THE INTRODUCTION OF A MODEL OF STATISTICAL DATA MATRICES ENCOMPASSING ORIGINAL VARIATE POPULATION METHODOLOGIES

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by

Charles R. Jeffords, 1950-

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A THESIS

Approved:

Gary Copus, Ph.D., Chairman

George G. Killinger, Ph.D.

Pauline Loveless, Ph.D.

Approved:

Robert G. Brooks, Ph.D.
Vice President of Academic Affairs

ABSTRACT

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Purpose

The purpose of this study was to examine the history, rationale, and sources of confusion of existing models and definitions of two and three dimensional statistical data matrices, and to lay a foundation for the acceptance of a standard model by suggesting clear definitions, recommendations, and modifications for a more robust model of statistical data matrices.

Methods

The methodology used was documentary research concerning statistical data matrix models, an insight into the problems of these, and innovation in the introduction of a modified model.

Findings

1. A major source of confusion in the field of data matrices is the lack of standard definitions for basic concepts.

2. A major cause of the lack of standard definitions of data matrices was a transition in the definition

of Q technique by its strongest proponent, William Stephenson, and his failure to acknowledge this transition. (a) The lack of a standard definition of Q contributed to the controversy of whether or not R technique and Q technique may exist in one and the same matrix. By using Stephenson's original definition of Q, R and Q may exist in the same matrix; by using Stephenson's revised definition of Q, restricting Q to a population of statements, R and Q may not exist in the same matrix. (b) Although Stephenson claimed to have divided the six techniques of Raymond Cattell into two methodologies, the properties of the techniques were not consistent within each methodology. Stephenson's revised definition of Q contributed to the inconsistencies of the properties between the techniques within each methodology.

3. Stephenson's transition was precipitated by the reciprocity principle of Sir Cyril Burt.

4. Measurement scales may be unique to a variate or common to at least two variates.

5. In terms of Cattell's techniques four equivalent descriptions exist for any re-standardization.

Recommendations

1. Cattell's six techniques should be divided into three methodologies grouped according to the

dimension of the original variate population sampled. The three original methodologies were designated as follows: (a) PPV--the statistical analysis of person population variates; (b) APV--the statistical analysis of attribute population variates; and (c) OPV--the statistical analysis of occasion population variates, respectively.

2. The basis of each methodology would not be the two variates of analysis, as in Cattell's techniques, but rather the population of the individual original variate.

3. The properties of measurement scale, conduciveness to a Sort, and population are consistent within each population methodology.

4. Different population methodologies may exist in the same matrix without re-standardization if common scaling exists through each dimension.

5. Although in terms of Cattell's techniques four equivalent descriptions exist for any re-standardization, a re-standardization can be expressed in but one manner in the original population model.

6. Re-standardization should be clearly distinguished in terms of its original and resulting variate populations.

7. In the case of an APV Sort, an additional axis was found necessary for the population of

instructions. An additional designation, IPV, the statistical analysis of instruction population variates, can only be the result of re-standardization from a previous APV Sort.

Supervising Professor

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TABLE OF CONTENTS

LIST OF TABLES

TABLE | PAGE

LIST OF FIGURES

FIGURE PAGE

CHAPTER I

INTRODUCTION

Criminology, a social science has explored several methodologies in order to understand criminal behavior. Social sciences, however, have traditionally fallen far short of the physical sciences in the predictability and formulation of scientific law. Using the success of physical science as a standard bearer, social science has attempted to create and substantiate theory with the aid of statistics, which is based on the pure science of mathematics.

The need for statistical information in criminal justice has commonly been noted. In 1931, the National Commission on Law Observance and Enforcement, better known as the Wickersham Commission, observed a "general agreement as to the importance of official and trustworthy statistics of crime, criminals, criminal justice, and penal administration" calling "adequate data ... the beginning of wisdom [National Commission on Law Observance and Enforcement, 1931, Vol. 1, No. 3, p. 3]." Yet clearly it is only the beginning as "There is no value in tables and masses of figures simply of themselves [*Ibid*., p. 4]."

"The function of statistics is, then, to focus attention on matters needing investigation and explanation [*Ibid*., p. 28]." More recently, for example, the *Task Force Report: Science and Technology* recognized a "much needed effort to correlate both individual characteristics and type of treatment to subsequent commission of crimes, rearrests, reconvictions, and recommitments [President's Commission on Law Observance and Administration of Justice, 1967b, p. 47]." The *Task Force Report: Crime and Its Impact--An Assessment* stated that:

> Statistics derived from the criminal justice system are necessary and important in dealing with crime. Modern statistical methods are not limited, however, to collecting data from official agencies. New methods of collection, new types of indicators of crime and of the effectiveness of the criminal justice agencies, new ways of looking at crime, special statistical studies, and in general a great deal more innovation in statistical efforts are all required if headway is to be made against crime [President's Commission on Law Enforcement and Administration of Justice, 1967a, p. 132].

There exists, it would seem, at least three methodological stages before statistics can be beneficial to criminology: (1) the collection of statistical data, (2) statistical analysis of the data, and (3) interpretation of the analysis in terms of its applicability to the realities of criminology.

One fundamental aspect of the collection and analysis of any statistical data is the placement of the data into a data matrix. The development of Q technique, a statistical technique, opened the way to new experimental possibilities by viewing data matrices in a different perspective. Additionally, however, the development of Q brought about alternative models of data matrices by William Stephenson, Sir Cyril Burt, and Raymond Cattell, none of which has gained universal acceptance in the scientific community. One reason for the existence of alternative models is the lack of standard definitions for some basic concepts of data matrices.

Purpose

It is the purpose of this study to examine the history, rationale, and sources of confusion of existing models and definitions of two and three dimensional data matrices and to lay a foundation for the acceptance of a standard model by suggesting clear definitions, recommendations, and modifications for a more robust model of statistical data matrices.

Methodology

The methodology to be used in this study is documentary research concerning statistical data matrix models, an insight into the problems of these, and innovation in the introduction of a modified model.

CHAPTER II

BASIC HISTORY OF TWO-SPACE DATA MATRICES

The scope of this study shall cover the examination of two- and three-space data matrices, the axes of which shall be the populations of persons, attributes, and occasions, as shown in Figure 1. It is the purpose of this chapter to present a basic historical background of the development of two-space data matrices. While there are naturally three unique two-space planes in three-space, the writer shall examine in this chapter the plane encompassing the population of persons and the population of attributes. This plane has been used in the majority of two-space studies and was the first to receive major attention historically.

While data matrices may be used for many methods of statistical analysis, their development has generally been associated with correlation and subsequent factor analysis. Thus, in order to present an accurate historical account, the terminology shall largely speak in terms of correlation and factor analysis.

Correlation of Attributes

Correlation, which according to Wiggins (1973) was initially suggested by Galton in 1888 and developed

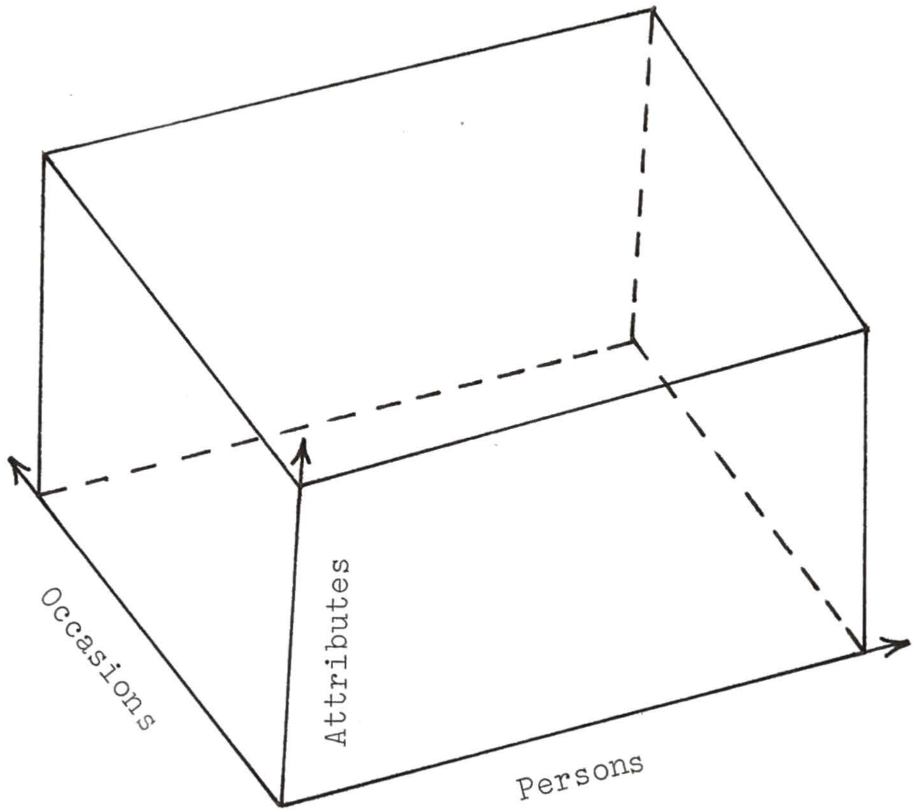

Figure 1

Three Dimensional Data Matrix

by Pearson in 1896, is generally thought of in terms of testing the relationship of attributes, such as tests, measurements, and personality traits, over a population of persons. Consider, for example, the two-dimensional matrix of a sample of persons taking a number of tests, as shown in Table 1. It is a simple procedure to compute the mean, standard deviations, and standard scores within each test, and the correlations between them, as shown in Tables 2 and 3, the correlation coefficient r_{jk} for a sample being found by the equation:

$$r_{jk} = \Sigma \; z_j z_k / N-1$$ (Glass and Stanley, 1970, p. 117).

