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Effect of Multiple Representations on Students' Performance on Interpretations and Techniques of Representation in Calculus

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Abstract

This study investigated the effect of the multiple representations approaches on students' representation interpretation in learning calculus. Pretest and posttest quasi-experimental design of non-equivalent groups was used. Three intact groups of size 53, 57, and 54 students from Jigjiga and Kebri-Dehar Universities in Ethiopia participated in this study. The groups included a GeoGebra-supported multiple representations approach (MRT) from Jigjiga University, a multiple representation approach (MR), and a conventional approach (CG), both from Kebri Dehar University. Representation interpretation problem pretest and posttest were administered compiled from pre and calculus contents, respectively. Students' performance on representation interpretation problems was assessed using rubric scores, and their interpretation techniques were labeled as local versus global and syntactic versus semantic. Results revealed no statistically significant mean difference among the three groups on representation interpretation from the posttest that was determined by one-way ANOVA ((F(2.161) = 2.232, P = .111 , Partial eta = .03). More students in each group demonstrated local and semantic interpretation than global and syntactic interpretation. After the treatment, many students from each group shifted towards the local and semantic interpretation. It is recommended that the study need to replicate other calculus contents with different participants to generalize the results of the study.

Keywords: GeoGebra, multiple representations, representation interpretations, calculus

Abstrak

Penelitian ini menyelidiki pengaruh pendekatan representasi ganda terhadap interpretasi representasi siswa dalam pembelajaran kalkulus. Penelitian ini menggunakan Pretest dan posttest pada desain kuasi-eksperimental pada kelompok non-ekuivalen. Tiga kelompok yang terdiri dari 53, 57, dan 54 mahasiswa dari Universitas Jigjiga dan Kebri-Dehar di Ethiopia berpartisipasi dalam penelitian ini. Kelompok-kelompok tersebut termasuk pendekatan multi representasi (MRT) yang didukung GeoGebra dari Universitas Jigjiga, pendekatan representasi ganda (MR), dan pendekatan konvensional (CG), keduanya dari Universitas Kebri Dehar. Soal interpretasi representasi pretest dan posttest yang diberikan merupakan gabungan dari masing-masing konten pra dan kalkulus. Performa siswa pada masalah interpretasi representasi dinilai menggunakan rubrik penskoran, dan teknik interpretasi mereka diberi label sebagai lokal versus global dan sintaksis versus semantik. Hasil penelitian menunjukkan tidak ada perbedaan rata-rata yang signifikan secara statistik antara ketiga kelompok pada interpretasi representasi dari posttest yang ditentukan oleh ANOVA satu arah ((F(2.161) = 2.232, P = .111 , Parsial = .03) Lebih banyak siswa di setiap kelompok mendemonstrasikan interpretasi lokal dan semantik daripada interpretasi global dan sintaksis. Setelah diberikan perlakuan, banyak siswa dari masing-masing kelompok beralih ke interpretasi lokal dan semantik. Disarankan agar penelitian perlu mereplikasi konten kalkulus lain dengan peserta yang berbeda untuk menggeneralisasi hasil penelitian.

Kata kunci: GeoGebra, Representasi Berganda, Interpretasi Representasi, Kalkulus

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INTRODUCTION

A mathematical entity can be represented in various ways using different representation types to bring into focus its multiple aspects. Panasuk and Beyranevand (2011), suggest that the emphasis on a single representation form can inhibit students' long-term mathematical development. Representation interpretation ability leads students to develop robust mathematical understandings (Huntley et al., 2007). Moreover, the meanings that students hold of representations and representational conventions may constrain their meanings of mathematical ideas (Bennett et al., 2011). The representation interpretation techniques have the nature of dualism on the mathematics reasoning spectrum. However, students experienced several obstacles either to develop correct understanding or owning the representation they hold. One of the obstacles, according to Rachma and Rosjanuardi (2021), is the epistemological obstacle due to their incomplete conceptual understanding and they were not accustomed to applying one mathematical concept into another, which could be tackled with the use of alternative representations. In this study, in addition to students' performance on representation interpretation problems, the students' representation interpretation techniques were analyzed along to dimensions of a "local versus global" (Leinhardt et al., 1990) and a "syntactic versus semantic" (Easdown, 2009) dimensions. The local versus global interpretation processes belong to one continuum, and the syntactic versus semantic processes belong to another continuum. The local and global representation interpretations involve interpreting a representation based on a distinct value and from the broad context, respectively. Similarly, the semantic and syntactic representation interpretation are interpreting the given particular representation type relating with the domain it represents and without relating to the domain it represents, respectively. As students interact with representations, they do so either locally or globally (Bossé et al., 2014; Kaput, 1989) in one dimension and syntactic or semantic (Easdown, 2009) on the other dimension. Our intuitive and experienced knowledge of mathematical ideas can be demonstrated, analyzed and interpreted mathematically using different representations. These representations are considered cognitive, semantic, syntactic and perceptual tools for problem-solving; conceptual understanding and disciplinary discourse (Rahmawati et al., 2017). By Rahmawati (2019), representation is declared as one of the five standards of mathematical processes besides problem-solving, reasoning, connections and communication. Inherently, mathematics is endowed with different types of representations that serve as a multi-focal lens to see complex mathematical concepts from different perspectives (Gagatsis & Elia, 2004). Many mathematical concepts are closely attached to different modes of representation. For instance, the concept of a function and its graph is hard to dissociate. In addition, calculus contents such as limits, continuity, derivatives and application of derivatives are among the challenging areas for student learning. In consequence, it is in the interest of mathematics educators to consider representational fluency as a "trademark" of students' success in calculus. Ainsworth (2008), emphasizes that the most basic competency that learners must develop is to understand the representational syntax. For example, by providing graphical representations, students can focus on the information content rather than the syntactic structure of sentences (Van Labeke & Ainsworth, 2001). Of course, the interpretation of representations belongs to structured systems, and the interpretation of their representing relationships is not objective or absolute but depends on the cognitive structure of the individual(s) making the interpretation (Andersen et al., 2009). Representation interpretation refers to all actions by which a student makes sense of or acquires meaning from a specific form of representation. Sometimes the person internalizes by means of interactions with the external physical structures of a notational system by reading, interpreting words and sentences, interpreting equations and graphs, and so on. Such interpretive acts can take place at an active, deliberate level subject to conscious efforts (Grossberg, 1982). Haciomeroglu and Andreasen (2013), conducted a research on calculus students understanding of derivative graphs and examined problems of representations in calculus. The participants' interpretations and representational schemes for derivative graphs differed because of their preference for mathematical processing. However, the representation knowledge and interpretation level significantly contribute to determining other learning outcomes in mathematics.

