Full-Wave Analysis of Shielded Cable Configurations by the FDTD Method

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***Abstract*—A numerical method is proposed to model transients in a shielded cable embedded in a three-dimensional field domain by using the finite-difference time-domain (FDTD) method. The coaxial shielded cable is assumed to be a multiconductor transmission line (MTL). The in cell voltage and the current on the external shield surface are calculated by a full-wave method, while the core current and the core-to-shield voltage are analyzed by assuming the validity of the quasi-TEM propagation mode inside the shield. The internal and external shield surfaces are coupled by the transfer admittance and by the transfer impedance of the cable shield. The solution is obtained by the FDTD method combining the MTL equations with the field equations. The proposed time-domain method takes into account the frequency-dependent parameters of the cable conductors by recursive convolution techniques. The validation of the procedure is performed in simple test configurations.**

***Index Terms*—Electromagnetic compatibility/interference (EMC/EMI), finite-difference time-domain (FDTD) method, multiconductor transmission lines (MTL), shielded cables.**

I. INTRODUCTION

SHIELDED cables are widely used to connect electrical and electronic apparatus in order to reduce possible electromagnetic interference (EMI). Nevertheless, the cable shield can collect the electromagnetic disturbance produced by external fields generating EMI in the apparatus. It is therefore important for electromagnetic compatibility (EMC) studies to develop software tools able to predict induced effects in shielded cable configurations.

Traditionally, the analysis of the induced effects in a shielded cable excited by an external field is performed by the transmission line (MTL) theory, which is valid assuming the quasi-TEM propagation mode [1]–[3]. For complex configurations of the cable layout or in the high frequency range, the hypothesis of the quasi-TEM approximation is not valid and a full-wave method is required.

The finite-difference time-domain (FDTD) method is a very popular full-wave technique to analyze transient fields in a three-dimensional (3-D) domain [4]. In the last years, a big research effort has been addressed to the developments of FDTD models to analyze wire structures whose size are less than the cell dimension [5]–[7]. Here, a prediction tool is proposed to combine the MTL theory with the full-wave FDTD procedure developed by Holland [5]. This method is significantly improved to analyze the performances of a lossy shielded cable against external fields. The frequency-dependent parameters produced by the skin effect in the cable conductors, i.e., internal resistances and inductances, are modeled in the time domain by convolution integrals that are recursively solved. The proposed method is validated in simple test configurations.

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II. MATHEMATICAL MODEL

A. Field Equations

Time-varying electromagnetic fields in a 3-D domain are described by Maxwell's curl equations

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{1a}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E} + \mathbf{J}_s \tag{1b}$$

where $\mathbf{E} = \mathbf{E}(x, y, z, t)$ and $\mathbf{H} = \mathbf{H}(x, y, z, t)$ are, respectively, the electric and magnetic vector fields, $\mathbf{J}_s = \mathbf{J}_s(x, y, z, t)$ the source current density, μ the permeability, ε the permittivity and σ the conductivity of the medium. Equation (1) can be numerically solved by the FDTD method in the Yee grid, i.e., a structured grid composed by uniform cubic cells ($\Delta x = \Delta y = \Delta z = \Delta$) [4]. According to the *leap-frog* scheme the time is discretized into equal time intervals Δt, and the electric and the magnetic fields are calculated at different time instants, $t = n\Delta t$ and $t = (n + 1/2)\Delta t$, respectively, where Δt is the time step and n is the iteration number. By applying the finite-difference scheme, (1) becomes [4]

$$\mathbf{H}^{n+1/2} = \mathbf{H}^{n-1/2} - (\Delta t/\mu)\nabla \times \mathbf{E}^n \tag{2a}$$

$$\mathbf{E}^{n+1} = \frac{\varepsilon - \sigma\Delta t/2}{\varepsilon + \sigma\Delta t/2}\mathbf{E}^n + \frac{2}{\varepsilon + \sigma\Delta t/2}\left(\nabla \times \mathbf{H}^{n+1/2} - \mathbf{J}_s^{n+1/2}\right) \tag{2b}$$

where the apices $n+1/2$ and $n+1$ refer to the time instants $t = (n+1/2)\Delta t$ and $t = (n+1)\Delta t$, respectively. According to the basic FDTD notation, the rectangular components of the vector fields $\mathbf{E}$ and $\mathbf{H}$ are calculated at different points of the FDTD structured grid as shown in Fig. 1 [4]. Any field component U function of space and time is denoted as $U^n(i, j, k) = U[x = i\Delta, y = j\Delta, z = k\Delta, t = n\Delta t]$.