For any two attributes, j and k, the standard error of estimate, $s_{k \cdot j}$, of attribute k as estimated from attribute j, may be found by the equation:

$$s_{k \cdot j} = s_k \sqrt{1-r^2_{jk}},$$ (McNemar, 1962, p. 125)

where s_k is the standard deviation of attribute k, and r_{jk} is the correlation coefficient of attributes j and k. Clearly, if $r_{jk} = 0$, then $s_{k \cdot j} = s_k$, and one can make no better estimate of k than its mean, even given knowledge of variable j. If $r_{jk} = 1$, then $s_{k \cdot j} = 0$, and one can estimate k from j exactly. Viewing s_k as a constant, the standard error of estimate increases in direct proportion to the expression $\sqrt{1-r^2_{jk}}$, termed the

TABLE 1

Exemplary Matrix for
Population of Persons

Persons	Tests 1	2	3
1	90	80	85
2	80	90	70
3	70	85	75
$\bar{X}j$	80	85	75
s_j	10	5	8.66

TABLE 2

Exemplary Standard Scores for Matrix with Population of Persons

Persons	Tests 1	2	3
1	+1.00	-1.00	+1.15
2	0.00	+1.00	-.58
3	-1.00	0.00	-.58
$\overline{X}j$	0.00	0.00	0.00
s_j	1.00	1.00	1.00

TABLE 3

Exemplary Correlation Matrix
from Population of Persons

Tests	Tests 1	2	3
1	1.00	-.50	+.87
2	-.50	1.00	-.87
3	+.87	-.87	1.00

coefficient of alienation.

The best estimate score, X'_{ik}, of individual i on attribute k from knowledge of his score on attribute j may be found by the equation:

$$X'_{ik} = r_{jk}s_k(X_{ij} - \overline{X}_j)/s_j + \overline{X}_k. \quad \text{(Hays, 1963, p. 503).}$$

Correlation of Persons

In 1911, however, according to Burt, Stern noted the possibility of "comparing two individuals ... in regard to a large series of characteristics [Burt, 1937, p. 60]." This would then be correlating persons as variables over a population of characteristics, or attributes. It is similarly simple to find the mean, standard deviations, and standard scores within each person, and the correlations between them, as demonstrated in Tables 4, 5, and 6. The reader will assume in his calculations for the sake of aesthetics that the attributes are merely a sample of a larger population of attributes. Those unfamiliar with finding an individual's mean score for a number of attributes or tests may recognize this as the technique used by many teachers in assigning course grades.

For any two individuals, i and h, the standard error of estimate of individual i as estimated from individual h may be found by the equation:

TABLE 4

Exemplary Matrix for
Population of Tests

Persons	Tests 1	2	3	$\bar{i}$	s_i
1	90	80	85	85	5
2	80	90	70	80	10
3	70	85	70	75	8.66

TABLE 5

Exemplary Standard Scores for Matrix with Population of Tests

Persons	Tests 1	2	3	$\bar{i}$	s_i
1	+1.00	-1.00	0.00	0.00	1.00
2	0.00	+1.00	-1.00	0.00	1.00
3	-.58	+1.15	-.58	0.00	1.00

TABLE 6

Exemplary Correlation Matrix
from Population of Tests

Persons	Persons 1	 2	 3
1	1.00	-.50	-.87
2	-.50	1.00	+.87
3	-.87	+.87	1.00

$$s_{i \cdot h} = s_i \sqrt{1-r^2_{hi}},$$

and the predicted raw score, or best estimate, of attribute j for individual i as estimated from individual h is found by the equation:

$$X'_{ij} = r_{hi} s_i (X_{hj} - \bar{X}_h)/s_h + \bar{X}_i.$$

Davies (1939) attributes the first actual use of correlation between persons to Burt in 1912, and its first examination in terms of factor theory to Burt in 1921. Burt used the terms P and T for the correlation of persons and the correlation of tests respectively, although he qualified:

> I plead guilty to using such shorthand phrases as P-axes, P-factors and even the P-method; but I do not care for the label P-technique, because in my view, there is no essential technical difference between the analysis by persons and by tests. P-factors and T-factors are convenient and colloquial laboratory abridgements for 'factors obtained by correlating persons' and 'factors obtained by correlating tests' [Burt, 1940, p. 177].

Stephenson, however, brought the correlation of persons to major attention. In his original article on the subject, entitled "Correlating Persons Instead of Tests," Stephenson claimed in the first paragraph:

> I am to propose a new general technique ... . The technique is a complete inversion of all previous factor techniques [Stephenson, 1935, p. 17].

Stephenson followed the terminology suggested by Thomson (1935), "using Q for the correlation between persons, restricting the letter r for the correlation between tests

[Stephenson, 1936d, p. 47]." "It is convenient to designate all previous factor analysis as r technique, and this new inverted form as Q technique [Stephenson, 1936c, p. 345]." Stephenson explained:

> So far as I know no one had previously used the technique completely or with full understanding. Prof. G. H. Thomson (1935) has obviously been very near to it, but saw difficulties instead of the flood of immediate applications [Stephenson, 1936c, p. 352].

Homogeneity of Units

The major difficulty seen by Thomson was that the units of the attributes of a matrix may be heterogeneous. Referring to a matrix with columns of persons and rows of attributes, he stated:

> When I standardize the matrix of raw scores by rows in the usual way, I am of course assuming that whatever the raw units may be, at any rate the same unit is used along any row on the different persons. .... .
>
> If I want to standardize the matrix of raw scores by columns, I can only do so if I can assume or ensure that the raw units used in any column are the same [Thomson, 1935, pp. 75-76].

Consider the example of the body measurements of a sample of females shown in Table 7. Each female is measured in different units, one in inches, one in centimeters, and one in uls, a unit of undetermined length. For any body part j, the mean over X_{1j} inches, X_{2j} centimeters, and X_{3j} uls can not be determined. Since correlation is an expression of variance and dependent upon a mean, one can not correlate body parts from this matrix.

TABLE 7

Exemplary Matrix with Heterogeneous Units for Persons

Females	Body Parts 1	2	3	$\bar{i}$	s_i
1	38 in.	24 in.	37 in.	33 in.	7.8 in.
2	91 cm.	64 cm.	88 cm.	81 cm.	14.8 cm.
3	51 ul.[a]	33 ul.	48 ul.	44 ul.	9.6 ul.

[a]measurement of undetermined length.

One can, however, correlate females, as it is commonplace to correlate variables measured in different units. For example, one may correlate intelligence measured in IQ with achievement measured in years of academic schooling satisfactorily completed. Similarly, for correlating persons, the units within each person must be constant, whereas the units between need not be constant. Therefore, for correlation, the units within a population must be constant, whereas the units of the variables need not be constant. If all the measurements have the same units, as demonstrated in Table 8, one can correlate both attributes and persons.

Four Factor Systems

The reader has just noted the difficulty of correlating persons over attributes unless the attributes have the same units. In an article entitled "The Foundations of Psychometry: Four Factor Systems," however, Stephenson (1936a) demonstrated the following manner by which correlation over originally heterogeneous units may be accomplished using standard scores.

The units of standard scores correspond to the number of standard deviations away from the mean which an individual lies with respect to the particular attribute projected over the entire population of persons. Therefore, if a person has different standard scores on different

TABLE 8

Exemplary Matrix with
Homogeneous Units Throughout

Females	Body Parts 1	2	3	$\bar{i}$	s_i
1	38 in.	24 in.	37 in.	33 in.	7.8 in.
2	36 in.	25 in.	35 in.	32 in.	6.1 in.
3	37 in.	24 in.	35 in.	32 in.	7.0 in.
$\bar{j}$	37.0 in.	24.3 in.	35.7 in.		
s_j	1.0 in.	.6 in.	1.2 in.		

attributes, he is highest with regard to the rest of the population on that attribute of which his standard score is highest. For example, one aspect of vocational guidance is finding those areas in which a person is particularly adept, where he would have a high standard score on some measurable quantity.

