Distributed Estimation of Stochastic Multi-Agent Systems for Cooperative Control with a Virtual Network

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Abstract

This paper proposes a distributed estimation algorithm that uses local information about the neighbors through sensing or communication to design an estimation-based cooperative control of the stochastic multi-agent system (MAS). The proposed distributed estimation algorithm solely relies on local sensing information rather than exchanging estimated state information from other agents, as is commonly required in conventional distributed estimation methods, reducing communication overhead. Furthermore, the proposed method allows interactions between all agents, including non-neighboring agents, by establishing a virtual fully-connected network with the MAS state information independently estimated by each agent. The stability of the proposed distributed estimation algorithm is theoretically verified. Numerical simulations demonstrate the enhanced performance of the estimation-based linear and nonlinear control. In particular, using the virtual fully-connected network concept in the MAS with the sensing/communication range, the flock configuration can be tightly controlled within the desired boundary, which cannot be achieved through the conventional flocking methods.

Index Terms

Multi-agent systems, distributed state estimation, consensus, rendezvous control, flocking control.

I. Introduction

Along with the increasing interest in autonomy, research on distributed estimation and control for a multi-agent system (MAS) has been carried out in various fields [1], [2]. This is primarily thanks to its clear advantages over the centralized approach in efficiency, robustness, flexibility, scalability, and reliability [3]. However, due to limited communication capability and the lack of a central administrator, the cooperative operation of a distributed MAS can be difficult. The following summarizes relevant research on the distributed estimation and control of the MAS.

A. Distributed estimation of MAS

Distributed estimation aims to estimate certain state information by sharing the estimated information among agents in the MAS in a distributed manner. Most of the distributed estimation is performed in the distributed Kalman filter framework [4]. In particular, observations and consensus averaging algorithms were integrated to estimate a large-scale distributed system in [5]. A robust unknown input observer for a linear MAS was applied to fault estimation in [6], and unknown inputs and states were estimated through distributed cooperative filters in [7]. In [8], a distributed fuzzy state observer was proposed to estimate the nonlinear function of dynamics. In addition, Kalman consensus filtering [9] and diffusion strategies for distributed Kalman filtering [10] were introduced to estimate stochastic systems. However, there are still unresolved issues resulting from a distributed framework approach, e.g., communication overhead [11].

B. Distributed control of MAS

Distributed control of the MAS is focused on achieving greater performance and efficiency during operations while addressing the topological constraints of MAS network. As the limited communication among agents restricts the capabilities of the MAS, some studies have focused on a distributed control protocol that can maximize or preserve the network connectivity [12], [13]. Besides, various studies have considered the consensus control of the MAS under the given network constraints. The fundamental idea of consensus is to reach a common agreement among agents by synthesizing the local control protocols with the shared information of the neighboring agents. Recent related studies investigated a consensus protocol in the nonlinear MAS [14], embedding distributed observers to follower agents [15], and convergence to consensus quickly in a different way from the conventional Laplacian approach [16]. Several cooperation problems use consensus in the control protocol, including rendezvous [17], formation [18], and flocking [19]. The stability of the consensus, as well as sufficient conditions for the

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existence of the desired consensus control protocol with noise and delay, have been verified using the Lyapunov theory [19]–[21]. In [22], consensus conditions are established for a stochastic approximation type algorithm that reduces the consensus gain in noisy measurement environments. Furthermore, distributed sliding mode control [23] and distributed PI control [24] have been studied for accomplishing consensus in stochastic MAS. Nevertheless, these methods can only be applied under certain conditions: [22] assumes that the consensus gain goes to zero as time goes to infinity; and [23] and [24] are working under the bounded disturbance condition. Thus, further research needs to be conducted on consensus control for stochastic systems.

Nomenclature

i	Agent index
N	Number of agents
N_i	Neighbor set of agent i
$\mathcal{L}$	Laplacian matrix
x_i	State vector of agent i
z_i	Measurement vector
u_i	Control input vector
ω_i	Process noise vector
ν_i	Measurement noise vector
A	State transition matrix
B	Control input matrix
H_i	Measurement matrix
Q	Covariance of process noise
R	Covariance of measurement noise
X	MAS state vector, $[x_1^T \cdots x_N^T]^T$
Z	MAS measurement vector, $[z_1^T \cdots z_N^T]^T$
U	MAS control input vector, $[u_1^T \cdots u_N^T]^T$
ω	MAS process noise, $[\omega_1^T \cdots \omega_N^T]^T$
F_1	MAS state transition matrix
F_2	MAS error input matrix
$\mathcal{Q}$	Covariance of MAS process noise, diag$\{Q, \cdots, Q\}$
$\hat{X}_i^-$	Prior estimation of X from the perspective of agent i, $[\hat{x}_{i,1}^{-T} \cdots \hat{x}_{i,N}^{-T}]^T$
e_i^-	Prior estimation error
Σ_i^-	Covariance of e_i^-
$\hat{X}_i$	Posterior estimation of X, $[\hat{x}_{i,1}^T \cdots \hat{x}_{i,N}^T]^T$
e_i	Posterior estimation error
Σ_i	Covariance of e_i
$\hat{U}_i$	Estimation of U
S_i	Residual covariance
G_i	Kalman gain
$\hat{X}$	Augmented estimation of X, $[\hat{X}_1^T \cdots \hat{X}_N^T]^T$
e	Augmented estimation error, $[e_1^T \cdots e_N^T]^T$
Σ	Covariance of e
Σ_{ij}^-	Prior cross covariance of e_i^- and e_j^-
Σ_{ij}	Posterior cross covariance of e_i and e_j

C. Distributed estimation of stochastic MAS for distributed control

For stochastic systems, a control strategy using the estimated information of the MAS state can improve the performance of the consensus protocol [25]. However, during the process of distributed estimation like Kalman consensus filtering, problems appear owing to a large amount of data to be transmitted among agents and the limited communication capability resulting in packet loss, delay, and large energy consumption [26]. To address these issues, a gossip-based approach [27] is proposed, in which a local information exchange occurs at random and an event-triggered consensus approach is proposed [28]. They can, however, only reduce the frequency of communication and not the size of the packets. By contrast, a proposed method in [29] for distributed estimation of the MAS state relies solely on neighbor information via local sensing and does not require communication. According to [29], by capturing the mutual influence from inter-agent cooperation, each agent can estimate the entire MAS state including non-neighboring agents (i.e., agents outside of the sensing range) by observing only the state of the direct neighbors. Although this approach can estimate MAS state successfully, estimated information was not used in the control protocol and thus could not contribute to the improvement of MAS coordination. Meanwhile, in [8], even though agents estimate the state to be used for control without exchanging the estimated information of others, each agent estimates its own state only (not the other agents' or entire MAS state) in a deterministic system.