Representation interpretation is a big concern in learning mathematics with multiple representations. People develop representations to interpret and remember their experiences to understand the world (Salkind & Hjalmarson, 2007). The information embedded in each representation is extracted through interpretation. Any representation needs to be interpreted for the information it conveys. Interpretation of representations is an inherently contextualized activity (Roth & Bowen, 2001). Interpretation refers to all actions by which a student makes sense of or acquires meaning from a specific form of representation. Palmer (1978), proposed five entities in the semantic interpretation of a representation: (1) the represented world, (2) the representing world, (3) the aspects of the world being represented (4) the aspects of the representing world doing the representing (5) the relation between these two worlds. This indicates that signification of a representation involves relating the signified and signifier.

The level of representation knowledge and interpretation has a great contribution to determining other learning outcomes in mathematics. For example, Niemi (1996) obtained that the level of representational knowledge which he measured by the ability to identify correct alternative representations of a fraction given in a particular representation is predictive of performance in problem-solving, justification, and explanation tasks. In the process of extracting meaning from representations, students interact with these representations with different dimensions that indicate their level of expertise and sophistication in representation fluency. Two categories in a continuum of representation interpretation dimensions were identified. These are local versus global representation interpretation and syntactic versus semantic representation interpretation. These two dimensions of representation interpretation mechanisms are used to explain students' success in problem solving.

A problem-solving task may progress from the problem that requires identifying specific points to that which requires relational reasoning and comparison. This kind of progress in problem-solving using representations is explained using the analogies local representation interpretation and global representation interpretation, respectively (Adu-Gyamfi et al., 2019). Local graph interpretation tasks require attention to specific details, while global interpretation tasks entail the identification of trends or patterns (Adu-Gyamfi et al., 2019). Chang et al. (2016), revealed that in the local representation interpretation, students consider function as a series of individual points, whereas in the global interpretation they treat function by assessing the order of the equation and its relation to the shape of a graph. Global interactions with representations are more cognitively complex, lead to a deeper understanding of representations, and to more students' success in doing translations between representations (Adu- Gyamfi et al., 2012; Bossé et al., 2014; Duval, 2006). Leinhardt et al. (1990), argue that global interpretation is general whereas local interpretation is specific.

Students make connections between representations in two recognized ways during the representation translation process. Some use isomorphic connections (locally considering characteristics of two representations and connecting similar ideas between the representations) (Adu-Gyamfi, et al., 2017). Others use transcendent connections (connecting ideas from both representations to more global mathematical concepts). Students who recognize and apply mixed connections (both isomorphic and transcendent connections) and transcendent connections over isomorphic connections perform translations with more understanding and are more successful in completing translations (Ainsworth, 1999).

The other dichotomy of representation interpretation dimension lies between syntax (form) and semantics (meaning) (Easdown, 2009). The syntactic interpretation (reasoning) is associated with the superficial end of the learning spectrum that involves very literal interpretations and superficial relationships, whereas semantic interpretation (reasoning) is more naturally associated with the deeper end of the learning spectrum and relies on solid intuition, insight or experience (Easdown, 2009). Therefore, we should expect (and possibly celebrate) an unresolvable tension between syntax and semantics. However, rather than regarding this as a nuisance or source of frustration in teaching mathematics, one can exploit the differences between syntactic and semantic reasoning to create opportunities to enhance learning and expose weaknesses or gaps in understanding. The previous studies indicated translation interpretation and the roles of each of the syntax and semantic presentations. These fail to include the other forms of representations that could help learners develop deeper understanding, and equally contribute to the gaps observed by using the syntax or semantics. Thus, this study attempted to answer the questions posed, that are signified to fill the gap observed by the following statement of the problem.

Representation interpretation is a decisive process in effective mathematics learning though most mathematics instructors do not emphasize it. They emphasize more in their classroom instruction. Students often fail to interpret the various mathematical representations properly in solving calculus problems, communicating calculus ideas with others and conceptual understanding in Ethiopian universities (Gemechu et al., 2018). Nguyen and Rebello (2009), found that university students in calculus-based physics courses were unable to process information from graphical representations as they could not read off values from the graph and were unable to correctly interpret the physical meaning of the graph, e.g., the area under the graph is the work done. This tendency seemed to be a barrier to successful problem-solving. Hence, this study intended to investigate the effect of the multiple representations approach on university students' performance on representation problems and their techniques of representation interpretation in calculus. Accordingly, the study tried to answer the following questions.

1. What is the effect of the multiple representations approach on students' performance of representation interpretation problems in calculus (limits, continuity, derivatives and application of derivatives)?
2. What is the students' labeling of representation interpretation with respect to local versus global interpretation in calculus due to the multiple representations approach?
3. What is the students' labeling of representation interpretation with respect to syntactic versus semantic interpretation in calculus due to the multiple representations approach?

METHODS

The study employed a multi-treatment pretest and post-test non-equivalent group quasi-experimental research design on purposefully selected groups of students who belong to two different universities in Ethiopia. The study intended to compare the effects of three differentiated approaches: GeoGebra supported multiple representations approach (labeled as MRT), multiple representations approach (labeled as MR) and the conventional approach (labeled as CG) on students' performance on representation interpretation problems. Furthermore, this study investigated students' representation interpretation techniques in solving representation interpretation problems in calculus. The students were grouped and labeled based on their representation interpretation techniques to two categories, each of which belongs to two different continuums (i.e., local versus global and syntactic versus semantic representation interpretation).

Participants

The study was conducted on first-year first-semester students of the social science stream for the course mathematics for social science at Jigjiga University (JJU) and Kebri-Dehar University (KDU) in 2019/20. These two universities are located in same region and use same curriculum and working in same socio-cultural settings. The first-year social sciences students at JJU and KDU were assigned to their section (labeled as A, B, C, etc.) based on the alphabetical order of their names. Each section contains, on average, 58 students. In the 2019/20 academic year, there were 23 sections in JJU and 12 in KDU, respectively. In addition, one section from JJU was selected and assigned to the GeoGebra supported multiple representations approach (MRT) (n = 53) group, and two sections from KDU were assigned to the multiple representations approach (MR) (n = 57) and comparison group (CG) (n = 54), respectively. The learning contents of the course covered limits, continuity, derivatives and application of derivatives.