In order to correlate persons over attributes with heterogeneous units, one can use as an homogeneous unit the relative ability of the person for each attribute in comparison with his ability on other attributes. Cattell (1951, p. 207) calls this the scale of saliency. Stephenson stated:

> If, then, any list of heterogeneous measurements or estimates can be arranged in an order of some kind, or in a scale, for their representativeness or significance for the individual, they may be held to be made homogeneous with respect to that individual. This ... opens the way to many applications of Q technique ... [Stephenson, 1936c, p. 346].

An individual's standard score for each attribute becomes the raw score for the new units across persons. By a quick glance at Table 11 (p. 22), one can see that these scores can not be standard scores across persons, because the sum of the scores do not necessarily equal 0, nor do the standard deviations necessarily equal 1. One can, then, re-standardize the matrix within each row, where $\overline{X}_{1j} = + .38$, $\overline{X}_{2j} = + .14$, and $\overline{X}_{3j} = -.53$, from which one may correlate rows using the equation:

$r_{hi} = \Sigma z_h z_i / N-1$ (Glass and Stanley, p. 117),

as shown in Tables 9-13.

Similarly, one may standardize the measurements of each individual across his population of attributes, then re-standardize these scores for each attribute across the population of persons, as shown in Tables 14-18.

There are, then, four systems of correlation, which Stephenson (1936a) named as follows:

1. System 1. Variates of attributes or tests, etc., entered in the matrix for standardization with respect to the population of persons.
2. System 2. Variates of persons entered into the matrix for standardization with respect to the population of attributes, etc.
3. System 3. Variates which have been standardized as at (1) entered into the matrix and re-standardized by way of system (2).
4. System 4. Variates which have been standardized as at (2) entered into the matrix and re-standardized by way of system (1).

Stephenson stated the following application of this terminology:

> Systems (1) and (4) are what I called r-technique elsewhere, since they deal with the correlations (r) between tests or separate traits etc., ... Systems (2) and (3) have been called Q-technique, their

<u>Process for Re-Standardization of Matrix from Population of Persons to Population of Tests</u>

Step 1.

TABLE 9

Matrix with Population of Persons

Persons	Tests 1	2	3
1	90	80	85
2	80	90	70
3	70	85	75
$\bar{X}j$	80	85	75
s_j	10	5	8.66

Step 2.

TABLE 10

Matrix with Standard Scores over Population of Persons

Persons	Tests 1	2	3
1	+1.00	-1.00	+1.15
2	0.00	+1.00	-.58
3	-1.00	0.00	-.58
$\bar{X}j$	0.00	0.00	0.00
s_j	1.00	1.00	1.00

Step 3.

TABLE 11

Matrix of Standard Scores Used As Raw Scores

Persons	Tests 1	2	3	$\bar{X}_i$	s_i
1	+1.00	-1.00	+1.15	+.38	+1.20
2	0	+1.00	-.58	+.14	+.80
3	-1.00	0	+.58	-.53	+.50

Step 4.

TABLE 12

Matrix of Re-Standardized Scores

Persons	Tests 1	2	3	$\bar{X}_i$	s_i
1	+.52	-1.15	+.64	0	1.00
2	-.18	+1.08	-.90	0	1.00
3	-.94	+1.06	-.10	0	1.00

Step 5.

TABLE 13

Correlation Matrix of Re-Standardized Scores

Persons	Persons 1	2	3
1	1.00	-.96	-.89
2	-.96	+1.00	+.70
3	-.89	+.70	+1.00

Process for Re-Standardization of Matrix from Population of Tests to Population of Persons

Step 1.

TABLE 14

Matrix with Population of Tests

Persons	Tests 1	2	3	$\bar{i}$	s_i
1	90	80	85	85	5
2	80	90	70	80	10
3	70	85	70	75	8.66

Step 2.

TABLE 15

Matrix with Standard Scores over Population of Tests

Persons	Tests 1	2	3	$\bar{i}$	s_i
1	+1.00	-1.00	0.00	0.00	1.00
2	0.00	+1.00	-1.00	0.00	1.00
3	-.58	+1.15	-.58	0.00	1.00

Step 3.

TABLE 16

Matrix of Standard Scores Used as Raw Scores

Persons	Tests 1	2	3
1	1.00	-1.00	0
2	0	+1.00	-1.00
3	-.58	+1.15	-.58
$\overline{X}j$	+.14	+.38	-.53
s_j	.80	1.20	.50

Step 4.

TABLE 17

Matrix of Re-Standardized Scores

Persons	Tests 1	2	3
1	+1.08	-1.15	+1.06
2	-.18	+.52	-.94
3	-.90	+.64	-.10
$\overline{X}j$	0	0	0
s_j	+1.00	+1.00	+1.00

Step 5.

TABLE 18

Correlation Matrix of Re-Standardized Scores

	Tests		
Tests	1	2	3
1	+1.00	-.96	+.70
2	-.96	+1.00	-.89
3	+.70	-.89	+1.00

> concern being with correlations (Q) between persons or whole aspects of persons, ... [Stephenson, 1936a, p. 198].

Stephenson claimed that one must:

> ... distinguish rigidly between the methodological bases of r- and Q-techniques. Laws in r-technique derive from large numbers of cases (persons) and from individual differences; laws in Q-technique derive from work on one person only, and are independent of individual differences [Burt and Stephenson, 1939, p. 273].

This is due to their difference in measurement scales.

Measurement Scales

The correlation of attributes uses a normative measurement scale. A normative scale has a sample of a population of persons distributed about the mean score of an attribute, the distribution of persons for the attribute relying upon the individual differences between persons for its variance. Each attribute has a separate scale. Correlations using a normative scale are therefore concerned with fundamental tendencies in a general sense for the population of persons. For example, one may deduce that, in general, a person with a long right index finger, in comparison with the rest of the population of persons, may also be expected to have a long left index finger.

The correlation of persons uses an ipsative measurement scale, a term coined by Cattell. "The term ipsative (Latin = he, himself) is ... a convenient one for designating scale units relative to other measurements on the person himself [Cattell, 1944, p. 294]." An ipsative scale has a sample of a population of a given individual's attributes distributed about the mean score of the attributes. The distribution of the attributes relies upon intra-individual differences of the given person for its variance. According to Burt, Stern called the two correlation approaches the "study of inter-individual and intra-individual variability [Burt, 1937, p. 60]." Each person has a separate scale for intra-individual variability. Correlations from ipsative measurements therefore compare specific and entire persons with regard to the attributes measured, neglecting the means of the individuals. One might, for example, correlate the attributes of body measurements of the August centerfold of Playboy Magazine with that of July, deducing that where Miss August has certain particularly large body parts in comparison with the rest of her body, one might expect, in general, largeness on the same body parts from Miss July.

Methodological Approaches

These different measurement scales leads to what Stephenson describes as "Two Methodological Approaches" (Stephenson, 1936a, p. 200). Stephenson views the comparison of entire persons, with respect to the population being examined, as exhibiting a Gestalt philosophy of inquiry. Using an example of body measurements, he stated:

> (Q) begins from the standpoint of the whole body-proportions of the person, and it can embrace all possible relations of the physical person in this respect. In a sense it can engulf the "total situation," for it takes all the physical measurements into account: but it always neglects the absolute features of all these physical measurements [Stephenson, 1936a, p. 200].

He viewed the correlation of attributes, on the other hand, as exhibiting an atomistic or elemental mode of inquiry, in that it compares the parts comprising the whole person.

Stephenson saw the comparison of persons in a Gestalt manner as having implications for typology. He explained that "a person has components, of traits and the like, disposed in a certain fashion; persons with similar dispositions will be of a type, ... [Stephenson, 1936b, p. 357]"

A typology examined by Q may be viewed as a syndrome. A psychopathic syndrone, for example, as described by McCord and McCord (1970), consists of such attributes as: being asocial, aggressive, impulsive, feeling little or no guilt, unable to form lasting bonds of affection, and possessing pleasing external manners. By using a Gestalt approach through Q an experimenter can encompass all the attributes of the syndrome into a single variate, thus measuring a specific individual over the total situation of the syndrome. An atomistic approach, in R however, can only examine one attribute of the syndrome in any given variate, and two attributes in any given correlation, expressing results in a general sense for a population of persons.

Stephenson further explained:

> Persons who correlate together, or are saturated with the same factor in Q-technique, obviously are alike in some respect. In a sense they approximate to a type. A factor can be regarded as a hypothetical 'typical person,' as shown by the system of correlations and the saturation of a person with this factor is merely a measure of the extent to which he resembles this hypothetical type. By Q-technique we can determine types of persons; ... [Stephenson, 1936b, p. 356].
>
> ... there are possibly only a few fundamental tendencies in the human being, and therefore only a few unitary traits are born out of system (1) analysis. But there are possibly millions of types, each a common factor (common that is to several or many persons, but not necessarily to all), in system (2) analysis. Type psychology comes into its own by way of system (2) [Stephenson, 1936a, p. 209].
>
> Factors in Q-technique will usually not be universal. One person may have one factor, and another have quite a different one. As Stern (1935) has put it, not everyone is a pick-pocket, and the traits of a pick-pocket may not be the same in system as those of the rest of us who are free from this manual deftness [Stephenson, 1936b, pp. 357-58].

