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Stochastic analysis of the vibrations of an uncertain composite truss for space applications

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Abstract

A hybrid technique to model the effects of mechanical uncertainties on the structural response of large composite trusses for space application is presented and discussed: the proposed method is based on the Monte-Carlo evaluation of finite element stochastic weighted integrals, which allows decoupling the structural discretization mesh from the stochastic one. A benchmark problem, regarding the modal analysis and the harmonic response of an uncertain composite truss, is studied by means of the proposed method: the full statistics of the truss response variables are calculated by Monte-Carlo based simulations and compared to those obtained by perturbative approximated approaches. The implications of the results here obtained onto the design strategy of structures affected by sensible uncertainty levels, as those made of composites, are discussed.

Keywords: (A) Fibre reinforced plastics (FRP); (B) Mechanical properties; (C) FEA; (C) Probabilistic methods; (C) Statistics

1. Introduction

Within the framework of a classical approach to rational continuum mechanics, it is usually assumed that the mechanical properties and the geometrical configurations featuring the structural members are fully deterministic. Nevertheless actual structures are always affected by uncertainties to some extents: the stiffness and strength modules of actual materials often exhibit a quite wide range of variation. Moreover the actual structural geometry can be slightly different from that assumed for design purposes, due to tolerances in assembling constructive members. Real structures are usually characterised by the following randomness sources [1]:

1. Uncertainties affecting the stiffness and strength modules of structural materials.
2. Uncertainties regarding the actual geometrical configuration of constructions.
3. Uncertainties about the actual constraints applied to the structure itself.
4. Uncertainties regarding the actual modules, directions and temporal dependence of the applied loads.

All the aforementioned uncertainty sources can contribute to deeply alter the structural response, especially regarding to the predictions obtained by a classical deterministic FEM based design. Deterministic design must be always verified by experimental testing. Nevertheless, for engineering purposes, it would be useful to model the potential effect of combined uncertainties on the global mechanical response before the structure itself is built.

The effect of random loads on a deterministic structure, whose materials properties, geometry and applied constraints are deterministic, can be described by classical FEM approaches involving Fourier transformation

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of the governing equations [2]. Nevertheless the basic requirement for analysing random loads within the framework of deterministic FEM is that the applied forces must result ergodic [3]. From a statistical point of view this latter requirement is often too strict, since it implies that the applied loads must be stationary with respect to time. This condition is rarely satisfied by actual applied forces, which are often far from being ergodic. Moreover, any additional source of uncertainties affecting either the mechanical properties or the geometrical configuration of applied constraints cannot be combined with random loads within the framework of a deterministic FEM approach [4].

From the mathematical point of view, the best way to model uncertainties in actual materials and structures is to introduce proper stochastic processes (or random fields), whose distribution and covariance properties are deduced from experimental tests. Statistical characterisation of the mechanical properties of actual materials would require extensive testing investigations, so some simplifying hypotheses must be introduced [5]: according to the central limit theorem, the random fields employed to model structural uncertainties can be assumed [6] to be normal or log-normal, while their auto-covariance structure is exponential. These latter hypotheses strongly simplify the digital simulation of random fields, since it allows reconstructing a stochastic process by the midpoint method [7].

Several stochastic FEM (SFEM) approaches have been presented in technical literature to deal with different uncertainty sources during the structure design stage: the simplest way of modelling this kind of problems relies upon Monte-Carlo brute force simulations [8], where a huge number of possible configurations for an assigned structure are digitally simulated and analysed. The statistics of mechanical response variables are obtained directly by calculating expected values onto the whole set of sample configurations previously generated. The Monte-Carlo method is very expensive and time-consuming from a computational point of view; therefore several approximated techniques [9] have been introduced to predict only a finite number of statistical moments – usually two, the mean value and the standard deviation – featuring the response variables of uncertain structures. These simplified approaches allow to reduce the simulation computational costs, even though they provide only limited statistical information if compared to full Monte-Carlo approaches [10]. As it will be shown later on, these partial data are not always sufficient to develop a reliable design for actual complex structures, thus care is required in employing approximated stochastic analysis techniques.

The main novelty element of this paper is the simulation of the harmonic response of an uncertain truss by means of the WIFEM method [11,12] (acronym for "weighted integrals finite element method") combined with a full Monte-Carlo simulation of the random fields representing the structural mass and elastic stiffness.

2. The WIFEM method for stochastic structural analysis

A key-matter in developing reliable stochastic structural analysis is represented by the choice of a proper discretization mesh: within the framework of deterministic finite element method, the local mesh refinement is essentially dependent on the expected stress gradients. Nevertheless, if the mechanical properties are assumed to be random, we must consider that the size of the "deterministic" mesh must comply with the covariance properties of the stochastic fields introduced. Taking into consideration a one dimensional exponentially correlated random field, this stochastic process can be simulated [13] by a set of discrete random variables having constant values over a sub-domain whose length is one half of the correlation distance. Considering a one dimensional mesh, if the element size were smaller than one half of the correlation length, the spatial fluctuations of the random field would be overestimated; if the element size were greater than one half of the correlation distance, the spatial variance of the stochastic process would be underestimated. Therefore the correlation length of the random fields provides a fixed scale for the allowable size of the finite elements: therefore the "geometric" mesh and the "stochastic" one are not independent. The WIFEM approach to stochastic structural mechanics provides an easy way to overcome the problems related to the coupling between the allowable mesh size and the random field correlation distances. Let us assume to deal with a material having random mechanical properties, which constitutes a three dimensional linearly elastic solid. Following the derivation of standard FEM approach, the stiffness matrix of a finite element constituting the solid can be expressed as