The MR group received a multiple representations approach, focusing on the verbal, numerical, graphical and algebraic representations. In the GeoGebra supported multiple representations treatment group, some classroom arrangement and classroom shifting were implemented during the intervention to a hall for the GeoGebra session and to a computer lab for learning with the GeoGebra worksheets and for practices with GeoGebra. Thus, for the appropriate implementation of the instructional design for the MRT group, three different classroom contexts and sessions were implemented. For multiple representations without technology sessions, chalk and board or pen and paper were used in the regular classroom. The time allocation for these group sessions was three credit hours of lectures and two tutorial hours per week. The students could also download and install the software into their electronic devices for use outside the classroom. In the MRT group, the teaching-learning was interactive using GeoGebra, where students explored calculus concepts with linked multi-representations that are often difficult using the chalk and board. The CG was taught based on the conventional approach, which was dominantly algebraic representation. The intervention lasted for about six weeks.

Table 1. Participants background information

		Gender			
		Female		Male	
Group	N	f	%	f	%
MRT	53	16	30.2	37	69.8
MR	57	21	36.8	36	63.2
CG	54	18	33.3	36	66.7

The MRT group comprised 16 (30.2%) female and 37 (69.8%) male students. Similarly, the MR group consisted of 21(36.8%) female and 36 (63.2%) male students. Likewise, the CG encompassed 18 (33.3%) female and 36 (66.7%) male students. In the three groups, female participants were fewer than their male counterparts. These ratios of female to male students in the universities mirror the proportions of female students to male students in the Ethiopian universities and across each discipline, as evidenced by Teferra et al. (2017). They witnessed a considerable gender gap in enrolment among male and female students in Ethiopian universities.

Data Collection Procedures

The data collection instrument for this study consisted of representation interpretation problems (pretest and posttest). Representation interpretation problems lend themselves to a rubric assessment, and the authors developed the rubric provided in Table 2. Hence, the rubric assessment technique quantified the student's score on the representation interpretation problems and associated representation type, and what each intended to elicit from the representation, as depicted in Table 2.

Table 2. Representation interpretation posttest items, representation type and intended to be elicited by each item.

Item	Representation type(s)	Intended to be elicited by the representation
1	Graphical	To be able to read the existence and nonexistence of both sided, one sided and in all points of an interval of a limit of a function from a graph
2	Graphical	To be able to arrange the numerical values of the derivative of a function provided graphically from the smaller to the larger
3. (a) - (d)	Combinations of numerical, algebraic and graphical	To be able to analyze the behavior of the function at infinity based on the information provided in combination of numerical, algebraic and graphical representations
4	Combinations of numerical, algebraic and graphical	To be able to compare which function increases slowly using ratio of the numerical values, looking at the positions of their graphs, and using derivative from their algebraic formula
5	Combination of verbal, realistic and graphical	Based on the information provided by the written text, the illustration provided graphical for the position of moving particles, to be able to determine when the particle is moving to the right, moving to the left, standing still and draw a graph of the velocity function using the concept of derivative from graph.
6	Numerical	To be able to explain verbally the numerical representation of limit of a function or any other appropriate representation(s)
7	Algebraic	Expected to discuss the solution of the algebraic equation using graphical interpretation or using any other appropriate representation(s)
8	Algebraic	To discuss the condition of the Mean Value Theorem (MVT) on an interval using written text (verbally) or any other appropriate representation(s)
9	Combination of numerical and realistic	To analyze a number that appears to be closer to the average velocities of a car over successful smaller intervals using appropriate representation(s)
10	Combinations of Verbal and Algebraic	To be able to derive the unit of a derivative of a physical quantity from the unit of the source physical quantity.

The representation interpretation test was intended to determine all groups' students' understanding of mathematical objects and concepts carried by different modes of representations. Similarly, it was designed to investigate whether they were familiarized with each representation's syntax, form and structure to use it as a tool to solve mathematical problems. In addition, their ways of interaction with the representations were labeled using two dimensions. These two dimensions were local versus global interpretation, and the other dimension was syntactic versus semantic interpretation.

Table 3. Rubric scoring of students' performance on representation interpretation problems

Unsatisfactory = 1	Proficient = 3	Excellent = 5
Attempts to construct the representation type, but work incomplete, unorganized, inappropriately scaled, or incorrectly labeled	Attempts made to construct the representation type with partially complete, organized, appropriately scaled or correct labeling.	Accurately construct and use the representation in a manner that supports a complete solution. The representation type is accurately and precisely labeled.

Based on the representation interpretation items indicated in Table 2 and the rubric (as seen in Table 3), students expected minimum and maximum scores on the representation interpretation pretest were 18 and 90, respectively. Similarly, students expected minimum and maximum scores on the representation interpretation post-test were 17 and 85, respectively.

Reliability and Validity of Instruments

Several efforts have been made to obtain reliable and valid information from the data collection instruments. Several types of validity were ensured: face validity, content validity, construct validity and criterion validity. For the validity of the representation interpretation problems, colleagues' comments were used, and the items were modified accordingly. In addition, faculty colleagues in the mathematics department at JJU were consulted to check the validity of concepts and appearances from the aspects it aimed to measure. They evaluated item appearances for each construct in terms of feasibility, readability, consistency of style and formatting, and the clarity of the language used to the level of the participants' experience. A panel of experts was also involved in evaluating the content validity of the constructs. They ensured whether each of the constructs incorporated all the items that were essential for the constructs. The literature review was also used to establish content validities of the constructs. Throughout the process of developing and validating the instruments, the cooperation and support of colleagues was phenomenal.

A pilot test was conducted on second-year mathematics department students to establish the reliability of the instruments of each construct. Thirty students (15 students for the pretest and 15 students for post-test) participated in the pilot test. The students' solutions to the construct were assessed using the rubrics of 1, 3 and 5-point scales. Two iterators were involved in determining the students' work using the predetermined rubric for scoring their solution for each item in each construct. As a result of this, the student's solution was analyzed separately by two faculty members at JJU, and the calculation of the reliability was computed manually using the formula Consensus/

(Consensus + Dissensus) X 100 recommended by Miles and Huberman (1994), cited in Rahmawati et al. (2017). Accordingly, the reliability of representation interpretation problems pretest was .81 and posttest was .75 indicating the instruments were in the acceptable range of reliability (Berry and Mielke, 1988).

Data Analysis

For the study both descriptive and inferential analyses were employed. Descriptive analysis by using frequency and percentages was used to determine the proportion of students that belong into the representation interpretation categories, and samples of students' works were used to demonstrate each of the categories. Inferential statistics such as one-way ANOVA as also used for the comparison of the groups in the study.