In the output consensus protocol in a stochastic system, fluctuations may occur in the state of agents due to sensor noise. In addition, since the distributed system can only use limited information from neighboring agents through a network, distributed optimal control is intractable under topological network constraints [30]. Although the aforementioned methods [22]–[24] have been proposed to deal with these issues, they can only be used in certain conditions. On the other hand, the consensus protocol using the estimated state information can effectively deal with above issues, but these consensus-based estimation algorithms, such as distributed Kalman filter method [4], require a frequent exchange of the estimated state information among neighbors, which can cause large communication overhead.

D. Main contributions

Building upon the study described in [29], this study proposes a distributed control protocol by promoting a distributed estimation algorithm to achieve effective cooperation in the stochastic MAS while addressing the aforementioned limitations. Individual agents in the proposed distributed estimation algorithm require only local observations on the state of neighboring agents obtained through either sensing or communication in contrast to most existing distributed strategies. This paper deals with two different MAS control problems: (i) linear rendezvous control and (ii) nonlinear flocking control, emphasizing that the proposed algorithm can be applied to various MAS control problems. The main contributions of this study are as follows:

1) The distributed estimation algorithm is developed in which each agent can estimate the state of the entire MAS using only local observations of the state of neighboring agents. The proposed approach has the advantage in that it is free from the exchange of the estimated information (including mean and covariance of the entire MAS state which could be a large amount of data) among agents which was required in the existing distributed estimation method; only local observations (or communication if allowed) of the state of neighboring agents are sufficient, which can significantly reduce the communication overhead;
2) The proposed estimation-based control protocol improves the coordination performance of the stochastic MAS for both linear and nonlinear MAS coordination problems;
3) A virtual fully-connected network can be established through the distributed estimation of the entire MAS states by using the same cooperative controller for all agents; this enables interactions even with non-neighboring agents, providing enhanced MAS capabilities particularly for nonlinear flocking control; and
4) Stability analysis of the estimation-based control protocol is verified using the Lyapunov theory.

The rest of this paper is organized as follows. In Section II, we present the graph theory, a description of the dynamics and sensor models of the MAS, and the linear rendezvous and nonlinear flocking problems. Section III presents the estimation-based control protocol for rendezvous, flocking control, and a detailed derivation of the proposed distributed estimation. Section IV analyzes the stability of the distributed estimation algorithm, followed by a numerical demonstration of the proposed distributed estimation and control protocol in Section V. Finally, some concluding remarks and areas of future study are provided in Section VI.

II. Preliminaries and Problem Formulation

A. Graph theory

A proximity graph (network) among N agents is used to describe the interconnections of the agents. The graph $\mathcal{G}$ is formally defined as a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a set of nodes $\mathcal{V} = \{1, \cdots, N\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The edge in graph $\mathcal{G}$, denoted by a pair $(i, j) \in \mathcal{E}$, indicates that the i-th agent can measure the state of the j-th agent. The set of neighbors N_i of the i-th agent on this proximity graph at time step k is defined as:

$$N_i[k] = \{j \in \mathcal{V} \backslash \{i\} : \|p_i[k] - p_j[k]\| < r\},$$

where $p_i[k] \in \mathbb{R}^n$ is the position vector of the i-th agent. If we suppose that all agents have the same sensing range $r > 0$, then $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$, i.e., an undirected graph. The set of edges is defined as $\mathcal{E}[k] = \{(i, j) : j \in N_i[k]\}$. The adjacency matrix $\mathcal{A} \in \mathbb{R}^{N \times N}$ is defined in an element-wise manner as:

$$[\mathcal{A}]_{ij} = \begin{cases} 1, & \text{if } j \in N_i, \\ 0, & \text{otherwise,} \end{cases}$$

where $\forall i, j \in \mathcal{V}$. The degree of nodes is a vector $d = \mathcal{A} \times 1_N \in \mathbb{R}^N$, where $1_N \in \mathbb{R}^N$ is a column vector with all elements having a value of 1. The degree matrix is defined as $\mathcal{D} = \text{diag}\{d\} \in \mathbb{R}^{N \times N}$, and the Laplacian matrix of the graph is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathbb{R}^{N \times N}$, which is a symmetric matrix.

B. Dynamics and sensor model

This study focuses on the behavior of mobile MAS applications, where each agent's state is a constituent of its position and velocity. The dynamics of the i-th agent can be written as:

$$x_i[k+1] = Ax_i[k] + Bu_i[k] + \omega_i[k], \tag{1}$$

where the state vector is denoted by $x_i[k] = [p_i^T[k]\ v_i^T[k]]^T \in \mathbb{R}^{2n}$, $v_i[k] \in \mathbb{R}^n$ is the velocity vector, $u_i[k] \in \mathbb{R}^n$ is the control input vector, and $\omega_i[k] \in \mathbb{R}^{2n}$ is the process noise vector. Hereafter, the time index is frequently omitted unless it is necessary. The state transition matrix A and control input matrix B have the following form:

$$A = \begin{bmatrix} I_n & \Delta t I_n \\ 0_n & I_n \end{bmatrix}, B = \begin{bmatrix} \dfrac{\Delta t^2}{2} I_n \\ \Delta t I_n \end{bmatrix},$$

where I_n and 0_n are the n-dimensional identity matrix and zero matrix, respectively and Δt is the sampling time.