$$\underline{\underline{K}}^{(e)} = \int_V \underline{\underline{B}}^{\mathrm{T}}(\underline{x})\underline{\underline{C}}\,\underline{\underline{B}}(\underline{x})\,\mathrm{d}V, \tag{1}$$

where V is the solid volume, $\underline{\underline{B}}(\underline{x})$ is a matrix containing the shape functions derivatives and $\underline{\underline{C}}$ is the Cauchy's constitutive tensor. Since the properties of the material are assumed to be random, without loss of generality, we can assume that the Cauchy's tensor itself can be split into a mean part and a random fluctuating term

$$\underline{\underline{C}} = \overline{\underline{\underline{C}}}(\underline{\underline{I}} + \underline{\underline{X}}), \tag{2}$$

where $\overline{\underline{\underline{C}}}$ is the mean Cauchy's tensor, $\underline{\underline{I}}$ is the identity matrix and $\underline{\underline{X}}$ is a matrix of suitable stochastic random fields. Following Eq. (2), also the stiffness matrix in Eq. (1) can be split into a mean part and a random one as

$$\underline{\underline{K}}^{(e)} = \int_V \underline{\underline{B}}^{\mathrm{T}}(\underline{x})\overline{\underline{\underline{C}}}\,\underline{\underline{B}}(\underline{x})\,\mathrm{d}V \qquad \Delta\underline{\underline{K}}^{(e)} = \int_V \underline{\underline{B}}^{\mathrm{T}}(\underline{x})\overline{\underline{\underline{C}}}\,\underline{\underline{X}}\,\underline{\underline{B}}(\underline{x})\,\mathrm{d}V. \tag{3}$$

The integrand in the second right hand side equation can be rearranged as

$$B_{ki}\overline{C_{kl}}X_{lm}B_{lj} = c_{ij}^{lm}x^p y^q z^r X_{lm}, \tag{4}$$

where c_{ij}^{lm} are deterministic coefficients and the exponents p, q, r are dependent on the index couples (i,j) and (l,m). From Eq. (4) the random part of the stiffness matrix can be rearranged as

$$\Delta K_{ij}^{(e)} = c_{ij}^{lm} I_{im}^{(e)}, \tag{5}$$

where $I_{im}^{(e)}$ is a weighted integral whose expression is

$$I_{im}^{(e)} = \int_V x^p y^q z^r X_{lm}\,\mathrm{d}V. \tag{6}$$

Eq. (6) shows that a weighted integral is the projection of a random field over a functional basis which is directly dependent on the elemental shape functions: the introduction of weight integrals allows obtaining a coordinate independent representation of the random fields here introduced to model the mechanical uncertainties. The weighted integral itself is a simple random variable, constant over the entire element, not a position dependent stochastic process: this means that the introduction of weighted integrals allows avoiding the coupling between the stochastic mesh and the geometric one. Similar considerations can be easily introduced for the mass matrix of a solid element, introducing a proper random field to describe the uncertainties featuring the materials density.

3. Hybrid WIFEM Monte-Carlo approach for modal analysis

As previously underlined, the Monte-Carlo brute force method for stochastic structural simulations is very time and cost consuming. The introduction of weighted integrals allows overcoming the coupling between the size of the geometric mesh and of the stochastic one. This feature contributes to strongly reduce the computational cost of a Monte-Carlo approach based on weighted integrals calculations and improves its geometrical flexibility.

Let us assume to consider the classical eigenvalue problem for normal vibration modes identification. If the structure is uncertain, the mass and stiffness matrix can be split into a mean deterministic part and a random fluctuating one, as well as the corresponding eigenvalues and the eigenvectors, thus yielding

$$\begin{aligned}&(\overline{\underline{\underline{M}}} + \Delta\underline{\underline{M}})^{-1}(\overline{\underline{\underline{K}}} + \Delta\underline{\underline{K}})(\overline{\underline{w}} + \Delta\underline{w}) \\ &\quad = (\bar{\lambda} + \Delta\lambda)(\overline{\underline{w}} + \Delta\underline{w}).\end{aligned} \tag{7}$$

Assuming $\underline{\underline{A}} = \underline{\underline{M}}^{-1}\underline{\underline{K}}$, Eq. (7) can be rearranged in the following form

$$\underline{\underline{A}}\,\underline{w} = \lambda\underline{w}, \tag{8}$$

where all the quantities involved are stochastic. Employing Neumann's expansion [14] we can write

$$\begin{aligned}\underline{\underline{M}}^{-1} &= (\overline{\underline{\underline{M}}} + \Delta\underline{\underline{M}})^{-1} = [\overline{\underline{\underline{M}}}(\underline{\underline{I}} + \overline{\underline{\underline{M}}}^{-1}\Delta\underline{\underline{M}})]^{-1} \\ &= (\underline{\underline{I}} + \overline{\underline{\underline{M}}}^{-1}\Delta\underline{\underline{M}})^{-1}\overline{\underline{\underline{M}}}^{-1} = \sum_{k=0}^{\infty}(-\underline{\underline{P}})^k\overline{\underline{\underline{M}}}^{-1},\end{aligned} \tag{9}$$

where the $\underline{\underline{P}}$ matrix is expressed as

$$\underline{\underline{P}} = \overline{\underline{\underline{M}}}^{-1}\Delta\underline{\underline{M}}. \tag{10}$$