To Stephenson, the thrust of Q, then, is the implications, the avenues that it opens for research. Besides type psychology, he stated that "The whole domain of psychology, ..., is opened for the technique to explore: introspective psychology in general, our sentiments, motives and ideals [Stephenson, 1935, p. 19]."

Q Sort

The manner which Stephenson devised to open research to introspective psychology was the Q Sort. In this respect the Q Sort must be regarded as one of the most important aspects of the methodology.

In the Q Sort individuals are given a number of statements or adjectives placed on individual cards and instructed to sort the cards into a continuum of piles according to the given condition of instructions. For example, the individual may be given a list of adjective personality descriptions with instructions to sort the descriptions into piles, per Table 19, from those least descriptive to those most descriptive of himself. The frequency or number of statements to be put into each pile is similarly pre-determined by the experimenter. The frequency distribution generally approximates normality. This sorting procedure differs from rank-order procedure in that one need rank the statements only to the degree necessary to sort into the specified piles.

Each pile is given a numerical score, as exemplified in Table 19, with each statement scored according to its placement, such that the raw score for each attribute is the numerical score of the pile in which it is placed. The units are the degree to which the sorter

TABLE 19

Exemplary Q Sort Distribution

	Least Significant										Most Significant
Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	2	4	7	10	12	10	7	4	2	1

[a]Data in this Table from Stephenson, 1935, p. 21.

believes the attribute to be descriptive of himself, or whatever else the conditions of instruction demand. Stephenson's general term for this unit under any given condition of instruction is the "significance" for the sorter. He stated that: "No matter how heterogeneous the units may be in rows, they can be rendered homogeneous with respect to the individual, by arranging them in an order of some kind for their representativeness or significance for the individual concerned [Stephenson, 1936b, p. 354]." These scores may then be correlated with a Sort of the same attributes made by another person, thus correlating persons via Q.

Stephenson also demonstrated that one may use Analysis of Variance within a Sort of an individual if the attributes of the Sort are from a "structured sample." "Structuring a sample consists of composing it artificially, instead of selecting it at random from a parent-universe [Stephenson, 1953, p. 66]." For Q Sorts, the population of statements are divided into sub-groups, with an equal number of statements from each sub-group being selected for the structured sample. The purpose is to test for a difference between the sub-groups. For example, one might test Sutherland's Theory of Differential Association by giving a known hardened criminal a Sort with a structured sample composed of one sub-group of statements favorable to the violation of law another sub-group

group of statements unfavorable to the violation of law. He might be given instructions to sort the statements according to the degree to which he agreed with the statements. A sample may be structured for one- or multi-way analysis, based upon the usual statistical considerations.

CHAPTER III

PERCEIVED CAUSE OF DATA MATRIX CONTROVERSY--TRANSITION OF Q DEFINITION BY STEPHENSON

The writer has largely presented to this point a non-controversial historical background of the development of two-space data matrices. This has been the textbook background before stepping into the real world of controversy. It is the purpose of this chapter to present an insightful historical account of the major cause, as the writer perceives it, of the controversy and confusion surrounding two and three dimensional data matrices.

Controversy in Two-Space Matrices

The model of two-space data matrices presented to this point has assumed that the variates of a dimension may become elements of the population of the perpindicular dimension and vice versa. It is the writer's hypothesis that a good deal of the confusion surrounding data matrices was due to a transition by Stephenson in terms of his acceptance of these assumptions and subsequently his definition of Q. Stephenson once asked "for the letter Q to stand for the system adumbrated at the outset [Stephenson, 1952a, p. 206]." The writer shall attempt to demonstrate that Stephenson violated this plea.

<u>Different Concepts of Q</u>

This transition by Stephenson, which he never acknowledged, concerning the definition of Q, resulted in the imprecise terminology of the same terms referring to different concepts. In the words of Mowrer:

> The expression of Q technique, as it has been used since 1935 is highly ambiguous, inexplicit and confusing. It refers, we find, to a number of different concepts and operations which need to be carefully distinguished and independently evaluated [Mowrer, 1953, p. 375].

Cattell found three distinct concepts of Q. He stated that: "Stephenson correlated persons when the measurements were expressed (a) as raw scores, (b) as standard scores, and (c) as self-estimates, e.g. of the order of 'importance' of a series of traits within the personality ... [Cattell, 1944, p. 295]." Briefly, (a) corresponds to System (2), (b) to System (3), and (c) to the Q Sort. Cattell found the three to have different measurement scales; (a) ipsative, (b) the re-standardization of normative, and (c) what he terms "solipsistic." While both ipsative and solipsistic have a population within the individual, the nature of raw data of ipsative and normative measurements is what he terms "behavioral," containing "dimensions of the external world," while the raw data of solipsistic measurement is "introspective," containing "dimensions of consciousness." In 1951, Cattell differentiated Q using behavioral data from Q using introspective data by designating the latter Qs, terming it "subjective Q" [Cattell, 1951, p. 204].

Not all writers, however, use the terms ipsative and solipsistic in a mutually exclusive sense. Kerlinger (1958), for example, designates all variates involving individual differences as using normative measurement and variates involving intra-individual differences as using ipsative measurement. Thus, solipsistic variates would be a subset of ipsative variates. This terminology shall be used throughout the course of this study, as the writer believes it to be the more widely held usage.

Reliability and Validity of Introspective Data

Although he has since reconsidered, Cattell did not initially grant validity to introspective data, stating that: "... introspective data cannot be truly integrated into scientific psychology [Cattell, 1952, p. 513]." According to Friel:

> The concept of validity deals with the question, 'What is being measured?' ... . one cannot determine the validity of a measuring system instrument until one has determined its reliability and the reliability is known to be substantial [Friel, n.d., p. 40].

Cattell objected to introspective data:

> ... taking as data that (which) cannot be witnessed by any second observer. In other words ... we cannot get a reliability coefficient calculated between data obtained by two distinct observers [Cattell, 1952, p. 513].

Cattell means scorers by observers, such that no matter how many experimenters give a given person a task involving introspective data, there can exist but one scorer, the person introspecting, for any variate. There can therefore be no test of inter-rater reliability, although test-retest reliability seems to be a plausible alternative. Frank (1956), in fact, found a test-retest reliability for Q Sorts of between .93 and .97.

The problem of reliability is not the only problem facing the validity of introspective data. Cattell's very argument in a section entitled "The Limitations of Solipsistic Continua" tips us to what the writer perceives to be the reason Stephenson saw introspective data as legitimate through Q. Cattell argued:

> Now, whether, measurement is in terms of behavior or of consciousness, the mathematical psychologist has a right to demand, as a condition of measurement in any reasonable sense of the word, (a) that the measurements shall lie in a single continuum, and (b) that this continuum shall be the same for all subjects. ... .
>
> Returning to Stephenson's ... self-reported variety of Q technique, we see that in fact it does not make the second of the above assumptions. ... indeed, the object of the correlation study is to discover how the divers directions of these continua in different persons are related [Cattell, 1944, p. 297].

The writer contends that assumption (b) should read: "That this continuum shall be the same for all elements of the population." Cattell is really arguing for the homogeneity of data throughout a population, in order to legitimate the statistical analysis and determine what is measured. We have previously demonstrated that the units between variates need not be constant for a correlation, such that the variates need not lie on the same continuum. The continuum, then, refers to each variate, and the continuum need be the same for all person subjects only in normative measurement. Stephenson seemingly believes that whatever basis of introspective continuum the individual uses in a Sort, the continuum would be the same throughout the Sort for that person.

Cattell, referring to the first assumption that the measurements lie on a single continuum, stated: "When subjects are asked to rank twenty diverse personality traits (abilities, sentiments, temperamental tendencies) with respect to 'importance for one's personality' one may well doubt whether such a dimension exists, or whether the subjects have any meaning of rating it if it does [Cattell, 1952, p. 297]." Simply, Stephenson believes that the statements form such a dimension and that sorters could rate it in a modified rank-order fashion. The writer believes that Stephenson saw Q as thus meeting these assumptions for introspective psychology, therefore legitimizing introspective psychology through Q.

Concerning the validity of introspective data, Cattell pointed additionally to the problem of interpretation of language, such that each subject and the experimenter may each have unique concepts of the item being scored and the conditions of instruction. The writer might add that for Sorts, which are ranking procedures, a difference of interpretation of any item also affects to a degree the scoring of every other item.

Furthermore, another problem with the validity of introspective data is that one can never be certain of the honesty of the subject. In particular, experimenters using prison inmates as scorers may expect some scorers to score according to what they believe casts them in the most favorable light. Stephenson was apparently not bothered by this problem, as he stated: "Clearly, no one is suggesting that we have to believe a persons self-descriptions: But it is a simple matter to show whether or not what he says is consistent and easy to show whether other regularities are to be observed for it as well, as when he says about himself correlates with what others say about him [Stephenson, 1961, p. 16]."