B. Lossy Shielded Cable Embedded in a Field Domain

The coupling of a uniform shielded cable with an external field can be studied by regarding the cable as a MTL in which only the shield is excited by the external field [2], [3]. In the

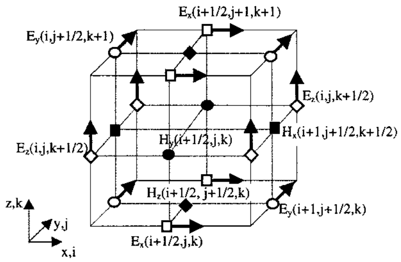

Fig. 1. Basic FDTD cubic cell of the 3-D discretization [4].

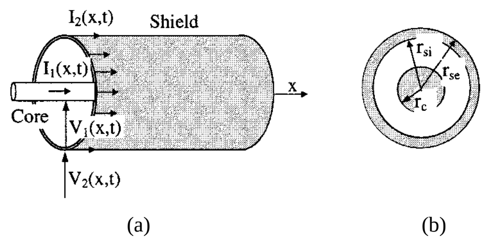

Fig. 2. Shielded cable representation of (a) a coaxial cable and (b) its cross section.

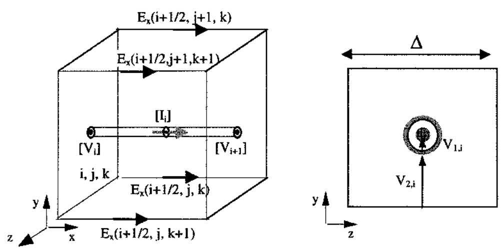

Fig. 3. FDTD cubic cell with a line section of a shielded cable running along (a) the x axis and (b) the cross section of the considered FDTD cell.

MTL approach, the cable is represented by two meshes: an internal mesh composed by the core and the internal part of the shield, and an external mesh composed by the shield and the reference plane as shown in Fig. 2. The internal and external meshes are coupled by the transfer impedance and by the transfer admittance of the cable shield [1]–[3].

In the proposed approach, the quasi-TEM mode is assumed as the dominant one in the region internal to the shield, while in the region outside the shield a full-wave approach is used. To combine the full wave and the MTL equations, the cable must be physically introduced in the FDTD field domain. The shielded cable is then discretized in sections of length Δ and any section is placed inside an electric field Yee cell as shown in Fig. 3. Inside the considered FDTD cell, a quasi-TEM approximation is assumed and the transversal dimension of the shielded cable is taken into account by modifying suitably the thin wire formalism [5].

The time-domain MTL equations describing the propagation of the current and voltage waves in the shielded cable running along the x axis inside the field domain can be written as

$$-\frac{\partial[V(x,t)]}{\partial x} = [L]\frac{\partial[I(x,t)]}{\partial t} + [\zeta(t)] * \frac{\partial[I(x,t)]}{\partial t} - [E(x,t)] \quad (3a)$$

$$-\frac{\partial[I(x,t)]}{\partial x} = [C]\frac{\partial[V(x,t)]}{\partial t} \quad (3b)$$

where $[V(x,t)]$, $[I(x,t)]$, $[E(x,t)]$ are, respectively, the voltage, current, and electric field vectors given by

$$[V]=\begin{bmatrix} V_1(x,t) \\ V_2(x,t) \end{bmatrix}, \quad [I]=\begin{bmatrix} I_1(x,t) \\ I_2(x,t) \end{bmatrix}, \quad [E]=\begin{bmatrix} 0 \\ E_{x2}(x,t) \end{bmatrix} \quad (4)$$