Therefore from (9) we obtain the following expansion for the $\underline{\underline{A}} = \underline{\underline{M}}^{-1}\underline{\underline{K}}$ matrix

$$\underline{\underline{A}} = \sum_{k=0}^{\infty}(-\underline{\underline{P}})^k\overline{\underline{\underline{M}}}^{-1}\underline{\underline{K}}. \tag{11}$$

Moreover the expansion (11) is equivalent to

$$\underline{\underline{A}} = \sum_{k=0}^{\infty}\underline{\underline{A}}_k, \tag{12}$$

where the expansion coefficients are related by the following set of recursive equations

$$\overline{\underline{\underline{M}}}\,\underline{\underline{A}}_k = \Delta\underline{\underline{M}}\,\underline{\underline{A}}_{k-1}. \tag{13}$$

The first term of the expansion (12) is simply

$$\underline{\underline{A}}_0 = \overline{\underline{\underline{M}}}^{-1}\underline{\underline{K}}. \tag{14}$$

Therefore, for an assigned structure, we can calculate the mean parts of the mass matrix and the stiffness matrix by a standard FEM approach and the random fluctuating ones by the WIFEM method. Once these calculations are performed, we can employ the recursive set of Eq. (13), starting with (14), to evaluate the coefficients of the Neumann's expansion (12) for the matrix $\underline{\underline{A}} = \underline{\underline{M}}^{-1}$ underline$\underline{K}$. The expansion itself can be truncated after a suitable number of terms Q: the truncation condition must rely upon the relative magnitude of the expansion terms, as for example

$$\frac{\underline{\underline{A}}_Q : \underline{\underline{I}}}{\underline{\underline{A}}_{Q-1} : \underline{\underline{I}}} < \delta, \tag{15}$$

where δ is a prescribed tolerance regarding to the trace of the expansion terms themselves. Therefore the $\underline{\underline{A}}$ matrix can be calculated for each simulated configuration complying with a prescribed tolerance, which makes possible to take into consideration also random fields featured by large stochastic variations. Once the matrix has been calculated for each simulated configuration of the structure, the corresponding eigenvalues problems (8) are solved and a set of samples of modal frequencies and eigenvectors are evaluated. Finally, a statistical analysis can be performed on the simulation data to obtain the actual distribution of the modal frequencies and displacements for a normalised set of eigenvectors. If S is the total number of configuration generated according to the Monte-Carlo hybrid approach, the mean values and covariance of modal vibration frequencies v_i are expressed as

$$\bar{v}_i = \frac{1}{2\pi}\sqrt{\frac{1}{S}\sum_{k=1}^{S}\lambda_i^{(k)}} \qquad E[v_i v_j] = \frac{1}{4\pi^2}\frac{1}{S}\sum_{k=1}^{S}\sqrt{\lambda_i^{(k)}\lambda_j^{(k)}}. \tag{16}$$

4. Hybrid WIFEM Monte-Carlo approach for harmonic response

The harmonic response of an assigned structure can be evaluated by means of the same simulation technique presented and discussed in Section 3. In this case the matrix representation of the structural problem is the following

$$-\omega^2\underline{\underline{M}}q^* + (1+jG)\underline{\underline{K}}q^* = \underline{Q}^*, \tag{17}$$

where $\underline{q}^*$ are the complex nodal displacements, G is the coefficient of structural damping, $\underline{Q}^*$ are the external harmonic forces and ω the pulsation of these latter. Considering a structural damping proportional to the mass matrix terms implies that all the terms in Eq. (17) are stochastic. Eq. (17) can be rearranged in the following form

$$\underline{\underline{K}}^*(\omega)\underline{q}^* = \underline{Q}^*, \tag{18}$$

where $\underline{\underline{K}}(\omega)$ is the stochastic imaginary structural stiffness. Solving Eq. (18) for complex displacements for a set of discretized values of pulsation requires the inversion of the complex stochastic stiffness matrix. This latter can be achieved by means of the Neumann's expansion, just discussed in Section 3. In fact the vector of unknown imaginary displacements can be expressed as

$$\underline{q}^* = \sum_{i=0}^{M}\underline{q}^{*(i)}, \tag{19}$$

where the expansion coefficients are determined by a set of recursive equation

$$\overline{\underline{\underline{K}}}^*\underline{q}^{*(i)} = \Delta\underline{\underline{K}}^*\underline{q}^{*(i-1)}, \tag{20}$$

where the random part of the complex stiffness matrix is calculated by means of the WIFEM approach. The expansion (19) is truncated to the Mth term, provided that the following conditions is verified

$$\frac{\left\|\underline{q}^{*(M+1)}\right\|}{\left\|\underline{q}^{*(M)}\right\|} < \delta, \tag{21}$$

where δ is a prescribed tolerance. Once the terms of the expansion (19) have been calculated by the recursive Eq. (20) for each simulated configuration, the statistics of magnitude and phase for each nodal degree of freedom can easily obtained. For the mean value of nodal displacements absolute magnitude and phase shift, calculated over N samples, we have

$$\|\overline{\underline{q}}^*\| = \frac{1}{N}\sum_{k=1}^{N}\|\underline{q}^*_{(k)}\| \qquad \bar{\varphi}_l = \frac{1}{N}\sum_{k=1}^{N}\arctan\frac{j\tilde{q}^*_{l\,(k)} - jq^*_{l\,(k)}}{q^*_{l\,(k)} + \tilde{q}^*_{l\,(k)}}, \tag{22}$$

the second equation being written for the lth degree of freedom. Similarly for the standard deviation of displacements absolute magnitude and phase shift

$$\sigma_{q^*_l} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\left\|q^*_{l\,(k)}\right\|^2 - \overline{q^*_l}^2\right)}$$
$$\sigma_{\varphi_l} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left\{\left[\arctan\frac{j\tilde{q}^*_{l\,(k)} - jq^*_{l\,(k)}}{q^*_{l\,(k)} + \tilde{q}^*_{l\,(k)}}\right]^2 - \overline{\varphi_l}^2\right\}}. \tag{23}$$

