A Cross-Check of the Reflectance Models to Be Used in Interpretation of Observations of Regolith-Like Surfaces

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Many current and proposed programs of satellite remote sensing of the Earth and other celestial bodies rely upon measurements of the intensity and polarization of light scattered by these bodies. These measurement data are interpreted by searching for the best fits to light-scattering characteristics precalculated with some theoretical models. For regolith-like surfaces, i.e., discrete densely packed random media, the light-scattering models are still under development and they work under different approaches. Here, to estimate the difference between the reflectance characteristics yielded by these procedures, we compare the results of simulations performed according to five frequently used approximate models of a semi-infinite particulate medium. Special attention is paid to taking into account the weak-localization effect. The models differ by the scattering matrixes of a volume element and the dependence of the imaginary part of the effective refractive index on the filling factor. The volume element is an individual spherical particle or a randomly oriented cluster of particles. The cases of modifying the scattering matrix by the static structure factor correction or by subtracting the contribution of the mean field are also considered. The values for the size parameter of particles or monomers in the clusters and the refractive index were assumed at 1.76 and $1.50 + i0.0001$, respectively; and two values for the filling factor (defined as a volume fraction occupied by particles in the medium), 20 and 10%, were considered. Our analysis shows that the angular dependences of the intensity and the linear polarization degree obtained with the considered models are rather close to each other. Moreover, they agree with the corresponding characteristics for a large cloud of particles (N is equal to or exceeds 10^6) with the filling factor up to 20%, which were obtained by approximate methods but well follow the trends found in rigorous simulations for smaller ensembles of particles (Penttilä et al., J. Quant. Spectrosc. Radiat. Transfer, 2021, 262, 107524). Hence, these approximate models are equally acceptable to the interpretation of the results of observations.

Keywords: light scattering, particulate random media, radiative transfer, opposition effects, weak localization

1 INTRODUCTION

To gain insight into the nature, origin, and evolution of various bodies in the Solar System, including the Earth, photometric and polarimetric data obtained in space-borne and ground-based observations are widely used. From the perspective of the present analysis of the light-scattering properties of regolith-like surfaces, it is worth mentioning, for example, the *Cassini–Huygens* space mission, which acquired a huge data amount while orbiting Saturn and studying the planet and its system for 13 years. To continue research of satellites of the giant planets, the Jupiter Icy Moons Explorer mission (JUICE) is planned to be implemented in the nearest future. However, for many celestial bodies, the surfaces of which are covered with a regolith—a powder-like material composed of grains of different sizes and packing density—the interpretation of the results of observations in terms of the sizes, refractive index, and packing density of regolith particles requires the use of an adequate model to determine the characteristics of electromagnetic radiation reflected from a densely packed discrete random medium.

To calculate the light scattering characteristics of a densely packed discrete random medium is an extremely complex problem that cannot currently be solved at the theoretical level. The difficulty in the theoretical consideration of this problem is mainly connected with peculiarities of the light scattering in the near field (Mishchenko et al., 2011; Tishkovets et al., 2013). The matter is that, as distinct from sparse media, densely packed media exhibit the effects, which cannot be described only by contributions of the ladder and cyclic diagrams to the scattered radiation, since these effects are caused by the interference of inhomogeneous waves of different scattering orders. The interference of this kind manifests itself in the mutual shadowing and may considerably influence the opposition effects (Tishkovets et al., 2013). While the description of the light scattering by densely packed media in terms of the ladder and cyclic diagrams has been considerably improved in recent years [see (Tishkovets et al., 2011; Doicu and Mishchenko, 2019b; Doicu and Mishchenko, 2019c) and references therein], the contribution of the diagrams corresponding to the interference of waves of different scattering orders has not been taken into account yet.

At the same time, as has been noted above, there is a pressing need for models that correctly describe the light scattering characteristics of densely packed media. Because of this, attempts are being made to develop such models on the base of the well-elaborated theory of light scattering by sparse media [see (Mishchenko, 1994; Barrowes et al., 2000; Tishkovets and Mishchenko, 2004; Mishchenko et al., 2006; Mishchenko et al., 2013; Tishkovets and Petrova, 2013; Muinonen et al., 2017; Ito et al., 2018; Muinonen et al., 2018; Tishkovets and Petrova, 2020; Penttilä et al., 2021) and references therein].

For sparse media, it is assumed that the waves propagating between scatterers in the medium are spherical, which essentially simplifies the theoretical analysis. Moreover, in this case, it is relatively easy to obtain the ensemble-averaged reflection matrix analytically for a plane-parallel medium (Tishkovets and Mishchenko, 2004; Mishchenko et al., 2006; Tishkovets et al.,

2011; Tishkovets and Petrova, 2013; Doicu and Mishchenko, 2019b; Doicu and Mishchenko, 2019c; Tishkovets and Petrova, 2020) or numerically with the Monte-Carlo technique for a spherical volume (Muinonen et al., 2017; Muinonen et al., 2018; Penttilä et al., 2021). To adapt these models to the problems of light scattering by densely packed media, the scattering characteristics of "a volume element" are modified. Specifically, it is suggested that the mean (coherent) field should be excluded [see (Barrowes et al., 2000; Muinonen et al., 2017; Muinonen et al., 2018; Penttilä et al., 2021) and the references therein], or the static structure factor correction should be made (Mishchenko, 1994; Mishchenko et al., 2013; Ito et al., 2018), or the scattering characteristics of randomly oriented clusters of particles should be considered as those of a volume element (Tishkovets and Petrova, 2013; Ito et al., 2018).

