Mobile Sensors Networks under Communication Constraints

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Abstract: The paper deals with the problem of computing optimal or suboptimal motion for a network of mobile sensors. The use of moving sensors means that for each point of the field under measurement asynchronous discrete time measures are given instead of continuous time ones, being possible to fix in advance the maximum time interval between two consecutive measures for the same point. The constraints here considered are on the full coverage of the field, with respect to the measurements, within the prefixed time interval and on the communication connections, between any pair of moving sensors, at any time. A solution, based on a local distributed approach, is proposed and compared with a centralized approach previously proposed, and here recalled, by the same authors. Some simulations show the effectiveness of the both solutions, putting in evidence advantages, disadvantages and differences.

Key–Words: Sensor network, dynamic configuration, optimal motion, network communications.

1 Introduction

Distributed sensors systems and networks are growing relevance in the scientific and engineering community as proved for example by [2, 16]. Several kinds of sensor networks are concerned with monitoring or surveillance of certain geographical areas, for example to measure ground humidity and solar radiation in farms or parks, temperature for fire prevention (buildings as well as woods), to detect presence and distribution of people in critical structures, and so on. In general an event can be detected or a measure can be made, on a certain point, if there are sensors close enough to operate, where *enough* is obviously strongly related to the sensors field of measure. The problem of maximize the numbers of detectable events or in general the field of measure of a sensor network in known in literature ([9]) as the *area coverage* problem. Using fixed positions for the sensors, the coverage problem can be faced in terms of collocation of sensors in the area under measurement. In other terms, the problem usually has been posed answering the question ""which are the *best* places to put the N sensors?", where *best* is considered with respect to area coverage according with energetic costs (for the deployment as well as for the communications) or number of sensors.

Such a problem has been well studied in a lot of works, such as [14, 10, 23, 17, 12, 18]. These approaches consider, in general, the use of a large number of low cost sensor devices; this choice makes expensive the sensors allocation task and limitates the possibility of reusing the same sensors system in another place or situation. At the same time, it makes also difficult to deal with faults of sensor nodes or communication links.

To face these problems, the idea of using mobile sensors has been approached.

In [20, 11, 5] the problem of self-deploying mobile sensors, able to configure according to the environment, is addressed and some solutions are proposed. There, motion is used only in the allocation and reconfiguration tasks.

From robotics ([4]) it comes the suggestion of a stronger use of movement capabilities with reduced number of sensor units moving continuously. This approach increases the flexibility and the reusabilty of the sensor network, the number of sensor is appreciably reduced and than it is easier and probably less expensive to cover large areas. On the other hand, this advantages are paid by the loose of a continuous measurement. Coverage, in fact, must be considered within a time interval by the requirement of a coordinated motion, for example to avoid collisions between sensors or to maintain the communication between agents in the network. The question to be answered in this case is "which are the *best* trajectories for the N moving sensors?", where *best* is considered with respect to several costs, first of all the field coverage. Dynamic, geometric and communication constraints must be also considered.

In [21, 3] the problem has been studied in the level set framework and some suboptimal solutions are proposed. An approach based on space decomposition and Voronoy graphs is proposed in [1]. A distributed multisensor approach is presented in [13]. An optimal control formulation for the problem of planning optimal trajectories for a single sensor in sense of area coverage is proposed in[22]. With a similar approach the authors proposed, in [8, 6, 7], a solution that takes in account dynamic and communication constraints on motion of sensors; the whole sensors network is modeled as a single dynamic system for which suboptimal covering trajectories are computed with a centralized algorithm.

In this paper a distributed approach to the coverage problem is proposed, and it is compared with the one in [7]. Each sensor node calculates the control inputs according to the coverage state of the environment, with the knowledge of the state of its neighbors, using a virtual force based method [15, 24, 19]. Distributed computation produces worse coverage performances with respect to the centralized one, but it can be used for online applications and for large networks that are very hard to handle with centralized approaches due the fast grow of the problem complexity when the network dimension became larger and larger. The paper is organized as follows. In Section 2 a general mathematical formulation of movement, sensing and communications is given. In Section 3 the centralized approach to the problem is described and simulations results are showed. The same is given in Section 4 for the distributed approach. Some final comments in 5 end the paper.

2 Mathematical Model

In the computation of the mathematical model the assumption of homogeneous sensor devices is assumed, that is all the sensors have the same characteristics. This is done only for sake of simplicity in the notations since, clearly, the proposed approach applies also to non homogeneous sensors systems: an additional index should be added to all variables and, somewhere, sums over such an index may be required.