where $V_1(x,t)$ is the voltage of the core respect to the internal shield, $V_2(x,t)$ is the in cell voltage of the shield, $I_1(x,t)$ and $I_2(x,t)$ are the internal and external mesh currents, respectively, $E_{x2}(x,t)$ is the tangential component of the electric field on the external shield surface, $[L]$ is the 2×2 per unit length (p.u.l.) inductance matrix, $[C]$ is the 2×2 p.u.l. capacitance matrix, $[\zeta(t)]$ is the 2×2 p.u.l. transient matrix derived by the inverse Laplace transform of the frequency-dependent impedance matrix $[Z(s)]$, i.e., $[\zeta(t)] = \mathrm{L}^{-1}[Z(s)/s]$, with $s = j\omega$, and the symbol $*$ represents the convolution operator. It should be noted that $[L]$ and $[C]$ are time-constant matrices, while $[\zeta(t)]$ is a time-dependent matrix, since its coefficients are derived by the frequency-dependent parameters of the shielded cable. The p.u.l. inductance and capacitance matrices are given by [1], [2], [5]

$$[L] = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix}, \qquad [C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix} \quad (5)$$

where

$$L_{11} = (\mu_0/2\pi)\ln(r_{si}/r_c) \quad (6a)$$

$$L_{22} = \frac{\mu_0}{2\pi} \frac{\iint_{\Delta y \Delta z,\, r>r_{se}} \ln(r/r_{se})\, dy\, dz}{\iint_{\Delta y \Delta z,\, r>r_{se}} dy\, dz} \quad (6b)$$

$$C_{11} = 2\pi\varepsilon_0\varepsilon_r/\ln(r_{si}/r_c) \quad (6c)$$

$$C_{22} = \mu_0\varepsilon_0/L_{22} \quad (6d)$$

where r_{se} is the external radius of the shield, r_{si} is the internal radius of the shield, r_c is the core radius, and ε_r is the relative permittivity of the cable insulation. The mutual inductance L_{12} of the transfer impedance of the cable shield is associated to the magnetic flux leakage in air and depends on the shield topology. The mutual capacitance C_{12} of the transfer admittance has a trend analogous to L_{12}. For solid tubular shields $L_{12} = 0$ and $C_{12} = 0$, while for braided shields L_{12} and C_{12} can be derived as function of the braid characteristics [1], [2].

The modified transient impedance matrix $[\zeta(t)]$ is given by a constant part, i.e., direct current (dc) resistance, and a series of time dependent terms as

$$[\zeta(t)] = [R_{dc}] + \sum_{k=1}^{\infty}[\zeta_k(t)]. \quad (7)$$

The coefficients of $[\zeta_k(t)]$ have exponential form and are therefore suitable for the recursive solution of the convolution integrals in (3a), as described in the Appendix.

Introducing (7) in (3a), it yields

$$-\frac{\partial[V(x,t)]}{\partial x} = [L]\frac{\partial[I(x,t)]}{\partial t} + [R_{dc}][I(x,t)] + \sum_{k=1}^{\infty}[\zeta_k(t)] * \frac{\partial[I(x,t)]}{\partial t} - [E(x,t)]. \quad (8)$$

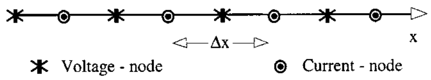

Fig. 4. One-dimensional FDTD discretization of a transmission line [2].

Applying the leap-frog scheme to (8) and (3b), the voltage $[V]$ and the current $[I]$ vectors are calculated at different grid points of the uniformly discretized line axis as shown in Fig. 4 [2]. The voltage vector $[V_i]$ is calculated at grid points $x = i\Delta x$, while the current vector $[I_i]$ is calculated at the grid points $x = (i + (1/2))\Delta x$. The updating expression for the current vector $[I_i]$ at generic ith grid point on the cable axis is given by

$$[I_i]^{n+1/2} = \left[\frac{[L]}{\Delta t} + \frac{[R_{dc}]}{2} + \sum_{k=1}^{\infty} \frac{[\zeta_k(0)]}{2}\right]^{-1} \cdot \left(\left[\frac{[L]}{\Delta t} - \frac{[R_{dc}]}{2} + \sum_{k=1}^{\infty} \frac{[\zeta_k(0)] - [\zeta_k(\Delta t)]}{2}\right] [I_i]^{n-1/2} - ([V_{i+1}]^n - [V_i]^n)/\Delta + [E_i]^n + \sum_{k=1}^{\infty} \left(\tfrac{1}{2}\,[\zeta_k(\Delta t)]\,[I_i]^{n-3/2} - [M_k]^{n-1}\right)\right) \tag{9}$$

where $\zeta_k(0) = \zeta_k(t = 0)$, $\zeta_k(\Delta t) = \zeta_k(t = \Delta t)$, $[M_k]^{n-1}$ is the matrix whose coefficients are given by the convolutions at the previous time instant, i.e., $t = (n-1)\Delta t$, as reported in the Appendix.