Unfortunately, the limits of applicability of the above specified models are unknown, since the data on the light-scattering characteristics measured in laboratory for the samples with thoroughly controlled parameters are still very rare (Mishchenko et al., 2013). Particularly, in the paper by Mishchenko et al. (2013), the solution of the vector radiative transfer equation was verified by the well-controlled experiment with a medium of monodisperse spherical particles suspended in water. It was found that, for the filling factor values below 0.1, all of the reflection matrix elements calculated with the static structure correction of a volume element very neatly fit the laboratory data. Some differences are observed only for the element R_{44}.

Given the laboratory data limitation, the quality of the model may be estimated with the methods that allow the scattering characteristics to be rigorously calculated for some cases. Such an attempt was recently made by Penttilä et al. (2021). They considered clusters composed of 10^3 to 10^5 spherical particles and compared their light-scattering characteristics obtained with the rigorous numerical method and with the model, according to which the mean field contribution was excluded (Muinonen et al., 2017; Muinonen et al., 2018; Penttilä et al., 2021). The calculations were performed for the backscattering domain, where the weak-localization effect reveals itself. For this relatively small clusters, the results of the rigorous simulations and the model approximation well agreed. For the clusters containing much more particles (from 10^6 to 10^9), the light-scattering characteristics were calculated within the frames of the assumed model approximation. It was found that, with growing the number of particles in a cluster, the intensity and the polarization of scattered light asymptotically tend to some limit that seems to be achieved when the number of particles in a cluster is 10^9. In the opinion of Penttilä et al. (2021), the backward-scattered properties of this cluster represent the properties of a macroscopic, almost infinite system. Because of this, it would be interesting to compare the results reported by Penttilä et al. (2021) to the intensity and the linear polarization degree of light reflected by a semi-infinite discrete medium, the calculation technique for which is well-developed and does not require large computational resources (Tishkovets and Petrova, 2020). This comparison would allow us to estimate the applicability of the model (Tishkovets and Petrova, 2020) to

densely packed media, the parameters of which are at least close to those used by Penttilä et al. (2021).

The purpose of this paper is to determine whether the differences between the reflectance characteristics yielded by different approximate procedures are substantial. For this, we compare the light-scattering characteristics in the back-scattering domain calculated by the models of a semi-infinite discrete medium with different versions of a volume element to each other and to the results reported by Penttilä et al. (2021). The parameters of all of the models considered below are assumed to be the same as those considered by Penttilä et al. (2021): the medium are composed of monodisperse spherical particles with the size parameter $x = 1.76$ ($x \equiv 2\pi a/\lambda$, where λ is the wavelength and a is the particle radius) and the refractive index $m = 1.50 + i0.0001$. As regards the filling factor (or the packing density) in the medium, which is a volume fraction occupied by particles in the medium, two values, $\xi = 20$ and 10%, were considered. The first value of ξ was used by Penttilä et al. (2021) for calculating the weak-localization contribution, while the second one, by Mishchenko et al. (2013) for verifying the applicability of the vector radiative transfer equation to densely packed media. (It is worth noting that Mishchenko et al. (2013) considered the other parameters of particles and the oblique radiation incidence). The simulations performed with two values of the parameter ξ will make it possible to follow the influence of the concentration of particles on the manifestation of the weak-localization effect in the models with different versions of a volume element of the medium.

2 MODELS OF A MEDIUM

In this paper we consider five models of a discrete random densely-packed medium (from A to E below), which differ by both the scattering characteristics of a volume element and the behavior of the extinction in a medium. The medium is considered as a semi-infinite layer, and the incident radiation is assumed to propagate perpendicularly to its boundary, since the theory for a more general case of the obliquely incident radiation is still at an early stage of development [see, e.g., (Tishkovets et al., 2011; Doicu and Mishchenko, 2019a)]. We compare the results of these models to each other and to the simulation results reported by Penttilä et al. (2021). To calculate the intensity and the linear polarization degree of light reflected by a semi-infinite discrete medium, we use the procedure and the fast algorithm described by Tishkovets and Petrova (2020). This procedure allows us to consider both the diffuse and coherent-backscattering components of the reflected light.

2.1 Model A

This model is simplest. The scattering matrix of a volume element in this model medium is equal to the scattering matrix of an individual particle of the medium. The size parameter and the refractive index of these particles were specified above (see the *Introduction*). To describe the propagation of radiation in a discrete random medium, a concept of the so-called complex
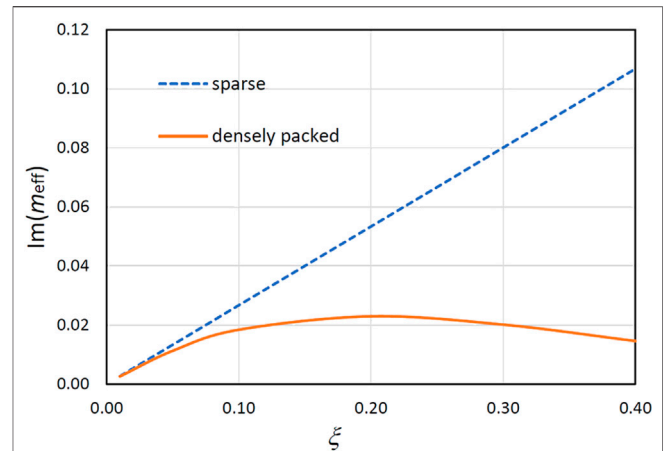

FIGURE 1 | The imaginary part of the effective refractive index of a medium composed of identical spherical particles with $x = 1.76$ and $m = 1.50 + i0.0001$ in dependence on the filling factor of particles according to **Eqs 1**, **2** for sparse (dashed line) and densely packed (solid line) media, respectively.