Stephenson found the problems of reliability and validity for introspective data to be trivial matters. Referring to Q-applications, Stephenson stated:

> ... nor is the concern ever with validity, reliability or other categorical presumptions. Storms are raised about these trivialities, but the reduction of all measurement to pure numbers -- thus freeing our science of countless individual units and countless dimensions -- is not even noticed or given a single glance. So the shoe is admired and the foot ignored [Stephenson, 1961, p. 15].

Stephenson apparently does not understand that numbers, pure or otherwise, can not be more meaningful than their validity. If it is unknown what a number measures, the number is useless. It is inconceivable to the writer how the questions of reliability and validity of measurements can ever be trivialities if the applications of the measurements are purported as other than trivial. Stephenson, in any event, saw the problems of introspective data largely solved by the Q Sort, or at worst trivialities compared to the possibilities opened by the Sort.

Q Synonymous with Sort

As the writer sees it, Stephenson was so enthralled with the Q Sort that the Sort became to him synonymous with Q, and he would not admit that he had ever envisioned a Q using behavioral data. It is revealing to compare portions of an article of Stephenson in 1952 with that from his book in 1953.

In 1952 he listed among principles for Q: " ... (a) The populations are statements, traits, or the like; (b) variates refer to operations of a single person, or about him, in one interactional setting; ... [Stephenson, 1952b, p. 484]." In 1953, he listed among Q-technique postulates:

> 1. The populations are groups of statements or the like. (2) Each variate has reference to an operation of a single person upon all the statements in one interactional setting [Stephenson, 1953, p. 58].

It is evident that in 1953 Stephenson no longer considered traits as possible populations for Q, even though they were charter members. Such was the result of the transition, but it was an evolving process.

<u>Precipitating Cause of Transition--The Reciprocity Principle</u>

The writer perceives the precipitating cause of the transition to be an article by Burt in 1937 in which he introduced an hypothesis termed the "reciprocity principle." This principle held the factors obtained by the two methods of correlation to be virtually identical, other than general factors, which Burt regarded as meaningless. Burt claimed that: "It seems clear, ..., that the two lines of approach should lead to consistent, and in the end, to identical conclusions [Burt, 1937, p. 60]." He submitted a mathematical proof demonstrating the fact.

Stephenson, however, claimed that "... Burt's algebraic proof is vitiated by untenable assumptions, and further, that even if it were true, the deductions drawn would be useless, since it would follow that it was waste of time to correlate persons as well [Burt and Stephenson, 1939, p. 278]." The "even if it were true" aspect seems to be vitiated by irrelevancy in terms of a defense for Stephenson. The procedure rendered useless clearly would not be the methods nor proof of Burt, but rather the methods of Stephenson. Since Burt viewed the factoring procedures of R and Q alternative for any matrix, there would be no reason to correlate persons "as well," for one may correlate either persons or tests. Stephenson, however, claimed the factors to be independent for the two techniques, and only in this case would it be useful to correlate persons "as well" in the first place.

Due to the reciprocity principle, Burt, in any event, did not view Q as "a complete inversion of all previous factor techniques." A joint article by Burt and Stephenson stated:

> The general character of (our) differences may be summed up by saying that Stephenson insists on a sharp opposition between r-technique and Q-technique, whereas Burt would regard them as involving much the same aims, methods, and theorems, and in principle, ... , as merely alternative ways of analyzing any rectangular table of figures [Burt and Stephenson, 1939, p. 274].

This same joint article tips us to the fact that Burt's review introducing the reciprocity principle was far from satisfactory to Stephenson. This article informs us that:

> (Burt's) conclusions implied certain tacit assumptions which seemed so untenable to Stephenson that, on reading the draft in manuscript, he considered that less confusion would be caused if its publication were postponed until 'a joint paper could be drawn up, either that we could all agree upon, or else giving our own versions of what we each felt we have done.'*
> *-Eventually it was agreed upon that this particular paper, dealing with the alleged reciprocity between correlating persons and traits, should be published; Stephenson's reply was added in a postscript to his article in this journal [Burt and Stephenson, 1939, pp. 270-271].

The article referred to here is "The Foundations of Psychometry: Four Factor Systems" in the journal Psychometrika, in which Stephenson's postscript reply was in the form of a "note added." Since the main section of the article was written before the transition, in terms of actual publications, Stephenson's transition began in the middle of the article.

Driving Force of Transition--Concept of Independent Matrices for R and Q

The driving force of the transition, as the writer sees it, was a counter to the reciprocity principle in the form of an argument that the factors in R and Q could not be identical because in actuality R and Q were from different matrices. By 1953

Stephenson, referring to Babington-Smith, was able to state:

> ... we find one of our most recent critics writing as follows:
>
> > ... If R-technique is concerned with analyzing the correlations or variances between columns in a matrix of scores, then Q-technique is concerned with analyzing the correlations of variance between rows. That is to say, where R-technique is concerned with the relationship between tests, Q-technique is concerned with those persons who take these tests ...
>
> One can merely say about this that it is in flagrante delicto. There never was a single matrix of scores to which both R and Q apply. Naturally, with such an Aunt Sally, our critic 'can see no reason for the controversy' that has developed 'round this complementary procedure' [Stephenson, 1953, p. 15].

This is adding the insult of being called an Aunt Sally to the mental injury of having believed Stephenson in the first place.

A proponent of Stephenson, Brown, is of the opinion that much of the confusion over Q is due to the fact that "Stephenson's stand ... has always been in terms of two data matrices, but all of those who have come to conclusions contrary to his own have used a single matrix [Brown, 1972, p. 58]." We shall attempt to demonstrate with the following evidence that Brown's analysis of Stephenson's stand "is in flagrante delicto." We shall attempt to demonstrate that before the publication of Burt's reciprocity principle, Stephenson held the belief that R and Q may exist in one and the same matrix.

In a section entitled "The Matrix of Data" from an introductory article titled "The Inverted Factor Technique," Stephenson presented the exemplary matrix reproduced in Table 20, and qualified:

> $_aX_1$ is the score obtained by a person a in test 1; and in general $_xX_y$ is the score of person x in test y. Let the scores be standardized in columns. Previous factorists have constructed their theorems about the correlations existing between columns of the above matrix, that is, about the correlations of the kind r_{12}.
>
> If, however, the scores in the matrix can be standardized in rows, we shall have correlations between persons, such as between persons a and b. Following Prof. G. H. Thomson's suggestion, I shall use Q as the sign for correlations between persons, so distinguishing them from correlations such as r_{12} between two tests. Q_{ab}, then, is the correlation between two rows in the matrix, for persons a and b [Stephenson, 1936c, p. 345].

Later in the article, in a section entitled "Experiment No. 4. Complementary r and Q Analysis," Stephenson stated:

> ... all adequate data previously used for r technique alone can be refactorized by way of Q technique. Nor is any analysis of data complete until it is examined in both ways, by r and Q technique [Stephenson, 1936c, p. 360].

In the main section of the divided Four Systems article, immediately before the perceived precipitating cause of transition, Stephenson demonstrated the possibilities of the four systems in a body measurement

TABLE 20

Stephenson's Original Q Matrix

	Tests							
Persons	1	2	3	4	...	....	....	M
a	${}_aX_1$	${}_aX_2$	${}_aX_3$	${}_aX_4$	...	....	....	${}_aX_M$
b	${}_bX_1$	${}_bX_2$	${}_bX_3$	${}_bX_4$	...	....	....	${}_bX_M$
c	${}_cX_1$	${}_cX_2$	${}_cX_3$	${}_cX_4$	...	....	....	${}_cX_M$
...	...	...	...					...
...	...	...	...					...
N	${}_NX_1$	${}_NX_2$	${}_NX_3$	${}_NX_4$				${}_NX_M$

*Data in this table from Stephenson, 1936c, p. 345.

example using one and the same matrix. He stated that: "There can be no doubt that any data can be examined by either system (1) or (2) [Stephenson, 1936a, p. 203]." (Recall that system (1) is the correlation of attributes and system (2) is the correlation of persons.) His first entry in the "Conclusion" section stated:

> The matrix of persons (row) and attributes (column) used by factorists can be employed in more ways than the one used almost exclusively in past factor analysis and psychometry generally. Four ways have been described and illustrated [Stephenson, 1936a, p. 205].

There simply can be little doubt from these examples that up to this point Stephenson believed that r and Q may come from the same matrix.

