Edge-Elements Modeling of Transmission Lines in Field Domain by Impedance Network Boundary Conditions

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***Abstract*—Impedance network boundary conditions (INBCs) are applied to the ports of a transmission line (TL) to simplify the finite-element-method modeling of a TL embedded in a field domain. The INBC-TL model is implemented in an edge-elements procedure to solve both time-harmonic and transient electromagnetic fields.**

***Index Terms*—Edge elements, finite-element method (FEM), impedance network boundary conditions (INBCs), transmission lines (TLs).**

I. Introduction

TRANSMISSION lines (TLs) are widely used for connection links between electronics. Traditionally, to analyze the electromagnetic (EM) field in a domain with a TL connection by a full-wave method based on partial differential equations (PDE), the physical domain of the TL must be discretized. To reduce the computational domain, a technique is here proposed to model the TL connection by an equivalent two-port network, that can be implemented in differential numerical techniques by impedance network boundary conditions (INBCs). In the recent past, the INBCs, which are coupled boundary conditions, have been introduced by the authors to model the field penetration through shielding panels [1]–[3]. In the present application, the TL domain can be eliminated from the computational domain and substituted by the INBCs applied to the ports of the eliminated TL. The application of the TL-INBC method to an edge elements procedure is presented in the following to model lossless, lossy, and field-excited TLs in frequency and time domains.

II. Mathematical Model

Let us consider two separate field domains ($\Omega_1,\ \Omega_2$) with two ports ($\Gamma_i,\ \Gamma_o$) connected by a transmission line (TL), as shown schematically in Fig. 1. The field domain is given by $\Omega=\Omega_1+\Omega_{\mathrm{TL}}+\Omega_2$, where Ω_{TL} is the TL domain region. The EM field in Ω, neglecting the source terms, can be described in frequency domain by the vector wave equation in terms of the electric field $\boldsymbol{E}$

$$\nabla\times\frac{1}{j\omega\mu}\nabla\times\boldsymbol{E}+j\omega\varepsilon\boldsymbol{E}+\sigma\boldsymbol{E}=0 \tag{1}$$

where μ, ε, and σ are the specific constants of the medium, and ω is the angular frequency.

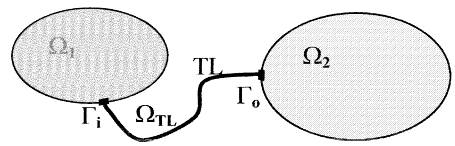

Fig. 1. System configuration.

A. Traditional Finite-Element Method (FEM) Solution by Whitney Elements

The Galerkin form of the vector wave equation (1) is given by

$$\int_\Omega\frac{1}{j\omega\mu}(\nabla\times\boldsymbol{u})\cdot(\nabla\times\boldsymbol{E})\,d\Omega+\int_\Omega(\sigma+j\omega\varepsilon)\boldsymbol{u}\cdot\boldsymbol{E}\,d\Omega+\oint_\Gamma\frac{1}{j\omega\mu}\,\widehat{\mathbf{n}}\times\nabla\times\boldsymbol{E}\cdot\boldsymbol{u}\,d\Gamma=0 \tag{2}$$

where $\boldsymbol{u}$ is the vector weighting function, and Γ is the boundary of Ω.

Adopting the *edge-elements* approximation, the electric field $\boldsymbol{E}$ in each finite element is locally given by

$$\boldsymbol{E}(\boldsymbol{r})\cong\sum_{k=1}^{N}\boldsymbol{w}_k(\boldsymbol{r})e_k \tag{3}$$

where $\boldsymbol{r}$ is the position vector, N is the number of the element edges, $\boldsymbol{w}_k$ is the vector trial function associated to the kth edge [4], and e_k is the circulation of the electric field along the kth edge of length ℓ_k, which can be given by

$$e_k=\int_{\ell_k}\boldsymbol{E}\cdot\vec{d\ell}. \tag{4}$$

Applying the FEM approximation (3) to (2), and assuming $\boldsymbol{u}=\boldsymbol{w}_k$ with $k=1,\,2,\,\ldots,\,N$, a linear equation final system is derived [4].

