Circuit-Oriented FEM: Solution of Circuit–Field Coupled Problems by Circuit Equations

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***Abstract*—A general circuit-oriented, full-wave, finite-element method (FEM) is proposed to analyze the coupled problem between circuits and fields both in frequency and in time domains. The electromagnetic field problem is modeled by an equivalent electrical network obtained by the Whitney finite-element equations. The presence of circuit components in the field domain is easily taken into account introducing the lumped circuit components directly in the field equivalent electrical network. Simple test configurations are analyzed by a CAD circuit simulator to show the performances of the proposed circuit-oriented method.**

***Index Terms*—Circuit–field coupled problem, circuit-oriented FEM, circuit parameter extractor, finite-element method (FEM).**

I. Introduction

Circuit-oriented techniques are very useful in the analysis of many electromagnetic (EM) problems such as the coupling of electrical circuits and EM fields, the extraction of circuit parameters from EM field configurations, the calculation of stray parameters, and the evaluation of the scattering matrix for a two-port network. This class of problems is relevant in the design of electrical/electronic devices and in electromagnetic compatibility (EMC) applications. In the past, different numerical techniques have been applied to solve field–circuits coupled problems in time domain or in frequency domain using low- and high-frequency formulations [1]–[11]. Recently, the authors of this paper presented a procedure to solve in frequency domain a circuit network equivalent to a field domain [2]. The aim of this paper is to improve the previous work extending the equivalent network approach to the analysis of field–circuit coupled problems introducing 1) lumped circuit elements into the field domain and 2) time domain analysis.

The electrical network equivalent to the field domain is derived by the edge element solution of the vector wave equation. Each edge of the finite-element method (FEM) mesh is modeled as a branch of an electrical network, which is coupled with other branches (i.e., edges) by current-controlled sources. The circuit components embedded in a field domain can be directly introduced in the equivalent electrical network, and the resulting network can be easily analyzed in both the time and frequency domains by a CAD circuit simulator.

It should be noted that the examined domain is not modeled through a unique network with lumped circuits, as in other methods [4], [7], but only the FEM final equations are modeled by equivalent circuits.

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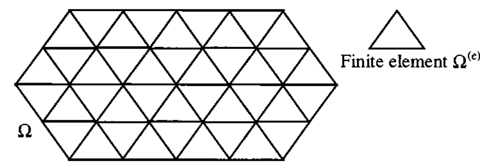


Fig. 1. FEM discretization of a field domain.

II. Mathematical Formulation

A. Whitney Element Solution of the Field Equation

Time-harmonic wave equation in terms of the vector electric field $\boldsymbol{E}$ is given by

$$\nabla \times \frac{1}{j\omega\mu} \nabla \times \boldsymbol{E} + \sigma \boldsymbol{E} + j\omega\varepsilon \boldsymbol{E} = -\boldsymbol{J} \tag{1}$$

where μ, ε, and σ are the specific constants of the medium, and $\boldsymbol{J}$ is the source current density. Equation (1) can be solved by the Galerkin method adopting Whitney elements [1], [2]. The considered field domain Ω is then discretized by homogeneous finite elements with arbitrary shape. An example of the finite-element discretization for a two-dimensional (2-D) domain is shown in Fig. 1, where a structured mesh is depicted for sake of simplicity. Using the edge element approximation for the electric field vector, the finite-element unknowns are the electric field circulations along the edges of the mesh.

The global system derived by the Whitney element solution of (1) is given by [1], [2]

$$\frac{[S_\mu]}{j\omega}[e] + j\omega[T_\varepsilon][e] + [T_\sigma][e] = -[I_S] \tag{2}$$

where $[e]$ is the global vector of the electric field circulations, and $[I_s]$ is the global source current vector. $[S_\mu]$, $[T_\varepsilon]$, and $[T_\sigma]$ are the global matrices obtained by assembling the local elemental matrices as

$$[S_\mu] = \sum_e \frac{1}{\mu^{(e)}} \left[S^{(e)}\right] \tag{3a}$$

$$[T_\varepsilon] = \sum_e \varepsilon^{(e)} \left[T^{(e)}\right] \tag{3b}$$

$$[T_\sigma] = \sum_e \sigma^{(e)} \left[T^{(e)}\right] \tag{3c}$$

where $\mu^{(e)}$, $\varepsilon^{(e)}$, and $\sigma^{(e)}$ are the specific constants of the homogeneous finite element $\Omega^{(e)}$, and the symbol Σ is referred to the