2.1 Motion Model

Under the simplifying hypothesis, each mobile sensor is modeled, from the dynamic point of view, as a material point of unitary mass, moving on $\mathbb{R}^n$ ($n = 2, 3$), called the *workspace*, under the action of an input force named $u^{(i)}(t)$, for the $i - th$ sensor. The motion equation is then

$$\ddot{x}^{(i)}(t) = u^{(i)}(t) \tag{1}$$

where $x^{(i)}T$ represent the position of the sensor in W at time t.

The linearity of 1 allows one to write the dynamics in the form

$$\begin{aligned} \dot{z}^{(i)}(t) &= Az^{(i)}(t) + Bu^{(i)}(t) \\ x^{(i)}(t) &= Cz^{(i)}(t) \end{aligned} \tag{2}$$

where

$$z^{(i)}(t) = \begin{pmatrix} \dot{x}_1^{(i)}(t) & x_1^{(i)}(t) & \dots & \dot{x}_n^{(i)}(t) & x_n^{(i)}(t)) \end{pmatrix}^T$$

represent the state vector.

Looking at the whole system it is possible to define

$$x(t) = \begin{pmatrix} x^{(1)}(t) & x^{(2)}(t) & \cdots & x^{(N)}(t) \end{pmatrix}^T$$

a generalized position,

$$z(t) = \begin{pmatrix} z^{(1)}(t) & z^{(2)}(t) & \cdots & z^{(N)}(t) \end{pmatrix}^T$$

a generalized state and

$$u(t) = \begin{pmatrix} u^{(1)}(t) & u^{(2)}(t) & \cdots & u^{(N)}(t) \end{pmatrix}^T$$

a generalized input.

It is very useful to define also a discrete time representation of the dynamical system. It can be easily obtained by discretization of (2):

$$\begin{aligned} z^{(i)}((k+1)T_s) &= A_d z^{(i)}(kT_s) + B_d u^{(i)}(kT_s) \\ x^{(i)}(kT_s) &= Cz^{(i)}(kT_s) \end{aligned} \tag{3}$$

where T_s is the sample time, $A_d = e^{AT_s}$ and $B_d = \int_0^{T_s} e^{A\tau} Bd\tau$

2.2 Sensing Model

Each mobile sensor at time t is assumed to take measures within a circular set of radius ρ_S around its current position $x^{(i)}(t)$. Such a set under sensor *visibility* will be denoted as

$$M^{(i)}(t) = \sigma(x^{(i)}(t), \rho_S^{(i)}) \tag{4}$$

The introduction of directional sensors can be modeled, within the present framework, by the simple change of 4 into

$$M^{(i)}(t) = \sigma(x^{(i)}(t), \rho_S^{(i)}, \theta_0^{(i)}, \Delta\theta^{(i)})$$

where $\theta_0^{(i)}$ denotes the main direction and $\Delta\theta^{(i)}$ the amplitude of the directional cone. Taking into account the whole system it is possible to define the *generalized visibility set* as:

$$M(t) = \bigcup_i M^{(i)}(t) \quad (5)$$

2.3 Communication Model

From the communications point of view, it is assumed that each mobile sensor can require to communicate with any other one. For this purpose, the sensors network can be seen as a communication network where a path starting from the sender node and ending to the addressee represents the communication channel. In this paper such a communication network is modeled as an Euclidean graph, that is a graph in which vertexes are points of an Euclidean space ($\mathbb{R}^n$).

Let's denote this graph with $\mathcal{G} =< V_\mathcal{G}, E_\mathcal{G} >$, where $V_\mathcal{G}$ represents the vertexes set and $E_\mathcal{G}$ the edges one. Each vertex represents the position of a sensor node. For every pair of nodes $(x^{(i)}, x^{(j)})$ there is an edge $e_{i,j}$ between them (and than they can communicate directly) if and only if $\| x^{(i)} - x^{(j)}\| \le \rho_C$. In the case of homogeneous sensors, that is the one considered in this paper, communication is always bidirectional and than $\mathcal{G}$ is undirected.

2.4 Coverage Problem

Given a time interval $\Theta = [0, t_f]$ and a generalized trajectory $x(t)$ for the sensor set, is possible to define the subset of Ω covered by the sensors fields of measure during the movement as the union of the measure fields at every configuration $x(t)$ with $t \in \Theta$:

$$M^\Theta = \bigcup_{t\in\Theta} M_(t) \quad (6)$$

The area covered by the sensors in the time interval Θ is than the measure $\mathcal{A}^\Theta = \mu(M^\Theta)$.

The coverage problem consists in maximizing $\mathcal{A}^\Theta$ according to the constraints.