effective refractive index m_{eff} is used [see, e.g., (Tishkovets et al., 2011; Doicu and Mishchenko, 2019a), and the references therein]. Its imaginary part $\mathrm{Im}\,(m_{\mathrm{eff}})$ is directly related to the extinction coefficient of the medium, and this dependence differs for sparse and densely packed media. Within model A, it is assumed that $\mathrm{Im}\,(m_{\mathrm{eff}})$ linearly depends on the concentration (or the number density) of particles in the medium η, analogously to that in a sparse medium model (Bohren and Huffman, 1983; Mishchenko et al., 2006; Tishkovets et al., 2011; Doicu and Mishchenko, 2019a):

$$\mathrm{Im}\left(m_{\mathrm{eff}}\right) = \frac{\eta}{2k_0}C_{\mathrm{ext}} = \frac{3\xi}{8\pi x^3}k_0^2 C_{\mathrm{ext}} = \frac{3\xi}{4x^3}\sum_l (2l+1)\mathrm{Re}\,(a_l + b_l).$$

$$(1)$$

Here, $k_0 = 2\pi/\lambda$, C_{ext} is the extinction cross-section of a particle, and a_l and b_l are the Mie coefficients (Bohren and Huffman, 1983), while $\xi = 4\pi\eta a^3/3$ is the filling factor. The single-scattering albedo is determined as usual, $\omega = C_{\mathrm{sca}}/C_{\mathrm{ext}}$, where C_{sca} is the scattering cross-section of a particle.

2.2 Model B

This model differs from model A only by the behavior of the extinction in dependence on the concentration of scatterers. In a densely packed medium, this dependence is nonlinear. If this medium is composed of identical spherical particles, $\mathrm{Im}\,(m_{\mathrm{eff}})$ can be determined from the following system of equations, which stems from the analysis of the mean field in a medium [see, e.g., (Tishkovets et al., 2011) and the references therein]

$$X_{Ln}^{(pn)} + \frac{3\xi}{2x^3}\sum_q \left(a_l + pqb_l\right)\sum_l (2l+1)X_{ln}^{(qn)}F_{Lln}^{(q)} = 0. \quad (2)$$

Here, $p, q = \pm 1$; n can take one of the values, ± 1;

$$F_{LlM}^{(q)} = \sum_s C_{LMl-M}^{s0} C_{Lql-q}^{s0} f_s,$$

where the quantities C with indexes are the Clebsch–Gordan coefficients (Varshalovich et al., 1988);

$$f_l = \int_\rho^\infty (g(y) - 1) h_l(y) j_l(y m_{\text{eff}}) y^2 \mathrm{d} y$$

$$- \frac{\rho^2}{1 - m_{\text{eff}}^2} \left[m_{\text{eff}} h_l(\rho) j_l'(\rho m_{\text{eff}}) - h_l'(\rho) j_l(\rho m_{\text{eff}}) \right]$$

where $\rho = 2x$, the j and h designations are for the Bessel and Hankel spherical functions and their derivatives, respectively, $g(y)$ is the pair correlation function for the system of identical solid spheres (Balescu, 1975).

The linear homogeneous system of **Eq. 2** has a non-trivial solution if its determinant is equal to zero. This condition allows us to determine the effective refractive index m_{eff}, which depends on the properties of the medium: the shape, sizes, refractive index, and filling factor of the particles. The dependences of $\text{Im}(m_{\text{eff}})$ on the filling factor of particles ξ calculated according to **Eqs 1, 2** for the particles' parameters specified above are shown in **Figure 1**.

As is seen from the figure, when the concentration of particles is high, $\text{Im}(m_{\text{eff}})$ of a densely packed medium may be substantially lower than the values predicted by **Eq. 1** that is valid for the sparse media approximation. For example, for a case of $\xi = 20\%$ and the particle parameters considered here, the value of $\text{Im}(m_{\text{eff}})$ in model A is more than twice that in model B: 0.0534 versus 0.0229.

2.3 Model C

In this model, the scattering matrix of a volume element of the medium coincides with that of a randomly oriented cluster of spherical particles. According to Tishkovets and Petrova (2013), when the number of particles in clusters is large, the intensity and the degree of linear polarization of light reflected by a semi-infinite medium composed of such clusters becomes almost independent on the further increase of this number. The imaginary part of the effective refractive index of the medium is assumed to be determined as

$$\text{Im}(m_{\text{eff}}) = \frac{\eta_1}{2k_0} C_{\text{ext}}, \tag{3}$$

where η_1 and C_{ext} are the concentration of clusters in the medium and the extinction cross-section of the clusters, respectively. The filling factor of particles (not clusters) in the medium is defined as

$$\xi = \frac{4}{3} \pi \eta_1 a_v^3, \tag{4}$$

where a_v is the radius of a volume-equivalent sphere of the cluster ($a_v^3 = N a^3$, where N is the number of particles with radii a in the cluster). The single scattering albedo is determined analogously to that in model A.

Earlier, model C was successfully applied to the media with relatively low values of the filling factor of particles in a medium ($\xi \leq 4\%$) (Tishkovets and Petrova, 2013).