The i-th agent is assumed to be able to measure to obtain the states of its neighboring agents. The measurement of the i-th agent including its own state is given by the following:

$$z_{i,x_j}[k] = x_j[k] + \nu_{ij}[k],\ j \in \overline{N}_i[k], \tag{2}$$

where $z_{i,x_j} \in \mathbb{R}^{2n}$ is the j-th agent's state measured by the i-th agent, $\nu_{ij} \in \mathbb{R}^{2n}$ is its noise, and $\overline{N}_i = N_i \cup \{i\}$. The measurement vector for the i-th agent is denoted as:

$$z_i[k] = H_i[k](X[k] + \nu_i[k]), \tag{3}$$

where $X = [x_1^T\ \cdots\ x_N^T]^T \in \mathbb{R}^{2nN}$ is the state vector of the MAS, $H_i \in \mathbb{R}^{2n|\overline{N}_i| \times 2nN}$ is the measurement matrix for the i-th agent, $|\overline{N}_i|$ is the cardinality of the set, and $\nu_i \in \mathbb{R}^{2nN}$ is the measurement noise vector. The measurement matrix H_i is given by the following:

$$H_i = [h]_{lm} = \begin{cases} I_{2n}, & \text{if } m = \overline{N}_{i,l}, \\ 0_{2n}, & \text{otherwise}, \end{cases}$$

where $\forall l \in \{1, \cdots, |\overline{N}_i|\}, \forall m \in \mathcal{V}$, and $\overline{N}_{i,l}$ is the l-th element of $\overline{N}_i$. This sensor model measures the neighbors and its own state. Notably this sensor model can also be regarded as a single-hop communication model within the communication range. The process noise ω_i and measurement noise ν_i are assumed to be independent and identically distributed (i.i.d.) white Gaussian random variables with $\omega_i \sim \mathcal{N}_{2n}(0, Q)$ and $\nu_i \sim \mathcal{N}_{2nN}(0, R)$. The following information on the agent dynamics and sensor model is assumed to implement the proposed distributed estimation.

Assumption 1. The MAS consists of homogeneous agents, representing that all agents have the common information about agent dynamics, process and measurement noise covariance as prior knowledge, i.e., A, B, Q, and R [29].

The following subsections sequentially describe the rendezvous and flocking control protocols that the MAS is designed to follow cooperatively.

C. Linear rendezvous control

The definition of the rendezvous in this study is that both position and velocity of all agents reach to a common value, i.e., $\lim_{k\to\infty} ||p_i[k] - p_j[k]|| = 0$ and $\lim_{k\to\infty} ||v_i[k] - v_j[k]|| = 0, \forall i, j \in \mathcal{V}$. Second-order consensus control for rendezvous follows [17], which is given as:

$$u_i(X) = K \sum_{j\in N_i} (x_j - x_i), \tag{4}$$
$$K = \begin{bmatrix} \rho_1 I_n & \rho_2 I_n \end{bmatrix},$$

where ρ_1, $\rho_2 > 0$ are the rendezvous control gains. A second-order consensus with agent dynamics (1) and the control protocol (4) can be achieved if the network contains a spanning tree (connectivity condition).

It is difficult to use the agent's exact state information for control because the model has uncertainty and the measurements are normally noisy. Thus, while retaining the structure of Eq. (4), the control input uses measurements applying the sensor model as:

$$u_i(z_i) = K \sum_{j\in N_i} (z_{i,x_j} - z_{i,x_i}). \tag{5}$$

D. Nonlinear flocking control

The definition of the flocking is based on the Reynolds rules [31] which agents fly densely without collision and velocity reach to a common value. For flocking control, this study uses the augmented Cucker-Smale model to align the velocity and achieve cohesion and separation [19], which are given as:

$$\begin{aligned} u_i(X) = \rho_3 &\sum_{j\in N_i} \phi_1(p_{ij})\big(v_j - v_i\big) \\ &+ \rho_4 \sum_{j\in N_i} \phi_2(p_{ij}, v_{ij})\big(p_j - p_i\big), \end{aligned} \tag{6}$$

where ρ_3 and $\rho_4 > 0$ are the flocking control gains, the relative position $p_{ij} = p_i - p_j$, and the relative velocity $v_{ij} = v_i - v_j$. In Eq. (6), the first term achieves the velocity consensus, whereas the second term keeps the relative distance among the agents. The ϕ functions are

$$\begin{aligned} \phi_1(p_{ij}) &= 1/(1 + \|p_{ij}\|^2)^{\beta}, \\ \phi_2(p_{ij}, v_{ij}) &= \frac{\rho_5}{2\,\|p_{ij}\|^2} \langle v_{ij}, p_{ij} \rangle + \frac{\rho_6}{2\,\|p_{ij}\|} (\|p_{ij}\| - \mathcal{R}), \end{aligned}$$

where $\beta \geq 0$, ρ_5, $\rho_6 > 0$ is the function gains, and $\langle \cdot, \cdot \rangle$ is the inner product. In addition, $\mathcal{R}$ is the desired distance between agents if $N = 2$, but if there are more than two agents, $\mathcal{R}$ is the upper bound of the agent's position from the center of all agents in a converged flock configuration [32]. It should be noted that this is only true if the network is fully-connected, which means that all agents interact with one other; otherwise, the $\mathcal{R}$ boundedness of the converged flock configuration is no longer guaranteed. However, in a general MAS operation, it is difficult for agents with limited sensing/communication range to maintain a fully-connected network topology at all times; this is why we introduce the concept of a virtual fully-connected network for tight flocking control by distributed estimation in Section III-B. The augmented Cucker-Smale model with agent dynamics (1) and a control protocol (6) is stable when $\beta < 1/2$ and the connectivity condition is satisfied.

Similar to the description in Section II-C, flocking control using measurements can be presented as:

$$\begin{aligned} u_i(z_i) = \rho_3 &\sum_{j \in N_i} \phi_1(z_{i,p_{ij}}) \big(z_{i,v_j} - z_{i,v_i} \big) \\ &+\ \rho_4 \sum_{j \in N_i} \phi_2(z_{i,p_{ij}}, z_{i,v_{ij}}) \big(z_{i,p_j} - z_{i,p_i} \big), \end{aligned} \tag{7}$$

where z_{i,p_j} and z_{i,v_j} are the position and velocity of the j-th agent measured by the i-th agent, respectively, and $z_{i,p_{ij}}$ and $z_{i,v_{ij}}$ are the relative position and velocity between the i-th and j-th agents as measured by the i-th agent.