RESULTS AND DISCUSSION

Pretest Results

The frequency and percentage result of local versus global and syntactic versus semantic representation interpretation of the pretest were described to identify the students' inclination.

Table 4. Frequency and percentage result of local versus global, and syntactic versus semantic representation interpretation of the pretest.

		Local		Global		Syntactic		Semantic	
Group	N	f	%	f	%	f	%	f	%
MRT	53	29	54.72	24	45.28	35	66.04	18	33.96
MR	57	31	54.39	26	45.61	32	56.14	25	43.86
CG	54	29	53.70	25	46.30	33	61.11	21	38.89

One way of labeling the representation interpretation dimensions was local versus global interpretation. As noted in Table 4, more than half students in each group demonstrated the behavior of local representation interpretation action before the intervention (54.72% of the MRT, 54.39% of the MR and 53.70% of the CG). The remaining students in each group demonstrated the behavior of global representation interpretation action (45.28% of the MRT, 45.61% of the MR and 46.30% of the CG). These results revealed that an almost equal percentage of students interacted with the representation through local interpretation before the intervention. Similarly, the percentage of students interacting with a representation through global interpretation was almost the same. However, the percentage of students who interact through global interpretation action was less than that of students who interact through local interpretation in each group.

The second type of representation interpretation action was syntactic versus semantic interpretation. The dimension of syntactic versus semantic representation interpretation is related to the interaction of students with the representation through the syntactic meaning of the representation, which is purely mathematics, and interpreting the representation by relating the representation with the domain it represents. As can be noticed from Table 4, more students were involved with the syntactic interpretation (66.04% MRT, 56.14% MR and 61.11% CG) before the intervention. The remaining students dealt with semantic interpretations (i.e., 33.96% of the MRT, 43.86% of the MR and 38.89 % of the CG). Hence, before the intervention occurred, the students in the three groups interacted with the representations through syntactic interpretation.

The descriptive statistics reported in Table 5 indicates that the MRT demonstrated superior performance on the representation interpretation problem pretest (M = 45.64, SD = 11.11) than the MR (M = 42.74, SD = 7.84) and the CG (M = 42.33, SD = 7.82). A one-way ANOVA was run to give conclusive evidence regarding the baseline difference between the three groups, which is given in Table 5.

Table 5. Descriptive statistics and One-way ANOVA result on the representation interpretation problem pretest.

Group	N	*M*	*SD*	Source	*SS*	*df*	*MS*	*F*	*P*
MRT	53	45.64	11.11	Between groups	349.46	2	174.73	2.15	.12
MR	57	42.74	7.84	Within Groups	13105.24	161	81.40		
CG	54	42.33	7.82	Total	13454.70	163			

To uncover any difference between the three groups during the pretest, the between-subjects one-way ANOVA was used. The result reported in Table 5 shows that the three groups had no significant mean difference on the representation interpretation problem pretest (F (2,161) = 2.15, P = .121).

Posttest Results

Representation Interpretation Posttest Result

Similar to the descriptive statistics of the pretest, the mean and standard deviation of the representation interpretation problem posttest were explored. The descriptive statistics are given in Table 6 below. Based on the output of the descriptive statistics, reported in Table 5, slight mean difference was detected on the representation interpretation problem posttest among the MRT (M = 39.30 SD = 7.70,), the MR (M = 36.61, SD = 6.35) and the CG (M = 38.31, SD = 6.19). The null hypothesis specified below was tested using a one-way ANOVA after verifying all assumptions for the underlined statistical tool.

$H_{[O1]}$: There is no statistically significant mean difference on the representation interpretation problem posttest among the three groups

This study intended to determine the effect of the multiple representations approach on students' performance on representation interpretation problems in calculus. The representation interpretation posttest of the three groups was examined. A one-way ANOVA was run to infer the effectiveness of the treatment type. The result in Table 6 shows that there was no statistically significant mean difference among the three groups on the representation interpretation problem posttest (F(2,161) = 2.232, P = .111, Partial = .03). This finding shows that alternative approaches to learning calculus did not show any statistically significant difference between the three groups. The reasons for this could espouse to various factors some of which could be background characteristics, dominance of one type of representation over others during their mathematics learning, or that the intervention never brought any impact. Yet, these need further study to explore determining predictors.

Table 6. Descriptive statistics and one-way ANOVA result on the representation interpretation problem posttest.

Group	N	*M*	*SD*	Source	*SS*	*df*	*MS*	*F*	*P*
MRT	53	39.30	7.70	Between groups	204.37	2	102.19	2.232	.11
MR	57	36.61	6.35	Within Groups	7370.33	161	45.78		
CG	54	38.31	6.19	Total	7574.70	163			

Representation Interpretation Techniques Posttest Results

Students' representation interpretations were analyzed by labeling two polarized dimensions (local versus global and syntactic versus semantic). Local interpretation is the act of emphasizing a specific aspect of the representation, whereas global representation is the act of holistic visualization of the representation (Duval, 2006). Local interpretation of representations is characterized, most of the time; by representing a function of continuous physical quantities by several discrete values. According to Easdown (2009), syntactic interpretation is associated with the shallow end, and semantic interpretation is associated with the more profound end in the learning spectrum. To this end, the students were labeled as local and global interpretation dimensions in the representation interpretation posttest. It can be noticed from Table 7 that more than half of the students demonstrated the behavior of local representation interpretation (64.15% MRT, 57.89% MR and 66.67% CG). The remaining students showed the behavior of global representation interpretation (35.85% of the MRT, 42.11% of the MR and 33.33% of the CG). These results revealed that most students in each group interact with the representation with local interpretation. When the pretreatment and post-treatment of local versus global interpretation were compared, the percentage of students in each group increased on the local interpretation. However, the percentage of students who interact through global interpretation was less than the percentage of students who interact through local interpretation in each group. As it can be noticed in Table 7, more percentage of students in the MR group than in the other two groups interact with the representations through global interpretation. In comparing the students' local versus global interpretation between the pre and post-treatments, 9.43% MRT, 3.5% MR and 12.97% CG shifted towards the local interpretation. Global interactions with representations are more cognitively complex, leading to a deeper understanding of representations. These results reveal that the percentage of students who interact with the representation through local interpretation exceeded that of students who interact with the representation in a global way even after the intervention. Students may focus on extracting meaning from the representation depending on the specific range of values without considering the overall variation of the representation. The level of sophistication in representation interpretation fluency can be manifested as the students are transcending from the local interpretation to the global interpretation (Adu-Gyamfi et al., 2012)**.**

Table 7. Frequency and percentage result of local versus global, and syntactic versus semantic representation interpretation of the posttest.