The FEM discretization of the region Ω_{TL} would lead to a great number of finite elements, and therefore to a nonefficient solution, especially if the skin effect in the TL conductors must

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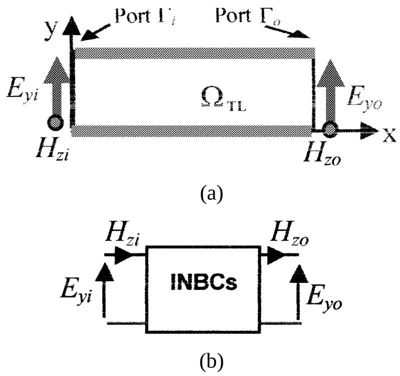

Fig. 2. (a) TL configuration. (b) Two-port network representation.

be taken into account. In the following, an original method is proposed to overcome this inconvenience.

B. Transmission Line Modeling by INBCs

Assuming for simplicity a simple TL, i.e., a parallel plate waveguide along the x axis as shown in Fig. 2(a), the field propagation in the TL characterized by a TEM (or quasi-TEM) mode is described by

$$-\frac{dE_y}{dx} = j\omega\mu H_z \tag{5a}$$

$$-\frac{dH_z}{dx} = (j\omega\varepsilon + \sigma)E_y \tag{5b}$$

where E_y and H_z are assumed to be the transverse electric and magnetic fields. The fields at the TL input port Γ_i in $x = 0$ (E_{yi}, H_{zi}) and at the output port Γ_o in $x = \ell$ (E_{yo}, H_{zo}) are derived from (5) as

$$\begin{bmatrix} H_{zi} \\ H_{zo} \end{bmatrix} = \begin{bmatrix} Y_0 & -Y_m \\ Y_m & -Y_0 \end{bmatrix} \begin{bmatrix} E_{yi} \\ E_{yo} \end{bmatrix} \tag{6}$$

where Y_0 and Y_m are the coefficients of the TL admittance matrix obtained by (5).

The proposed method consists in the imposition of the input–output (I/O) relations (6) on Γ_i and Γ_o. Equation (6) can be then introduced in the boundary integral of (2) by substituting

$$\frac{\hat{\mathbf{n}} \times \nabla \times \boldsymbol{E}}{j\omega\mu} = [Y_0 E_{yi} - Y_m E_{yo}]\hat{\mathbf{t}} \qquad \text{on } \Gamma_i \tag{7a}$$

$$\frac{\hat{\mathbf{n}} \times \nabla \times \boldsymbol{E}}{j\omega\mu} = [Y_m E_{yi} - Y_0 E_{yo}]\hat{\mathbf{t}} \qquad \text{on } \Gamma_o \tag{7b}$$

where $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$ are the unit vectors normal and tangential to the boundary Γ. Equation (7) represents the application of the INBCs for TL modeling.

Considering generic TL configurations, the field propagation can be described in terms of voltage and currents. The voltage V between two points (P_1, P_2) in a transverse plane is given by the line integral of the transverse electric field $\boldsymbol{E}_t$ between those two points

$$V = -\int_{P_2}^{P_1} \boldsymbol{E}_t \cdot d\vec{\ell}. \tag{8}$$

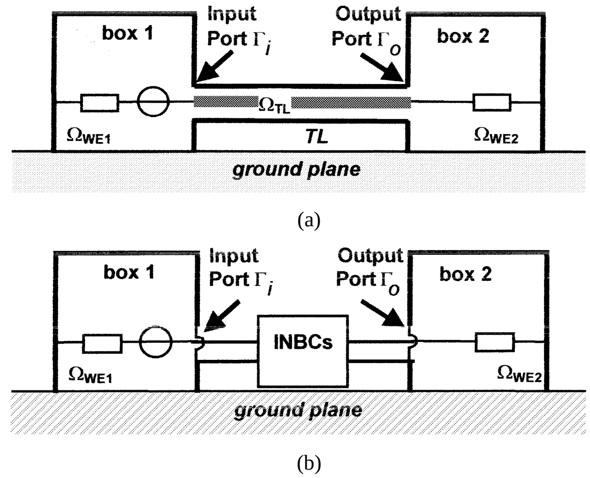

Fig. 3. (a) Initial field domain. (b) The reduced domain after the INBC application.