The voltage vector $[V_i]$ at generic ith point on the cable axis is updated by

$$[V_i]^{n+1} = [V_i]^n - \frac{\Delta t}{\Delta x}\,[C]^{-1} \left([I_i]^{n+1/2} - [I_{i-1}]^{n-1/2}\right). \tag{10}$$

C. FDTD Combined Solution

By the FDTD method with the leap-frog scheme, the explicit solution of the field equations (2a) and (2b) can be combined with the explicit solution of the MTL telegraphers' equations (9) and (10). Obviously appropriate boundary conditions and load terminal conditions must be respectively applied to the field and MTL equations. The magnetic field $\mathbf{H}$ and the cable current vector $[I]$ are explicitly updated at time $t = (n+1/2)\Delta t$, while the electric field $\mathbf{E}$ and the cable voltage vector $[V]$ are explicitly updated at time $t = (n+1)\Delta t$. The main steps of the iterative procedure can be summarized as follows.

Time $(n + 1/2)\Delta t$*:*

a) update $\mathbf{H}^{n+1/2}$ by standard FDTD expressions derived by (2a) in the 3-D field domain [4];
b) evaluate the tangential electric field vector $[E_i]^n$ on the cable shield surface at any ith point of the cable axis by interpolating the field values $\mathbf{E}^n$ obtained by (2b) [5];
c) update $[\mathbf{I}_i]^{n+1/2}$ by (9);
d) evaluate the source current density $\mathbf{J}_s^{n+1/2}$ (only in the FDTD cubic cells with embedded cable sections) by partitioning the current values $[I_i]^{n+1/2}$ obtained by (9) under the assumption that the current density in the cell is parallel to the cable axis ($\mathbf{J}_s = J_x\mathbf{x}$) [5].

Time $(n + 1)\Delta t$*:*

e) update $\mathbf{E}^{n+1}$ by standard FDTD expressions derived by (2b) [4];

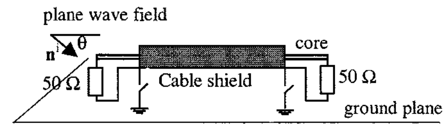

Fig. 5. Configuration of the field-excited shielded cable.

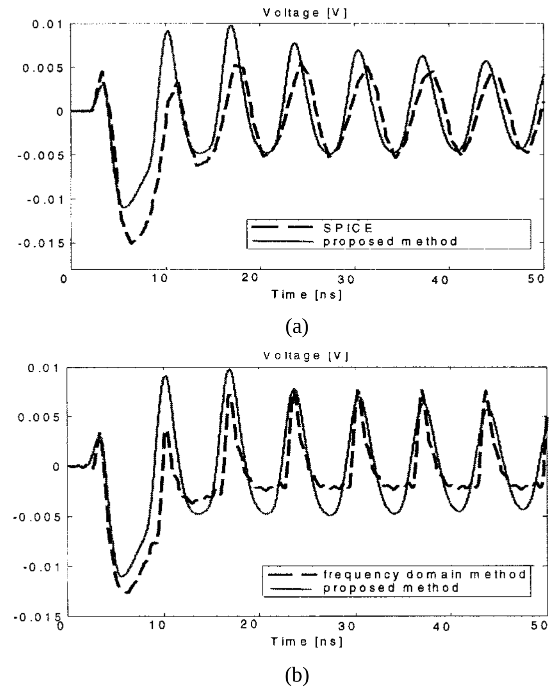

Fig. 6. External shield-to-ground voltages calculated at point $x = 0$; comparison with (a) frequency domain solution and (b) SPICE results.