2.4.1 Geometric Constraints

The workspace Ω is supposed to be a box subset of $\mathbb{R}^n$. Then the trajectory must satisfy the constraints

$$x_{min} \le x^{(i)}(t) \le x_{max}$$

If needed, it is possible to constrain the starting and/or the final state (positions and/or speeds):

$$z(0) = z_{start}$$
$$z(t_f) = z_{end}$$

A particular case is the periodic trajectories constraint, useful in tasks in which measures have to be continuously repeated necessarily in periodic way:

$$z(0) = z(t_f)$$

It is also necessary to avoid collisions between sensors. Then, for any time t

$$\|x^{(i)}(t) - x^{(j)}(t)\| \ge \rho_B$$

for $i \ne j$

2.4.2 Dynamic Constraints

Physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) suggest the introduction of the following additional constraints

$$v_{min} \le \dot{x}(t) \le v_{max}$$
$$u_{min} \le u(t) \le u_{max}$$

that introduce a maximum velocity considered acceptable both for motors speed of the mobile platforms and to guarantee enough time for a sensor detection of any field point to perform the measurements correctly.

2.4.3 Communication Constraints

In order to assure communication between sensors, a full connection of the sensors network is required. This can be obtained choosing a network topology that sensors must maintain. The desired network topology is represented by an *adjacency matrix* $\mathcal{A} \in \mathbb{R}^{N\times N}$. As well known, $\mathcal{A}(i,j) = 1$ if a direct link between node i and node j does exist, while $\mathcal{A}(i,j) = 0$otherwise. In order to maintaining topology, the following constraints on the distance between sensors must be introduced

$$\mathcal{A}(i,j) = 1 \Rightarrow \|x^{(i)}(t) - x^{(j)}(t)\| \le \rho_C \; \forall t \in \Theta$$

3 Centralized Approach

The first solution to the coverage problem proposed in this paper is to plan off-line trajectories for the whole system. To do that, the coverage problem is modeled as an optimal control problem.

3.1 Coverage Evaluation

The computation of the area $\mathcal{A}^\Theta$ covered by the sensors measures during Θ can be a very hard task. To evaluate the coverage performances of a generalized trajectory $x(t)$ a different functional is than defined. It is based on the distance $d(x(t), p)$ between a point of the workspace and the generalized sensors trajectory that is defined as:

$$d(x(t), p) = \min_{t \in \Theta, j \in \{1,2,\dots N\}} \left|\left|p - x^{(j)}(t)\right|\right| \quad (7)$$

Making use of the function

$$\text{pos}(\xi) = \begin{cases} \xi & \text{if} \quad \xi > 0 \\ 0 & \text{if} \quad \xi \leq 0 \end{cases} \quad (8)$$

that fixes to zero any nonpositive value, the function

$$\hat{d}(x(t), p, \rho_S) = \text{pos}\left(d(y(t), p) - \rho_S\right) \geq 0$$

can be defined.

A measure of how the generalized trajectory $x(t)$ produces a good coverage of the workspace can then be given by

$$J(x(t)) = \int_{p \in W} \hat{d}(x(t), p, \rho_S) \quad (9)$$

Smaller is $J(x(t)$, better is the coverage. If $J(x(t)) = 0$ than $x(t)$ covers completely the workspace.

3.2 Optimal Control Formulation

Making use of the element introduced in previous subsections, the Optimal Control Problem can be formulated in order to find the best trajectory $x^*(t)$ that maximizes the area covered by measurement of the N moving sensors during the time interval Θ, and satisfies the constraints. Then a constrained optimal control problem is obtained, whose form is ([8])

$$\begin{gathered} \min J(x(t)) \\ f(x(0), x(t_f)) \leq 0 \\ x_{min} \leq x^{(i)}(t) \leq x_{max} \\ v_{min} \leq \dot{x}(t) \leq v_{max} \\ u_{min} \leq u(t) \leq u_{max} \\ \|x^{(i)}(t) - x^{(j)}(t)\| \geq \rho_B \\ \mathcal{A}(i,j) = 1 \Rightarrow \|x^{(i)}(t) - x^{(j)}(t)\| \leq \rho_C \end{gathered} \quad (10)$$

The optimal solution $u^*(t)$ is given by the control inputs that produce the optimal trajectory $x^*(t)$.

Unluckily this problem is, in general, very hard to solve analytically. In next section a solvable discrete approximation is then defined and solved.

3.3 Non Linear Programming Approximation

In order to overcome the difficulty of solving a problem as (10) due to the complexity of the cost function $J(\cdot)$, a discretization is performed, both with respect to space W, and with respect to time, in all the time dependent expressions.