In the postscript of the same Four Systems article, however, immediately after the perceived precipitating cause of transition, i.e., the publication of the reciprocity principle, Stephenson stated:

> The important systems are (1) and (2), the other two being of minor immediate interest. I have clearly stated that systems (1) and (2) can be quite independent, but it will be better to state explicitly that I regard the two as, by very definition, statistically independent of one another in general. ... .
>
> System (2) in general is not supposed to be regarded as the direct observe or mere transpose of data already analyzable by way of system (1); nor is it the case that these systems are merely complementary ways of analyzing one and the same

> original matrix of data, the results being no less complementary or deducible one from the other. Such complementary conditions, and the possibility of direct analysis either in rows or in columns, is only possible under the special case of universality of unit. ... .
>
> In the general case, however, the data is by very definition distinct for analysis by way of system (1) or (2) respectively. ... .
>
> ... . But even if one and the same unit is used throughout, for all persons and all attributes, the respective standardizations to which the data is submitted (in effect) when either rows or columns are correlated, radically change the original data. In effect, therefore, one is not dealing with the same data in the two systems, for factor analysis only concerns standardized material and not crude data. It would have been better to have stated this explicitly or more emphatically [Stephenson, 1936a, pp. 206-207].

It is obvious that Stephenson is changing from viewing R and Q as aspects of the same matrix to originating from separate matrices. There are three prongs of that change in evidence; a), downgrading the importance of re-standardization, b), stressing the general non-universality of unit of systems (1) and (2), and c), implying separate matrices because factor analysis is only concerned with standardized material. We shall examine each of these prongs.

a). Stephenson downgrades re-standardization, i.e. systems (3) and (4), which unite R and Q in any matrix, by calling it of minor importance. This is the beginning of a phasing out of (3) and (4) in R and Q. By 1953 Stephenson would state:

> (the premises) for systems 3 and 4 always involve

> doubly standardized data. Those for R- and Q- methodologies, instead, are for data standardized in one way only [Stephenson, 1953, p. 57].

Thus, while Stephenson originally defined systems (2) and (3) as aspects of Q and (1) and (4) as aspects of R, R and Q became insteads of (3) and (4).

b). By phasing out re-standardization from R and Q, only matrices with universal units could be analyzed by both R and Q. In the postscript note, he stressed that this is the exception rather than the rule, such that without systems (3) and (4) one would generally not be able to analyze a matrix by both R and Q.

c). Theoretically, then, Stephenson's only remaining problem would be the case of universal units, which he attempted to overcome by claiming that factor analysis concerns only standardized data rather than crude data. This argument seems particularly weak and contrived, as standardized data, after all, is but a subservient dependent and an algebraic manipulation of crude data. Furthermore, Burt demonstrated the reciprocity of a matrix using raw data, and Stephenson seems to be begging the question by claiming factor analysis to be unconcerned with crude data.

The writer takes exception to Stephenson's statement that it would have been better to have stated the "standardized data only" hypothesis explicitly, because the implication is that he always believed as such. Considering his previous articles, however, the writer can not accept this implication.

In the joint article of Burt and Stephenson, Stephenson presented another tactic with the similar purpose of implying independent matrices by denying raw data the use in both R and Q. He stated that: "The view put forward by Burt and accepted by Vernon, namely, that the self-same traits can change in the twinkling of an eye into chameleon-like items of a statistical population when correlating persons, appears to Stephenson a gratuitous assumption [Burt and Stephenson, 1939, p. 276]." Stephenson, himself, seems to have changed "in the twinkling of an eye." As Burt noted, Stephenson's "original exposition of Q-technique was based on this very assumption [Burt, 1940, p. 188]." Stephenson, then, was attempting to push onto Burt what he considered the gratuitous assumptions which he had made four years earlier. Burt, who had in fact done work on the subject previous to Stephenson, was never given credit for these assumptions in Stephenson's articles portraying the assumptions as cause for a new technique.

The writer shall present one more quotation of Stephenson concerning the question of independent matrices, in order to demonstrate his concept of R and Q after the transition, while presenting further evidence of his lack of credibility. The reader shall find, additionally, that Stephenson has redefined the four factor systems, resulting in the elimination of all ipsative measurement other than solipsistic from the systems.

In a section entitled "The Matrices of Data for R and Q" from <u>The Study of Behavior</u>, Stephenson stated:

> We ... wrote a paper for Psychometrika (1936a) called "Foundations of Psychometry: Four Factor Systems," ... defining two independent systems or matrices of data, one for R and the other for Q, upon which to base any discussion of the matters at issue. In R, individual differences, with all their assumptions, warranted or not, are basic to all else. In Q, intra-individual 'significances' alone are postulatory, replacing the role of individual differences completely. These matters were clearly stated originally, but our critics Burt, Thomson, Cattell, and recently Babington-Smith have continued to suppose that only one matrix is ever at issue, involving individual differences either directly, indirectly, or fundamentally, which looked at down its columns is R, and along its rows is Q.
>
> The four systems initially defined in the 1936 paper were as follows:
>
> System 1.-(r). Tests are applied to a sample of persons, and the correlations between the tests are factored. Individual differences are at issue.
>
> System 2.-(Q). Persons are applied to a "sample" of statements or the like, and the correlations between the person-arrays are factored. Intra-individual "significance" is involved.
>
> System 3.- The transpose of 1. Data which

> have been standardized in columns for purposes of 1 are now standardized along the rows, and the correlations between persons are factored. . . . .
>
> System 4.- The transpose of system 2. Data which have been standardized in columns for purposes of 2 are now standardized along the rows, and the correlations between "statements" are factored [Stephenson, 1953, pp. 51-52].

As one views the quotation in terms of Stephenson's credibility and the independent matrices concept, he must say, contrary to Stephenson, that the matters of independent matrices were not "clearly stated originally" in the *Psychometrika* article, because: a) the main section of the article presented the four systems in one matric, and b) the *Psychometrika* article was not his original article on the subject. Earlier articles, such as "The Inverted Factor Technique," clearly presented the possibility of R and Q existing in the same matrix. Furthermore, the four systems were not "initially defined in the 1936 paper" as they were in *The Study of Behavior*. Initially, Q was not limited to statements and encompassed system (3), and R encompassed system (4).

Effect of Transition--Restriction of Q to Introspective Data

The effect of this revised "initial definition" for Stephenson was to limit R to behavioral data using normative scales and limit Q to introspective data using solipsistic scales, such that the two methods could not exist in the same matrix using the same data. One could correlate persons from R data and statements or attributes from Q data only by re-standardization, but Stephenson had eliminated re-standardization from R and Q.

An additional effect of this definition, however, was the elimination of the possibility of ipsative measurement other than solipsistic, not only from R and Q but from all of the four systems. One could not correlate persons over body measurements without re-standardization, or the standardization of this ipsative measurement using these four systems. This led to an inaccurate portrayal by Stephenson of Burt's P-technique, which could not be presented by Stephenson's revised four system definition. In _The Study of Behavior_ Stephenson stated:

> We therefore find the following statement by Burt fairly acceptable about these matters:
>
> (i) Where a project is designed in terms of tests which are applied to a sample of subjects and where the correlations between tests are subjected to analysis, one may talk of R-technique.
>
> (ii) If with the same data, one runs correlations between persons instead of between tests, then one has P-technique.
>
> (iii) The innovation, which Stephenson claims as Q-technique, is to design an experiment in terms of people ... then to assess qualities of performance with respect to each person in turn, and then to make correlations between people.

> The P-technique to which reference is made, is merely our system (3) again; that is, data put together for R-purposes can also be analyzed in the reciprocal manner ... [Stephenson, 1953, p. 16].

Stephenson is simply incorrect, as P-technique is not system (3), but rather system (2) using behavioral data.

The reader can see that Burt does not view P as a re-standardization from the following statement: "... so long as every person is assessed by the same observer, the best and quickest method is still to proceed by correlating traits, and to calculate factor-measurements for the several persons (so-called 'T-technique'); if, however, each person is assessed by a different observer, and particularly if the persons are fewer than the tests or traits, then the direct method is also the most practical, namely, to proceed by correlating persons (so-called 'P-technique') [Burt, 1940, p. 177]." The reason to correlate persons with different observers, rather than correlating tests, is that there is a question of the homogeneity of units of the different observers. For example, a particular marking by one observer on a test may not mean the same thing as the marking by another observer. Thus, to correlate tests with different observers for each person may use heterogeneous units over the population of persons. Therefore, the direct method of proceeding "by correlating persons (so-called P-technique)" can not be a re-standardization of data standardized by tests, hence can not be system (3).

Unique and Common Scaling

As the writer has previously stressed, the question of independent matrices stems, as the writer sees it, from the different definitions of Q. Burt uses Stephenson's initial definition of Q, the correlation of persons over a population of attributes, while Stephenson uses his "revised 'initial definition,'" the correlation of person-arrays over a population of statements. The writer shall attempt to demonstrate that the different definitions yield opposing results on the question of independent matrices.

The crux of the differences, as the writer sees it, lies in the fact that there are really three types of scaling, which the psychometrists accept differently. In the writer's terms, there is; a) scaling which is unique to a particular person, such that each individual measures in a unique manner the elements from a population of attributes, b) scaling which is unique to a particular test, such that each test measures in a unique manner the individuals of a population of persons, and c) scaling which is common to more than one variate.

Briefly, a) involves solipsistic data, b) involves normative data, and c) involves either normative or ipsative data, depending upon what is correlated.