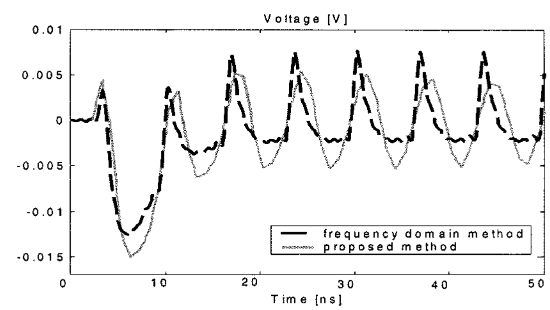

Fig. 7. Internal shield-to-core voltage calculated at point $x = 0$.

f) update $[V_i]^{n+1}$ by (10).

It should be noted that the coupling of the field and MTL equations are given by points b) and d).

III. Applications

To validate the proposed method a simple configuration has been considered as shown in Fig. 5: a uniform TL excited by a plane wave field has been examined since for this configuration analytical results are available in frequency and time domains assuming the validity of the quasi-TEM propagation mode [2], [3], [9]. The geometrical configuration of the braided shielded cable above a PEC ground plane is: length $l = 1$ m, height $h = 0.15$ m, $r_c = 0.451$ mm, $r_{si} = 1.397$ mm, $r_{se} = 1.524$ mm, strand diameter of the braid $d = 0.127$ mm, number of strands per belt $W = 9$, number of belts $B = 12$, weave angle $\phi = 27.7°$ [1]–[3]. The copper inner core is matched to the shield by a 50 Ω resistance at both ends, the shield is opened at both ends to avoid the influence of the wire ground connection, which should be physically introduced in the full-wave method producing reflection and reirradiation. The shielded cable is excited by a vertically polarized plane wave field with double exponential waveform [$E^i = E_0(e^{-\alpha t} - e^{-\beta t})$ with $E_0 = 1$ V/m, $\alpha = 4 \cdot 10^8$ s^{-1}, $\beta = 5 \cdot 10^8$ s^{-1}] impinging on the line axis with incidence elevation angle $\theta = 30°$ and azimuthal angle $\psi = 0$. The proposed full-wave FDTD analysis is carried out in a computational domain of dimensions 3×2×2 m assuming $\Delta = 2.5$ cm and $\Delta t = \Delta/(2c)$, where c is the free space velocity.

The external shield-to-ground voltage at point $x = 0$ computed by the proposed method is reported in Fig. 6 and compared with the exact solution obtained by the frequency domain method via the inverse Laplace transform [3] and with the exact SPICE time-domain solution for lossless TLs [9]. The internal shield-to-core voltage calculated at point $x = 0$ is shown in Fig. 7. The comparisons show the accuracy of the proposed method, which is also very efficient since it presents an explicit solution scheme.

IV. Conclusion

A full-wave FDTD model has been proposed to analyze cable configurations embedded in a field domain. The method proposed in [5] has been significantly improved to study lossy MTLs excited by external fields. The proposed method is useful to analyze nonuniform shielded cable configurations taking into account the skin effect in the cable conductors and shield. The method has been validated by comparison with other techniques and has been revealed to be efficient. The extention of the proposed method to multiconductor shielded cables is straightforward.

Appendix

The coefficients of the symmetric 2×2 matrix $[R_{dc}]$ in (7) are

$$\begin{aligned} R_{11,\,dc} &= R_{c,\,dc} + R_{si,\,dc} \\ R_{12,\,dc} &= K_{12}/(2\pi r_{si}\sigma_s(r_{se} - r_{si})) \\ R_{22,\,dc} &= K_{22}/(2\pi r_{se}\sigma_s(r_{se} - r_{si})) \\ R_{c,\,dc} &= 1/(\pi r_c^2 \sigma_c) \\ R_{si,\,dc} &= K_{11}/(2\pi r_{si}\sigma_s(r_{se} - r_{si})) \end{aligned}$$

where σ_c and σ_s are, respectively, the conductivity of the core and of the shield, and K_{11}, K_{22} and K_{12} are some constants depending on the nonmagnetic cable shield characteristics (i.e., tubular, braided, ...) [1]–[3].