2.4 Model D

This model is analogous to that considered by Mishchenko (1994), Mishchenko et al. (2013), and Ito et al. (2018); i.e., the scattering characteristics of a volume element of the medium are corrected with accounting for the so-called static structure factor. According to the Percus–Yevick approximation (Balescu, 1975), the structure factor is given by (Balescu, 1975; Mishchenko, 1994; Ito et al., 2018)

$$S(\vartheta) = \frac{1}{1 - C(u)},$$

where $u = 4x \sin \frac{\vartheta}{2}$, ϑ is the scattering angle, and

$$C(u) = 24\xi \, W(u).$$

If $u \neq 0$,

$$W(u) = \frac{\alpha + \beta + \delta}{u^2} \cos u - \frac{\alpha + 2\beta + 4\delta}{u^3} \sin u - \frac{2(\beta + 6\delta)}{u^4} \cos u +$$

$$+ \frac{2\beta}{u^4} + \frac{24\delta}{u^5} \sin u + \frac{24\delta}{u^6} (\cos u - 1).$$

If $u = 0$,

$$W(0) = -\left(\frac{\alpha}{3} + \frac{\beta}{4} + \frac{\delta}{6} \right).$$

The coefficients α, β, δ are

$$\alpha = \frac{(1 + 2\xi)^2}{(1 - \xi)^4},$$

$$\beta = -6\xi \frac{(1 + \xi/2)^2}{(1 - \xi)^4},$$

$$\delta = \alpha\xi/2.$$

Then, with accounting for the structure factor, the normalized scattering matrix of a particle in the medium $\mathbf{F}(\vartheta)$ is connected with the matrix of an individual isolated particle $\mathbf{Z}(\vartheta)$ by the following relationship (Ito et al., 2018)

$$\mathbf{F}(\vartheta) = \frac{4\pi}{k_0^2 C_{\text{sca}}^{\text{s}}} \mathbf{Z}(\vartheta) S(\vartheta), \tag{5}$$

where

$$k_0^2 C_{\text{sca}}^{\text{s}} = \int_{4\pi} Z_{11}(\vartheta) S(\vartheta) \mathrm{d}\Omega = 2\pi \int_0^\pi Z_{11}(\vartheta) S(\vartheta) \sin \vartheta \mathrm{d}\vartheta. \tag{6}$$

The imaginary part of the effective refractive index of the medium is determined as

$$\text{Im}(m_{\text{eff}}) = \frac{\eta}{2k_0} \left(C_{\text{sca}}^{\text{s}} + C_{\text{abs}} \right) = \frac{3\xi}{8\pi x^3} k_0^2 \left(C_{\text{sca}}^{\text{s}} + C_{\text{abs}} \right), \tag{7}$$

where C_{abs} is the absorption cross-section of a particle calculated according to the Mie theory, while the scattering cross-section $C_{\text{sca}}^{\text{s}}$ is obtained from **Eq. 6**, which takes into account the structure factor. The single-scattering albedo of particles in this model is determined as

$$\omega = \frac{C_{sca}^s}{C_{sca}^s + C_{abs}}. \qquad (8)$$

The results of this modeling for the noncoherent part of the reflected radiation, which is determined by the radiative transfer equation, were compared to those of laboratory measurements. This comparison showed that, for the concentrations up to 10%, all of the elements of the reflectance matrix of the medium (except R_{44}) well agree with the measurement results (Mishchenko et al., 2013).

2.5 Model E

This model corresponds to that developed by the researchers from the University of Helsinki [see (Muinonen et al., 2017; Muinonen et al., 2018; Penttilä et al., 2021) and references therein]. In this model, the contribution of the coherent (or mean) field is subtracted from the scattering characteristics of a volume element. The field scattered by some volume containing particles (a cluster of particles) in the medium $\mathbf{E}^{(s)}$ is represented by a sum of the incoherent $\mathbf{E}^{(in)}$ and coherent $\mathbf{E}^{(c)}$ parts [see, e.g., (Barrowes et al., 2000; Muinonen et al., 2017; Muinonen et al., 2018; Penttilä et al., 2021), and the references therein]

$$\mathbf{E}^{(s)} = \mathbf{E}^{(in)} + \mathbf{E}^{(c)}.$$

Since the locations of scatterers in the medium are random, the equality $<\mathbf{E}^{(in)}> = 0$ should be fulfilled. Here the angle brackets mean the ensemble-averaging over a large number of volume-element realizations. Then, the average of the second moment of the field $\mathbf{E}^{(in)}$ is determined by the relationship

$$<\left|\mathbf{E}^{(in)}\right|^2> = <\left|\mathbf{E}^{(s)}\right|^2> - <\left|\mathbf{E}^{(c)}\right|^2>. \qquad (9)$$

The terms in the right part of **Eq. 9** can be averaged numerically by generating clusters with sufficiently large number of particles with the concentration and parameters of particles required. In this averaging procedure, some function $H(\vartheta)$ is derived; by multiplying this function by the scattering matrix of an individual particle, the scattering matrix of a volume element is obtained. The whole technique is described at length by Muinonen et al. (2017). The procedure of calculating the scattering characteristics, which are required to find the reflectance matrix of a medium (or a cluster containing a large number of particles), is the same as that for relationships **Eqs 5–8** in the previous model. The only difference is that the normalized function $H(\vartheta)$ should be used instead of the structure factor.