The workspace is divided into square cells $c_{i,j}$ with resolution (size) l_{res}, and the trajectories are discretized with sample time T_s according with (3).

Representing the generalized input sequence from time $t = 0$ to time $t = NT_s$ as:

$$U_N^{(i)} = \begin{pmatrix} u^{(i)}(0) \\ u^{(i)}(T_s) \\ \vdots \\ u^{(i)}((N-1)T_s) \end{pmatrix}$$

and defining the following vectors

$$V_N^{(i)} = \begin{pmatrix} z^{(i)}(0) \\ U_N^{(i)} \end{pmatrix} \qquad h_n^{(i)} = \begin{pmatrix} A_d^n \\ A_d^{n-1} B_d \\ \vdots \\ B_d \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

it is possible to write state and output values at time $nT_s \leq NT_s$ as:

$$z^{(i)}(nT_s) = h_n^{(i)T} V_N^{(i)} \quad (11)$$

and

$$x^{(i)}(nT_s) = Cz^{(i)}(nT_s) = Ch_n^{(i)T} V_N^{(i)} \quad (12)$$

State and output sequences, from time $t = 0$ to time $t = NT_s$, can be represented by the following vectors

$$Z_N^{(i)} = \begin{bmatrix} z^{(i)}(0) \\ z^{(i)}(T_s) \\ \vdots \\ z^{(i)}(NT_s) \end{bmatrix} \qquad X_N^{(i)} = \begin{bmatrix} x^{(i)}(0) \\ x^{(i)}(T_s) \\ \vdots \\ x^{(i)}(NT_s) \end{bmatrix}$$

Defining the following matrices:

$$H_N^{(i)} = \begin{pmatrix} {h_0^{(i)}}^T \\ {h_1^{(i)}}^T \\ \vdots \\ {h_N^{(i)}}^T \end{pmatrix} \qquad C_N^{(i)} = \begin{pmatrix} C & 0 & \cdots & 0 \\ 0 & C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C \end{pmatrix}$$

and according to 11 and 12, the relations between these sequences and the input ones are described by:

$$Z_N^{(i)} = H_N^{(i)} V_N^{(i)} \tag{13}$$

and

$$X_N^{(i)} = C_N^{(i)} H_N^{(i)} V_N^{(i)} \tag{14}$$

Considering the whole system it is possible to define generalized input, state and output (positions) sequences

$$V_N = \begin{bmatrix} V_N^{(1)} \\ V_N^{(2)} \\ \vdots \\ V_N^{(m)} \end{bmatrix}$$

$$Z_N = \begin{bmatrix} Z_N^{(1)} \\ Z_N^{(2)} \\ \vdots \\ Z_N^{(m)} \end{bmatrix} \qquad X_N = \begin{bmatrix} X_N^{(1)} \\ X_N^{(2)} \\ \vdots \\ X_N^{(m)} \end{bmatrix}$$

These sequences are related by:

$$Z_N = H_N V_N \tag{15}$$

and

$$X_N = C_N H_N V_N \tag{16}$$

where

$$H_N = \begin{pmatrix} H_N^{(1)} & 0 & \cdots & 0 \\ 0 & H_N^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_N^{(m)} \end{pmatrix}$$

and

$$C_N = \begin{pmatrix} C_N^{(1)} & 0 & \cdots & 0 \\ 0 & C_N^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_N^{(m)} \end{pmatrix}$$

Using the notation above it is possible to approximate the Coverage Problem (10) with a nonlinear programming problem on variables V_N:

$$\min_{V_N} J(V_N) = \sum_i \sum_j \hat{d}(C_N H_N V_N, c_{ij}, \rho_S)$$

$$z_{min} \le {h_n^{(i)}}^T V_N^{(i)} \le z_{max}$$
$$f(V_N) \le 0$$

$$\| C{h_n^{(i)}}^T V_N^{(i)} - C{h_n^{(j)}}^T V_N^{(j)} \| \ge d_{col}$$

$$\mathcal{A}(i,j) = 1 \ \Rightarrow \ \| C{h_n^{(i)}}^T V_N^{(i)} - C{h_n^{(j)}}^T V_N^{(j)} \| \le \rho_C$$

Suboptimal solutions can then be computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) method has been applied. The obtained model can be customized according to the specific task, as shown in the following section.

3.4 Simulation Results

In this section simulations results are reported in order to put in evidence the capabilities and the effectiveness of the proposed solution, and to show how different topology constraints influence the coverage performances. The values of parameters used in all the simulations are:

$$u_{max} = 0.5N,$$
$$v_{max} = 1.5\tfrac{m}{sec},$$
$$T_s = 0.5sec$$
$$t_f = 15sec$$

Ring Network

In the *ring topology* (figure 1 for $N = 4$) each node is directly connected with two other nodes; with this structure the network maintain connection even with the fault of one sensor node. The solutions for a ring network of three moving nodes are shown in figure 2.