The coefficients of the symmetric 2×2 matrix $[\zeta_k(t)]$ in (7) are

$$\begin{aligned} \zeta_{11,\,k}(t) &= \zeta_{c,\,k}(t) + \zeta_{si,\,k}(t) \\ \zeta_{12,\,k}(t) &= 2R_{12,\,dc}(-1)^k e^{-\alpha_{sk} t} \\ \zeta_{22,\,k}(t) &= 2R_{22,\,dc} e^{-\alpha_{sk} t} \\ \zeta_{c,\,k}(t) &= 2R_{c,\,dc} e^{-\alpha_{ck} t} \\ \zeta_{si,\,k}(t) &= 2R_{si,\,dc} e^{-\alpha_{sk} t} \end{aligned}$$

with $\alpha_{ck} = k^2\pi^2/(\mu_0\sigma_c r_c^2)$, $\alpha_{sk} = k^2\pi^2/(\mu_s\sigma_s(r_{se} - r_{si})^2)$.

Since the coefficients $[\zeta_k(t)]$ have exponential behavior, the convolution in (8) can be recursively evaluated by

$$\begin{aligned} [M_k]^n &= [\zeta_k(t)] * \frac{\partial[I(x,\,t)]}{\partial t} \\ &= [M_k]^{n-1} + \int_{t-\Delta t}^{t} [\zeta_k(t-\tau)] \frac{\partial[I(x,\,\tau)]}{\partial \tau}\, d\tau \end{aligned}$$

with

$$[M_k]^{n-1} = \begin{bmatrix} e^{-\alpha_{ck}\Delta t} M_{c,\,k}^{n-1} + e^{-\alpha_{sk}\Delta t} M_{si,\,k}^{n-1} & e^{-\alpha_{sk}\Delta t} M_{12,\,k}^{n-1} \\ e^{-\alpha_{sk}\Delta t} M_{12,\,k}^{n-1} & e^{-\alpha_{sk}\Delta t} M_{22,\,k}^{n-1} \end{bmatrix}$$

with $M_{c,\,k}^{n-1}$, $M_{si,\,k}^{n-1}$, $M_{12,\,k}^{n-1}$and $M_{22,\,k}^{n-1}$ the convolutions of the exponential terms $\zeta_{c,\,k}(t)$, $\zeta_{si,\,k}(t)$, $\zeta_{12,\,k}(t)$ and $\zeta_{22,\,k}(t)$, respectively, computed at time $t = (n-1)\Delta t$.

References

[1] E. F. Vance, *Coupling to Shielded Cables*. New York: Wiley, 1978.

[2] S. Celozzi and M. Feliziani, "FDTD analysis of the interaction between a transient EM field and a lossy shielded cable," in *Proc. 10th Int. Zurich Symp. EMC*, Zurich, Switzerland, Mar. 9–11, 1993.

[3] M. D'Amore and M. Feliziani, "EMP coupling to coaxial shielded cables," in *Proc. IEEE Int. Symp. EMC*, Seattle, WA, Aug. 2–4, 1988.

[4] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. 14, pp. 302–307, May 1966.

[5] R. Holland and L. Simpson, "Finite-difference analysis of EMP coupling to thin struts and wires," *IEEE Trans. Electromag. Compat.*, vol. 23, pp. 88–97, May 1981.

[6] K. R. Umashankar, A. Taflove, and B. Becker, "Calculation and experimental validation of induced currents on coupled wires in an arbitrary shaped cavity," *IEEE Trans. Antennas Propagat.*, vol. 35, pp. 1248–1257, Nov. 1987.

[7] M. Feliziani and F. Maradei, "Field-to-wire coupling using the finite element-time domain (FE-TD) method," *IEEE Trans. Magn.*, vol. 31, pp. 1586–1589, May 1995.

[8] J. O. Bérenger, "A multiwire formalism for the FDTD method," *IEEE Trans. Electromag. Compat.*, vol. 42, pp. 257–264, Aug. 2000.

[9] S. Celozzi and M. Feliziani, "Time-domain solution of lossless field-excited transmission lines equations," *IEEE Trans. Electromag. Compat.*, vol. 37, pp. 421–432, Aug. 1995.