III. Distributed State Estimation and Control Protocol for Coordination of Stochastic Multi-agent System

This section describes the development of a distributed estimation algorithm that can track the entire agent belonging to the MAS and design a control protocol based on it. Here, the distributed estimation algorithm is developed for both linear and nonlinear MAS control and improves the coordination performance of the MAS as exemplified by the rendezvous and flocking problems, respectively.

A. Linear systems

This subsection presents a distributed estimation algorithm and estimation-based rendezvous control protocol in linear systems while only states of neighboring agents through sensing or communication are processed as local information. It is assumed that computation time for the estimation process is less than the data sampling time (i.e., sensing frequency).

1) Distributed state estimation: First, we derive a distributed estimation algorithm in a linear MAS that applies a collective behavior using the state of the MAS. All agents follow the same feedback control strategy given as Eq. (5) to achieve consensus through coordination. Concatenating the agent dynamics, the MAS dynamics is expressed as:

$$X[k+1] = (I_N \otimes A) X[k] + (I_N \otimes B) U(Z[k]) + \omega[k], \tag{8}$$

where $Z[k] = [z_1^T[k] \cdots z_N^T[k]]^T$ is the vector of measurements of all agents at time step k, $U(Z) = [u_1^T(z_1)\ \cdots\ u_N^T(z_N)]^T \in \mathbb{R}^{nN}$ is the control input of the MAS, $\omega = [\omega_1^T\ \cdots\ \omega_N^T]^T \in \mathbb{R}^{2nN}$, and $\otimes$ denotes the Kronecker product. Because the control input of each agent is a linear function of X with noise, Eq. (5) can be decomposed using Eq. (2) as:

$$\begin{aligned} u_i(z_i) &= K \sum_{j \in N_i} \big((x_j + \nu_{ij}) - (x_i + \nu_{ii}) \big) \\ &= -K(\mathcal{L}_i \otimes I_{2n}) X - K(\mathcal{L}_i \otimes I_{2n}) \nu_i, \end{aligned} \tag{9}$$

where $\mathcal{L}_i \in \mathbb{R}^{1 \times N}$ is the i-th row of the Laplacian matrix. Then, using the Laplacian matrix $\mathcal{L}$, the control input $U(Z)$ of the MAS can be rewritten in a compact matrix form as

$$U(Z) = -(\mathcal{L} \otimes K) X - (I_N \otimes K) \bar{\nu}, \tag{10}$$

where $\bar{\nu} = \big[\big((\mathcal{L}_1 \otimes I_{2n}) \nu_1 \big)^T \cdots \big((\mathcal{L}_N \otimes I_{2n}) \nu_N \big)^T \big]^T \in \mathbb{R}^{2nN}$. Substituting Eq. (10) into Eq. (8), the MAS dynamics takes the state feedback as

$$\begin{aligned} X[k+1] &= (I_N \otimes A) X[k] - (\mathcal{L}[k] \otimes BK) X[k] \\ &\quad -\ (I_N \otimes BK) \bar{\nu}[k] + \omega[k] \\ &= F_1[k] X[k] -\ (I_N \otimes BK) \bar{\nu}[k] + \omega[k], \end{aligned} \tag{11}$$

where $F_1 = (I_N \otimes A) - (\mathcal{L} \otimes BK) \in \mathbb{R}^{2nN \times 2nN}$.

The Kalman filter method is used to recursively estimate the total state of the MAS from the perspective of an individual agent, using a Bayesian approach. Let $z_i^{0:k} = \{z_i[0], \cdots, z_i[k]\}$ denote the set of measurements collected by the i-th agent up to time step k. Based on $z_i^{0:k}$, the updated state and covariance of the MAS, estimated by the i-th agent at time step k, are defined as:

$$\begin{aligned}\hat{X}_i[k] &:= \mathbb{E}[X[k]|z_i^{0:k}],\\ \Sigma_i[k] &:= \mathbb{E}[e_i[k]e_i^T[k]],\end{aligned}$$

where $\mathbb{E}[\cdot|\cdot]$ is the conditional expectation and $e_i = X - \hat{X}_i \in \mathbb{R}^{2nN}$. Suppose the state of the MAS is estimated using a centralized approach. In this case, the exact control action for each agent can be calculated by synthesizing all of the agents' measurements, resulting in concurred control actions from the entire MAS viewpoint. However, because each agent only knows its own measurement in a distributed fashion in this paper, it is difficult for each agent to know precisely the control actions exerted on the other agents. Accordingly, at best, each agent estimates the entire MAS control input, $U(Z)$, based on its estimated information of the MAS state. Using Eq. (10), the estimated control input of the MAS by the i-th agent is defined as:

$$\hat{U}_i[k] := \mathbb{E}[U(Z[k])|z_i^{0:k}] = -(\mathcal{L}[k] \otimes K)\hat{X}_i[k]. \tag{12}$$

Note that unlike in a centralized approach, $\hat{U}_i$, $\forall i \in \mathcal{V}$, might be different concerning each agent's specific viewpoint. Through a comparison with Eq. (10), Eq. (12) can be represented as:

$$\hat{U}_i[k] = U(\hat{X}_i[k]). \tag{13}$$

Then, from the MAS dynamics (8), the state of the MAS predicted by the i-th agent is defined as:

$$\begin{aligned}\hat{X}_i^-[k+1] &:= \mathbb{E}[X[k+1]|z_i^{0:k}]\\ &= (I_N \otimes A)\hat{X}_i[k] + (I_N \otimes B)U(\hat{X}_i[k])\\ &= F_1[k]\hat{X}_i[k].\end{aligned} \tag{14}$$

and the predicted error covariance is computed using Eqs. (11) and (14)

$$\begin{aligned}\Sigma_i^-[k+1] &:= \mathbb{E}[e_i^-[k+1]e_i^{-T}[k+1]]\\ &= F_1[k]\Sigma_i[k]F_1^T[k] + (I_N \otimes BK)\\ &\quad \times \mathbb{E}[\bar{\nu}[k]\bar{\nu}^T[k]](I_N \otimes BK)^T + \mathcal{Q},\end{aligned} \tag{15}$$

where $e_i^- = X - \hat{X}_i^- \in \mathbb{R}^{2nN}$, $\mathcal{Q} = \mathrm{diag}\{Q, \cdots, Q\} \in \mathbb{R}^{2nN \times 2nN}$ and $\mathbb{E}[\bar{\nu}\bar{\nu}^T] = \mathrm{diag}\{(\mathcal{L}_1 \otimes I_{2n})R(\mathcal{L}_1 \otimes I_{2n})^T, \cdots, (\mathcal{L}_N \otimes I_{2n})R(\mathcal{L}_N \otimes I_{2n})^T\} \in \mathbb{R}^{2nN \times 2nN}$.