		Local		Global		Syntactic		Semantic	
Group	N	f	%	f	%	f	%	f	%
MRT	53	34	64.15	19	35.85	37	69.81	16	30.19
MR	57	33	57.89	24	42.11	32	56.14	25	43.86
CG	54	36	66.67	18	33.33	36	66.67	18	33.33

The local versus global interpretation labeling are more viable in representation translation. Many researchers postulate a connection between the nature of student errors in translations and the type of interpretive activity required in such translations; students have more difficulty and perform more errors in respect to translations requiring global as opposed to local interpretive actions (Dreyfus & Eisenberg, 1996; Dunham & Osborne, 1991; Gagatsis & Shiakalli, 2004).

The semantic versus syntactic representation interpretation labeling is related to the interaction of students with the representation through the syntactic meaning of the representation, which is purely mathematics, and interpreting the representation by relating the representation with the domain it represents. As can be observed in Table 7, more students were involved with the semantic interpretation (69.81% MRT, 56.14% MR and 66.67% CG) after the intervention. The remaining percentage of students in each group dealt with syntactic representation interpretations in their interaction with the representation. Regarding the semantic versus syntactic dimension of representation interpretation, the type of treatment implemented did not significantly influence the students' variation of representation interpretation techniques. In comparing the pre and post-treatments, students' semantic versus syntactic interpretation status showed that 3.85% MRT and 5.56% CG were shifted to the semantic interpretation action. However, no variation between the pre and post-treatment was observed on the MR.

In conclusion, more students in the three groups demonstrated the local interpretation behavior in the local versus global labeling of representation interpretation. Likewise, most students in the three groups tended to incline towards semantic interpretation in the syntactic versus semantic dimension of interpretation.

Labeling of Individual Student's Work on Representation Interpretation

The local versus global and semantic versus syntactic interaction with a representation can be observed through the sample of some students' work. As an example, the students were given the problem below, a type of graphical representation, to see if the students were able to arrange the numerical values of the derivative of a function provided graphically from the smaller to the larger.

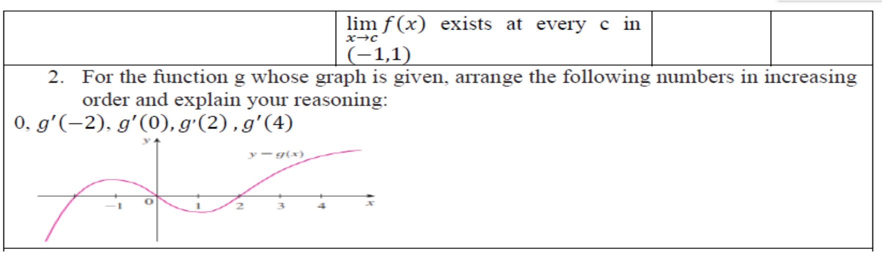

Figure 1. The problem

Examples of students' responses are as given below that describe a sample for each university identified as MR, MRT and CG.

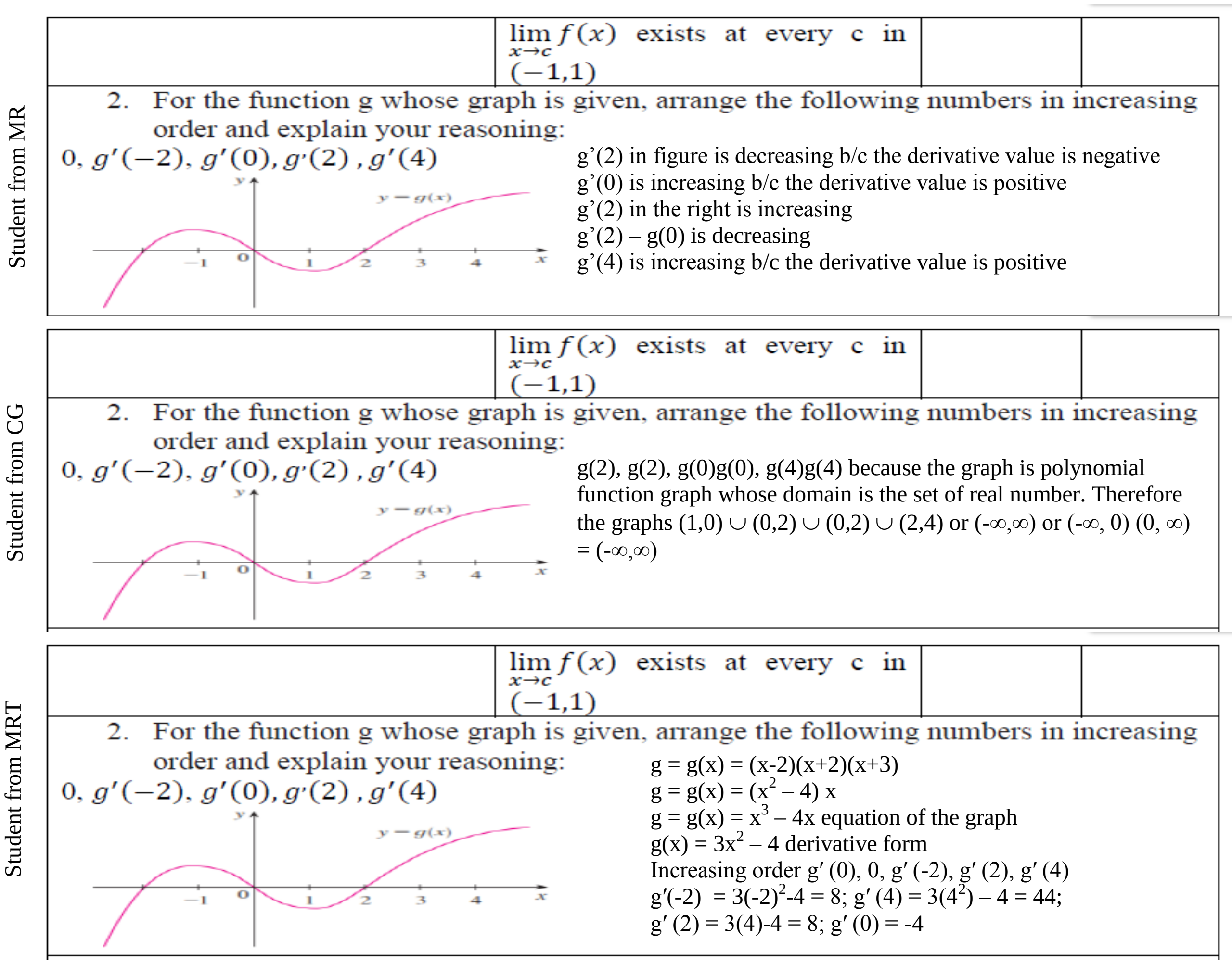

Figure 2. Student's global and syntactic, and local and syntactic interpretations of derivative value of a function from its graphical representation