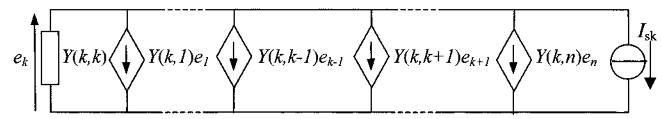


Fig. 2. Equivalent circuit of the kth edge for the frequency domain analysis of the field problem.

assembling operator. The generic coefficients of the local matrices $[S^{(e)}]$ and $[T^{(e)}]$ using Whitney one-form finite elements are given by

$$S_{ij}^{(e)} = \int_{\Omega^{(e)}} \nabla \times \mathbf{w}_i \cdot \nabla \times \mathbf{w}_j \, d\Omega \tag{4a}$$

$$T_{ij}^{(e)} = \int_{\Omega^{(e)}} \mathbf{w}_i \cdot \mathbf{w}_j \, d\Omega \tag{4b}$$

where $\mathbf{w}_i$ and $\mathbf{w}_j$ are the Whitney 1-form vectorial trial functions associated with the ith and jth edges of the finite element $\Omega^{(e)}$ [13].

The source current global vector $[I_s]$ is given by assembling the local vector $[I_s^{(e)}]$, i.e., $[I_s] = \Sigma_e [I_s^{(e)}]$. The generic coefficient of the local vector $[I_s^{(e)}]$ is given by

$$I_{si}^{(e)} = \int_{\Omega^{(e)}} \mathbf{w}_i \cdot \boldsymbol{J} \, d\Omega. \tag{5}$$

It should be noted that (5) is nonzero only when $\boldsymbol{J}$ is nonzero in the considered finite element $\Omega^{(e)}$.

B. Field Solution by Equivalent Circuits

Global equation (2) can be written in compact form as

$$[Y][e] = -[I_s] \tag{6}$$

where $[Y]$ is a square symmetric sparse matrix, and $[e]$ is the vector of the unknown electric field circulations. The kth row of system (6) is

$$Y(k,\, k)e_k + Y(k,\, 1)e_1 + \cdots + Y(k,\, n)e_n = -I_s(k) \tag{7}$$

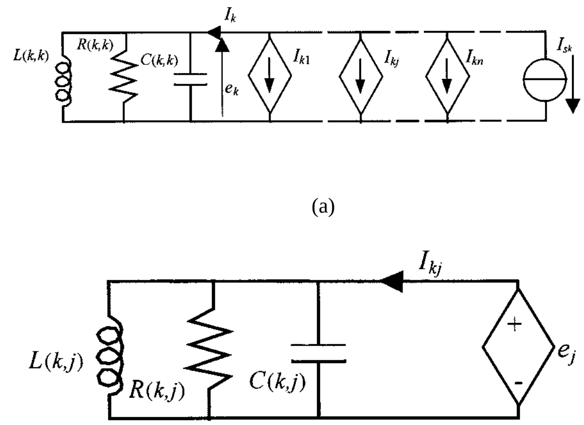
where the generic coefficient $Y(k,\, j)$ of the admittance matrix $[Y]$ is given by

$$Y(k,\, j) = \frac{S_\mu(k,\, j)}{j\omega} + T_\sigma(k,\, j) + j\omega T_\varepsilon(k,\, j). \tag{8}$$

Equation (7) was modeled in [2] by an equivalent circuit constituted by the parallel connection of the admittance $Y(k,\, k)$ and of the voltage controlled current sources $Y(k,\, j)e_j$, as shown in Fig. 2 [2]. This model presents two limitations: 1) It is valid only for a single frequency since the admittances are frequency dependent, and 2) a complex value of the control parameter $Y(k,\, j)$ for the controlled sources is not permitted in many commercial CAD circuit simulators.

In order to overcome these problems, (7) can be also written as

$$Y(k,\, k)e_k + I_{k1} + \cdots + I_{kn} = -I_s(k) \tag{9}$$



(a)

(b)

Fig. 3. Equivalent circuit of the kth edge for the time domain analysis of (a) the field problem and (b) the control circuit.

where

$$I_{kj} = Y(k,\, j)e_j. \tag{10}$$

Equation (9) can be modeled for the time domain analysis of the field problem by the equivalent circuit shown in Fig. 3, where the generic admittance $Y(k,\, j)$ is modeled by the parallel connection of resistance $R(k,\, j)$, inductance $L(k,\, j)$, and capacitance $C(k,\, j)$, given by

$$\frac{1}{L(k,\, j)} = S_\mu(k,\, j) \tag{11a}$$

$$R(k,\, j) = \frac{1}{T_\sigma(k,\, j)} \tag{11b}$$

$$C(k,\, j) = T_\varepsilon(k,\, j). \tag{11c}$$

Furthermore, the current-controlled current sources in Fig. 3(a) are controlled by the current I_{kj} of the circuit shown in Fig. 3(b), where the voltage-controlled voltage source e_j has a unit transconductance.

