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CRRA utility and the sustainability of cooperation in infinitely-repeated games

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ABSTRACT

In a symmetric infinitely-repeated game, where players have constant relative risk aversion (CRRA), or constant elasticity of intertemporal substitution, utility functions, it is shown that the critical discount factor required to sustain full cooperation is decreasing in the coefficient of relative risk aversion (increasing in the elasticity of intertemporal substitution). An application to cooperation in international environmental agreements (IEA) is presented and it is shown that the limit of the critical discount factor as the number of countries goes to infinity is equal to one (zero) if the coefficient of intertemporal inequality aversion is less (greater) than one.

1. Introduction

In the literature on experimental tests of the infinitely-repeated prisoners' dilemma, surveyed by Bó and Fréchette (2018), the link between personal characteristics and cooperation has been tested. Sabater-Grande and Georgantzis (2002) find that risk aversion of players is negatively related to cooperation whereas Dreber et al. (2014), Davis et al. (2016), and Proto et al. (2018) do not find a relationship between risk aversion of players and cooperation.

This note analyses cooperation in a symmetric infinitely-repeated game where the players have constant relative risk aversion (CRRA) utility functions. The parameter of the CRRA utility function can be interpreted as the coefficient of relative risk aversion or, more relevantly in a deterministic game of complete information, as the reciprocal of the elasticity of intertemporal substitution. It is shown that the critical discount factor required to sustain cooperation as a subgame-perfect Nash equilibrium using Nash-reversion trigger strategies is decreasing in this parameter. Hence, the more risk averse are the players then the easier it is to sustain cooperation. An application to international environmental agreements (IEA) with many countries is presented.

2. CRRA utility and the critical discount factor

Consider a symmetric game with $J \geq 2$ players where a stage (or constituent) game is repeated infinitely. In the stage game, the monetary payoff to the identical players is a function of their actions $y_j(\boldsymbol{\sigma})$ where $\boldsymbol{\sigma} = \left(\sigma_1, \ldots, \sigma_J\right)$ and $j = 1, \ldots, J$. The monetary payoff (or income) of a player is assumed to be continuous and strictly quasi-concave in its own action, which will ensure the existence of a Nash equilibrium. The utility of money (or income) for the players is given by the constant relative risk aversion (CRRA) utility function $u\left(y_j\right) = y_j^{1-\gamma}/(1-\gamma)$ where $\gamma \geq 0\ (\gamma \neq 1)$ is the coefficient of relative risk aversion (or the reciprocal of the elasticity of intertemporal substitution), which is defined as $\gamma = -u''y/u'$ and is more generally a measure of the concavity of the utility function. If $\gamma = 1$ then the CRRA utility function is $u\left(y_j\right) = \ln\left(y_j\right)$. Note that this is in line with the usual assumptions of game theory, as utility is a function of the actions of the players, $u_j(\boldsymbol{\sigma}) = u\left(y_j(\boldsymbol{\sigma})\right)$. In the stage game, a player maximising utility is equivalent to maximising the monetary payoff y_j since $u\left(y_j\right)$ is an increasing monotonic transformation of y_j. If the players cooperate to maximise joint welfare then they all play σ_C and receive the monetary payoff y_C. If the players behave non-cooperatively then the outcome will be the Nash equilibrium (assumed to be unique) where they all play σ_N and receive the monetary payoff y_N. If one player deviates by playing σ_D when all the others are playing σ_C then the deviator

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receives the monetary payoff y_D. Obviously, except in perverse examples, it will be the case that $y_D > y_C > y_N$.[1]

