6) and (7) can be solved numerically with the initial boundary condition at

$\xi = 0, f_1 = f_2 = 1$ and $\frac{df_1(z)}{d\xi} = \frac{df_2(z)}{d\xi} = 0$.

3 Result and discussion

In Fig. 2, (a), (c) and (e) show the variation of f_1 with ξ for different values of the modulation parameter b by keeping $(a = 0.00)$, $(a = a_{cr})$ and $(a = 1.50)$ respectively. While, (b), (d) and (f) show the variation of f_2 with ξ under the variation of modulation parameter (b) by keeping $(a = 0.00)$,

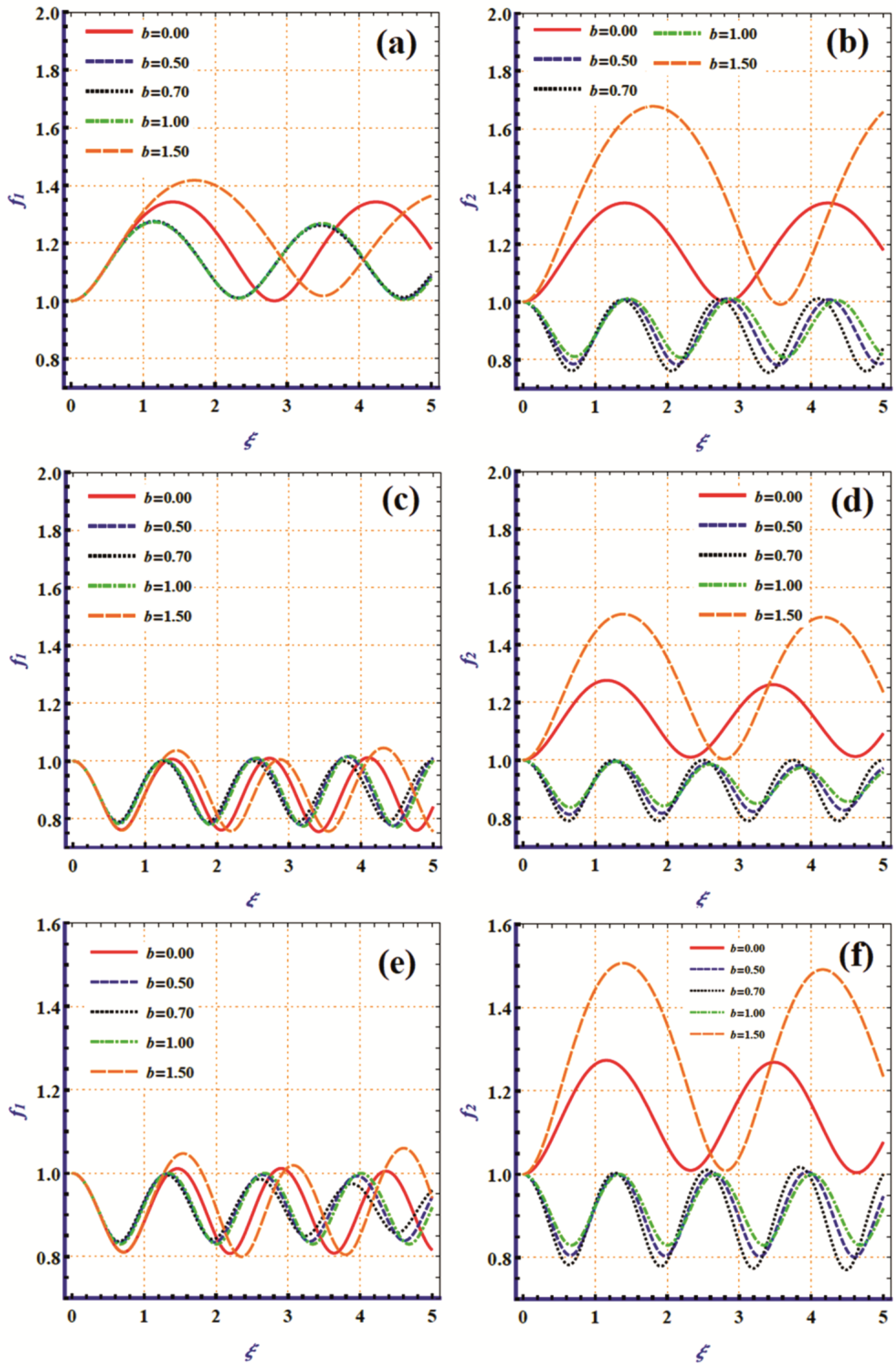

Fig 2 — Variation of f_1 with ξ for different b values at $p = p_{cr}$ for (a) $a = 0.00$, (c) $a = 0.70$ and (e) $a = 1.50$. The corresponding variation of f_2 with ξ for same b values at $p = p_{cr}$ for (b) $a = 0.00$, (d) $a = 0.70$ and (f) $a = 1.50$.