The area covered with measures is the 75% of the total one. The same results are showed in figure 3 for a ring network with four moving nodes on a larger workspace

The area covered with measures is the 76% of the total one.

Line Network

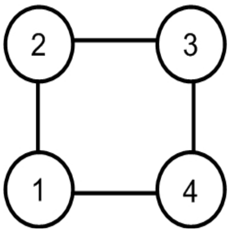

Figure 1: Ring Topology with four nodes

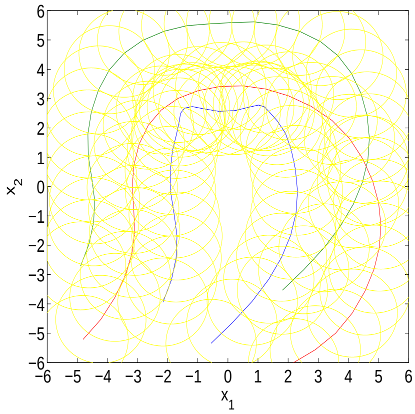

Figure 2: Suboptimal trajectory for a moving sensor network with three nodes and ring topology ($x_{max} = y_{max} = 6m$, $x_{min} = y_{min} = -6m$).

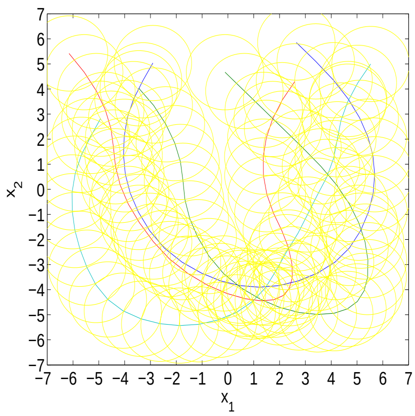

Figure 3: Suboptimal trajectory for a moving sensor network with four nodes and ring topology ($x_{max} = y_{max} = 7m$, $x_{min} = y_{min} = -7m$).

The *line topology* (figure 4), is the less constraining topology, and the one who allows the best coverage performances. The problem of this network structure is that it is not directly fault tolerant, because the fault of one of the internal nodes cause the loss of network connection if no recover maneuver is performed. Solutions for ring network of 3 moving nodes are shown in 5

The area covered with measures is the 83% of the

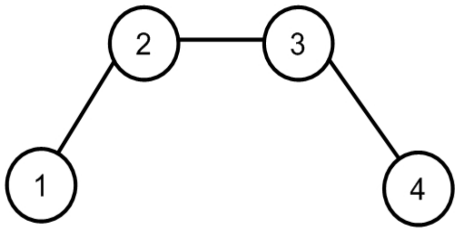

Figure 4: Line Topology with four nodes

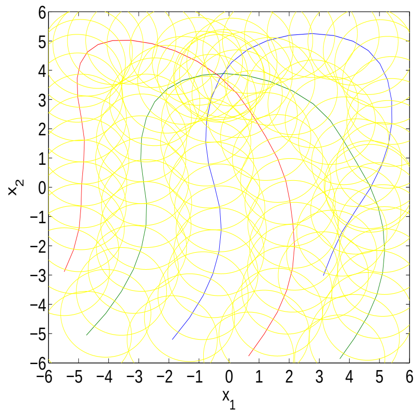

Figure 5: Suboptimal trajectory for a moving sensor network with tree nodes and line topology ($x_{max} = y_{max} = 6m$, $x_{min} = y_{min} = -6m$).

total. The same results are showed in figure 6 for a ring network with four moving nodes on a larger workspace

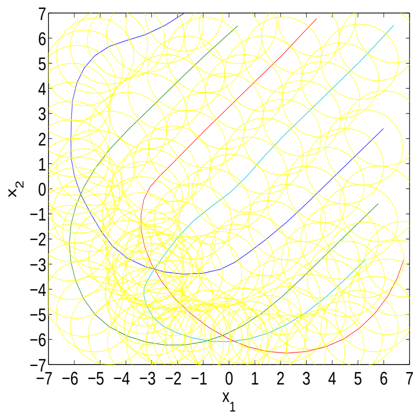

Figure 6: Suboptimal trajectory for a moving sensor network with four nodes and line topology ($x_{max} = y_{max} = 7m$, $x_{min} = y_{min} = -7m$).