The updated state is calculated from the predicted state and the new measurement at time step $k+1$. Using Eq. (3), the measurement residual, denoted by $\tilde{z}_i \in \mathbb{R}^{2n|\overline{N}_i|}$, is represented as:

$$\begin{aligned}\tilde{z}_i[k+1] &= z_i[k+1] - H_i[k+1]\hat{X}_i^-[k+1]\\ &= H_i[k+1]e_i^-[k+1] + H_i[k+1]\nu_i[k+1].\end{aligned} \tag{16}$$

The residual covariance is defined as:

$$\begin{aligned}S_i[k+1] &:= \mathbb{E}[\tilde{z}_i[k+1]\tilde{z}_i^T[k+1]]\\ &= H_i[k+1]\Sigma_i^-[k+1]H_i^T[k+1]\\ &\quad + H_i[k+1]RH_i^T[k+1].\end{aligned} \tag{17}$$

The updated estimation of $X[k+1]$ is given by the following:

$$\hat{X}_i[k+1] = \hat{X}_i^-[k+1] + G_i[k+1]\tilde{z}_i[k+1], \tag{18}$$

$$G_i[k+1] = \Sigma_i^-[k+1]H_i^T[k+1](S_i[k+1])^{-1}, \tag{19}$$

where G_i is the Kalman gain. The update of the state estimation error covariance is the same as the conventional Kalman filter method, which can be written compactly as

$$\Sigma_i[k+1] = \big(I_{2nN} - G_i[k+1]H_i[k+1]\big)\Sigma_i^-[k+1]. \tag{20}$$

Remark 1. The above distributed estimation process can be considered an alternative derivation of the sensing-based distributed estimation algorithm described in [29].

Remark 2. The developed system is designed in discrete-time but can be readily extended to continuous-time considering a difference in the state and covariance update between discrete-time and continuous-time systems in the Kalman filter [33].

Remark 3. Assumption 1 requires that the information on the MAS such as the dynamic model and process and measurement noise characteristics (which is utilized in the prediction stage of the estimation) should be given as the prior knowledge. Such knowledge is usually considered as the fundamental information for implementing general state estimators like the Kalman filter. If the information on the heterogeneous MAS is given as a priori, the proposed estimation algorithm can be readily extended for the heterogeneous MAS as well.

2) Distributed estimation-based rendezvous control: Next, we propose distributed estimation-based rendezvous control to enhance the performance for a stochastic MAS. Notably, the estimation algorithm derived in the previous subsection (as well as in [29]) only accounts for the distributed estimation side. When the estimation and control influence each other, i.e., a control action is based on the estimated information, a re-derivation of the distributed estimation algorithm is needed to account for the estimation-based control loop.

The estimation-based control input of the i-th agent is represented as

$$u_i(\hat{X}_i) = K \sum_{j \in N_i} (\hat{x}_{i,j} - \hat{x}_{i,i}), \tag{21}$$

where $\hat{x}_{i,j}$ is the j-th agent state estimated by the i-th agent. The MAS dynamics with the estimation-based control is then given by

$$X[k+1] = (I_N \otimes A)X[k] + (I_N \otimes B)U(\hat{X}[k]) + \omega[k], \tag{22}$$

where $\hat{X}[k] = [\hat{X}_1^T[k] \cdots \hat{X}_N^T[k]]^T$ is the vector of the estimated state of the MAS at time step k and $U(\hat{X}) = [u_1^T(\hat{X}_1) \ \cdots \ u_N^T(\hat{X}_N)]^T$ is the control input of the MAS. Eq. (21) can be reformulated using a Laplacian matrix, and accordingly the control input of the MAS can be expressed as:

$$u_i(\hat{X}_i) = -K(\mathcal{L}_i \otimes I_{2n})\hat{X}_i, \tag{23}$$

$$U(\hat{X}) = [u_1^T(\hat{X}_1) \ \cdots \ u_N^T(\hat{X}_N)]^T = -(I_N \otimes K)\hat{X}_{\mathcal{L}}, \tag{24}$$

where $\hat{X}_{\mathcal{L}} = \left[\left((\mathcal{L}_1 \otimes I_{2n})\hat{X}_1\right)^T \cdots \left((\mathcal{L}_N \otimes I_{2n})\hat{X}_N\right)^T\right]^T \in \mathbb{R}^{2nN}$. By substituting Eq. (24) into Eq. (22), the MAS dynamics with the distributed estimation-based rendezvous control becomes

$$X[k+1] = (I_N \otimes A)X[k] - (I_N \otimes BK)\hat{X}_{\mathcal{L}}[k] + \omega[k], \tag{25}$$

which can be rewritten using the estimation error as:

$$\begin{aligned} X[k+1] = &(I_N \otimes A)X[k] - \ (\mathcal{L}[k] \otimes BK)X[k] \\ &+ \ (I_N \otimes BK)(\mathcal{L}[k] \otimes I_{2n})X[k] \\ &- \ (I_N \otimes BK)\hat{X}_{\mathcal{L}}[k] + \omega[k] \\ = &F_1[k]X[k] + (I_N \otimes BK)e_{\mathcal{L}}[k] + \omega[k]. \end{aligned} \tag{26}$$