($a = a_{cr}$) and ($a = 1.50$) respectively. The variation of f_1 and f_2 with respect to ξ are different from each other due to the asymmetry in the modulation parameters a and b. It is clearly realized from the beam width parameter differential Eqs. (6) and (7) that, propagational characteristics of f_1 in the plasma is only depends on the value of modulation parameter a and for f_2 it is only depend upon the modulation parameter b. Due to coupling of these beam width differential equations, the variation in one of the

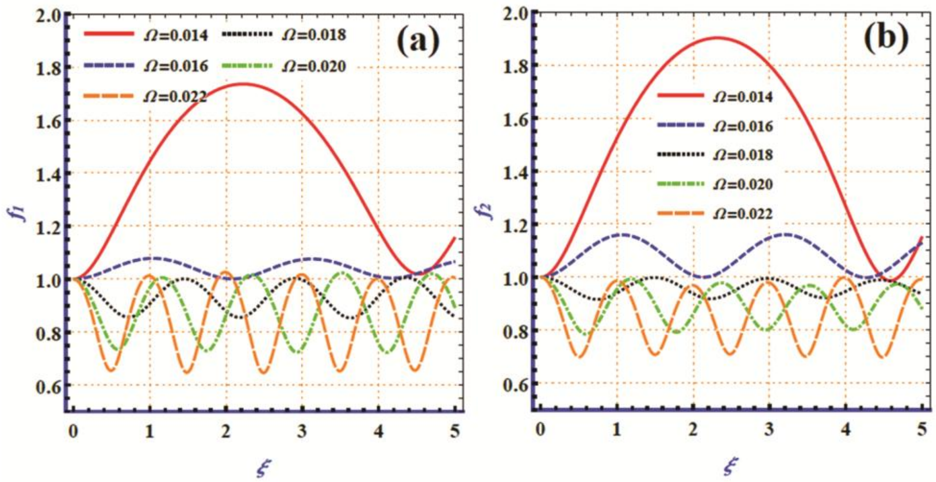

Fig. 3 — Effect of Ω on (a) f_1 and (b) f_2 with ξ for $a = a_{cr}$, $b = 1.00$ and $p = p_{cr}$.

modulation parameter $(a \text{ or } b)$ affects the propagational characteristics of both transverse beam width parameters f_1 and f_2 in the plasma.

From above figures it is clearly observed that, the self-focusing length of asymmetric finite AiG beams decreases with increase in the modulation parameter b up to critical value (b_{cr}) of modulation parameter and then increase with further increases in the modulation parameter. The lowest possible value of beam width parameters f_1 and f_2 under self-focusing is varies with modulation parameter b, it is lowest for critical value of modulation parameter (b_{cr}) when a is fixed and vice versa. The minimum value of waist widths $r_0 f_1$ and $r_0 f_2$ is minimum for b_{cr} and it is also depends upon the value of a. The minimum value of $r_0 f_1$ and $r_0 f_2$ is decreases with increase in the value of a, then it is lowest for critical value of a $(a = a_{cr})$ and again increases with further increase in the value of a. The lowest possible value of the waist width is observed for $a = a_{cr}$ and $b = b_{cr}$. From this, we can say that the self-focusing length and waist-width of the asymmetric finite AiG laser beams can be controlled by appropriate value of modulation parameters.

In Fig. 3, author investigated the variation of beam width parameters for finite AiG laser beam for asymmetric modulation parameters under the variation of density of plasma $(i.e. \omega_p/\omega_0)$. These variations are carried out for the condition of under-dense plasma, where plasma frequency is much smaller than that of the frequency of the asymmetric finite AiG laser beam $(\omega_p/\omega_0 \ll 1)$. It is seen that the finite AiG laser beams shows oscillatory self-focusing under asymmetric modulation parameters with decreasing self-focusing length with increasing density of plasma.

The finite AiG laser beam under asymmetric modulation parameter gets self-focused earlier as compared to that of finite AiG laser beam under symmetric modulation parameter. The periodicity of beam width parameters f_1 and f_2 are also decreases under the asymmetry in the modulation parameters. The asymmetry in the modulation parameters self-focuses the finite AiG laser beam at smaller distance, so it is very advantageous to take into consideration finite AiG beam under asymmetric modulation parameter. The modulation parameter b is dominant in f_2 so that is get defocusses with larger amplitude. The self-focusing length and periodicity of beam width parameter for f_1 is less than f_2 due to the asymmetry in the modulation parameter

4 Conclusions

- The propagation of asymmetric finite AiG laser beam inside the isotropic collisionless plasma is strongly affected by the asymmetry in the modulation parameters.
- The self-focusing length and periodicity of the finite AiG laser beam are significantly affected by the asymmetry in the modulation parameters. The self-focusing length and periodicity of the finite AiG laser beam are decreases with in the domain of the modulation parameter.
- The asymmetry in the modulation parameters supports the earlier self-focusing of the finite AiG laser beam.

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