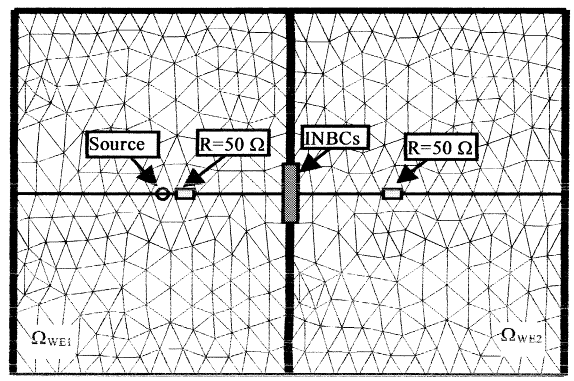

Fig. 4. Finite-element mesh of the computational domain after the elimination of Ω_{TL} of the field-to-circuit coupled problem with INBCs.

The current I is given by the line integral of the transverse magnetic field $\boldsymbol{H}_t$ around any closed contour lying solely in the transverse plane

$$I = \oint_{\ell} \boldsymbol{H}_t \cdot d\vec{\ell}. \tag{9}$$

The I/O relations of a TL in terms of voltage and currents are

$$\begin{bmatrix} I_i \\ I_o \end{bmatrix} = \begin{bmatrix} Y_0 & -Y_m \\ Y_m & -Y_0 \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix}. \tag{10}$$

By (7)–(9), it is possible to substitute a TL domain Ω_{TL}, which is eliminated from the FEM computational domain, by a two-port network as shown schematically in Fig. 3.

The proposed procedure is suitable to accurately model the propagation along the TL and permits a significant reduction of the computational domain where the EM field is numerically solved by the FEM. Also, the generation of the FEM mesh is facilitated as shown in Fig. 4.

The lumped elements embedded in the FEM computational domain and representing the TL terminations are modeled according to [5] and [6]. Finally, it should be noted that the full-wave simulation of the TL terminations permits to take into account the parasitic elements, i.e., stray capacitances and inductances.

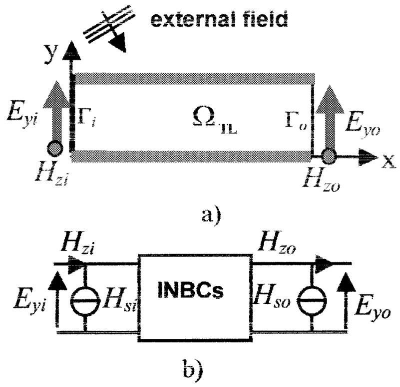

Fig. 5. (a) Field-excited TL. (b) Active two-port network representation in terms of field quantities.

In the following, the two-port network equations in terms of input and output voltages and currents (10) are reported and then used in the application tests.

1) Lossless Transmission Lines: The two-port network equations for a lossless TL are

$$\begin{bmatrix} I_i \\ I_o \end{bmatrix} = \begin{bmatrix} \dfrac{\cos(\beta\ell)}{jR_c\sin(\beta\ell)} & -\dfrac{1}{jR_c\sin(\beta\ell)} \\ \dfrac{1}{jR_c\sin(\beta\ell)_c} & -\dfrac{\cos(\beta\ell)}{jR_c\sin(\beta\ell)} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} \tag{11}$$

where $R_c = (L/C)^{1/2}$ is the characteristic resistance, $\beta = \omega(LC)^{1/2}$ is the phase constant, ℓ is the length of the TL, and L and C are the per-unit-length (p.u.l.) series-inductance and shunt-capacitance of the TL.

2) Lossy Transmission Lines: The two-port network equations for a lossy TL are

$$\begin{bmatrix} I_i \\ I_o \end{bmatrix} = \begin{bmatrix} \dfrac{\cosh(\gamma\ell)}{Z_c\sinh(\gamma\ell)} & -\dfrac{1}{Z_c\sinh(\gamma\ell)} \\ \dfrac{1}{Z_c\sinh(\gamma\ell)} & -\dfrac{\cosh(\gamma\ell)}{Z_c\sinh(\gamma\ell)} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} \tag{12}$$

where $Z_c = [Z/Y]^{1/2}$ is the characteristic impedance, $\gamma = [ZY]^{1/2}$ is the propagation constant, and Z and Y are the p.u.l. series-impedence and shunt-admittance of the TL.