This model yields the results that rather well agree with those of rigorous calculations of the intensity and the degree of linear polarization of light scattered by clusters of spherical particles, the number of which is $N \leq 10^5$ (note that the clusters were generated by filling a spherical volume by small constituents with the packing density specified) (Penttilä et al., 2021).
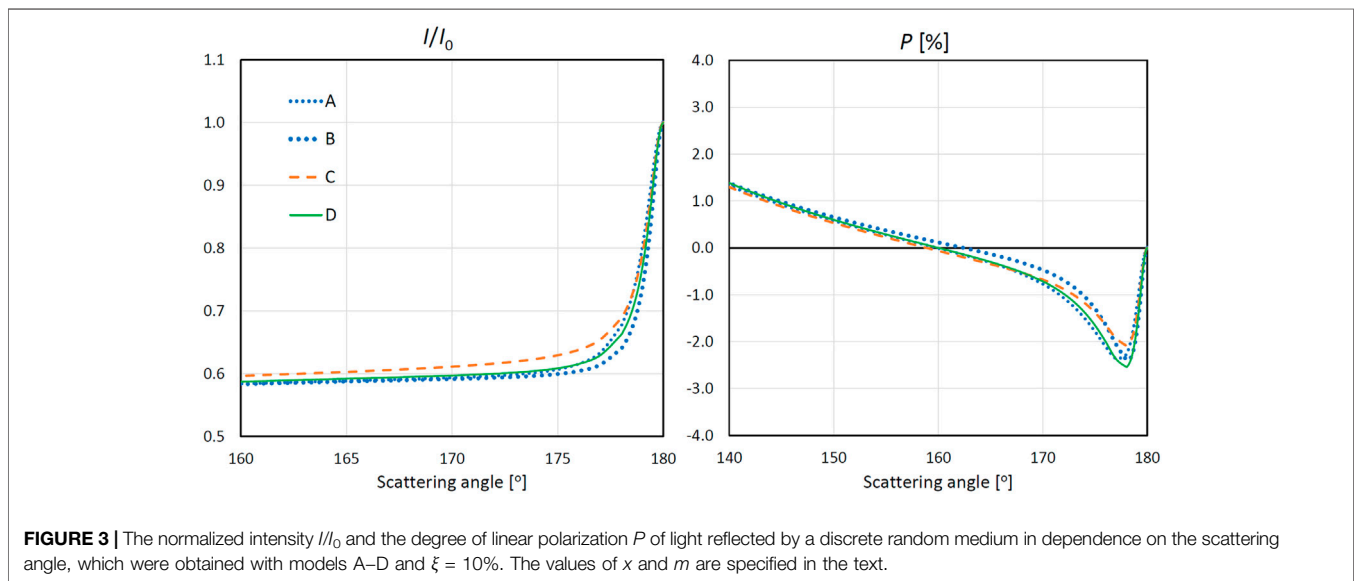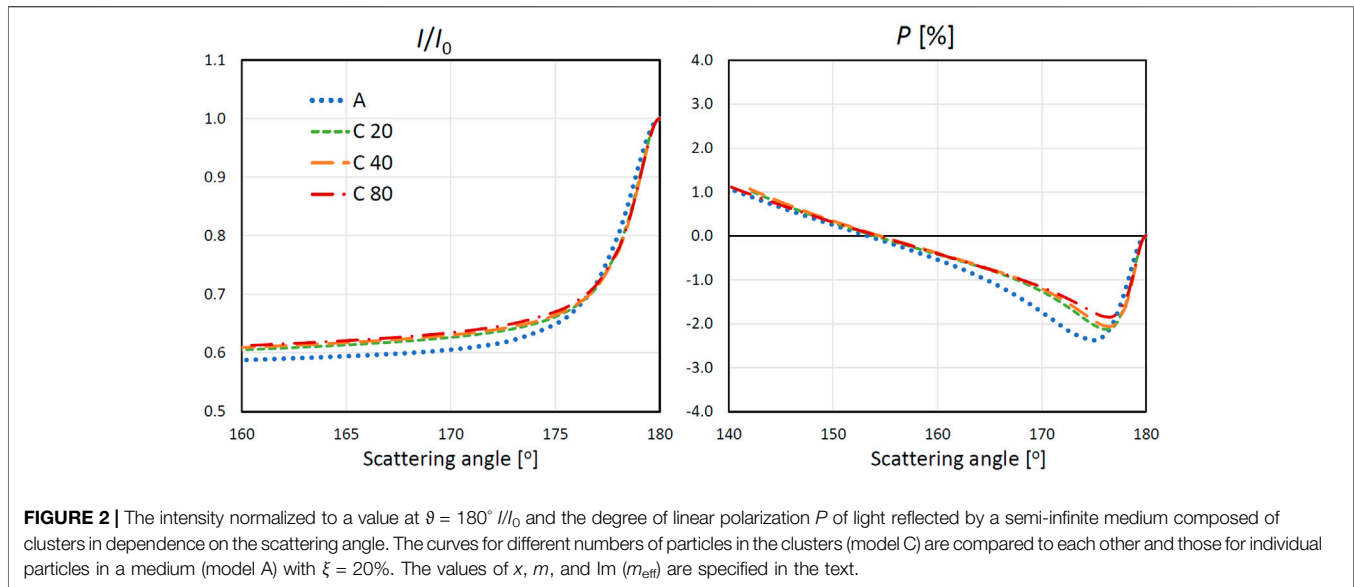
3 RESULTS AND DISCUSSION

To solve the radiative transfer and weak localization equations, the scattering matrix of a volume element of the medium is usually presented as a series expansion in generalized spherical functions (Mishchenko et al., 2006). The algorithms and codes to solve these equations by using these expansions are available on the web-sites https://www.giss.nasa.gov/staff/mmishchenko/brf/ and http://rian.kharkov.ua/index.php/en/software-en, respectively. Here we use these codes to calculate the normalized intensity and the degree of linear polarization of radiation reflected by a semi-infinite medium.

The simulations according to model C required that clusters with the assumed filling factor should be constructed. For this, we randomly placed N identical non-overlapping spherical particles of the specified size into a spherical volume, the size of which provides the required filling factor ξ for a given value of N. To calculate the single-scattering matrix of clusters, we used the publicly available FORTRAN code (MSTM) (Mackowski and Mishchenko, 2011), which is based on the superposition T-matrix method, one of the most versatile and efficient direct computer solvers of the macroscopic Maxwell equations for an arbitrary multi-sphere configuration in random or fixed orientation. The parameters of clusters' constituents and the filling factor are the same as those considered by Penttilä et al. (2021): $x = 1.76$, $m = 1.50 + i0.0001$, and $\xi = 20\%$. The values of $\text{Im}(m_{eff})$ were derived according to **Eqs 3, 4**; and they are 0.0798, 0.0771, and 0.0720 for $N = 20$, 40, and 80, respectively.