In the infinitely-repeated game, the stage game is played indefinitely and the players maximise the discounted sum of utility where the common discount factor is $\delta \in [0,1)$. The discount factor may be purely the discount factor of the player or the probability of the game continuing to the next round in which case the players are maximising expected utility. As is well known, from Friedman (1971), cooperation can be sustained as subgame-perfect Nash equilibrium using Nash-reversion trigger strategies if the discount factor is sufficiently large.[2] Each player plays σ_C provided all players have always played σ_C. If any player deviates by playing σ_D then all players play σ_N forever thereafter. Hence, players will cooperate if:

$$\frac{1}{1-\delta}u(y_C) \geq u(y_D) + \frac{\delta}{1-\delta}u(y_N) \tag{1}$$

Cooperation can be sustained as a subgame perfect Nash equilibrium if the discount factor is greater than the critical value:

$$\delta_N^* \equiv \frac{u(y_D)-u(y_C)}{u(y_D)-u(y_N)} = \begin{cases} \dfrac{y_D^{1-\gamma}-y_C^{1-\gamma}}{y_D^{1-\gamma}-y_N^{1-\gamma}} = \dfrac{1-\lambda_C^{1-\gamma}}{1-\lambda_N^{1-\gamma}} & \gamma \neq 1 \\ \dfrac{\ln(y_D)-\ln(y_C)}{\ln(y_D)-\ln(y_N)} = \dfrac{\ln(\lambda_C)}{\ln(\lambda_N)} & \gamma = 1 \end{cases} \tag{2}$$

where $\lambda_C \equiv y_C/y_D < 1$, and $\lambda_N \equiv y_N/y_D < \lambda_C < 1$.

Proposition 1. *The critical discount factor required to sustain full cooperation using Nash-reversion trigger strategies δ_N^* is decreasing in γ.*

Proof. Differentiating the critical discount factor δ_N^* with respect to the parameter γ yields:

$$\frac{\partial \delta_N^*}{\partial \gamma} = \frac{(\lambda_N/\lambda_C)^\gamma}{\left(\lambda_N-\lambda_N^\gamma\right)^2}\left[\lambda_C\left(-\lambda_N+\lambda_N^\gamma\right)\ln(\lambda_C) - \left(-\lambda_C+\lambda_C^\gamma\right)\lambda_N\ln(\lambda_N)\right] \qquad \gamma \neq 1 \tag{3}$$

This derivative will be negative if the expression in square brackets is negative, which will be the case if:

$$\frac{\lambda_N\ln(\lambda_N)}{\lambda_N-\lambda_N^\gamma} < \frac{\lambda_C\ln(\lambda_C)}{\lambda_C-\lambda_C^\gamma} \qquad \gamma \neq 1 \tag{4}$$

Since $\lambda_C > \lambda_N$, this will be case if $Z = \lambda\ln(\lambda) \big/ (\lambda - \lambda^\gamma)$ is increasing in λ. Differentiating Z with respect to λ yields:

$$\frac{dZ}{d\lambda} = \frac{\lambda - \lambda^\gamma\left(1+(1-\gamma)\ln(\lambda)\right)}{(\lambda-\lambda^\gamma)^2} \qquad \gamma \neq 1 \tag{5}$$

This will be positive if $\lambda^{1-\gamma} > 1+\ln\left(\lambda^{1-\gamma}\right)$ for $\lambda \in [0,1)$, $\gamma \geq 0$ and $\gamma \neq 1$, or equivalently $\Omega \equiv x - (1+\ln(x)) > 0$ where $x = \lambda^{1-\gamma}$ for $x \geq 0$ and $x \neq 1$ (since this implies that $\gamma = 1$). Since Ω is convex, $d^2\Omega/dx^2 = 1/x^2 > 0$, and has a minimum at $x=1$, $d\Omega/dx = (x-1)/x$, where $\Omega = 0$, it follows that $\Omega > 0$ for $x \geq 0$ and $x \neq 1$. Hence, Z is increasing in λ, and the derivative (3) is negative for $\gamma \geq 0$ and $\gamma \neq 1$.

Finally, note that taking the limit of (3) as γ goes to one yields that $\lim_{\gamma\to 1}\left(\partial\delta_N^*/\partial\gamma\right) = \ln(\lambda_C)\ln(\lambda_N/\lambda_C)/2\ln(\lambda_N) < 0$.■

The more risk averse are the players (the larger is γ) then the lower is the critical discount factor δ_N^* and the easier it is to sustain cooperation even though the game is deterministic.

