Produce Basket: The Effect of Experiential Fraction Pedagogy on Preservice Teacher Learning

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Abstract: *Formal notation of fractions is a critical stumbling block for students, impeding their progress to acquire flexible number sense beyond integers and greatly impacting their success in algebra. Conflicting and vague definitions of fractions are a major cause of confusion and frustration among students and their teachers. A team consisting of a veteran fourth-grade teacher, a professional development leader/university instructor, a math educator, and a mathematician developed an experiential learning module called Produce Basket (PB) for elementary grades fraction learning. Since its inception 5 years ago, PB has been taught in about a dozen elementary classrooms in north central Ohio and is woven into the preservice teaching program at Ohio State University Mansfield. Given the high stakes and strong claims for this approach, there is now a pressing need to assess the strengths and weaknesses of Produce Basket's approach. The aim of this article is to report research conducted with two classes (total $n = 22$) of preservice teachers that were instructed over an 8-week period with PB. The study finds initial evidence that PB improves student understanding of fractions. The authors used both quantitative and qualitative instruments to measure student understanding.*

Keywords: *fractions, teacher preparation, Algebra Project*

Introduction

Operations with fractions is one of the leading struggles that students (and adults) have with mathematics. The confusion of fractions begins early in elementary education when students are given the definition of fraction as "part of a whole" which implies that a fraction is always less than one. While students discover how to work with "parts of a whole," they are given new and unusual fraction notation where their familiar $0 - 9$ digits suddenly mean something other than counting how many. The new notation of $\frac{a}{b}$ is an abstract representation with no single concrete basis; instead, there are numerous descriptions that appear mutually incompatible (Wu, 1999). To add to the struggle, many teachers lack confidence and are unable to offer a better pathway.

In response to these struggles, a team consisting of a veteran fourth grade teacher, a professional development leader/university instructor, a math educator, and a mathematician developed an experiential learning module called Produce Basket (PB) for elementary fraction learning (Adams, et. al., 2022). This module is structured under the premise that a better definition of fraction is *a number that tells us "how many" and "what kind."* The parts of a fraction are numerator, meaning "to count" (how many), and denominator, meaning "name" (what kind). Before students are given the traditional mathematical language and notation, they should have experience identifying **how many** and **what kind**.

Since its inception 5 years ago, PB has been taught in about a dozen elementary classrooms in north central Ohio and is woven into the preservice teaching program at Ohio State University Mansfield. Given the high stakes and strong claims for this approach, there is now a pressing need to assess the strengths and weaknesses of Produce Basket's approach to learning fractions. This article reports on a research study that was conducted with preservice teachers using the PB module.

Methodology and Study Procedures

Research Questions

The goal of this work is to test the efficacy of the Produce Basket model to improve the understanding of fractions among elementary education majors at Ohio State University. The Produce Basket module was taught to two classes (n=22) of OSU students in the elementary education program. We sought to answer these questions:

1. What evidence do we see that experience with PB supports students' conceptual understanding of fractions, including standard fraction notation?
2. What evidence do we see that PB helps students grasp the meaning and use of fraction notation within operations?
3. What evidence do we see that PB supports student understanding of fractions as numbers?

Study Procedures

The research team developed and implemented a pre/post-test that requires extended answers, including explicit use of explanatory models and justification. Additional anecdotal evidence and classroom work was collected for analysis, including video, interviews, and classroom observations. Both classes were team-taught by two of the authors of this paper, with observations by a third member. Follow-up data will be collected by the remaining author when these students enter math methods courses.

Research Subjects and Course Methodology

Most of the participants were traditional first-year college students. There were a few (3) non-traditional students. The course prerequisites are "A grade of C- or above in 1075 (second-semester precollege algebra); or credit for 1074, 75, or 104 (college algebra); or Math Placement Level R or above; or ACT math subscore of 22 or higher that is less than 2 years old." Students may have tested into this class, or they may have taken Math 1050 and/or Math 1075. There were no unusual student selection criteria, and no one who was normally placed in the course was turned away.

Each of the two course sections had 11 participants. One of the classes also had 2 or 3 students who opted out of the research. One section had 2 self-identified males and the other section had 1. The rest identified as female.