In **Figure 2**, as an illustration of the results of model C, we present the intensity normalized to a value at opposition (the scattering angle is $\vartheta = 180°$) I/I_0 and the degree of linear polarization P of light reflected from a medium composed of randomly oriented clusters. These quantities are shown in dependence on the scattering angle, and the curves for different numbers of particles in the clusters are compared. For each of the assumed values of N, we actually generated several configurations of the clusters, but the angular dependences of I/I_0 and P obtained for the medium composed of these clusters turned out to be very close under a given N. Consequently, we show here the phase curves only for one of the configurations for a specified N. Moreover, as it became clear from the model calculations, the growth of N to the values exceeding ~50 does not influence much the angular behavior of the considered quantities (**Figure 2**).

Let us turn to comparison of the results of different models. The normalized intensity and the linear polarization degree calculated by the above described models A–D for the packing density of the medium $\xi = 10\%$ are shown in **Figure 3** in dependence on the scattering angle ϑ. To avoid overloading the diagrams, we present here models C only by the results for clusters with 40 constituents. The reason is that, as has been noted above, the further growth of the clusters induces only very weak changes in the angular dependences of the intensity and the linear polarization degree of light scattered by a medium composed of these clusters (**Figure 2**). As is seen from **Figure 3**, all of the models with the packing density $\xi = 10\%$ yield rather close results. It should be reminded that, when verifying the applicability of the radiative transfer equation to a densely packed medium, model D perfectly fitted the phase curves measured in the laboratory (Mishchenko et al., 2013). Since the weak-localization equation was derived under the same conditions as the classic radiative

FIGURE 2 | The intensity normalized to a value at $\vartheta = 180°$ I/I_0 and the degree of linear polarization P of light reflected by a semi-infinite medium composed of clusters in dependence on the scattering angle. The curves for different numbers of particles in the clusters (model C) are compared to each other and those for individual particles in a medium (model A) with $\xi = 20\%$. The values of x, m, and Im (m_{eff}) are specified in the text.

FIGURE 3 | The normalized intensity I/I_0 and the degree of linear polarization P of light reflected by a discrete random medium in dependence on the scattering angle, which were obtained with models A–D and $\xi = 10\%$. The values of x and m are specified in the text.

transfer equation (Tishkovets et al., 2011), a good agreement between models A–D in **Figure 3** suggests that all of them may be used to estimate the parameters of a medium at least in the cases, when the properties of particles in a medium are close to those specified here and the packing density of particles is less than 10%.

Let us consider the results of simulations with the packing density $\xi = 20\%$ according to all of the models, including model E, in more detail. The obtained phase curves of the normalized intensity and the linear polarization degree are shown in **Figure 4**.

The values of Im (m_{eff}) in each of the models and the parameters of the phase curves of the normalized intensity I/I_0 and the polarization P are listed in **Table 1**. These parameters are the angular halfwidth of the intensity peak $\Delta\vartheta$ at a level of 0.8 of its

maximum, the inversion angle ϑ_{inv}, at which the linear polarization changes its sign, the polarization value P_{min} at the minimum of the so-called negative branch observed at high scattering angles (or low phase angles $\varphi = 180° - \vartheta$), and the angular position of this minimum ϑ_{min}.

There are two characteristic features in the phase curves of the characteristics of radiation reflected by a medium, which are reproduced by each of the considered models. They are a narrow interference peak in the intensity, which is rigorously centered at the backscattering direction, and a branch of negative polarization, the minimum of which is close to the opposition. The dependence of these features on the properties of a scattering medium was analyzed at length in many papers [see, e.g. (Tishkovets et al., 2011; Tishkovets and Petrova, 2013), and
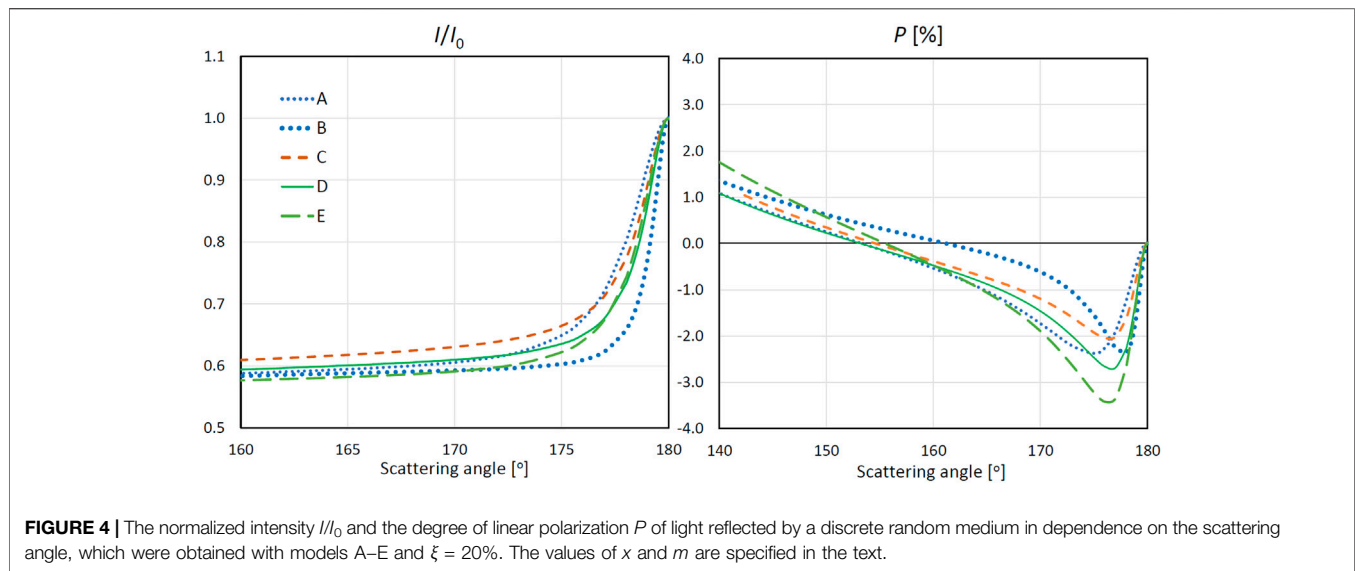
FIGURE 4 | The normalized intensity I/I_0 and the degree of linear polarization P of light reflected by a discrete random medium in dependence on the scattering angle, which were obtained with models A–E and $\xi = 20\%$. The values of x and m are specified in the text.