The $e_{\mathcal{L}}$ in (26) is defined as:

$$e_{\mathcal{L}} = \begin{bmatrix} (\mathcal{L}_1 \otimes I_{2n})(X - \hat{X}_1) \\ \vdots \\ (\mathcal{L}_N \otimes I_{2n})(X - \hat{X}_N) \end{bmatrix} = \mathcal{L}' e, \tag{27}$$

where $\mathcal{L}' = \text{diag}\{\mathcal{L}_1 \otimes I_{2n}, \cdots, \mathcal{L}_N \otimes I_{2n}\} \in \mathbb{R}^{2nN \times 2nN^2}$ and the augmented estimation error $e = [e_1^T \ \cdots \ e_N^T]^T \in \mathbb{R}^{2nN^2}$. Substituting Eq. (27) into Eq. (26) with $F_2 = (I_N \otimes BK)\mathcal{L}' \in \mathbb{R}^{2nN \times 2nN^2}$, Eq. (26) can be compactly expressed as

$$X[k+1] = F_1[k]X[k] + F_2[k]e[k] + \omega[k]. \tag{28}$$

The estimation process addressed in Section III-A1 is modified to reflect the control schemes based on the estimated state information, i.e., (21). Without sharing the estimated state information of the MAS, it is difficult for each agent to know the conditional expectation of the control input of the MAS (i.e., $\mathbb{E}[U(\hat{X}[k])|z_i^{0:k}]$) because it is subject to the estimated state of the MAS perceived from the individual agent's viewpoint. Thus, using the estimated information known by each agent, the control input of the MAS estimated by the i-th agent is defined as Eq. (12), which results in the predicted state of the MAS being equal to Eq. (14). Before deriving the estimation error covariance, the initial augmented estimation error covariance is assumed to satisfy the following.

Assumption 2. The initial augmented estimation error covariance, $\mathbb{E}[e[0]e^T[0]] \in \mathbb{R}^{2nN^2 \times 2nN^2}$, is identically set.

Along with Assumption 2, the predicted error covariance is computed using Eqs. (28) and (14) as:

$$\begin{aligned}\Sigma_i^-[k+1] &= \mathbb{E}[e_i^-[k+1]e_i^{-T}[k+1]] \\ &= F_1[k]\Sigma_i[k]{F_1}^T[k] + F_2[k]\mathbb{E}[e[k]e^T[k]]F_2^T[k] \\ &\quad + F_1[k]\mathbb{E}[e_i[k]e^T[k]]F_2^T[k] \\ &\quad + F_2[k]\mathbb{E}[e[k]e_i^T[k]]{F_1}^T[k] + \mathcal{Q},\end{aligned} \tag{29}$$

where $\mathbb{E}[ee^T]$ and $\mathbb{E}[e_ie^T]$ are given as

$$\mathbb{E}[ee^T] = \begin{bmatrix}\Sigma_1 & \Sigma_{12} & \cdots & \Sigma_{1N} \\ \Sigma_{21} & \Sigma_2 & \cdots & \Sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{N1} & \Sigma_{N2} & \cdots & \Sigma_N\end{bmatrix} = \Sigma, \tag{30}$$

$$\mathbb{E}[e_ie^T] = \begin{bmatrix}\Sigma_{i1} & \cdots & \Sigma_i & \cdots & \Sigma_{iN}\end{bmatrix}, \tag{31}$$

where Σ_{ij} is the cross-covariance between the i-th agent and the j-th agent. Substituting Eq. (30) into Eq. (29),

$$\begin{aligned}\Sigma_i^-[k+1] &= F_1[k]\Sigma_i[k]{F_1}^T[k] + F_2[k]\Sigma[k]F_2^T[k] \\ &\quad + F_1[k]\mathbb{E}[e_i[k]e^T[k]]F_2^T[k] \\ &\quad + F_2[k]\mathbb{E}[e[k]e_i^T[k]]{F_1}^T[k] + \mathcal{Q}.\end{aligned} \tag{32}$$

The updated state from the predicted state follows Eqs. (16)–(20).

For a recursive iteration of the estimation process, each agent needs to update $\Sigma[k]$ to $\Sigma[k+1]$, representing the estimation error covariance of all agents and its cross-covariance. This is required in the predicted error covariance of the next step, which was not needed for the estimation process described in Section III-A1. As the reason for this, each agent uses its estimated information for control, so the expected estimation error of other agents from the perspective of the i-th agent is considered to the estimation of the MAS state. To this end, the cross-covariance is computed as:

$$\begin{aligned}\Sigma_{ij}[k+1] :=&\ \mathbb{E}[e_i[k+1]e_j^T[k+1]] \\ =&\ -G_i[k+1]H_i[k+1]\Sigma_{ij}^-[k+1] \\ &- \Sigma_{ij}^-[k+1]H_j^T[k+1]G_j^T[k+1] \\ &+ G_i[k+1]H_i[k+1]\Sigma_{ij}^-[k+1] \\ &\times H_j^T[k+1]G_j^T[k+1] + \Sigma_{ij}^-[k+1],\end{aligned} \tag{33}$$

where the predicted cross-covariance is

$$\begin{aligned}\Sigma_{ij}^-[k+1] &:= \mathbb{E}[e_i^-[k+1]e_j^{-T}[k+1]] \\ &= F_1[k]\Sigma_{ij}[k]{F_1}^T[k] + F_2[k]\Sigma[k]F_2^T[k] \\ &\quad + F_1[k]\mathbb{E}[e_i[k]e^T[k]]F_2^T[k] \\ &\quad + F_2[k]\mathbb{E}[e[k]e_j^T[k]]{F_1}^T[k] + \mathcal{Q}.\end{aligned} \tag{34}$$

Remark 4. The updated Σ calculated by each agent is always the same insofar as it begins with same initial condition based on Assumption 2.

Remark 5. As in [34], many studies are often carried out under the assumption that the initial covariance of each sensor node is the same. Similarly, introducing Assumption 2 is merely motivated by the brevity of Eqs. (29)–(34), which can be readily rederived with additional notations for different Σ of each agent. Note that the different initial setup does not affect the stability guarantee of the proposed algorithm as will be discussed in Section IV.

Remark 6. In Eq. (21), the state estimates of direct neighbors and those outside the sensing range can be used for control action, making interaction possible between all agents including non-neighboring agents.

B. Nonlinear systems

This subsection expands on a distributed estimation algorithm used for linear systems for application to nonlinear systems and proposes an estimation-based nonlinear flocking control protocol. The system's nonlinearity comes from the control input, which is a nonlinear function of the state of the MAS.

