

ANALYSIS OF STUDENT ABILITY IN FUNCTIONAL THINKING THROUGH THE FUNCTION TABLE: THE CASE OF SQUARE FUNCTION

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Abstrak

Salah satu cara untuk menelusuri kemampuan mahasiswa dalam memahami konsep fungsi dapat dilakukan dengan cara menganalisis kemampuan berpikir fungsionalnya. Penelitian ini bertujuan untuk mengidentifikasi dan mengetahui kemampuan berpikir fungsional mahasiswa melalui tabel fungsi. Penelitian ini merupakan penelitian kualitatif yang bersifat eksploratif. Subjek penelitian adalah 2 dari 56 mahasiswa semester 5 Jurusan Pendidikan Matematika Universitas Islam Negeri Suska Riau. Subjek penelitian dipilih secara *purposive* yaitu mahasiswa yang menyelesaikan secara benar lembar tes dan memiliki kemampuan komunikasi yang lancar. Data dikumpulkan melalui tes dan wawancara. Hasil lembar jawaban tes dianalisis berdasarkan kerangka kerja berpikir fungsional dan hasil wawancara dianalisis untuk mendalami dan mengklarifikasi berpikir fungsional mahasiswa yang belum terungkap pada lembar jawaban. Hasil analisis data menunjukkan bahwa ada dua tahapan penyelesaian yaitu tahap pertama, menentukan pola bilangan dan berpikir tentang pola bilangan sampai ke- n , menentukan beda dari X_n , menentukan beda dari Y_n , menentukan perubahan nilai antara X_n dan Y_n , menggeneralisasikan barisan X_n , dan menggeneralisasikan barisan Y_n . Sedangkan tahap kedua, menentukan perubahan nilai antara X_n dan Y_n , menggeneralisasikan barisan X_n , menggeneralisasikan barisan Y_n , dan menggeneralisasikan hubungan antara X_n dan Y_n . Berdasarkan hasil penelitian menunjukkan bahwa mahasiswa mampu memenuhi semua kerangka kerja berpikir fungsional dalam menyelesaikan masalah melalui tabel fungsi.

Kata kunci: Berpikir fungsional, fungsi kuadrat, pemecahan masalah, tabel fungsi.

Abstract

One way to track students' ability to understand the concept of function can be done by analyzing their functional thinking skills. This study aims to identify and determine students' functional thinking skills through function tables. This research is an exploratory qualitative research. The research subjects were 2 out of 56 5th semester students of the Department of Mathematics Education, Suska Riau State Islamic University. The research subjects were selected *purposively*, namely students who correctly completed the test sheet and had fluent communication skills. Data were collected through tests and interviews. The results of the test answer sheets were analyzed based on the functional thinking framework and the results of the interviews were analyzed to explore and clarify students' functional thinking that had not been revealed on the answer sheets. The results of data analysis show that there are two stages of completion, namely the first stage, determining the number pattern and thinking about the number pattern up to the n th, determining the difference from X_n , determining the difference from Y_n , determining the change in value between X_n and Y_n , generalizing the X_n sequence, and generalizing Y_n sequence. While the second stage, determine the change in value between X_n and Y_n , generalize sequence X_n , generalize sequence Y_n , and generalize the relationship between X_n and Y_n . Based on the results of the study, it showed that students were able to fulfill all the frameworks for solving problems through the function table

Keywords: Functional thinking, function table, mathematical problem solving, quadratic function



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INTRODUCTION

At the level of junior high school education up to college the function material is taught. However, understanding the function is not a concept that is easily understood by students. Many students experience errors in representing and interpreting functional forms. These difficulties and misunderstandings will have an impact on student learning outcomes, if a solution is not found. So that teachers/lecturers need to provide practice questions related to function material and familiarize students with functional thinking from an early age. Functional thinking itself is a mental activity in generalizing the relationship between covariant quantities that can be represented through words, algebra, tables and graphs. This is in line with the opinion of Blanton, dkk (2015) stating that functional thinking is a generalization of the relationship between covariant quantities and can be represented through words, algebraic notation, tables and graphs.