3. Application to international environmental agreements

Now consider cooperation between countries (an IEA) in an infinitely-repeated environmental game with many countries.[3] In the stage game, each country has an identical endowment of labour, L, and uses labour and emissions of pollution, σ_j, to produce a consumption good, y_j. The welfare of each country is given by the CRRA utility function, $u\left(y_j\right)$, where γ in this context can be interpreted as a coefficient of intertemporal inequality aversion. The production of the consumption good is negatively affected by global emissions of pollution, $\sum_{i=1}^J \sigma_j$, where the production function of the jth country is assumed to be:

$$y_j = e^{-\beta\sum_{i=1}^J \sigma_i} L_{1-\alpha}\sigma_j^\alpha \tag{6}$$

where $\alpha \in (0,1)$ and $\beta > 0$.

In the stage game, each country can choose its emissions of pollution by setting the quantity of tradeable emissions permits, σ_j, to maximise its output of the consumption good, y_j, which is equivalent to maximising its CRRA utility, $u\left(y_j\right)$. If the countries cooperate then they choose a common quantity of tradeable emissions permits to maximise joint welfare. Setting $\sigma_j = \sigma$ for all countries, $j = 1,\ldots,J$ then choosing σ to maximise y_j yields the cooperative quantity of tradeable emissions permits $\sigma_C = \alpha/\beta J$, and the corresponding output of the consumption good:

$$y_C = e^{-\alpha}L^{1-\alpha}\sigma_C^\alpha \tag{7}$$

If the countries set the quantity of tradeable emissions permits non-cooperatively then each country will choose σ_j to maximise y_j given the quantities of tradeable emissions permits chosen by all the other countries. This will yield the Nash equilibrium quantity of tradeable emissions permits $\sigma_N = \alpha/\beta$, and the corresponding output of the consumption good:

$$y_N = e^{-\alpha J}L^{1-\alpha}\sigma_N^\alpha \tag{8}$$

If when all the other countries are setting the cooperative quantity of tradeable emissions permits, σ_C, a country deviates then it will set σ_j to maximise y_j given that all the other countries are setting σ_C. This yields the quantity of tradeable emissions permits when a country deviates $\sigma_D = \alpha/\beta$, which is equal to σ_N since the best-reply function of the jth country does not depends upon the quantities of tradeable emissions permits chosen by the other countries. The corresponding output of the consumption good is:

$$y_D = e^{-\alpha(2-1/J)}L^{1-\alpha}\sigma_D^\alpha \tag{9}$$

Dividing (7) and (8) by (9) yields λ_C and λ_N:

$$\lambda_C \equiv \frac{y_C}{y_D} = e^{\alpha(1-1/J)}J^{-\alpha} < 1, \quad \lambda_N \equiv \frac{y_N}{y_D} = e^{-\alpha(J-1)^2/J} < 1 \tag{10}$$

To show that λ_C is greater than λ_N, divide λ_C by λ_N then take logs, which yields:

$$\frac{\lambda_C}{\lambda_N} = e^{\alpha(J-1)}J^{-\alpha} \Rightarrow \quad \ln\left(\frac{\lambda_C}{\lambda_N}\right) = \alpha\left[(J-1)-\ln(J)\right] > 0 \tag{11}$$

Hence, as assumed in Section 2, $\lambda_C > \lambda_N$. An IEA is sustainable if cooperation is sustainable as a subgame perfect Nash equilibrium using Nash-reversion trigger strategies, and this will be the

[1] There are games where the Nash equilibrium and the cooperative outcome coincide so this assumption does not hold, but sustaining cooperation is not an issue in such a game.

[2] Nash-reversion trigger strategies are considered for the sake of simplicity. With optimal-punishment strategies, as in Abreu (1986, 1988), the analysis would be complicated as the punishment would depend upon γ.

[3] For a survey of the literature on sustaining cooperation in IEA, see Barrett (2005).