5. Benchmark problem: an uncertain composite truss for space applications

The theoretical presentation of the hybrid WIFEM Monte-Carlo technique for stochastic structural mechanics, developed in the previous sections, is suitable for modelling a wide set of problems. As an example of this method, let us consider a reticular structure, as sketched in Fig. 1. Each tubular filament wound member has an internal radius of 0.08 m and a thickness of 3 mm and it is assumed to be manufactured by a standard carbon/epoxy composite whose mean density is 1700 kg/m^3. Let us consider an angle-ply lamination sequence for the truss members, having a ±45° characteristic orientation to the beam axis: this laminate is supposed [15] to have a mean Young's modulus of 8.13 GPa, while the shear stiffness is 15.4 GPa. The whole truss is 6 m long and 1 m wide and it has 25 members with the same cross-section: longitudinal and transversal beams are 1 m long, while the diagonal members are 1.4142 m long. The layout of the truss here proposed is typical of reticular structures for space applications: on the international space station, ISS, trusses constitute the primary supporting skeleton for manned modules and solar panels. The truss is supposed to be fixed at the x axis edges; a summary of the tubular beams cross-section properties is presented in Table 1.

No uncertainties featuring either the geometric configuration of the structure or the applied constraints are taken into account, while random mechanical properties for the carbon/epoxy composites are introduced:

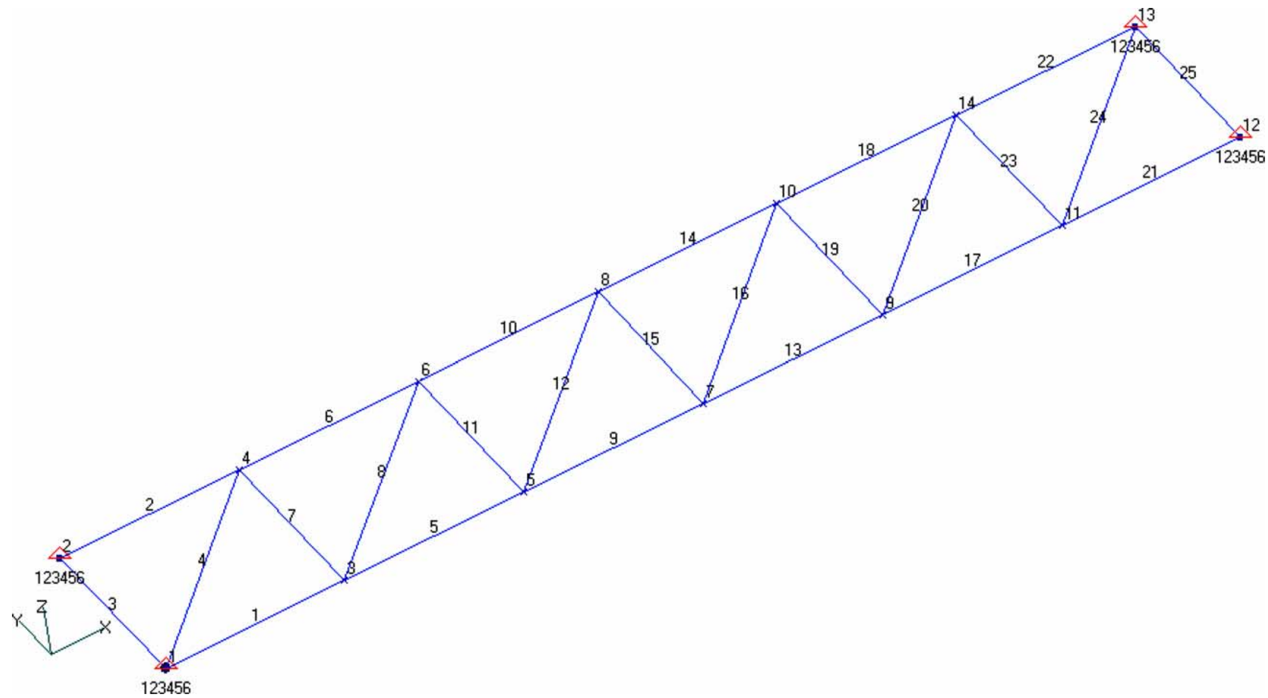

Fig. 1. Geometric configuration of the truss.

Table 1
Cross-sectional properties of filament wound carbon/epoxy tubes

A (m^2)	I_{xx} (m^4)	I_{yy} (m^4)	J (m^3)	A_x (m^2)	A_y (m^2)
1.96×10^{-3}	5.97×10^{-6}	5.97×10^{-6}	1.19×10^{-5}	1.04×10^{-3}	1.04×10^{-3}

A, cross-section area; I_{xx},I_{yy}, inertia moments; J, polar inertia moment; A_x,A_y, shear areas.

we assume that both the beam axial Young's modulus and shear stiffness can be modelled by a Gaussian distributed and exponentially correlated random field. The same assumption is valid for the mass density of the material; statistical independence is supposed to hold between the stiffness and mass random fields.