The two sections moved through the material at very similar rates. The course contained the same topics and spent the same amount of time on each topic as other non-treatment semesters, but the topics were presented using Produce Basket instead of other activities. For example, the standard course topic "Make a Ruler" activity was replaced with comparing Produce Basket pieces.

Test Procedures

The pre-test was administered at the beginning of the course, and again, unaltered, at the end of the course. Test results were never shared with students, and no feedback about the questions was given. Tests were independently graded by two evaluators. The pre-test had strong inter-rater reliability;

the raw pre-test scores had a correlation of $+0.78$, which increased when two outliers were reconsidered. The post-test had somewhat higher variability between the two raters but was still strong ($+0.76$).

In addition to looking for conceptual understanding, the pre/post also collected some attitude information via two short tasks at the start of the instrument:

1. "In one sentence describe your experience as a mathematics student in elementary school."
2. "In one sentence express your feelings about teaching elementary mathematics."

Results from this portion of the data were scored separately from the conceptual questions. We found a weak-moderate positive correlation between attitude scores and conceptual understanding (correlation coefficient $+0.38$).

Sample Pre-Test Questions and Student Responses

Three sample questions and pre-test student responses are presented here, along with a discussion of how these were scored.

Item 1.
Compare each of the following by inserting the correct symbol (<, >, =) and explain your reasoning: (a) $\frac{15}{14} \quad \frac{13}{12}$

The first item in the test contained 8 problems of this type, designed to detect various conceptual strengths and weaknesses. For the example shown here it is useful to be able to decompose a fraction into a whole number and a proper fraction. This problem also tempts students to consider the relative sizes of the integers without regard to their roles in the fraction.

Each problem in item 1 was scored using this rubric:
1 = Incorrect answer with no explanation
2 = Incorrect answer with flawed reasoning
3 = Correct answer with no reasoning or calculated common denominator
4 = Correct answer with correct logic/explanation of larger/smaller or more /less than

Pre-test Responses

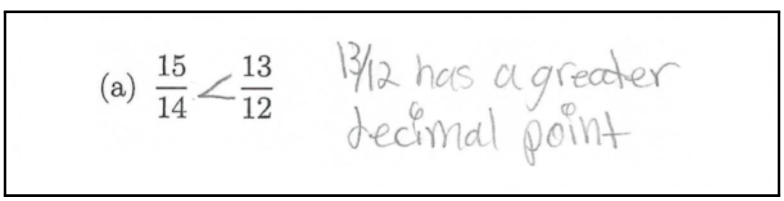

Figure 1: *Student A Response (Score: 3).*

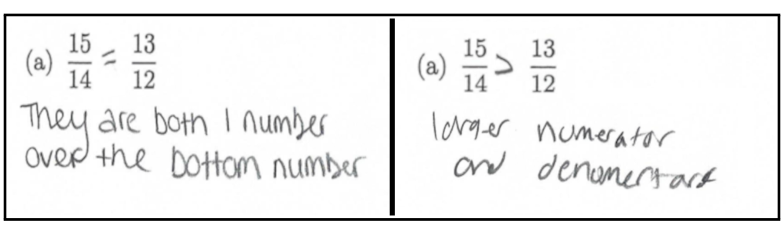

Figure 2: *(Left) Student B Response (Score: 2); (Right) Student C Response (Score: 2).*

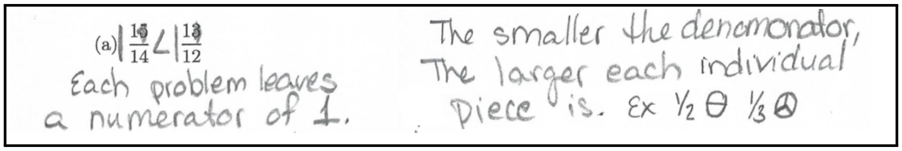

Figure 3: *(Left) Student D Response (Score: 4).*

Student A has a correct response but appears to have used a calculator. Students B and C are trying to reason about the problem by considering properties of the integers involved in the fraction, but without understanding their true roles. The response of Student B (this reasoning occurred more than once) indicates understanding that the numerator is a kind of counter but does not understand what the denominator counts. Student D is able to reason about fractions and clearly understands what the denominator means.