TABLE 1 | The values of Im (m_{eff}) in models A–E and the parameters of the phase curves of the normalized intensity and the linear polarization degree for $\xi = 20\%$.

Model	Im (m_{eff})	$\Delta\vartheta$ [°]	ϑ_{inv} [°]	P_{min} [%]	ϑ_{min} [°]
A	0.0534	~2	~154	−2.38	~175
B	0.0229	~0.9	~161	−2.37	~178
C	0.0771 ($N = 40$)	~1.7	~155	−2.04	~176
D	0.0237	~1.4	~153	−2.69	~177
E	0.0252	~1.5	~156	−3.42	~176

references therein]. Specifically, it was shown that the halfwidth of the interference peak and the phase angle at the polarization minimum ($\varphi_{\mathrm{min}} = 180° - \vartheta_{\mathrm{min}}$) under nonzero polarization of the noncoherent component are in direct proportion to the concentration of scatterers (or the filling factor ξ). When the concentration of particles decreases, the angular range, within which the weak-localization effect manifests itself, also decreases, while the inversion scattering angle ϑ_{inv} grows (compare **Figures 3**, **4**). As might be expected, with increasing the packing density, the difference between the model curves becomes more noticeable, though all of them exhibit qualitatively the same behavior. Moreover, the models C, D, and E well agree in terms of the inversion angle of polarization and the position of the polarization minimum.

As is also seen from **Figure 4** and **Table 1**, the decrease in the value of Im (m_{eff}) in model B relative to that in model A (0.0229 versus 0.0534) results in narrowing the intensity peak ($\Delta\vartheta$ becomes ~0.9° instead of ~2°) and in moving the polarization minimum to opposition (ϑ_{min} becomes ~178° instead of ~175°). This is explained by the fact that the decrease in the imaginary part of the effective refractive index Im (m_{eff}) under the same characteristics of a volume element (an individual spherical particle) of a medium yields the equivalent decrease (by a factor of ~2.3) in the concentration of scatterers. According to the above mentioned studies, more porous media should exhibit

narrower opposition features due to the weak-localization effect.

Unfortunately, since the experimental data on the scattering by discrete random densely-packed media with thoroughly controlled parameters (the sizes and shape of particles, the packing density, and the other parameters of a medium) are limited, it is currently impossible to determine unambiguously which of these models more correctly describes the light scattering by a densely packed particulate media. Because of this, we compare the results of these models to those reported by Penttilä et al. (2021). They describe the simulations for the intensity and the linear polarization degree of light scattered by a large spherical volume randomly filled with spherical particles, the sizes, the refractive index, and the filling factor (or the packing density) of which are equal to the parameters considered here. In the cited paper, the number of particles N in the volume was varied within 10^3–10^9. For ensembles containing more than 10^5 particles, the calculations were performed with an approximate method under the assumption that the waves propagating between scatterers in the medium are spherical, while numerically rigorous methods were used for smaller ensembles. In addition, in the approximate method, the contribution of the mean field to the scattering matrix of a volume element and the imaginary part of the effective refractive index was removed (Muinonen et al., 2017; Muinonen et al., 2018; Penttilä et al., 2021). The results of the approximate and exact calculations for $N < 10^5$ turned out to be in satisfactory agreement.

To facilitate the comparison of our models (**Figure 4**) with the results of Penttilä et al. (2021), we show them together in **Figure 5**. To avoid overloading the diagrams, only the data for the particles' number $N = 10^5$, 10^6, and 10^9 from Penttilä et al. (2021) are given in **Figure 5**.