Basing on Philippidis' experimental results [15], it has been demonstrated that carbon/epoxy filament wound composites can present a sensible dispersion of elastic modules: for a ±45° angle-ply laminate the longitudinal stiffness standard deviation is about 10% of the corresponding mean values. Moreover a variable void content can cause randomness in the composite material density: usually this effect is sensibly smaller than that affecting the elastic modules, since void content can be easily controlled during the manufacturing process. In the following analyses we will assume a fixed 5% variance coefficient for the material mass density.

Since the data reported by Philippidis are limited only to a single material and a single manufacturing process, they cannot be employed directly for general probabilistic design purposes, still they provide a general estimation for expected mechanical properties dispersions: so it is worth to investigate the effects of a finite variation range of the mechanical properties on final structural response variables. The sensitivity of these latter on uncertainties sources can provide further information about the construction performances: thus three different values of Young's modulus and shear stiffness variances have been selected, namely 5%, 10% and 15%; from now on we will denote these variance values, respectively, as case A, case B and case C. Since Philippidis has tested samples 12.5 cm long, the correlation distance for both the mass density and stiffness stochastic fields as been assumed equal to 25 cm. This assumption will lead to a correct reconstruction of the random fields by the midpoint method.

The beam elements have been modelled by standard Hermite's cubic shape functions for bending, and linear shape functions for extension/compression and torsion: each node has six degrees of freedom. A probabilistic WIFEM-Monte-Carlo simulation algorithm has been implemented in the MATLAB environment: the proposed code, denoted as WIFEMTRUSS, performs both the calculation of the deterministic solution and the further statistical analysis. The deterministic part of the code has been validated comparing the results obtained by NASTRAN code numerical solution: a summary of

Table 2
Deterministic modal frequencies for the truss

Mode	NASTRAN: frequency (Hz)			WIFEMTRUSS: frequency (Hz)
	1 element/beam	2 element/beam	4 element/beam	1 element/beam
I	10.168	10.161	10.158	10.162
II	26.719	26.849	26.835	26.913
III	30.929	36.315	37.958	38.721
IV	39.698	39.723	39.738	39.916
V	48.749	50.080	50.088	50.577
VI	55.654	66.194	68.629	70.043
VII	71.013	76.875	77.053	78.732
VIII	75.162	81.584	81.686	83.373
IX	81.532	91.225	93.804	96.153
X	87.550	101.930	102.647	106.193

σ_ξ, Young's and shear modulus standard deviation; σ_η, mass density stan-

the modal frequencies deterministic values is reported in Table 2. The introduction of one single element for each beam member is sufficient to capture accurate values of the deterministic modal frequencies.

5.1. Analysis of natural vibration modes probability distribution

The random fluctuating parts of the mass and stiffness matrixes are calculated by means of the WIFEM method discussed in Section 3. The number of samples which must be generated to obtain accurate results for response statistics depend both on the variance of the random fields and on the geometrical configuration of the structure: the statistical convergence of the hybrid method here proposed must be checked by verifying that at least second order moments have reached a stable asymptotic value after N runs [10]. To validate the stochastic part of the code we have considered a simple composite cantilever 1 m long, whose cross-section and random material properties are identical to those featuring the case B truss beam members. This simpler validation strategy has been adopted since a brute-force Monte-Carlo analysis of the whole truss would have required a huge computational time: in Fig. 2 the convergence of the Monte-Carlo simulations for the single beam example is clearly shown versus the total number of generated configurations (iterates). In Fig. 3 we present a plot of the values of first modal frequency standard deviation versus the variance of mass density and longitudinal stiffness. The three curves there reported are referred to a WIFEM hybrid approach, a brute-force Monte-Carlo analysis and a stochastic Rayleigh–Ritz's method (SRRM) [16]: the agreement of the numerical results is clearly pointed out.

In Tables 3 and 4 the output data for the statistical analysis of the truss natural vibration frequencies are reported: the standard deviations affecting the modal frequencies are increasing functions of the random field variances and they are also dependent on the modes shapes. These latter results can be explained by observing

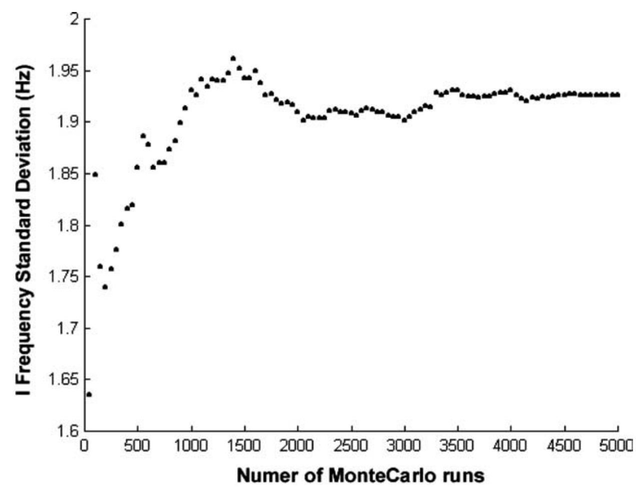

Fig. 2. Convergence of Monte-Carlo simulations for the single beam example.