3) Lossy Transmission Lines With Distributed Sources: When the TL is illuminated by an external EM field, distributed voltage and current sources appear in the TL propagation equations [7]. In this case, the active two port network model shown in Fig. 5 can be derived for the field-excited lossy TL

$$\begin{bmatrix} I_i \\ I_o \end{bmatrix} = \begin{bmatrix} \dfrac{\cosh(\gamma\ell)}{Z_c\sinh(\gamma\ell)} & -\dfrac{1}{Z_c\sinh(\gamma\ell)} \\ \dfrac{1}{Z_c\sinh(\gamma\ell)} & -\dfrac{\cosh(\gamma\ell)}{Z_c\sinh(\gamma\ell)} \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} + \begin{bmatrix} I_{si} \\ I_{so} \end{bmatrix} \tag{13}$$

where

$$\begin{bmatrix} I_{si} \\ I_{so} \end{bmatrix} = \begin{bmatrix} -\dfrac{E_s}{Z_c\sinh(\gamma\ell)} \\ J_s + \dfrac{\cosh(\gamma\ell)}{Z_c\sinh(\gamma\ell)}\,E_s \end{bmatrix} \tag{14}$$

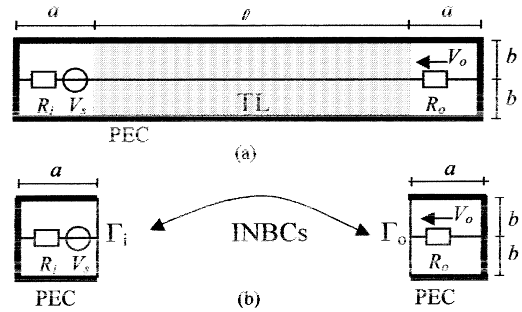

Fig. 6. (a) Computational domain in a traditional FEM application (b) In the proposed FEM-INBC procedure ($\lambda = 1$ m, $a = 10$ cm, $b = 1$ cm, $R_i = R_o = 50\ \Omega$).

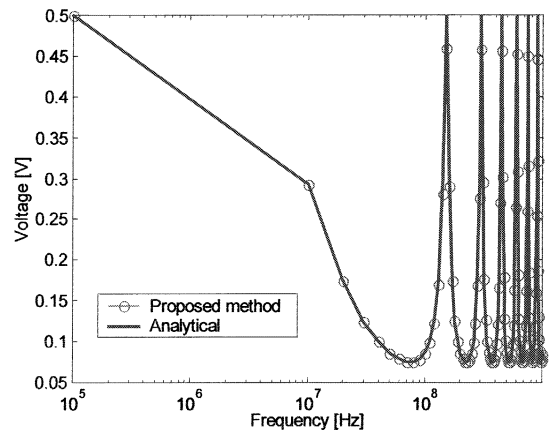

Fig. 7. Voltage V_o versus frequency considering a lossless TL. ($R_i = R_o = 50\ \Omega$, $V_s = 1$ V).

and

$$E_s = \int_0^\ell \cosh\left[\gamma\left(\ell - x\right)\right] V_s(x)\,dx - \int_0^\ell Z_c \sinh\left[\gamma\left(\ell - x\right)\right] I_s(x)\,dx \tag{15a}$$

$$J_s = \int_0^\ell \sinh\left[\gamma\left(\ell - x\right)\right] V_s(x)/Z_c\,dx - \int_0^\ell \cosh\left[\gamma\left(\ell - x\right)\right] I_s(x)\,dx \tag{15b}$$

where $V(s)$ e $I(s)$ are distributed voltage and current sources given by the coupling with the external field [7]. The active INBCs (13) are easily implemented in the FEM formulation as third kind boundary conditions [4].

4) Lossless Transmission Lines in Time Domain: Equation system (6) for a lossy transmission line in time domain is given by

$$\begin{bmatrix} I_i(t) \\ I_o(t) \end{bmatrix} = \begin{bmatrix} Y_0(t) & -Y_m(t) \\ Y_m(t) & -Y_0(t) \end{bmatrix} * \begin{bmatrix} V_i(t) \\ V_o(t) \end{bmatrix} \tag{16}$$

where the transient admittances $Y_0(t)$ and $Y_m(t)$ are given by

$$Y_0(t) = L^{-1}\left[Y_0(s)\right], \qquad Y_m(t) = \left[Y_m(s)\right] \tag{17}$$

being L^{-1} the inverse Laplace transform and s the Laplace variable. Equation (17) can be analytically obtained as series of ex-

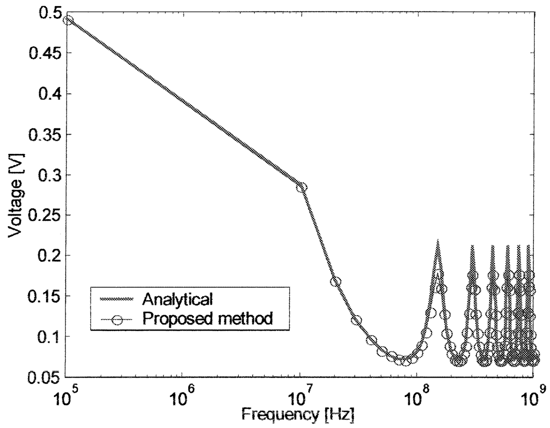

Fig. 8. Voltage V_o versus frequency considering a lossy TL with a p.u.l. series resistance of 1 Ω/m ($R_i = R_o = 50\ \Omega, V_s = 1$ V).