By the proposed model, the circuital and the control parameters of the equivalent circuit are not frequency dependent. The circuit can be analyzed in both frequency and time domains. It should be noted that the equivalent circuit is suitable to perform the frequency domain analysis for a frequency range compatible with the FEM discretization.

Finally, since the FEM global system is sparse, I_{kj} in (9) is nonzero only when the kth and the jth edges belong to the same finite element; otherwise, I_{kj} is zero.

C. Coupled Problem of Fields and Circuits

To analyze the coupling between EM fields and circuits, the lumped circuit components must be embedded in the field domain. The currents flowing into the circuit components are taken into account by modifying the FEM global matrices via the introduction of circuit elements [1]. The procedure states the following.

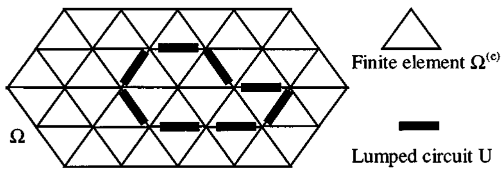


Fig. 4. Introduction of lumped circuits into a FEM discretized field domain.

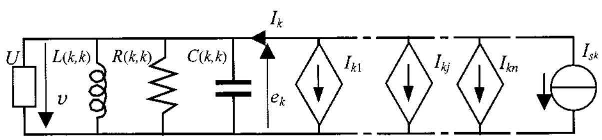


Fig. 5. Equivalent circuit of the kth edge with a parallel connection of a lumped load U for the time domain analysis of the field–circuit problem.

— The circuit element where the current flows is overlapped to a finite-element edge.
— The voltage across the circuit element coincides with the electric field circulation along the edge where the circuit element is located.

The lumped circuits are therefore placed inside the considered field domain discretized by finite elements, as shown in Fig. 4, where the lumped circuits are overlapped to the edges of the FEM mesh. Assembling the circuit elements with the finite elements, an equation system is derived where the unknowns are only the electric field circulations along the FEM edges [1].

The analysis of the field–circuit coupled problem can be easily carried out by the proposed circuit-oriented method. Let us consider a lumped circuit U whose current–voltage (i–v) relation is known. By the assumption that the voltage v across a lumped circuit element overlapped to the kth edge is related to the electric field circulation e_k as [1], [2]

$$v = -e_k \tag{11}$$

the lumped circuit element U can be simply connected in parallel with the equivalent circuit of the kth edge reported in Fig. 3(a); therefore, the final circuit shown in Fig. 5 is obtained.

III. Applications

The proposed circuit-oriented, full-wave, FEM has been applied to analyze simple field–circuit problems by a CAD circuit simulator [12].

First, the full-wave circuit-oriented method is validated by analyzing in the time domain the simple test configuration shown in Fig. 6. The field–circuit domain is given by a parallel plate configuration in free space with a load (a series connection of a diode D and a resistance $R = 1\ \Omega$) embedded in the field domain. The excitation is given by a time-harmonic voltage source $v_s(t)$ of amplitude 10 V at the frequency $f = 400$ MHz. The field–circuit configuration is matched at both ends by the characteristic impedance $R_c = 7.54\ \Omega$ of the parallel plate configuration, which is modeled by lumped circuit elements used as boundary conditions of the field domain. The considered domain was discretized by a structured mesh composed by right triangles having the short side length equal to 1 cm. The voltage

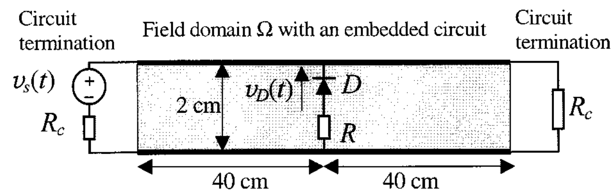


Fig. 6. Parallel plate configuration.