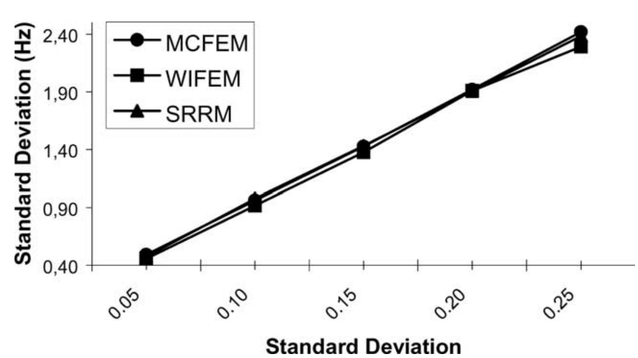

Fig. 3. Standard deviation of the I modal frequency for the single beam example.

Fig. 4, where the whole set of frequencies corresponding to the ensemble of Monte-Carlo runs for case B are presented: due to the uncertainties featuring the mechanical properties of the material, the actual modal vibration frequencies can span wide intervals. As an example, for case B scenario (Fig. 6), the probability of having a third vibration frequency lower than its deterministic counterpart is

Table 3
Mean values of modal frequencies versus uncertainties levels

Mode	WIFEMTRUSS: frequency (Hz)			
	Deterministic	Mean values $\sigma_\xi = 5\%$, $\sigma_\eta = 5\%$	Mean values $\sigma_\xi = 10\%$, $\sigma_\eta = 5\%$	Mean values $\sigma_\xi = 15\%$, $\sigma_\eta = 5\%$
I	10.162	10.162	10.113	10.086
II	26.913	26.841	26.062	25.232
III	38.721	36.954	35.271	34.510
IV	39.916	41.167	42.118	42.668
V	50.577	50.588	50.858	51.553
VI	70.043	70.047	69.974	69.959
VII	78.732	78.698	78.335	77.965
VIII	83.373	83.390	83.944	84.879
IX	96.153	96.153	96.059	96.101
X	106.193	106.192	106.081	106.044

σ_ξ, Young's and shear modulus standard deviation; σ_η, mass density standard deviation.

Table 4
Standard deviations of modal frequencies versus uncertainties levels

Mode	WIFEMTRUSS: standard deviations (Hz)		
	$\sigma_\xi = 5\%$, $\sigma_\eta = 5\%$	$\sigma_\xi = 10\%$, $\sigma_\eta = 5\%$	$\sigma_\xi = 15\%$, $\sigma_\eta = 5\%$
I	0.027	0.549	0.546
II	0.590	2.948	4.058
III	2.995	4.587	5.073
IV	3.092	4.599	5.107
V	0.212	1.976	3.178
VI	0.184	2.044	2.075
VII	0.328	2.528	2.909
VIII	2.320	4.683	5.814
IX	0.277	2.811	2.885
X	0.340	3.122	3.178

σ_ξ, Young's and shear modulus standard deviation; σ_η, mass density standard deviation.

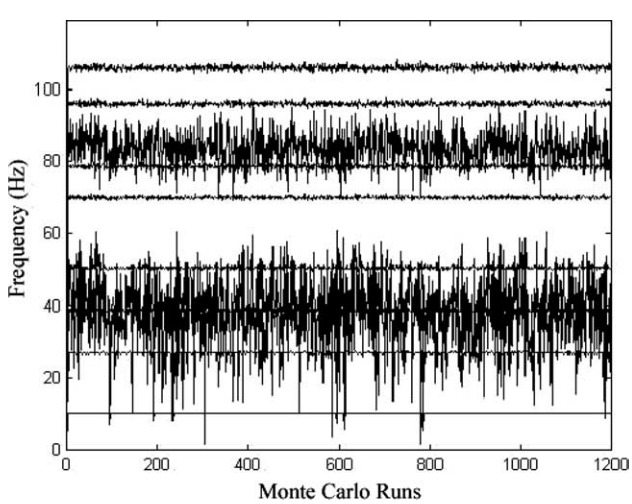

Fig. 4. Ensemble of modal frequencies versus Monte-Carlo runs.

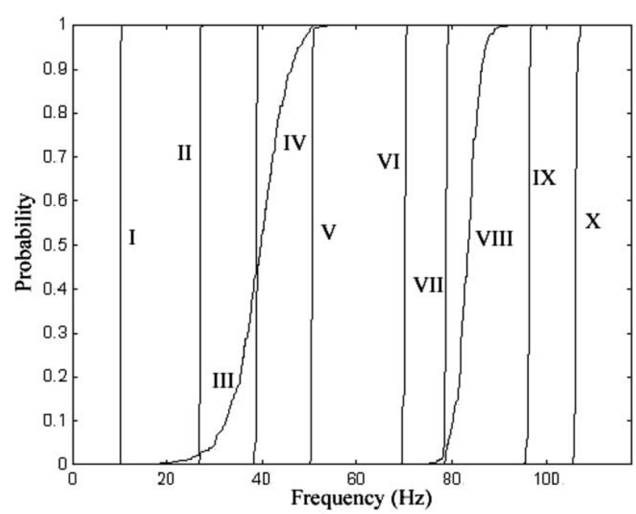

Fig. 5. Modal frequencies distributions: case A.

about 33%. Moreover, as shown in Fig. 4, permutation of actual vibration modes can occur: for example the fourth mode, corresponding to in-plane bending, tends to exchange with the first three ones, which are, respectively, represented by single and double-wave out-of-plane bending and pure torsion.