From the sample work of a student from the CG group (SCG), it can be observed that the student described the graph of the function based on the behavior of a polynomial function. Furthermore, the student tried to interpolate to determine the functional behavior between two specific values and extrapolate to extend and forecast the functional behavior by inferring from the given particular values. Thus, the student can be said to be interacting with the representation through global interpretation. Conceptually, the student dealt purely with the graphical structure without relating to some context using some analogies. Hence, the student's conceptual interpretation of the representation was syntactic.

It can be noticed from another sample of students' work from the MR group. Rather than analyzing the graphical representation by considering intervals, the students treated the rate of change of the graphical representation by considering particular points. Hence, the student interacted with the representation through local interpretation. The student's conceptual interpretation of the representation was also syntactic interpretation.

The other sample of a student's work from the MRT group indicates that the student tried to determine the interpolation and extrapolation behavior of the graphical representation of the function by deriving its algebraic formula. This student interacted with the representation through global interpretation. But the student did not relate the concept of the representation with some context using an analogy. Hence, the conceptual interpretation of the representation was syntactic. Even though the representation was something that stands for a referent (i.e., what the representation was meant to depict) in the mathematical domain, the relation between the representation and its referent was often opaque (syntactic interpretation). The students' local and syntactic interpretation behavior leads to difficulties in learning with MRs.

Mathematics learning is associated with five skills (NCTM, 2000, 2014) that include: problem solving, reasoning, communication, connection making, and representation. The multiple representations may facilitate all these learning abilities (Ernaningsih & Wicasari, 2017). Demetriadis (2004) indicated that the efficient use of multiple representations ensures the quality of learning and transfer. It is widespread to use multiple representations in mathematics classrooms for comprehending concepts. However, instructors use static representations available in hardcopy. With the advent of learning technologies, the opportunity of employing dynamic, immersive, adaptive, interactive, and collaborative supporting representations for learning is at a higher stage (Demetriadis, 2004). Classroom instruction may also effectively use multiple representations to complement each other, constrain misinterpretations of single representations, and support learner's in constructing deeper (more abstract) domain knowledge (Ainsworth, 1999). Hence, the optimum benefits of learning mathematics with multiple representations are achieved through meaningful interaction with each representation. One of the significant interactions with representations to extract meaning from each representation and their combination is representation interpretation. After accomplishing the treatments, a representation interpretation problem posttest was administrated, and the groups were compared on their scores. One-way ANOVA was used to compare their score on the representation interpretation problem, and frequency and percentage were used to label representation interpretations. No statistically significant difference was obtained among the three groups on the representation interpretation problem as it was determined by one-way ANOVA ($F(2,161) = 2.232$, $P = .111$) of the posttest.

The ability to drive meaning from mathematical representations is the other manifestation of students' sophistication in representational fluency. Interpretation of facts and concepts within each representation and across representations can be accomplished in a "local versus global" dimension (Leinhardt et al., 1990) and/or "syntactic versus semantic" dimensions (Easdown, 2009). Perceiving graphical representation as a series of individual points is local interpretation, and perceiving as a concrete value is a global interpretation (Chang et al., 2016). Local interpretation of graphical representation is a one-to-one association between order pairs and points, and global interpretation is recognizing the overall variation occurring among order pairs of a set (Duval, 2006). In local interpretation, one can perceive and confirm parts independent of each other, and global interpretation uses the relationships to construct a consistent structure (Huang, 2015). In the local interpretation interaction, students discretize continuous values. In this study, the students' representation interpretation dimensions were analyzed along two dimensions: a "local versus global" dimension (Leinhardt et al., 1990) and a "syntactic versus semantic "dimension (Easdown, 2009). The syntactic representation interpretation refers to the form of the representation, and semantic representation interpretation refers to the meaning of the representation. The syntactic versus semantic dimension of representation interpretation tends to lie at the polar ends in the mathematics reasoning spectrum (Easdown, 2009). Syntactic representation interpretation is associated with the shallow end of the learning spectrum and relies on simple or naive, incremental rules, literal interpretations and superficial relationships. On the other hand, semantic representation interpretation is more naturally associated with the deeper end of the learning spectrum and relies on solid intuition, insight or experience. Easdown (2009), argues that rather than regarding this as a nuisance or source of frustration in teaching mathematics, one can exploit the differences between syntactic and semantic reasoning to create opportunities to enhance learning and expose weaknesses or gaps in understanding.

As students interact with representations, they do so either locally (focusing only on particular characteristics of a representation without considering it through a broader context) or globally (recognizing generalized notions associated through broader contexts encoded in the representation) (Bossé et al., 2014; Kaput, 1987). According to the frequency and percentage results, more than half of the students in each group demonstrated the behavior of local representation interpretation (64.15% MRT, 57.89% MR and 66.67% CG). In contrast, the remaining students in each group demonstrated the behavior of global representation interpretation. These results revealed that most students in each group interact with the representation with local interpretation. When the pretreatment and post-treatment of local versus global interpretation were compared, the percentage of students in each group increased on the local interpretation. However, the percentage of students who interact through global interpretation was lesser than the percentage of students who interact through local interpretation in each group. More percentage of students in the MR group than in the other two groups shows they interact with the representations through global interpretation. Comparing students' local versus global interpretation between the pre and post-treatments showed that 9.43% of students in the MRT, 3.5% of the MR and 12.97% of the CG shifted towards the local interpretation. Students might focus on extracting meaning from the representation depending on the specific range of values without considering the overall variation of the representation. The level of sophistication in representation interpretation fluency can be manifested as the students are transcending from the local interpretation to the global interpretation (Adu- Gyamfi et al., 2012). Interacting through global representation interpretation is more cognitively complex as it leads to a deeper understanding of representations than the local representation interpretation.