The area covered with measures is the 82% of the total one. The growth of coverage performance with respect to the ring topology is evident.

4 Distributed Approach

In this section a distributed approach to the area coverage with moving sensors is proposed. At each sample time kT_s the control input to the $i-th$ sensor is calculated according to the sensor position, the positions of the sensor neighbors and the map coverage state, that can be obtained exchanging data with other sensors over the communication network that is maintained connected. Communication delays are not considered in this paper.

$$u^{(i)}(k) = u_C^{(i)}(k) + u_I^{(i)}(k) \tag{17}$$

4.1 Coverage

The first addendum of (17) describes the *virtual force* that attracts the sensors to the unmeasured cells and drives them to cover the workspace with measures. The functional structure, inspired to the electric field, for this virtual force is :

$$u_C^{(i)}(k) = \sum_n \sum_m \gamma_{m,n} \frac{d_{m,n}^{(i)}}{\|d_{m,n}^{(i)}\|^3} \tag{18}$$

where

$$d_{m,n}^{(i)} = c_{m,n} - x^{(i)}$$

$$\gamma_{m,n} = \begin{cases} 0 & \text{if } c_{m,n} \text{ has been measured} \\ 1 & \text{otherwise} \end{cases}$$

The intensity of the virtual force generated by a generic cell on a sensor vary inversely with respect to the cell measure reliability and to the distance between the sensor and the cell. In this manner sensors are directed to the nearest unmeasured cells as it is reasonable. When a generic cell has been measured it stops to contribute to the virtual force and then $u_C^{(i)}(k)$ remains limited.

Like all the potential based controls, u_C has undesired equilibria. Let $x^{(e)}$ be one of this equilibria, that is $u_C^{(i)}\Big|_{x^{(i)}=x^{(e)}} = 0$: it is possible to see that it is not stable. The following simple case show well this fact.

Let c_1 and c_2 be two unmeasured cells. The virtual force generated by them has an equilibrium point $x^{(e)}$ on the segment $\overline{c_1 c_2}$. Defining the vectors

$$\begin{array}{ll} d_1 = c_1 - x & d_1 = c_2 - x \\ d_1^{(e)} = c_1 - x^{(e)} & d_1^{(e)} = c_2 - x^{(e)} \\ \Delta x = x - x^{(e)} & \end{array}$$

it's possible to see (figure 7) that if

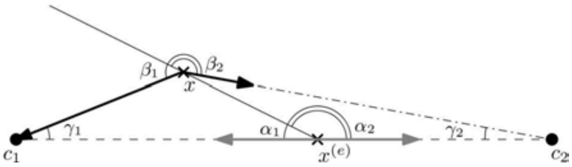

Figure 7: Force generated by two unmeasured cells

- $\alpha_1 \leq \frac{\pi}{4}$
- $\| \Delta x\| \leq \| d_1^{(e)}\| \left(\cos(\alpha_1) - \sin(\alpha_1)\right)$ and then $\beta_1 \leq \frac{\pi}{4}$

$\Delta x^T \; u_C^{(i)}\Big|_{x^{(i)}=x} \geq 0$ and then the sensor escapes from q_e. In fact

$$\begin{gathered} \Delta x^T \; u_C^{(i)}\Big|_{x^{(i)}=x} = \\ = \frac{\Delta x^T d_1}{\| d_1\|^3} + \frac{\Delta x^T d_2}{\| d_2\|^3} = \\ = \frac{\| \Delta x\| \, \| d_1\| \, \cos\beta_1}{\| d_1\|^3} + \frac{\| \Delta x\| \, \| d_2\| \, \cos\beta_2}{\| d_2\|^3} = \end{gathered}$$

Applying the sines theorem

$$\begin{gathered} = \frac{\| \Delta x\| \, \sin\beta_1 \cos\beta_1}{\| d_1\| \, \| d_1^{(e)}\| \, \sin\alpha_1} + \frac{\| \Delta x\| \, \sin\beta_2 \cos\beta_2}{\| d_2\| \, \| d_2^{(e)}\| \, \sin\alpha_2} = \\ = \frac{\| \Delta x\| \, \sin(2\beta_1)}{2\| d_1\| \, \| d_1^{(e)}\| \, \sin(\alpha_1)} + \frac{\| \Delta x\| \, \sin(2\beta_2)}{2\| d_2\| \, \| d_2^{(e)}\| \, \sin(\alpha_2)} \geq \\ \geq \frac{\| \Delta x\| \, \sin(2\alpha_1)}{2\| d_1\| \, \| d_1^{(e)}\| \, \sin(\alpha_1)} + \frac{\| \Delta x\| \, \sin(2\alpha_2)}{2\| d_2\| \, \| d_2^{(e)}\| \, \sin(\alpha_2)} = \\ = \frac{\| \Delta x\| \, \cos(\alpha_1)}{\| d_1\| \, \| d_1^{(e)}\|} + \frac{\| \Delta x\| \, \cos(\alpha_2)}{\| d_2\| \, \| d_2^{(e)}\|} \geq \end{gathered}$$