The distribution of the vibration modes for the three scenarios reported in Figs. 5–7, are far from being Gaussian, especially for modes affected by reciprocal permutations: therefore the distributions featuring structural response variables can hugely differ from Gaussian ones, even though the uncertainties are

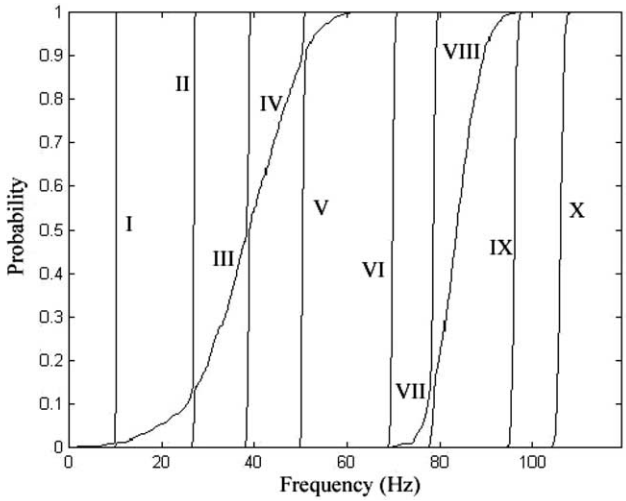

Fig. 6. Modal frequencies distributions: case B.

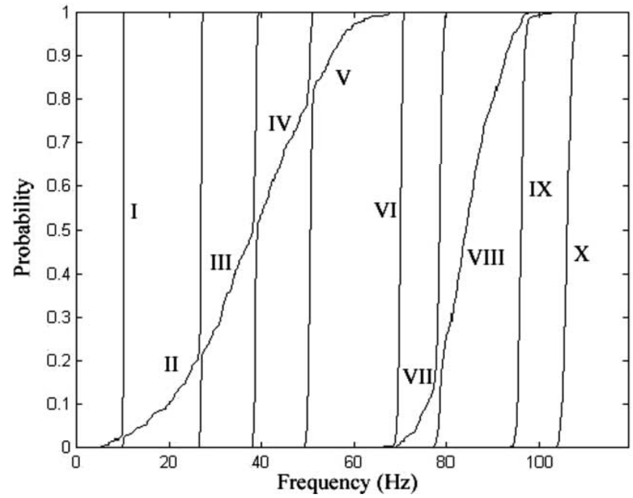

Fig. 7. Modal frequencies distributions: case C.

modelled by normally distributed random fields. Thus approximated stochastic FEM methods, which provide predictions about a limited number of statistical moments featuring the response variables of an uncertain complex structure, are not always suitable for probabilistic design purposes, since actual probability distributions are very difficult to be inferred.

5.2. Analysis of harmonic response

The set of applied loads selected for the harmonic response of the truss structure are presented in Fig. 8: this set of applied forces and torques has been chosen to provide an excitation of both the in-plane and the out-of-plane dynamics of the truss itself. The absolute value of each applied force is 5 N, while it is 3 N m for nodal torques. The deterministic solution for the harmonic response problem is reported in Fig. 9: this plot refers to the y translation of the node D in Fig. 8. The maximum displacement magnitude is centred on the frequency value which also identifies the fourth modal vibration frequency; this latter corresponds to single-wave in-plane bending. In Fig. 10 we present the case A node D harmonic response curves for each Monte-Carlo simulated

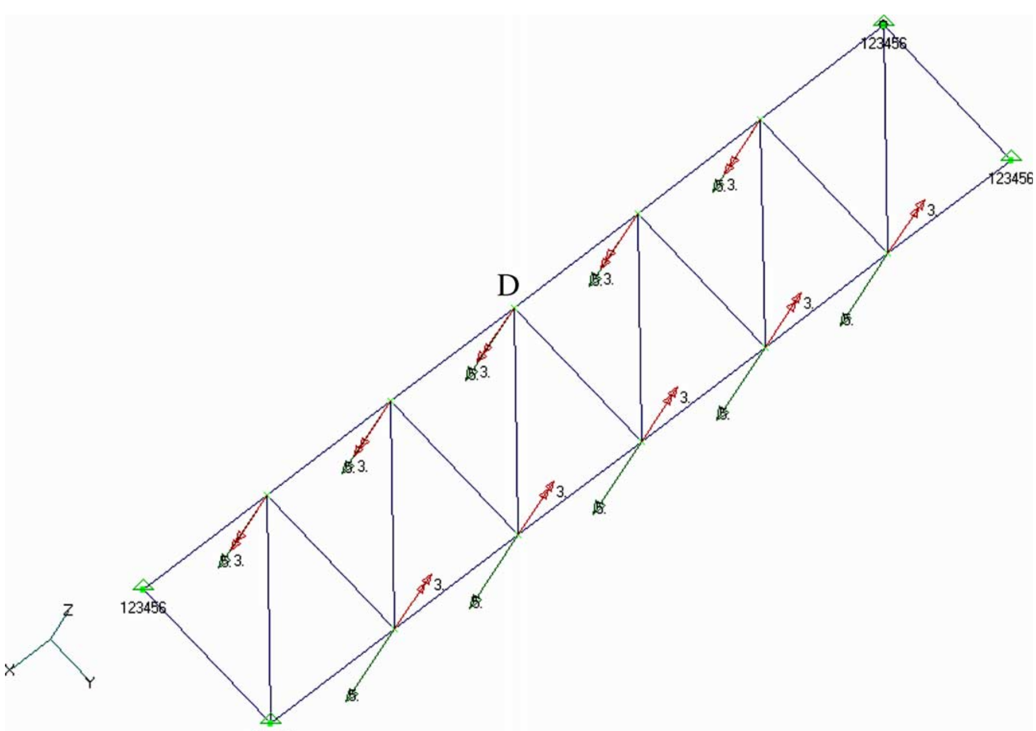

Fig. 8. FEM model for harmonic response analysis.