The students in each group drastically shifted from syntactic representation interpretation before the intervention to the semantic representation interpretation after the intervention. The syntactic representation interpretation was characterized by reading representations on a syntax level without considering the underlying meaning. When the students tried to compare two different representations of the same calculus concepts, they related them only by syntax and superficial features. On the other hand, in semantic interpretation, students connect surface features of representations to the represented content and consider the meaning of the representation. Students involved in this category compare different representations based on the represented meaning and are, thus, able to recognize the shared underlying meaning of several different representations of the same calculus concepts and transfer information between these representations, which is foundational for the development of problem-solving capability. As noted by Widodo et al. (2020), it is necessary to prepare learning tools that can accommodate the problem-solving process into a heuristic in learning. These can be hold through the development of representation interpretation which gives students alternative ways of viewing the concept with which they can gain deeper understanding. Pambudi et al. (2020), further consolidated this by stating "mathematical connections play an important role, namely as a tool for students to use in solving mathematical problems where students who have good mathematical connection skills succeed in solving mathematical problems well, while poor mathematical connection skills cause students to fail in solving mathematical problems" (p.129); and mathematical connection can be improved through the development of representational fluency and representation interpretation.

The semantic versus syntactic representation interpretation dimension is related to the interaction of students with the representation through the syntactic meaning of the representation, which is purely mathematics, and interpreting the representation by relating the representation with the domain it represents, respectively. Hence, semantic interpretation was related to the domain, which leads to deep understanding, whereas syntactic interpretation is pure mathematics at the symbolic level. The analysis indicated that more students in each group were involved with the semantic interpretation (69.81% MRT, 56.14% MR and 66.67% CG) after the intervention. However, the remaining students in each group dealt with syntactic representation in their interaction with the representation. Regarding the semantic versus syntactic dimension of representation interpretation, the type of treatment implemented did not bring any influence on the students. In comparing the pre and post-treatments, students' semantic versus syntactic interpretation status showed that 3.85% of the MRT and 5.56% of the CG shifted to the semantic interpretation action. However, no variation between the pre and post-treatment was observed on the MR. Hence, semantic interpretation relates to the domain, which leads to deep understanding, whereas syntactic interpretation is pure mathematics at the symbolic level. However, the interplay between the indicated dimensions is useful for holistic development and capability to solve problems. This is so, because Inzunza (2006) cited in Yusuf, Rahim, and Eu (2021) stated that difficulty faced by students in interpreting and using correct terms will disrupt the problem-solving process, and these can be mitigated through the development of representation interpretation and fluency.

In conclusion, more students in the three groups demonstrated the local interpretation behavior in the local versus global dimension of representation interpretation. Similarly, in the syntactic versus semantic dimension of interpretation, most students in the three groups tended towards semantic interpretation.

CONCLUSION

In this study, the students' performance on representation interpretation problems was invariant due to the multiple representations approach in calculus. The multiple representations approach did not bring a statistically significant variation in the students' performance on the representation interpretation problem in calculus. The GeoGebra supported multiple representation approach, and the conventional approach groups demonstrated a greater ability to decode the structural relationship that the representation conveys a mathematical concept and connect to the domain through semantic interpretation than the multiple representations approach only. The GeoGebra might have helped students to see alternatives, and the conventional approach supported by the instructors was an accustomed experience. Likewise, lack of experiences with the multiple representations approach might have negatively contributed to develop greater ability. The percentage of students labeling local representation interpretation was greater than the percentage of students labeling global representation interpretation in each group before and after the intervention. The percentage of students labeled local interpretations increased after the intervention with a greater number of students from the CG, MRT and MR, respectively, in their order. Students may focus on extracting meaning from the representation depending on the specific range of values without considering the overall variation of the representation. Hence, the treatments did not bring more variation to the representation interpretation techniques labeling local interpretation and global interpretation in solving representation interpretation problems. Likewise, the percentage of students labeling semantic interpretation is greater than the percentage of students labeling syntactic interpretation in each group after the intervention. A magnificent percentage of students also shifted from labeling syntactic interpretation to labeling semantic interpretation. Hence, the treatment types were equally likely to have equivalent influences on labeling syntactic and semantic representation interpretation in calculus. Recommended is also conducting further research on why the intervention failed to bring significant changes on representation interpretation, and on why syntactic and semantic representation to have been influenced equivalently. It would be also wise to check similar study on other mathematical concepts to check if the result is persistent and generalizable.

LIMITATION OF THE STUDY

As this study was an experimental type, limitations in terms of implementation of the interventions and those caused by the background of students, including the attempt to implement within the tight schedule of the university were challenges for proper implementation of the intervention. Some of the results also indicate the need for further study to uncover issues that might arise as a consequence.

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REFERENCE

Adu-Gyamfi, K., Stiff, L., & Bossé, M. J. (2012). Lost in translation: Examining translation errors associated with mathematical representations. *School Science and Mathematics,* 112(3), 159-170. https://doi.org/10.1007/s10763-016-9730-1.

Adu-Gyamfi, K., Bossé, M. J., & Chandler, K. (2017). Student connections between algebraic and graphical polynomial representations in the context of a polynomial relation. *International Journal of Science and Mathematics Education, 15*(5), 915-938, https://doi.org/10.1007/s10763-016-9730-1.

Adu-Gyamfi, K., Bossé, M. J., & Lynch- Davis, K. (2019). Three types of mathematical representational translations: Comparing empirical and theoretical results. *School Science and Mathematics, 119*(7), 396-404, https://doi.org/10.1111/ssm.12360.

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education*, *33*(2-3), 131-152, https://doi.org/10.1016/S0360-1315(99)00029-9.

Ainsworth, S. (2008). The educational value of multiple-representations when learning complex scientific concepts. *Visualization: Theory and Practice in Science Education* (pp. 191-208): Springer, https://doi.org/10.1007/978-1-4020-5267-5_9.

Andersen, C., Scheuer, N., Echeverría, M. d. P. P., & Teubal, E. (2009). *Representational systems and practices as learning tools*: Sense Publishers Rotterdam. https://doi.org/10.1163/9789087905286

Bennett, A., Moore, T., & Nguyen, X. (2011). *A longitudinal study on students' development and transfer of the concept of integration.* Paper presented at the Conference & Exposition, American Society for Engineering Education, Vancouver, British Columbia, Canada. https://doi.org/10.18260/1-2--17338

Berry, K. J., & Mielke Jr, P. W. (1988). A generalization of Cohen's kappa agreement measure to interval measurement and multiple raters. Educational and Psychological Measurement, 48(4), 921-933.