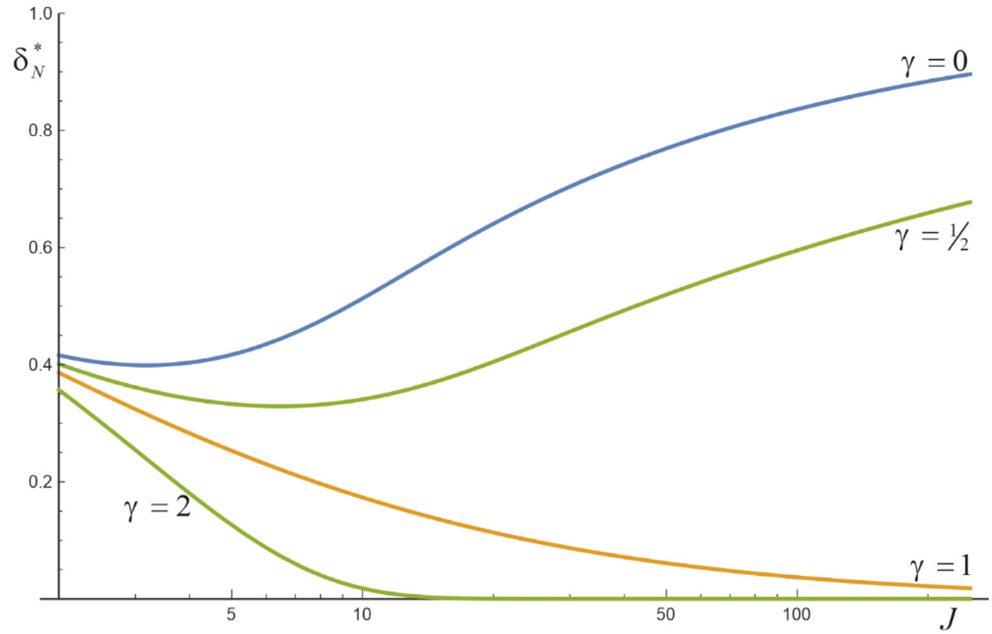

Fig. 1. The critical discount factor versus the number of countries.

case if the discount factor is greater than the critical discount factor in (2). Substituting from (10) into (2) yields:

$$\delta_N^* = \begin{cases} \dfrac{1-\lambda_C^{1-\gamma}}{1-\lambda_N^{1-\gamma}} = \dfrac{1-e^{\alpha(1-\gamma)(1-1/J)}J^{-\alpha(1-\gamma)}}{1-e^{-\alpha(1-\gamma)(J-1)^2/J}} & \gamma \neq 1 \\ \dfrac{\ln\lambda_C}{\ln\lambda_N} = \dfrac{J(\ln J-1)+1}{(J-1)^2} & \gamma = 1 \end{cases} \quad (12)$$

This depends upon α, γ and J, but not on β and L, and, from Proposition 1, it is decreasing in γ. For an IEA, an important question is can cooperation be sustained when the number of countries becomes very large especially when there are global externalities. This can be answered with this model by looking at the limit of the critical discount factor as the number of countries goes to infinity.

Proposition 2. *The limit of the critical discount factor required to sustain full cooperation using Nash-reversion trigger strategies as the number of countries goes to infinity is equal to one if $\gamma < 1$ and zero if $\gamma \geq 1$.*

Proof. If $\gamma < 1$ or $1-\gamma > 0$ then the limit of the numerator and the limit of the denominator in (12) as the number of countries goes to infinity are equal to one. Hence, the limit of the critical discount factor is equal to one as the number of countries goes to infinity. If $\gamma = 1$ then the numerator and the denominator both go to infinity as the number of countries goes to infinity, but the denominator goes to infinity more rapidly than the numerator so the limit of the critical discount factor is equal to zero. If $\gamma > 1$ or $1-\gamma < 0$ then the numerator and the denominator both go to minus infinity as the number of countries goes to infinity, but the denominator goes to minus infinity more rapidly than the numerator so the limit of the critical discount factor is equal to zero. ■

To illustrate this proposition, Fig. 1 shows the critical discount factor versus the number of countries for $\gamma = 0$, $\gamma = \frac{1}{2}$, $\gamma = 1$, and $\gamma = 2$ when $\alpha = 1/2$.

4. Conclusions

Proposition 1 is a quite general result, but Proposition 2 applies only to the specific application, although it also holds for a Cournot oligopoly with linear demand and many firms. Hence, one might conjecture that the result is more general.

Data availability

No data was used for the research described in the article.

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