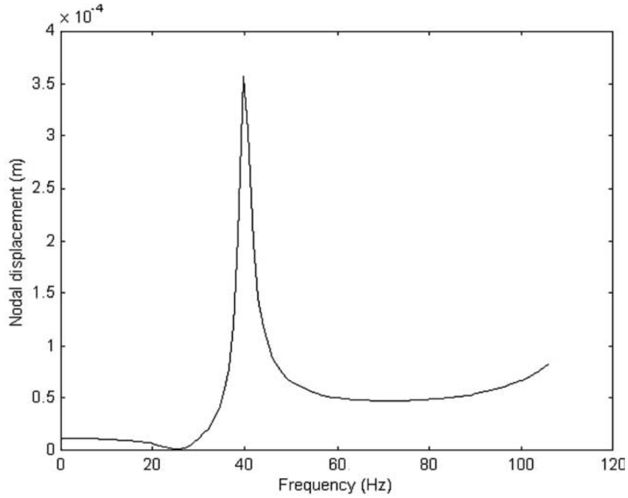

Fig. 9. Deterministic harmonic response of the truss.

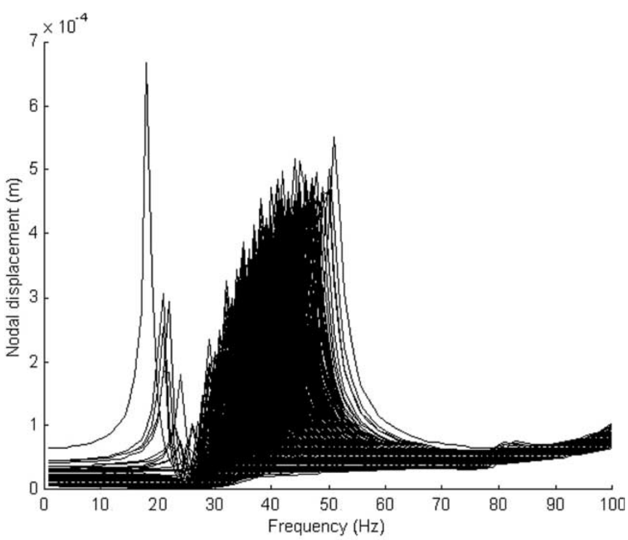

Fig. 10. Ensemble of harmonic response curves for case A analysis.

configuration: the mode permutation phenomenon is clearly highlighted, since the peak of maximum displacement tends to shift towards frequency values lower than the deterministic ones. This result confirms that the fourth mode tends to exchange with the first three ones, as already shown by probabilistic natural vibration modes analysis. Moreover a second low frequency response peak appears around 80 Hz for some simulated configurations: this effect depends on the mode exchanging between the seventh and the eighth modal frequencies. This peculiar behaviour of uncertain structures has deep implications about both passive and active control systems, since the vibration suppression capabilities of deterministic synthesised control devices can result to be out of the frequency range of actual natural modes.

6. Conclusions

According to the results of experimental investigations [15], the mechanical properties of fibre reinforced composites show a sensible statistical dispersion around the mean values which are usually employed in conventional deterministic design. The effects of mechanical uncertainties, especially regarding to the stiffness modules, can be considered negligible if we take into consideration structures characterised by simple geometrical configurations, such as a single beam or a plate. Nevertheless the structural response variability due to mechanical uncertainties of composites can attain significant levels for complex geometrical configurations, as in the case of reticular structures. Several approximated techniques, all hinged upon stochastic finite element methods, have been proposed to predict the structural response variability of complex uncertain structures: nevertheless these perturbations based approaches are able to provide information only about a finite number of statistical moments, usually two – mean value and standard deviation –, featuring the response variability. Full Monte Carlo simulations allow obtaining the complete statistical distributions of random response variable: the hybrid technique here proposed combines stochastic weighted integrals with a Monte-Carlo simulation strategy, avoiding coupling between the finite elements size and the correlations distances featuring the random fields. The main contribution of the authors is hinged on applying the outlined simulation approach to the harmonic response of reticular structures.

The hybrid Monte-Carlo simulation technique has been implemented in the MATLAB environment, developing a stochastic finite element code which is able to analyse the vibration response of large three-dimensional composite made trusses with random material mechanical properties. A benchmark problem, involving the modal analysis and the harmonic response of a typical reticular structure for space application has been studied; the structure here considered is assumed to be manufactured by composites tubes having a circular cross-section.

The simulations have pointed out that the vibration modes sequence for actual structures can results in being considerably different from that predicted by standard deterministic FEM analyses. In fact the actual vibration modes can appear in a permutated order with respect to those featuring the deterministic solution. The effects of mechanical uncertainties are clearly pointed out also by the harmonic response analysis of the same structure and have deep implications on the final performances and on the control of actual constructions.

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