1) Distributed state estimation: The agent's nonlinear control input using measurements is expressed as:

$$u_i(z_i) = f_i(X, \nu_i), \tag{35}$$

where $\forall i \in \mathcal{V}$, $f_i \in \mathbb{R}^n$ is a nonlinear function. Thus, the dynamics of the MAS is the same as in Eq. (8) except that $U(Z)$ is the nonlinear feedback control input of the MAS, which is represented as:

$$\begin{aligned} X[k+1] =& (I_N \otimes A)X[k] + (I_N \otimes B)U(Z[k]) + \omega[k] \\ =& f(X[k], \nu[k]) + \omega[k], \end{aligned} \tag{36}$$

where $f(X, \nu) = (I_N \otimes A)X + (I_N \otimes B)U(Z) \in \mathbb{R}^{2nN}$ is a nonlinear function and $\nu[k] = [\nu_1^T[k] \cdots \nu_N^T[k]]^T$.

Because noises are all assumed to be a zero mean Gaussian, the predicted state estimate is given from Eq. (36) as:

$$\hat{X}_i^-[k+1] := f(\hat{X}_i[k], 0). \tag{37}$$

Along with (37), the corresponding predicted error covariance needs to be computed for the estimation process. Because f is a nonlinear function, the predicted error covariance is calculated through the linearization of f as in the extended Kalman filter. The predicted error covariance Σ_i^- is given by the following:

$$\Sigma_i^-[k+1] = \mathcal{F}_i[k]\Sigma_i[k]\mathcal{F}_i^T[k] + \mathcal{Q}, \tag{38}$$

where $\mathcal{F}_i = \left.\dfrac{\partial f(X,\nu)}{\partial X}\right|_{(\hat{X}_i,0)}$ is the Jacobian matrix of f. Because the observation dynamics remains linear, the updated state from the predicted state is the same as in Eqs. (16)–(20).

2) Distributed estimation-based flocking control with virtual fully-connected network: The main idea of this study, distributed estimation-based nonlinear flocking control, is presented in this section. Regardless of the proximity network topology, the proposed method allows each agent to interact with not only its direct neighbors but also non-neighbors. This can be accomplished by introducing the concept of a virtual fully-connected network. With distributed estimation of the entire MAS states, facilitated by using the same cooperative controller (i.e., the nonlinear flocking controller in Eq. (7)) for all agents, each agent can utilize the state information of all other agents as if they are obtained through a fully-connected network. Note that the proposed approach only utilizes the observation of neighbors like other conventional distributed systems. However, the conventional distributed systems do not allow the interaction with non-neighboring agents due to the limited sensing/communication range.

As a benefit of using the proposed virtual fully-connected interactions, the performance of the proposed flocking algorithm in terms of the converged configuration is similar to the conventional flocking algorithm (Eq. (7)) with a fully-connected network. Under a fully-connected network, the boundary of the flock configuration can be more tightly controlled, as described in Section II-D. Figure 1 shows the concept of the proposed flocking algorithm using the virtual fully-connected network with distributed estimation.

With the basic structure following the flocking model described in Section II-D, the distributed estimation-based flocking control input with a virtual fully-connected network is represented as:

$$\begin{aligned} u_i(\hat{X}_i) = \rho_3 &\sum_{j=1, j\neq i}^{N} \phi_1(\hat{p}_{i,ij})\big(\hat{v}_{i,j} - \hat{v}_{i,i}\big) \\ &+ \ \rho_4 \sum_{j=1, j\neq i}^{N} \phi_2(\hat{p}_{i,ij}, \hat{v}_{i,ij})\big(\hat{p}_{i,j} - \hat{p}_{i,i}\big), \end{aligned} \tag{39}$$

where $\hat{p}_{i,j}$ and $\hat{v}_{i,j}$ are the position and velocity of the j-th agent estimated by the i-th agent, respectively, and $\hat{p}_{i,ij}$ and $\hat{v}_{i,ij}$ are the relative position and velocity between the i-th and j-th agents estimated by the i-th agent. In Eq. (39), unlike with the previous control protocol, the estimated state information of all agents are used for the control protocol regardless of the proximity network topology, as mentioned in Remark 6. The MAS dynamics with the proposed control protocol is the same as in Eq. (22) except that $U(\hat{X})$ consists of Eq. (39).

The prediction process is expressed the same as in Eq. (14), i.e.,

$$\begin{aligned} \hat{X}_i^-[k+1] &:= f'(\hat{X}_i[k]) \\ &= (I_N \otimes A)\hat{X}_i[k] + (I_N \otimes B)U(\hat{X}_i[k]), \end{aligned} \tag{40}$$

where $f'(X) = (I_N \otimes A)X + (I_N \otimes B)U(X)$ and $U(X) = [u_1^T(X) \cdots u_N^T(X)]^T$. By calculating the Jacobian matrix of f' in Eq. (40), the predicted error covariance can be calculated as Eq. (38), i.e., $\mathcal{F}_i = \left.\dfrac{\partial f'(X)}{\partial X}\right|_{\hat{X}_i}$. The update step is the same as in Eqs. (16)–(20).