Many researches on functional thinking have been carried out, one of the experts who conducted research on functional thinking is Blanton et al. (2016) who found that there were eight levels of functional thinking of students. Doorman, dkk (2012); Stephens (2017); Stephens, dkk (2017); Wilkie (2015); Wilkie & Clarke (2015); Wilkie & Clarke (2016) design and develop learning for teachers so as to improve students' functional thinking. Research Yuniati et al. (2019) found some representations of students in functional thinking, but in general the representations that appear are algebraic representations. Furthermore Yuniati et al (2020); Yuniati et al. (2020); Yuniati & Suparjono (2021); Yuniati & Suparjono (2019) found that students

were able to think partially functionally and use arithmetic sequence formulas. On the other hand Stephens et al. (2017) dan Stephens et al. (2017) found that through intervention can improve students' ability to determine recursive patterns and covariance relationships between variables. In this study, looking for student responses that are sometimes different from certain tasks (tasks given in the form of linear functions and quadratic functions). But from several previous studies, no one has conducted research that uses function tables to determine the functional thinking ability of students who can generalize the form of quadratic functions

The quadratic function is a mathematical material that must be studied by high school (SMA)/Madrasah Aliyah (MA) students even up to college. The material for quadratic functions has many applications in everyday life and is a prerequisite for studying other mathematics, such as derivatives, integrals, linear programming and geometry. Given the importance of students understanding the material quadratic function, it is necessary to explore the extent to which students' ability to understand the material of quadratic functions. Therefore, the purpose of this study is to explore students' abilities in functional thinking through function tables involving quadratic functions.

RESEARCH METHODS

This research is a qualitative research that is exploratory. There were 56 students who participated in this study in the fifth semester of Mathematics Education at the State Islamic University of Suska Riau. The student is given a test via google meet and the time given is 60 minutes. The

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tests given are non-routine mathematical task. Based on the results of the answer sheet analysis, two groups of different answers were obtained. Then from each group one student's answer was chosen, so there were two student answers that were used as research subjects. The research subjects were selected purposively, namely students who had completed correctly and had fluent communication skills.

Data collection uses tests and interview guidelines to identify students' functional thinking abilities. The test was adopted from Tanişli (2011) which uses a function table to explore students' functional thinking. In solving test questions, students are expected to be able to: 1) determine recursive patterns, 2) determine covariational changes, and 3) generalize correspondence. The indicators used to analyze functional thinking are outlined in Table 1.

Table 1. Indicators for analyzing functional thinking

Functional Thinking Framework	Description	Indicators for Analyzing Functional Thinking	Code
Define a recursive pattern	Observing objects in tabular form and thinking about the next unknown object	➤ Determine the number pattern of the given object and think about the number pattern up to the n th.	PL1
		➤ Determine the difference of X_n	PL2
		➤ Determine the difference from Y_n	PL3
Determine the covariational relationship	Determine the change in the value of the relationship between variations in quantity in a given problem	➤ Determines the value change in the sequence X_n	PN1
		➤ Determines the value change in the sequence Y_n	PN2
		➤ Determines the change in value between X_n and Y_n	PN3
Determine correspondence	Generalizing the relationship between quantity variations on a given problem	➤ Generalizing the sequence X_n	GB1
		➤ Generalizing the sequence Y_n	GB2
		➤ Generalizing the relationship between X_n and Y_n	GB3

The interviews used were unstructured interviews. Because the questions in the interview guide used are still very likely to develop according to the conditions or characteristics of the respondents. This activity was documented with an audiovisual recording device and interviews were conducted via chat via WhatsApp. The next step is triangulation which is used to test the validity of the data. Creswell

(2012) states that triangulation is the process of corroborating evidence based on different individuals, types of data (observation records and interviews), or data collection methods (documents and interviews) in descriptions and themes in qualitative research. Researchers examine each source of information and find evidence to support a theme. This ensures that the research is accurate because the information refers to an

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information source, individual, or process. The triangulation method used in this research is to compare student answer sheets and interviews.