Observing that $\| d_1\| \leq \| d_1^{(e)}\|$ and $\| d_2\| \geq \| d_2^{(e)}\|$

$$\begin{gathered} \geq \frac{\| \Delta x\| \, \| d_1^{(e)}\| \, \cos(\alpha_1)}{\| d_1^{(e)}\|^3} + \frac{\| \Delta x\| \, \| d_2^{(e)}\| \, \cos(\alpha_2)}{\| d_2^{(e)}\|^3} = \\ = \frac{\Delta x^T d_1^{(e)}}{\| d_1^{(e)}\|^3} + \frac{\Delta x^T d_2^{(e)}}{\| d_2^{(e)}\|^3} = (\Delta x)^T \; u_i^C\Big|_{x=x^{(e)}} = 0 \end{gathered}$$

In figure 8 the u_i^C field is displayed for a more complicated case.

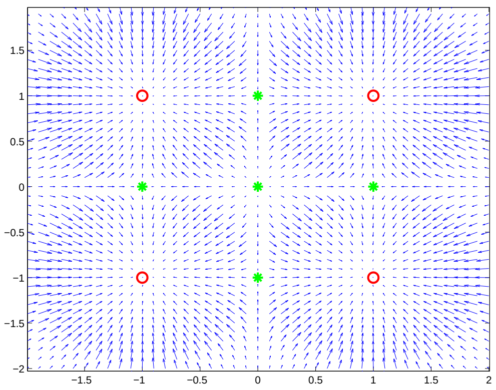

Figure 8: Force field generated by four unmeasured cells (red circles). Five equilibria are starred in green, is easy to see that they are not stable

4.1.1 Interaction

The second addendum of (17) describes the *interaction virtual force*. This force depends from the distances and relative velocities between the sensor and its neighbors. Defining

$$d_{i,j} = x^{(j)} - x^{(i)} \quad v_{i,j} = \dot{x}^{(j)} - \dot{x}^{(i)}$$
$$\rho_{i,j} = \| d_{i,j} \| \quad \dot{\rho}_{i,j} = \frac{d_{i,j}^T v_{,j}}{\rho_{i,j}}$$

the functional structure of the interaction force is given by

$$u_i^I(k) = \sum_j \left(e^{(\rho_{i,j}-\rho_B)^{-2}} \frac{d_{i,j}}{\rho_{i,j}} + e^{(\dot{\rho}_{i,j}+v_B^{(i,j)})^{-2}} v_{i,j} \right) - \sum_{y|\mathcal{A}(i,y)=1} \left(e^{(\rho_{i,y}-\rho_C)^{-2}} \frac{d_{i,y}}{\rho_{i,y}} - e^{(\dot{\rho}_{i,j}-v_C^{(i,y)})^{-2}} v_{i,y} \right) \tag{19}$$

where

$$v_B^{(i,j)} = \sqrt{2u_{max}(\rho_{i,j} - \rho_B)}$$
$$v_C^{(i,j)} = \sqrt{2u_{max}(\rho_C - \rho_{i,j})}$$

represents the limit velocities that, considering the actuators limits, allows the system to avoid collisions (v_B) or to maintain topology (v_C). The virtual force between two sensors with zero relative velocity is shown in figure 9. If the distance between sensors is far from critical values (ρ_B,ρ_C) the virtual force is zero and the sensor can freely explore the workspace under the action of $u_i^C(k)$. If the distance between sensors became near to ρ_B or ρ_C the interaction force grows, becoming larger than $u_i^C(k)$ and the sensor moves to avoid collisions or the breaking of network connections.

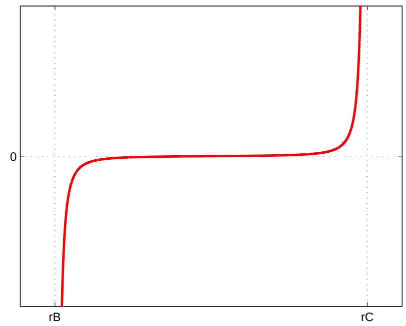

Figure 9: Attractive interaction force between two sensors

4.2 Simulation Results

Using distributed approach is possible to work online on large groups of sensors. However, coverage performances are worse than the ones of the centralized approach. In figure 10 solutions of the same scenario of figure 3 are displayed. With the distributed approach the area covered is the 70% of total one versus the 76% obtained using the centralized approach. In figure 11 the coverage of an area with 9 sensors maintaining mesh topology is shown. In figure 12 the same scenario is shown for a less constraining topology.