When analyzing the phase curves in **Figure 5**, it is important to have in mind that the geometry of the light-scattering simulations for a spherical cloud of particles and a semi-infinite layer differs, i.e., the scattering media are different in shape. Because of this, the agreement between the angular profiles for
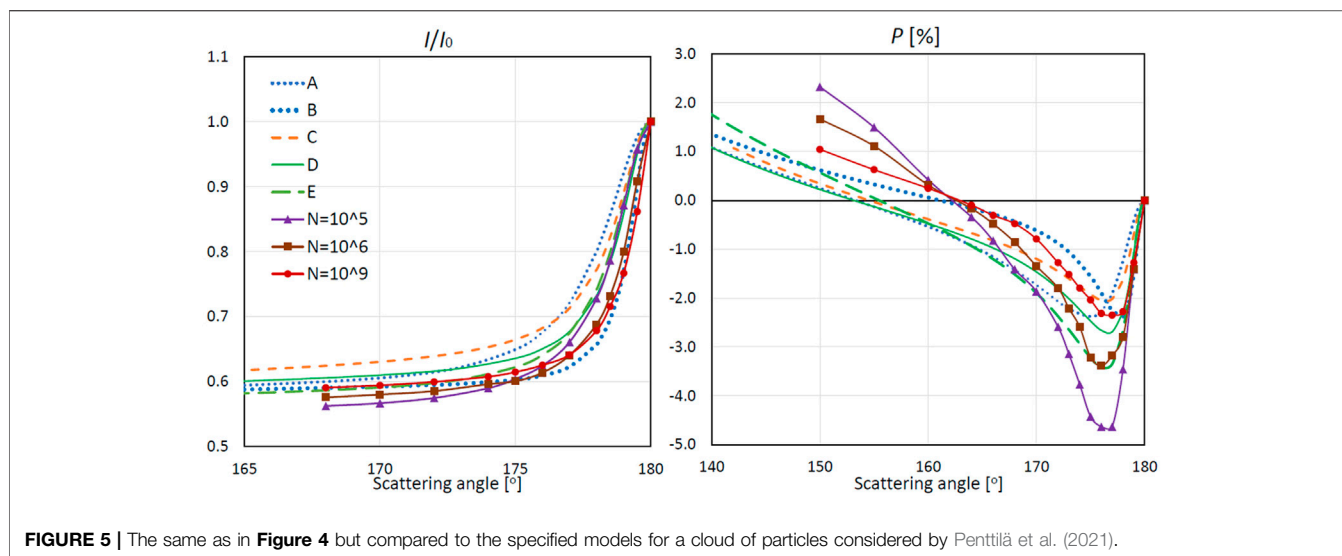
FIGURE 5 | The same as in **Figure 4** but compared to the specified models for a cloud of particles considered by Penttilä et al. (2021).

model E and a cloud of 10^9 particles (Penttilä et al., 2021), for which the light-scattering characteristics of a volume element are the same, is worse than it could be expected. In addition, an assumption on the exponential decrease of the intensity with depth in a medium, which was made when deriving the weak localization equations for a semi-infinite layer (Tishkovets and Petrova, 2020), may play a certain role in this discrepancy. Consequently, the fact that, among models A–E, model B appears to agree best of all with the data for a cloud of 10^9 particles should not be considered as a decisive factor in favor of using this model in the interpretation of measurements. Most likely, this agreement resulted from a particular combination of the parameters of particles and might not be relevant to the other parameters' values.

The comparison of the phase curves of polarization produced by models C–E and those for a large cloud of particles suggests that the angular position of the polarization minimum ϑ_{min} turns out to be almost independent of the model type. This circumstance makes this parameter most reliable for the interpretation of the measured phase curves of polarization.

4 CONCLUSION

At present, it is a challenge to describe theoretically the light scattering process for densely packed random media, where particles are not in far zones of each other, as in sparse media. Because of this, to estimate the light scattering properties of densely packed media, researchers turn to approximate methods and models that stem from the light-scattering theory for sparse media. However, it is not clear yet whether the results of this approximate modeling are correct, since the experimental data for the samples with thoroughly controlled parameters are limited so far.

To estimate the difference between the results yielded by the approximate procedures, we considered five approximate models, which are most frequently used to calculate the reflection matrix

of a densely packed semi-infinite medium, and compared their results to each other and to those of the rigorous and approximate simulations for a large spherical cloud of particles (Penttilä et al., 2021). In our models, the incident radiation is assumed to propagate perpendicularly to its boundary. The parameters of particles were chosen according to those assumed by Penttilä et al. (2021). The input parameters of our models differ by the scattering matrix of a volume element of the medium and the dependence of the extinction on the concentration of particles. For our computations, we used the fast algorithm proposed by Tishkovets and Petrova (2020).

It was found that the reflectance characteristics obtained with different models for a semi-infinite medium (**Figure 4**) qualitatively agree with each other and with those for a cloud containing 10^6 and more particles (Penttilä et al., 2021; **Figure 5**). Hence, these approximate models are equally acceptable to the interpretation of the results of measurements. It should also be noted that the approximate simulations performed both in this study and by Penttilä et al. (2021) are based on the same assumption that the waves propagating between scatterers in the medium are spherical. Consequently, the qualitative agreement between their results could be expected. However, the difference between the intensity and linear polarization profiles obtained here and by Penttilä et al. (2021) may be caused by different shapes of the media considered.

One of the purposes of developing the reflectance models for densely packed particulate media with accounting for the weak-localization effect is to apply them to the interpretation of different measurement results, in particular, numerous observations of atmosphereless bodies of the Solar System. Consequently, the used model should allow a lot of parameters' cases to be tested rather quickly. For example, even in the simplest model A, there are four input parameters (the size of particles, the real and imaginary parts of their refractive index, and the filling factor) that should be varied when fitting the experimental data with the model. Because of this, together with the reliability and effectiveness of the model, the time required to calculate the reflectance matrix of a medium

becomes one of the key parameters. In this regard and given the results of the present analysis, the model and the fast algorithm presented by Tishkovets and Petrova (2020) are particularly appealing for the use in interpretation of the data concerning the media composed, at least, of weakly absorbing particles comparable to the wavelength in size with the filling factor less than 20%.

It is worth noting that, as any approximate model, the above models may be used only within some particular ranges of the parameters describing the properties of a medium. However, at present, to determine these ranges is impossible, since laboratory measurements of samples with thoroughly controlled characteristics, which could serve as a reference, are still severely lacking. We may expect that the present models will naturally work correctly for rather loosely packed media (i.e., for $\xi < 20\%$), while their applicability to denser media or those containing particles with the other parameters should additionally be verified. We are planning to estimate the discrepancy between the parameters of the observed objects resulted from fitting their light-scattering characteristics with different models in our future studies.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

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AUTHOR CONTRIBUTIONS

VT: Conceptualization, Methodology, Investigation, Software. EP: Methodology, Validation, Investigation, Visualization.

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