In general, the process of data analysis in qualitative research includes: data reduction, data categorization, synthesis, and ends with the preparation of working hypotheses (Creswell, 2012). Qualitative data analysis is carried out interactively and takes place continuously until complete, so that the data is saturated. The size of the data saturation is indicated by the absence of new data or information. The stages of data analysis in this qualitative study were modified from Creswell (2012) as follows: 1) transcribing interview data, scanning student answer sheets, and arranging the data into certain types based on the characteristics of the data; 2) reducing data, namely removing unnecessary data and organizing raw data obtained from the field; 3) analyze the data in more detail by coding or categorizing the data. At this stage, the coding of these categories with special terms can be seen in table 1; 4) drawing the structure of students' functional thinking in solving problems based on data categorization; and 5) drawing conclusions based on the results of data analysis, both obtained by using tests and interviews.

RESULTS AND DISCUSSION

Based on the research subject's answer sheet S1, the first step is to understand the given function table by reading and looking at the test sheet. Then determine the number pattern on the x variable, where the number pattern is the addition of 1 and the number pattern on the y variable is a multilevel number, namely +2, +3, +4, +5, and so on. While the subject of S2 did not determine the number pattern on

the variable x and variable y . The answer sheets for S1 and S2 subjects in determining number patterns can be seen in Figure 1.

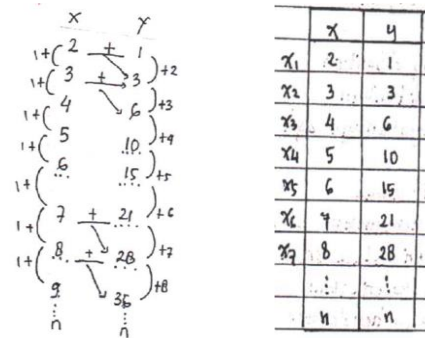


Figure 1. Answer sheets for subjects S1 and S2 determine the number pattern

In determining the covariational relationship, the subject of S1 stated that the relationship between many numbers x and many numbers y is if $x = 2$ then $y = 1$, if $x = 3$ then $y = 3$, if $x = 4$ then $y = 6$, if $x = 5$ then $y = 10$, and so on. While the subject of S2 stated that there is a relationship between many numbers x and y numbers, namely to determine y in the second row and so on is the sum of the numbers x and y in the previous row or $x = x_1$, $y = y_1$, $y_2 = x_1 + y_1$, $y_3 = x_2 + y_2$, $y_4 = x_3 + y_3$, $y_5 = x_4 + y_4$, $y_6 = x_5 + y_5$, $y_7 = x_6 + y_6$, and so on. The answer sheets for subjects S1 and S2 in determining the covariational relationship can be seen in Figure 2.

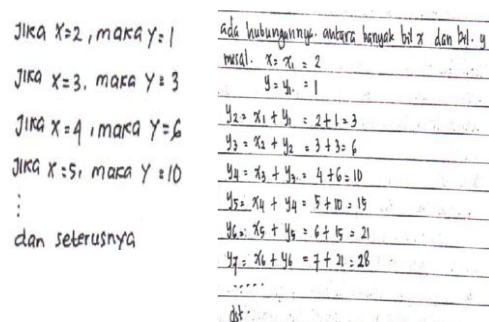


Figure 2. Answer sheets for subjects S1 and S2 determine the covariational relationship

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Generalizing the relationship between covariance variations (correspondence), subjects S1 and S2 generalize many numbers x using the arithmetic sequence formula $U_n = a + (n - 1)b$ so that the formula for many numbers x is $U_n = n + 1$. The answer sheets for subjects S1 and S2 in determining the correspondence of many x numbers can be seen in Figure 3.