Here are two other sample problems. The point of these examples is merely to indicate the kind of understanding the research is looking for.

Item 5.
In the space below, draw your own number line and show the locations of $\frac{5}{7}$ *and* $\frac{8}{5}$.

Sample Pre-test Response

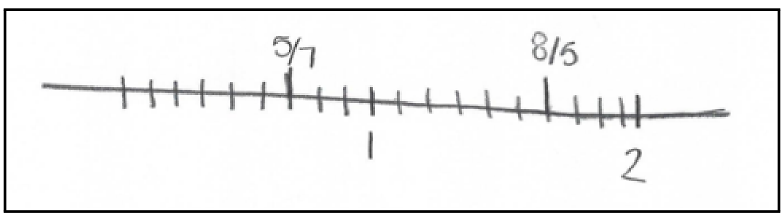

Figure 4: *Student E Response (Score: 3).*

Student E used a decimal system for the line and placed $\frac{8}{5}$ correctly. In particular, the student was not troubled by a fraction greater than one. The placement of $\frac{5}{7}$ is likely based on the decimal approximation $\frac{5}{7} \approx 0.7$, instead of using appropriate subdivisions of the interval.

Item 9.
Using drawings (squares, rectangles, groups), show that $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

Sample Pre-test Response

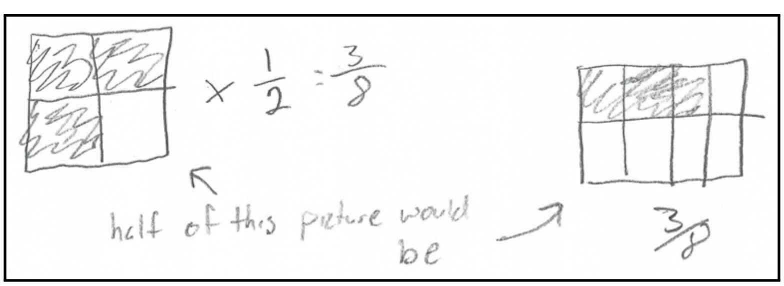

Figure 5: *Student F Response (Score: 4).*

The student made sense out of the problem, even though their answer indicates half of three-fourths, rather than three-fourths of one half.

Overall Pre-Test Results

The pre-test set a benchmark for student understanding of fractions, which was unsurprisingly low. The average score for 22 students on the pre-test (conceptual portion) was 60.6%. We saw gains in the post-test, which had an average score of 75.2%.

Classifying Problems

To try to understand the students' conceptual understanding a bit better, we classified the problems into three broad types:

1. **Fractions are Numbers.** Evidence that student can reason about the relative size of a fraction or its position on a number line.
2. **Standard Fraction Notation (SFN) Makes Sense.** Evidence that student can decode standard fraction notation, such as the different meanings of numerator, denominator, and reference to a whole.
3. **Standard Fraction Notation (SFN) and Operations.** Evidence that student can connect standard fraction notation to integer operations.

These categories are not mutually exclusive. Of the 10 items on the test, three were classed as predominantly category (1), two in category (2), and one in category (3). The four remaining items sought to gain evidence that students could model fractions to reveal their quantitative meaning. These items fell in the intersection of the categories, requiring some aspects of all of them. These examples are representative of the latter type:

Item 8. Using drawings (squares, rectangles, groups), show that $\frac{7}{4} + \frac{2}{8} = 2$
Item 10. Using drawings (squares, rectangles, groups), show that $\frac{3}{2} \div \frac{1}{4} = 6$.

Two of these four items (requiring students to model fraction multiplication and division) caused the most difficulty. The average pre-test score on those two problems was 1.8 and 1.9 (out of 4).

Data Results and Analyses

As Table 1 suggests, we saw growth in all categories. Recall that the maximum score is 4 on each item.