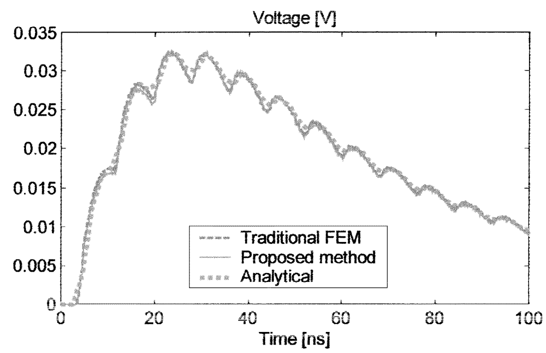

Fig. 9. Voltage V_o versus time considering a lossless TL. [$R_i = R_o = 50\ \Omega$, $V_s = \exp(-10^8 t) - \exp(-2.10^8 t)$ V].

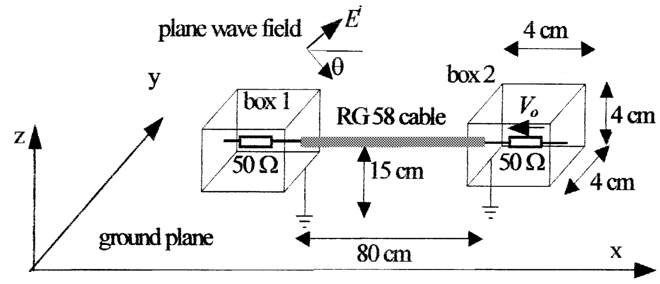

Fig. 10. Configuration of a field excited RG58 coaxial cable connecting two boxes.

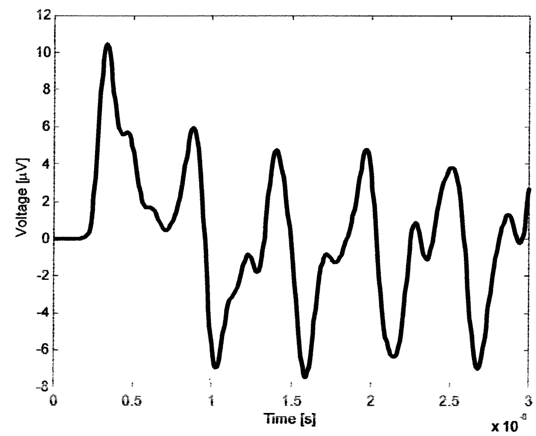

Fig. 11. Voltage V_o versus time considering a field-excited RG58 coaxial cable [$E^i = \exp(-10^8 t) - \exp(-5.10^8 t)$ V/m, $\theta = 30°$].

ponential terms and an efficient recursive convolution technique can be applied [2].

III. Applications

To test the validity of the FEM-INBC method to model TLs, a simple two-dimensional (2-D) configuration has been examined as shown in Fig. 6. The circuit elements embedded in the field domain have been taken into account by modifying adequately the field equations as described in [2]. The results obtained by the proposed method have been compared with those obtained by the FEM discretization of the TL domain and with the analytical solutions as shown, respectively, in Figs. 7–9 for a lossless TL, a lossy TL, and a lossless TL in time domain.

Finally, the configuration shown in Fig. 10 is analyzed where the two identical boxes with circuits inside are connected by a RG58 coaxial cable excited by a plane wave EM field. The numerical solution has been performed by the two steps procedure described in [9]. The transient behavior of the voltage V_o obtained considering the field-excited RG58 coaxial cable is shown in Fig. 11.

IV. Conclusion

An efficient procedure has been presented to analyze field domains connected only by TLs.

Lossless TLs, lossy TLs, field-excited TLs have been modeled by INBCs in frequency and time domains.

The proposed model has been implemented in a Whitney elements procedure but it can be easily applied to other numerical techniques.

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