A matrix composed entirely of similar common scaling yields homogeneous data throughout, and in the words of Burt; "... the distinction between variables and populations ... is an ad hoc rather than ... an absolute distinction ... [Burt and Stephenson, 1939, p. 275]." In this case, R and Q may exist in one and the same matrix. Burt and Cattell each recognized the possibility of common scaling and saw ipsative measurement other than solipsistic as part of Q, and thus accepted the possibility of R and Q occurring in the same matrix.

After his transition, however, Stephenson did not recognize common scaling as he regarded measurement by common scaling "a gratuitous assumption" and the resulting items "chameleon-like [Burt and Stephenson, 1939, p. 276]," leaving only solipsistic data from statements for the correlation of persons. One can not correlate attributes from the solipsistic data of a sort without re-standardization, since a Sort, in effect, is a standardization, as the measurement of each cell can only be determined in regard to other statements, or cells, in the sort. Similarly, although the measurement of any cell in unique scaling may be independent of the other cells, in any given matrix consisting entirely of unique scaling, each data cell may be used in only one variate without re-standardization. As Stephenson had phased re-standardization out of R and Q, in this manner R and Q could not exist in the same matrix using the same data.

Since, however, our body measurement example in Table 8, page 18, in Chapter II exemplifies the possibility of common scaling, the writer must accept the possibility of common scaling. The writer would admit that common scaling is rarer than unique scaling, but this fact is irrelevant to the question of its existence.

Controversy in Three-Space Matrices

This study has largely been restricted, thus far, to an examination of the background and controversy of two-space data matrices. We shall now extend our examination to the development and controversy surrounding three-space matrices.

Covariation Chart of Cattell

To go from two-space to three-space another dimension is needed, and in 1946 Cattell brought major attention to the possibility of three-space by adding the dimension of occasions. He noted:

> It is necessary to stress this, and to point out in particular that, the occasions axis is not a time axis. The occasions of testing need not be separated by equal intervals of time or even be in temporal sequence. The occasions axis is a series of occurrences of test-person events, with some changing external or internal conditions which distinguish the events [Cattell, 1946, p. 96].

There existed in Cattell's model, then, three types of variates, (attributes, persons, and occasions), each of which would use either of the other two as populations, with the remaining referent remaining constant. This gave a possibility of six correlation techniques, which Cattell named as follows:

R--the correlation of attributes over a population of persons on one occasion.

Q--the correlation of persons over a population of attributes on one occasion.

P--the correlation of attributes over a population of occasions for one person (not to be confused with Burt's P-technique, although Cattell termed the technique after Burt's P, recognizing that both involve intra-individual differences.)

O--correlation of occasions over a population of attributes for one person.

S--correlation of persons over a population of occasions for one attribute.

T--correlation of occasions over a population of persons concerning one attribute (not to be confused with Burt's T-technique, although they both use a population of persons.)

As a two-space matrix involving two techniques can be viewed as a rectangle, a three-space matrix involving six techniques can be seen as a cube. Cattell represented the six techniques by what he termed the "covariation chart," commonly referred to as the covariation cube, shown in Figure 2. Each axis represents the universal population of one of the three referents, (persons, attributes, occasions), with each axis neither ordered nor continuous, but rather representing a discrete series. Every measurement of a person can be seen as a cell in this three-dimensional space, with any series of cells parallel to any axis representing a correlatable series. Keeping any referent constant reduces three-space to two-space. Each of the six techniques is represented in two-space with one referent constant, one referent a population of variants, and one referent a population from which to draw a sample series.

There are three unique faces on the cube, each containing two techniques. The techniques are thus paired: R-Q, T-S, and P-O. Cattell believes the

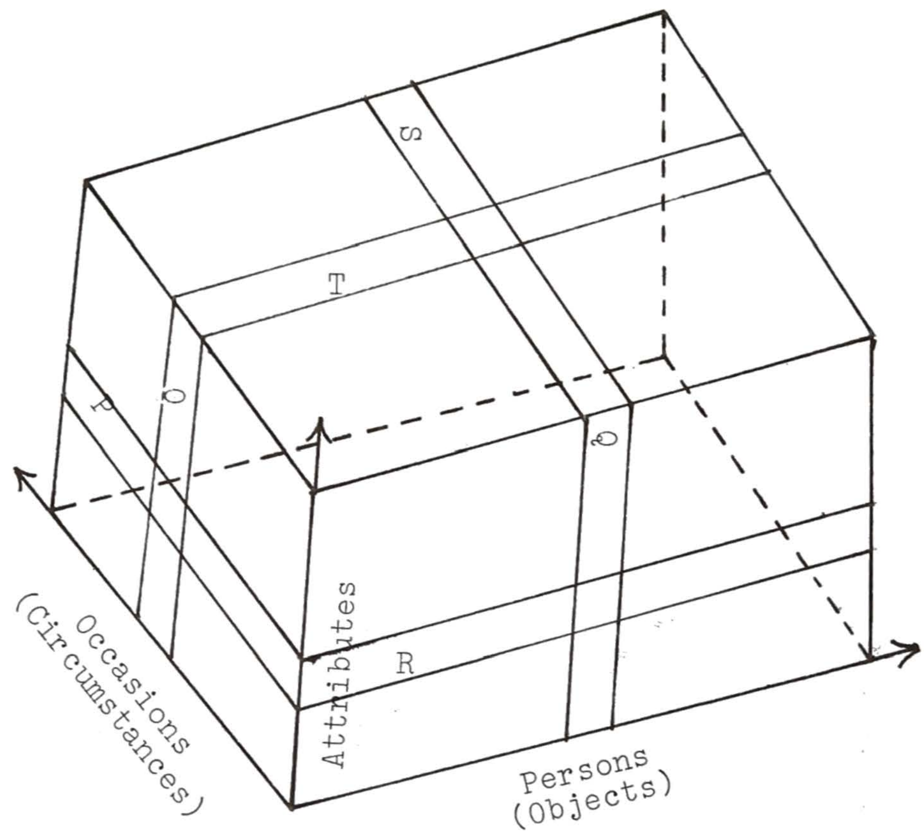

Figure from Cattell, 1951, p. 208.

Figure 2

Cattell's Covariation Chart

reciprocity principle to hold in two-space, such that the two techniques sharing a face share the same data. Just as the reciprocity principle finds identical factors for R and Q, which share the same raw data, so it finds identical factors for the pairs T-S and P-O. Thus, Cattell stated: "... there are basically only three independent factor-analytic experimental designs, namely, R, T, and P techniques [Cattell, 1952, p. 506]." The reciprocity principle does not extend to the six techniques in three-space, as they do not share the same data.

The six techniques of the cube can also be arranged into pairs of techniques sharing edges of the cube. These pairs would be: R-T, Q-O, and P-S. The relationship of the members of these pairs is that any correlatable series of data may be a variate of either of the two techniques sharing the edge on which the series lies. For example, the correlatable series of scores of a sample of persons on a given test on a given occasion, represented in the cube by a line segment, can be used either as a test variate in R or as an occasion variate in T, as the series may be correlated with either the scores of the same persons on another attribute, or the scores of the same persons on the same attribute on another occasion.

A pair of techniques sharing an edge may use the same series of data because the members of each pair use the same measurement scale; R and T use normative scales, having populations of persons; Q and O use ipsative scales, having a population of attributes; and P and S use what Cattell terms abative measurement scales, having populations of occasions. "Abative is, of course, from 'ab'--away--meaning standardization over sets of variances away from those in the main score matrix [Cattell, 1966, p. 116]." In the conventional cube placing the population of persons along the rows, attributes along the columns, and occasions along the files, ipsative scales are across the rows of the cube, normative scales down the columns, and abative scales through the files. Since abative scaling in P and S techniques is concerned with intra-individual differences, using one person per variate, abative scaling is a subset of ipsative scaling.

Reaction of Stephenson to Chart

Stephenson had several points of disagreement with this three dimensional model of Cattell. Stephenson, in fact, had alluded to the possibility of correlating one person on different occasions earlier than Cattell. In 1939, he stated: "The use to which

I have put Q-technique in the present paper accepts the conditions of a person as variable: if need be, for instance, we could vary one individual, testing him when he is drunk, sober, drugged, or depressed, and determining the influence of these conditions on his ability ... [Stephenson, 1939, p. 34]."

In response to the occasions axis of Cattell, Stephenson noted:

> ... we can produce two further schemes, along Cattell's lines, superordinate to his, one called H, and the other D. In the former everyone in Cattell's designs could be in a state of hypnosis, all the measurements being repeated for this condition. In the other all the subjects could be in a state of intoxication (D) or confusion or both [Stephenson, 1952a, p. 206].

The two conditions H and D being internal conditions distinguishing the events implied to Stephenson the needed expansion of Cattell's model by the addition of further axes, as each internal condition can also be measured at given temporal points. Cattell (1966), in fact, did later expand to a ten dimensional model, but the examination of these additional dimensions is beyond the scope of this study. Stephenson, then, saw the addition of an occasions axis as opening a needless source of confusion which could be better handled merely as variate designs within the two axes of persons and attributes.