Table 1: *Student growth by item from pre-test to post-test.*

Item Number(s)	Problem Category	Average Change from pre to post
1, 4, 5	Fractions are Numbers (FAN)	0.4
2, 3	SFN Usable for Operations	0.6
6	SFN Understanding	0.4
7, 8, 9, 10	SFN Understanding, Ops, FAN	0.8

In percentage terms, these changes range from 10% (0.4) to 20% (0.8) increases in raw scores. Students appeared to become better at modeling fraction operations using geometric models.

Anecdotal evidence

Produce Basket was used to address a substantial amount of course material, and several students wearied of it. Several students questioned the need for an extended immersion in one topic, or indeed the need for models at all.

Some students appreciated the module; this included two of the students viewed by the instructors as among the most mathematically struggling.

Summary and Final Thoughts

The gains exhibited by the students using Produce Basket provide encouraging evidence that Produce Basket is on the right track for clarifying the mathematical basis for fractions for prospective teachers. In-service teachers who have adopted Produce Basket have been strong advocates for continued use and development of the PB module in their schools and provide us with steady demand for training. More research is needed to document the effectiveness of this module and approach and provide more evidence that a metaphor for fraction learning built along the lines of Moses' experiential philosophy can lead to a mathematically precise definition of a critical concept and help students learn.

In total, this data appears to support the hypothesis that authentically taught Produce Basket can significantly improve pre-teachers' understanding of fractions.

It is useful to note that the materials that are used in Produce Basket are inexpensive and easily obtained. All supporting documents, such as game boards, rules, and teaching guides and goals are available without cost. Physical materials, namely produce tile and dice, are available online or adaptable from existing resources.

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Debe Adams (adams.2167@osu.edu) teaches college-level math courses including courses for pre-service teachers at The Ohio State University, Mansfield. In 2009, she began teaching The Algebra Project pedagogy in the high school classroom and currently provides professional development grounded in the pedagogy of The Algebra Project for K-12 in-service math teachers through OSU Mansfield's Math Literacy Initiative (MLI).

Terri Bucci (bucci.5@osu.edu) is an associate professor and researcher in mathematics education. Terri's research interests focus on mathematics education and international teacher education, with a focus on the professional preparation of primary grades teachers in Haiti. Dr. Bucci has worked collaboratively with Haitian and US colleagues for many years and is currently involved in a collaborative project which connects the human, intellectual, and fiscal resources between US faculty of education and Haitian colleagues.

Lee McEwan (mcewan@math.ohio-state.edu) is professor emeritus of mathematics at Ohio State University's Mansfield Campus. Lee's areas of expertise include algebraic geometry and the topology of algebraic singularities. A long-time advocate of the Algebra Project, Lee co-founded the Mathematics Literacy Initiative (MLI) at OSU-Mansfield with Terri Bucci and collaborated with Kevin Reinthal to develop Produce Basket.

Kevin Reinthal (koreinthal@gmail.com) taught mathematics in fourth grade for 30 years in the Lucas Local Schools in Loudonville, Ohio. He is the lead developer of Produce Basket. As a Math Teacher Leader, Kevin has worked with dozens of in-service teachers to learn and adapt Produce Basket to their classrooms. He led the intervention in Ms. Adams' classroom.

A Note about the Authors

Because of the distribution of strengths of our team we opted to list the authors in alphabetic order. The context of this work is the Mathematics Literacy Initiative at the Ohio State University, a multi-year effort led by Dr. Terri Bucci. The MLI grew out of work that Bucci and McEwan did with Bob Moses and the Algebra Project starting in 2009. Kevin Reinthal is a veteran elementary school teacher who undertook professional development training through the MLI. He identified the issues with teaching fractions addressed in this work and is the primary developer of Produce Basket. He co-taught the intervention for this research. Debe Adams has a background with the Algebra Project also dating to 2009, having taught Algebra Project aligned high school mathematics classes under a 5-year NSF grant in Illinois before coming to OSU as a PD specialist with the MLI. She is the primary instructor for the courses in this intervention and aided in data collection and analysis. Lee McEwan is a retired mathematician at OSU who has worked with the MLI and the Algebra Project for professional development of K-12 mathematics teachers and served as Mr. Reinthal's partner in developing Produce Basket.