Bossé, M. J., Adu-Gyamfi, K., & Chandler, K. (2014). Students' Differentiated Translation Processes. *International Journal for Mathematics Teaching & Learning*, Retrieved from http://www.cimt.org.uk/journal/bosse5.pdf.

Chang, B. L., Cromley, J. G., & Tran, N. (2016). Coordinating multiple representations in a reform calculus textbook. *International Journal of Science and Mathematics Education*, *14*(8), 1475-1497. https://doi.org/10.1007/s10763-015-9652-3.

Demetriadis, S. (2004). Interaction between learner's internal and external representations in multimedia environment: a state-of-the-art.

Dreyfus, T., & Eisenberg, T. (1996). Mathematical Thinking. *The nature of mathematical thinking*, 253-284.

Dunham, P. H., & Osborne, A. (1991). Learning How to See: Students Graphing Difficulties. *Focus on learning problems in mathematics, 13*(4), 35-49.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1-2), 103-131, http://dx.doi.org/10.1007/s10649-006-0400-z.

Easdown, D. (2009). Syntactic and semantic reasoning in mathematics teaching and learning. *International Journal of Mathematical Education in Science and Technology, 40*(7), 941-949, https://doi.org/10.1080/00207390903205488.

Ernaningsih, Z., & Wicasari, B. (2017). *Analysis of Mathematical Representation, Communication, and Connection in Trigonometry.* Paper presented at the the 2017 International Conference on Research in Education.

Gagatsis, A., & Elia, I. (2004). The Effects of Different Modes of Representation on Mathematical Problem Solving. *International Group for the Psychology of Mathematics Education, URL:* https://gnosis.library.ucy.ac.cy/handle/7/38008.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology*, *24*(5), 645-657, https://doi.org/10.1080/0144341042000262953.

Gemechu, E., Kassa, M., & Atnafu, M. (2018). MATLAB Supported Learning and Students' Conceptual Understanding of Functions of Two Variables: Experiences from Wolkite University. *Bulgarian Journal of Science and Education Policy, 12*(2), 314-344.

Grossberg, S. (1982). How does a brain build a cognitive code? *Studies of Mind and Brain* (pp. 1-52): Springer.

Haciomeroglu, E. S., & Andreasen, J. B. (2013). Exploring calculus with dynamic mathematics software. *Mathematics and Computer Education, 47*(1), 6.

Huang, C. H. (2015). Calculus students' visual thinking of definite integral. American Journal of Educational Research, 3(4), 476-482.

Huntley, M. A., Marcus, R., Kahan, J., & Miller, J. L. (2007). Investigating high-school students' reasoning strategies when they solve linear equations. *The Journal of Mathematical Behavior, 26*(2), 115-139, doi:10.1016/j.jmathb.2007.05.005.

Inzunza, S. (2006). Student's errors and difficulties for solving problems of sampling distributions by means of computer simulation. ICOTS-7, 1–4.

Kaput, J. (1989). Linking representations in the symbol systems of algebra', Wagner, S. & Kieran, C.(Eds.), Research Issues in the Learning and Teaching of Algebra: Lawrence Erlbaum, Hillsdale, NJ.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1-64, URL: https://doi.org/10.3102/00346543060001001.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. Reston VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (2014). Principles to actions: Ensuring mathematical success for all. Reston VA: NCTM.

Nguyen, D. H., & Rebello, N. S. (2009). *Students' difficulties in transfer of problem solving across representations.* Paper presented at the AIP Conference Proceedings.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89*(6), 351-363, https://doi.org/10.1080/00220671.1996.9941339.

Palmer, S.E. (1978). Fundamental aspects of cognitive representation. In Eleanor Rosch & Barbara Lloyd (eds.), *Cognition and Categorization*. Lawrence Elbaum Associates. pp. 259-303 (1978).

Pambudi, D. S., Budayasa, I. K., & Lukito, A. (2020). The role of mathematical connections in mathematical problem solving. *Jurnal Pendidikan Matematika,* 14(2), 129-144, https://doi.org/10.22342/jpm.14.2.10985.129-144.

Panasuk, R., & Beyranevand, M. (2011). Middle school algebra students' preferred representations when solving problems. *The Mathematics Educator, 13*(1), 32-52.

Rachma, A. A., & Rosjanuardi, R. (2021). Students' obstacles in learning sequence and series using onto-semiotic approach. *Jurnal Pendidikan Matematika,* 15(2), 115-132, https://doi.org/10.22342/jpm.15.2.13519.115-132

Rahmawati, D. (2019). Translation Between Mathematical Representation: How Students Unpack Source Representation? *Matematika dan Pembelajaran, 7*(1), 50-64, https://doi.org/10.33477/mp.v7i1.1045.

Rahmawati, D., Purwantoa, S., Hidayanto, E., & Anwar, R. B. (2017). Process of Mathematical Representation Translation from Verbal into Graphic. *International Electronic Journal of Mathematics Education,* https://doi.org/10.29333/iejme/618.

Roth, W.-M., & Bowen, G. M. (2001). Professionals read graphs: A semiotic analysis. *Journal for Research in Mathematics Education*, 159-194, https://doi.org/10.2307/749672.

Salkind, G. M., & Hjalmarson, M. (2007). Mathematical representations. *http://mason.gmu. edu/~ gsalkind/portfolio/products/857LitReview.pdf. Di akses tanggal, 15*.

Teferra, T., Asgedom, A., Oumer, J., & Assefa, B. (2017). Ethiopian Education Development Roadmap (2017-30), Retrieved from: https://moe.gov.et.

Van Labeke, Nicolas and Ainsworth, Shaaron E. (2001). Applying the DeFT Framework to the Design of Multi-Representational Instructional Simulations. In: AIED'01 - 10th International Conference on Artificial Intelligence in Education, 19-23 May 2001, San Antonio, Texas, IOS Press, pp. 314–321.

Widodo, S.A., Pangesti, A.D., Istiqomah, I., Kuncoro, K.S., & Arigiyati, T.A. (2020). Thinking process of concrete student in solving two-dimensional problems. *Jurnal Pendidikan Matematika*, 14(2), 117-128, https://doi.org/10.22342/jpm.14.2.9460.117-128.

Yusuf, M., Rahim, S. S. A., & Eu, L. K. (2021). Obstacles faced by college students in solving probability word problems. *Jurnal Pendidikan Matematika*, 15(1), 83-90, https://doi.org/10.22342/jpm.15.1.12801.83-90.