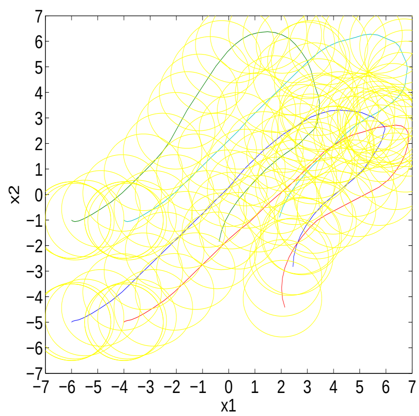

Figure 10: Solution of the same scenario of figure 3 obtained with the distributed approach

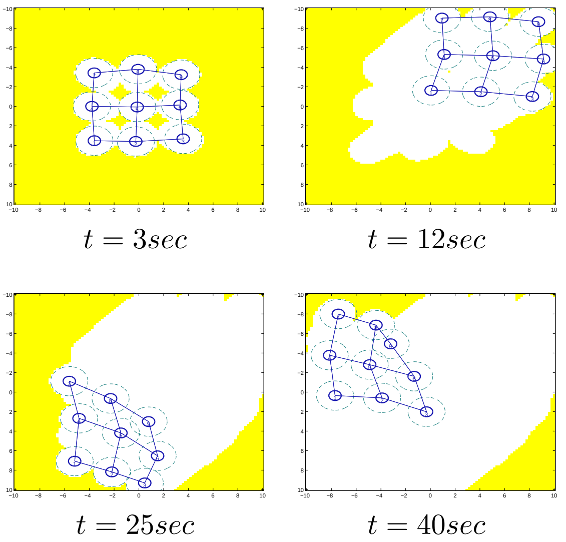

Figure 11: Coverage of a square area ($x_{max} = y_{max} = 10m$, $x_{min} = y_{min} = -10m$) wiht 9 moving sensors maintaining mesh topology

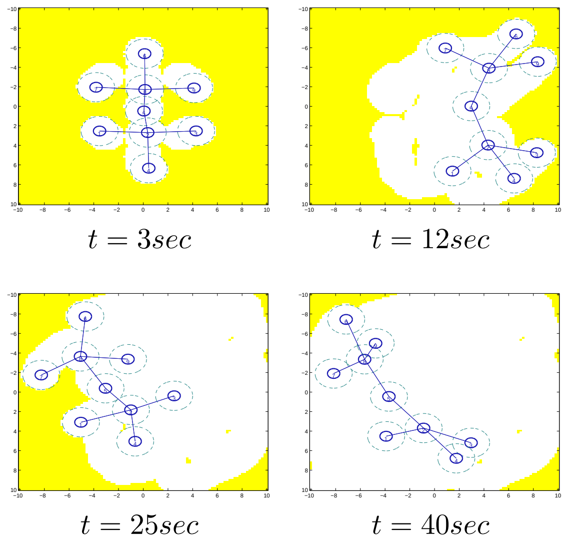

Figure 12: Coverage of the same area of figure 11 with the same number of sensors maintaining a less constraining topology

5 Conclusions

In the present paper mobile sensors networks have been addressed. Two mail problems have been here faced. The first is the computation of optimal trajectories for each mobile sensor in order to perform measurements all over a large area of interest, with a repetition of the measurement at each point within a prefixed time. The second one is the preservation of communication connections between any pair of mobile units, problem equivalent to the maintenance of graph connection.

The problem, formulated according to optimal control approaches, has been solved by space and time discretization, so yielding suboptimal solutions. In addition to a centralized approach, where a single entity elaborate all the informations and evaluate trajectories, offline, for the whole network, a decentralized solution has been proposed, where every sensor compute by itself, online, the motion according with the coverage state of the workspace and the states of its neighbors.

Simulation results have been reported to show the effectiveness of the solutions proposed and to allow a comparison between global (centralized) and local (distributed) approaches. As far as this second aspect, it is evident that global solutions give better results but with a strong computational effort and, not seen by simulations but obvious due to the hypothesis, a large amount of data transmissions. On the other hand, fast computations and few data transmissions in decentralized approach is payed with a contained reduction of performances.

Then, present and future work will deeply study the possibility of increasing the decentralized approach performances in terms of field coverage without loosing the velocity and the simplicity of computations.

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