Bilangan x diketahui (n untuk x)	Rumus ke- n Aritmatika (a)
$a = 2$	$a = 2$
$b = 1$	$b = 3 - 2 = 1$
$U_n = a + (n-1)b$	$U_n = a + (n-1)b$
$U_n = 2 + (n-1)1$	$= 2 + (n-1)1$
$U_n = 2 + n - 1$	$= 2 + n - 1$
$U_n = n + 1$	$U_n = n + 1$

Figure 3. Answer sheets for subjects S1 and S2 determine the correspondence of many numbers x

Next, subject S1 generalizes many numbers y using the formula $U_n = an^2 + bn + c$, while subject S2 generalizes many numbers y using the formula $U_n = \frac{a}{0!} + \frac{(n-1)b}{1!} + \frac{(n-1)(n-2)c}{2!}$. So the result of generalizing many numbers y is $U_n = \frac{1}{2}n^2 + \frac{1}{2}n$. The answer sheets for S1 and S2 in determining the correspondence of many y numbers can be seen in Figure 4.

$U_n = an^2 + bn + c$	<table border="0"> <tr> <td>n</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> <td>13</td> <td>14</td> <td>15</td> <td>16</td> <td>17</td> <td>18</td> <td>19</td> <td>20</td> <td>21</td> <td>\dots</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	\dots																								
n		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	\dots																									
$U_n = \frac{1}{2}n^2 + \frac{1}{2}n + 0$	Rumus ke- n Aritmatika Untuk y																																															
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$U_n = \frac{1}{2}n(n+1)$																																																

$U_n = \frac{a}{0!} + \frac{(n-1)b}{1!} + \frac{(n-1)(n-2)c}{2!}$	$a = 1$
$U_n = \frac{1}{0!} + \frac{(n-1)2}{1!} + \frac{(n-1)(n-2)1}{2!}$	$b = 2$
$= \frac{1}{1} + \frac{(n-1)2}{1} + \frac{(n-1)(n-2)}{2}$	$c = 1$
$= 1 + (n-1)2 + \frac{(n-1)(n-2)}{2}$	
$= 1 + 2n - 2 + \frac{n^2 - 3n + 2}{2}$	$\dots (x 2)$
$2U_n = 2 + 4n - 4 + \frac{n^2 - 3n + 2}{1}$	
$2U_n = n^2 + n$	
$U_n = \frac{n^2 + n}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$	

Figure 4. Answer sheets for subjects S1 and S2 determine the correspondence of many numbers y

Then the subject of S2 generalizes many numbers x and many numbers y by trial and error so that the formula $y_n = \frac{x_n^2 - x_n}{2}$ is obtained. The answer sheet for the subject of S2 in determining the correspondence of many x numbers and many y numbers can be seen in Figure 5.

$$y_n = \frac{x_n^2 - x_n}{2}$$

$$y_2 = \frac{3^2 - 3}{2}$$

$$= \frac{9 - 3}{2} = \frac{6}{2} = 3$$

Figure 5. The answer sheet for subject S2 determines the correspondence of many numbers x and y

The results show that the stages of the S1 subject in solving the problem can be described as follows: 1) The subject determines the number pattern of the given object and thinks about the number pattern up to the n th, 2) The subject determines the difference from X_n , 3) The subject determines the difference from Y_n , 4) The subject determines the change in value between X_n and Y_n , 5) The subject generalizes the X_n sequence, and 6) The subject generalizes the Y_n sequence. While the solution stages of the S2 subject in solving the problem through the function table can be described as follows: 1) The subject determines the change in value between X_n and Y_n , 2) The subject generalizes the X_n sequence, 3) The subject generalizes the Y_n sequence, and 4) The subject generalizes the relationship between X_n and Y_n . The scheme of the two subjects in solving the problem can be seen in Figure 6.