Stephenson further argued:

> ... in place of the two independent systems R and Q defined in 1935, there are currently appearing many additional letters of the alphabet (P, T, O, and the like, of Cattell). But the latter merely adumbrate variate designs, and only R and Q have the status of methodologies. It is only too easy for the unwary, however, to place these techniques P, T, O, etc. on a par with the basic systems R and Q [Stephenson, 1953, p. 33].

> ... we would like to state categorically that there exist only two factor-systems in the domain of multi-variate analysis, one called R, and one Q, and that these cannot be reduced in a single system. Roughly, the former deals with nomothetic postulates ... , and the latter with idiographic postulates ... . The two are under no circumstances merely two ways of looking at the same facts [Stephenson, 1952a, p. 206].

Nomothetic postulates refer to universal laws, while idiographic postulates apply to a population within the individual. It would seem, then, that R Methodology merely incorporates all variates using individual differences, while Q Methodology incorporates all variates using intra-individual differences.

As one may easily have guessed, another criticism given by Stephenson regarding Cattell's model was the use of Q in the same matrix with R, Stephenson claiming:

> ... (R and Q) are under no circumstances merely two ways of looking at the same facts. Since we claim prior use of the designation of Q, dating back to 1935, it seems ironical for Cattell to borrow the term to perpetuate the very misconception against which Q was directed in the first place. We resist any such peculiar displacements, and would ask for the letter Q to stand for the system adumbrated at the outset [Stephenson,

1952a, p. 206].

"It seems somewhat ironical," although by now redundant, to point out that in fact, Cattell was using the term Q as it had been "adumbrated at the outset," using it in the same matrix with Q, while Stephenson, using R and Q in separate matrices, was not.

This redefinition of Q by Stephenson, furthermore, adds confusion to his claim of the existence of only two methodologies. Although Stephenson inferred the distinction between these methodologies to be the involvement of R with individual differences and the involvement of Q with intra-individual differences, Stephenson's redefinition also limited Q to sorts. However, as demonstrated in Table 21, of the four techniques, O, P, Q, and S, involving the intra-individual differences of ipsative scales, only two, O and Q, are conducive to a Sort. P and S, then, involve intra-individual differences, but are not conducive to a Sort, and P even involves the correlation of attributes. If, as Stephenson contends, R and Q are the only methodologies, then P and S are either hybrids or in limbo.

Stephenson adds further confusion by listing "Cattell's P, T, O, etc." under "R-Variate designs" in a diagram in <u>The Study of Behavior</u> (Stephenson,

TABLE 21

Properties of Cattell's Techniques

Technique	Variable	Population	Measurement Scale	Conducive to Sort
O	Occasions	Attributes	Ipsative	Yes
P	Attributes	Occasions	Ipsative (Abative)	No
Q	Persons	Attributes	Ipsative	Yes
R	Attributes	Persons	Normative	No
S	Persons	Occasions	Ipsative (Abative)	No
T	Occasions	Persons	Normative	No

1953, p. 42). P and O involve individual differences, O is conducive to a Sort, and yet Stephenson listed both under R-Variate designs. Additionally, as noted by Cattell, "Some alleged instances of Q technique are really O technique ... [Cattell, 1952, p. 303]." In fact, in The Study of Behavior, Stephenson, referring to Q-sort arrays, stated: "... our purpose was to correlate and factor analyze such arrays, for different purposes, or for the same persons under different conditions of experiment [Stephenson, 1953, p. 9]." As one would assume different conditions of experiment to include different occasions, Stephenson definitely seems to include a Sort using O as part of Q, yet once again, he listed O under R. Stephenson, then, has not clearly demonstrated R and Q as two consistent methodologies within which all six techniques fall and has brought confusion to the field of data matrices by his inconsistencies.

CHAPTER IV

CONCLUSIONS AND RECOMMENDED MODIFIED MODEL

A major aspect of this study has been the attempt to demonstrate: (1) a lack of standard definitions in the field of data matrices, and (2) that this lack of standard definitions contributed to the confusion and largely to the existence of alternate models of data matrices. It is the purpose of this chapter to attempt to lay a foundation for the acceptance of a standard model by suggesting clear definitions, recommendations, and modifications for a more robust model of statistical data matrices.

Rather than retaining the six techniques of Cattell, or dividing them into the two methodologies of Stephenson, the writer recommends a division into three methodologies, grouped according to the dimension used as the original population sampled. The methodologies would thus consist of the pairing of Cattell's techniques sharing the edge of a cube, i.e., R and T, Q and O, P and S. Subsequently, as may be seen by referring to Table 21, page 67, Chapter III, the properties of measurement scale, conduciveness to Sort, and population would then be consistent within each methodology.

Since the writer has attempted to demonstrate that defining the same terms in different manners has been a source of confusion, rather than designing the new methodologies R, Q, and P, the writer shall define them in the following manner: (a) PPV--the statistical analysis of person population variates, (b) APV--the statistical analysis of attribute population variates, and (c) OPV--the statistical analysis of occasion population variates, respectively.

Grouped in this manner, the distinction between each methodology would not be the two variates of analysis, as is the case with Cattell's techniques, but rather the population of the individual original variate. While each variate may be used in two of Cattell's techniques, when grouping the methodologies by populations each variate can be used in but one original methodology. Furthermore, the definition of Cattell's techniques in terms of the two variates of analysis leads to the possibility of hybrids in Cattell's model. Consider, for example, (a) the correlation over a population of persons, of one attribute on one occasion with another attribute on another occasion, thus mixing R and T techniques, (b) the correlation over a population of attributes, of one person on one occasion with another person on another occasion, thus mixing Q and O techniques, and (c) the correlation over a population of occasions, of one person on one attribute with another person on another attribute, thus mixing P and S techniques. Generally, one may only correlate variates using the same population, and each of these mixed techniques uses a specific population and thus falls nicely into its respective methodology in the population model.

Note that a correlation involving a mixed technique, or a correlation matrix involving the two techniques of the methodology, uses data cells from all three dimensions. This is significant, as a correlation matrix involving all three dimensions may encompass all three of Cattell's plane factor designs. This fact does not indicate, however, whether or not the reciprocity principle holds or whether the same factors can be found using the different methodologies. Using the previous logic of each, Burt, the author feels sure, would argue that the factors would be the same in each methodology; Cattell would see them the same in a matrix covering all dimensions thoroughly; and Stephenson would argue specifically that a methodology with a population of persons could not have the same factors as a methodology with a population of attributes.

Since, however, the writer has accepted the possibility of common scaling, the three population methodologies can exist in one and the same matrix using the same raw data without re-standardization, if common scaling exists through all three dimensions of the particular matrix. Furthermore, a PPV variate generally has common scaling with an OPV variate examining the same attribute or attributes across a population of occasions, and thus PPV and OPV generally may exist in the same matrix using the same data. Only APV, however, is conducive to the unique scaling of a Sort, and in the case of an APV Sort the other two methodologies can not exist in the same matrix without re-standardization.

This population methodological model is additionally an improvement in terms of re-standardization. Since in Cattell's model any variate can be used in either of his techniques which utilizes its population, no given variate is inherently a part of only one of his techniques. Thus, in describing any re-standardization there exists four equivalent descriptions in terms of Cattell's techniques. For example, a variate with a population of attributes re-standardized over a population of persons may be the re-standardization of any of the following: (1) Q into R, (2) Q into T, (3) O into R, or (4) O into T. However, in the population methodological model each re-standardization can be expressed in but one manner, as each variate is part of but one methodology. The previous example would be termed the re-standardization of APV into PPV. Rather than viewing re-standardization as part of the resulting methodology, as did Stephenson in his original four systems, or designating it of minor importance, as Stephenson later did, the writer recommends clearly noting any re-standardization in terms of the original and resulting variate populations, such as the re-standardization of an APV variate into a PPV variate, or APVrPPV.

In the case of an APV Sort, place must additionally be made on the covariation chart for conditions of instruction, distinct from the three axes of persons, attributes, and occasions, since clearly, the results of any given Sort by a given person on a given occasion depend largely on the conditions of instruction. The simple solution is to add an axis for conditions of instruction, the axis becoming meaningful only in APV. Since our basic methodological unit is the variate, we should not be confused by the possibilities of correlating one person under different conditions of instruction, one person on two occasions under different conditions of instruction, or two persons on one or different occasions under different conditions of instruction. These all clearly fall under APV.

If, as the writer has suggested, conditions of instruction is granted an axis, it should be possible to have a variate using a population of instructions, and indeed one can correlate two items over a population of instructions. We must, then, make an additional designation, i.e., IPV, the statistical analysis of instruction population variates. We must note, however, that an IPV variate can only be the result of re-standardization, as the measurement of each data cell in its population can only have meaning in regard to the rest of its elements in a previous APV Sort.

The writer, then, has presented a modified model of data matrices with concise definitions in the hopes of laying a foundation for the acceptance of a standard model of statistical data matrices.

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