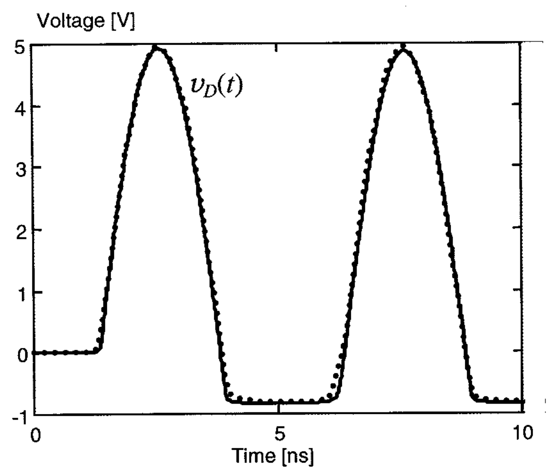


Fig. 7. Transient voltage $v_D(t)$ calculated by the proposed full wave equivalent network (solid line) and by the TL approach (dashed line).

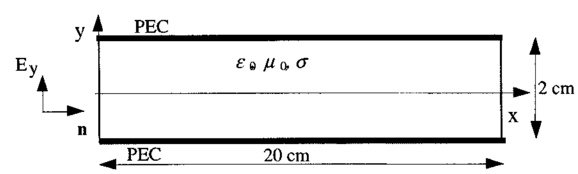
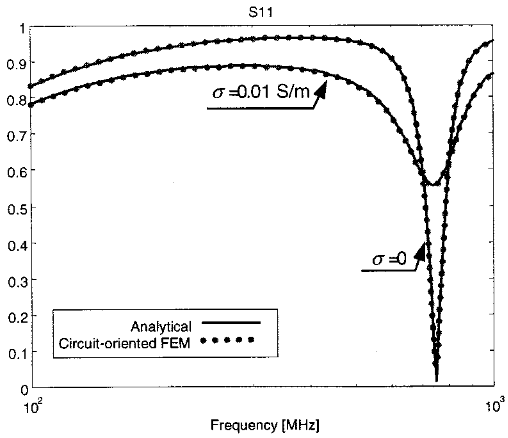


Fig. 8. Configuration of a parallel plate waveguide.

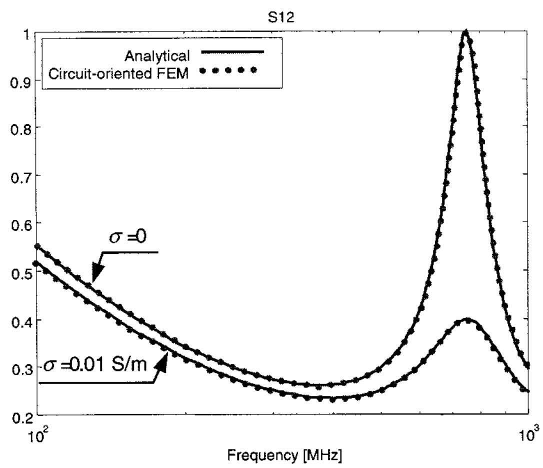
drop $v_D(t)$ along the diode D calculated by the full wave proposed method is compared with the transmission line (TL) solution obtained by a Spice simulation, as shown in Fig. 7.

Second, the proposed method is applied to extract circuit parameters from the field domain shown in Fig. 8. The problem consists in the calculation of the scattering parameters in the frequency domain for the two-port network that is equivalent to the considered parallel plate waveguide of characteristics μ_0, ε_0, and σ. By imposing unit current sources at the ports of the waveguide and calculating the voltage drop at the ports in short-circuited and open-ended conditions, it is possible to determine the scattering matrix by simple manipulations. It should be noted that the equivalent network derived by the proposed approach is valid at any frequency; therefore, the circuit simulator can be used to carry out with one run the complete frequency domain characterization of the waveguide.

In the traditional FEM, different simulations would have been necessary to compute the scattering parameters at each frequency. The scattering parameters S_{11} and S_{12} obtained



(a)



(b)

Fig. 9. Scattering parameters (a) S_{11} and (b) S_{12} of the parallel plate waveguide shown in Fig. 8.

by the proposed procedure for lossless and lossy waveguide with $\sigma = 0.01$ S/m are shown in Fig. 9. The comparison of the numerical results with the analytical solution shows a very good agreement.

IV. Conclusion

A circuit-oriented full-wave FEM has been proposed to analyze the coupled problem between circuits and fields.

The method is a significant extension of previous works since it permits easily the analysis of lumped circuits embedded in an EM field domain and the analysis in the time domain or in the frequency domain by a commercial CAD circuit simulator.

The introduction of lumped circuits in a field domain problem can be useful to study the coupling between fields and circuits to extract circuit parameters from the field domain and to model boundary conditions.

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