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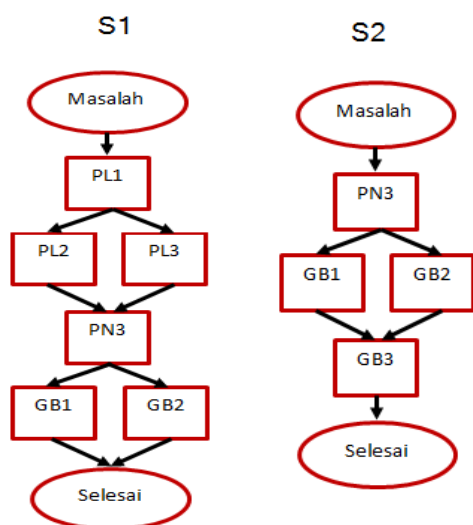


Figure 6. Schematic of the two subjects in solving problems

Empirical data shows that students' functional thinking in solving problems through function tables starts from determining the number pattern, namely the variable number pattern x , then the variable number pattern y . This was done by the subject of S1, while the subject of S2 in solving problems through a function table without looking for number patterns. This number pattern is usually called a recursive pattern which is described in the research findings of Lannin, et al. (2006) and Warren et al. (2006). This approach is used by students to see patterns in the dependent variable in the function table without considering the independent variable. Then otherwise determine the pattern of independent variables in the function table without considering the dependent variable. In this case, the recursive pattern is done partially, namely determining the recursive pattern on the variable x and determining the recursive pattern of the variable y . Next, subjects S1 and S2 both determine the change in value (covariance) between X_n and Y_n . Changes in value are usually referred to as the relationship between two

quantities or a covariational relationship Tanişli (2011). The relationship between the two quantities in this study there are two ways, namely first, the relationship between the independent and dependent variables using the implication of "if...then...". Second, using mathematical examples and operations (ie, +). This is in line with research findings Tanişli (2011) that the relationship between two quantities is explained in a semi-symbolic form, using familiar mathematical symbols for numbers (1, 2, 3, etc.) and mathematical operations (+, -, x).

In generalizing the relationship between variations in the quantity of S1 subjects solved separately, so did S2 subjects. However, subject S2 can generalize the relationship between X_n and Y_n . Generalizing the relationship between quantity variations is usually called the Smith correspondence (Pinto and Cañadas, 2012) and Tanişli (2011). In this case, students have difficulty in generalizing the relationship between X_n and Y_n , namely $y_n = \frac{x_n^2 - x_n}{2}$. This is in accordance with the findings of Tanişli (2011), namely students have difficulty in completing the general form $y = 2x - a$ and $y = 3x - a$. Thus, it can be concluded that students have difficulty in generalizing the relationship between quantities in the general form of functions in subtraction operations. Thus, the results of this study indicate that students can solve problems by fulfilling all functional thinking frameworks through two stages of completion.

Another finding, students prefer numerical representations in determining recursive patterns and use algebraic representations in determining relationships between covariations and generalizing correspondences. This is in line with the research of Yuniati et al.

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(2019) which states that most students use algebraic representations in functional thinking. In this study, it has a drawback, namely questions are given to students in the form of assignments that are done at home, this is due to the presence of covid-19. so that there is no strict supervision when students work on questions.

CONCLUSION AND SUGGESTIONS

This study analyzes students' ability in functional thinking through a table of functions involving quadratic functions. We found two stages of solution in solving the problem. The first stage, 1) students determine the number pattern of the given object and think about the pattern of numbers up to the n th, 2) students determine the recursive pattern, 3) students determine the covariational relationship between X_n and Y_n , 5) students generalize the sequence X_n , and 6) students generalize the sequence Y_n . While the second stage, 1) students determine the covariational relationship between X_n and Y_n , 2) students generalize the X_n sequence, 3) students generalize the Y_n sequence, and 4) students generalize the relationship between X_n and Y_n . From the results of this study, it can provide knowledge for lecturers that by providing routine and non-routine practice questions, students can explore functional thinking. In addition, lecturers can also determine students' abilities in representing and interpreting functional forms. However, this study has limitations, namely the provision of tests via google meet and interviews conducted via whatsapp, due to the covid-19 pandemic so that it is not optimal.

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