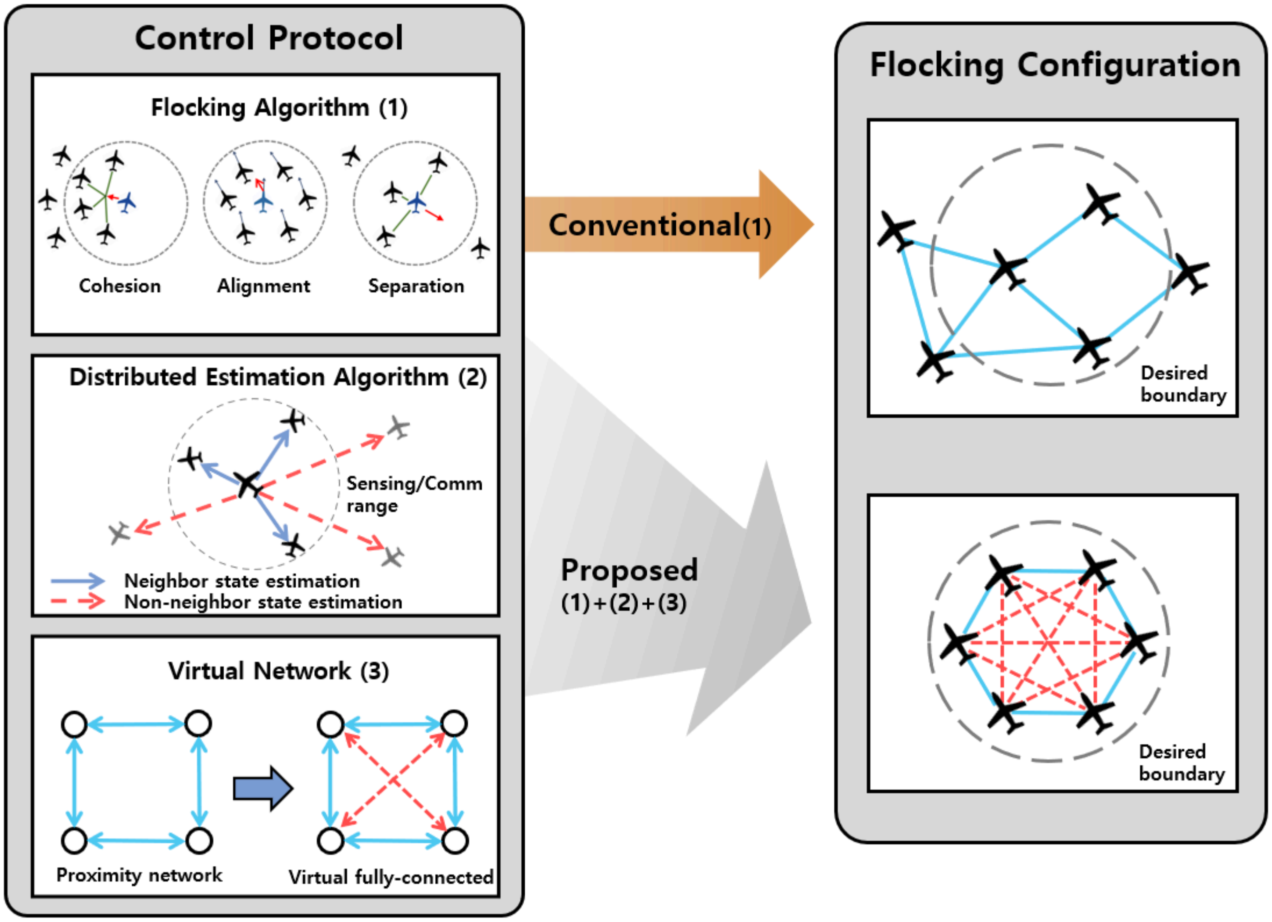


Fig. 1. Concept of an estimation-based flocking algorithm.

IV. Stability Analysis

In this section, a stability analysis of the proposed estimation algorithm in a linear system is described. The stability analysis is conducted through a Lyapunov theory in discrete-time, and focuses on finding a suitable Lyapunov function candidate of the state estimation error e_i to be globally asymptotically stable in the sense of the Lyapunov. In the stochastic MAS considered in this paper, the estimation error can be regarded as a super martingale of the Lyapunov functions [35], which should satisfy the following conditions:

$$\begin{cases} V(e_i[k],k) = 0, \text{iff } e_i[k] = 0 \\ V(e_i[k],k) > 0, \text{iff } e_i[k] \neq 0 \\ V(e_i[k],k) \to \infty, \text{iff } e_i[k] \to \infty \end{cases} \quad \forall k \tag{41}$$

$$\Delta V(k+1,k) < 0, \ \forall k \tag{42}$$

where $\Delta V(k+1,k) := V(\mathbb{E}[e_i[k+1]|e_i[k]], k+1) - V(e_i[k],k)$. Note that the augmented estimation error, e, in the MAS dynamics (28) couples the estimation error dynamics of each agent, e_i, with e. To verify the estimation stability of e_i, let us investigate the relationship between e in the error dynamics and e_i. For the investigation, the proposition of the estimation analysis in the linear cooperative MAS is first presented as

Proposition 1. The linear cooperative MAS distributed estimation error covariance $\Sigma_i[k]$ is positive definite and bounded for all $k > N$ if the following system is observable [29]:

$$\begin{aligned} X[k+1] &= F_1[k]X[k], \\ z_i[k] &= H_i[k]X[k]. \end{aligned} \tag{43}$$

Lemma 1. Suppose the system given in (43) is observable and $C_i[0], \forall i \in \mathcal{V}$, exists. Then, given the agent dynamics (1) and the control protocol (21), there exists a linear mapping between the estimation error of the i-th agent, e_i, and the augmented estimation error, e, as:

$$e[k+1] = C_i[k+1]e_i[k+1] + \alpha_i[k+1], \forall i \in \mathcal{V}, \forall k \geq 0 \tag{44}$$

$$\begin{aligned}
C_i[k+1] &= \mathcal{F}[k+1]C_i[k](\mathcal{F}_{i,2}[k+1]\mathcal{F}_{i,1}[k])^{-1}, \\
\alpha_i[k+1] &= \mathcal{F}[k+1]\alpha_i[k] + \gamma[k+1] - C_i[k+1] \\
&\quad \times \big(\mathcal{F}_{i,2}[k+1]F_2[k]\alpha_i[k] + \zeta_i[k+1]\big), \\
\text{where } \mathcal{F}_{i,1}[k] &= F_1[k] + F_2[k]C_i[k], \\
\mathcal{F}_{i,2}[k+1] &= I_{2nN} - G_i[k+1]H_i[k+1], \\
\zeta_i[k+1] &= \mathcal{F}_{i,2}[k+1]\omega[k] \\
&\quad - G_i[k+1]H_i[k+1]\nu_i[k+1], \\
\text{and } \mathcal{F}[k+1] &= \text{diag}\{\mathcal{F}_{1,2}[k+1]\mathcal{F}_{1,1}[k], \\
&\quad \cdots, \mathcal{F}_{N,2}[k+1]\mathcal{F}_{N,1}[k]\}, \\
\gamma[k+1] &= \begin{bmatrix} \mathcal{F}_{1,2}[k+1]F_2[k]\alpha_1[k] + \zeta_1[k+1) \\ \vdots \\ \mathcal{F}_{N,2}[k+1]F_2[k]\alpha_N[k] + \zeta_N[k+1] \end{bmatrix}.
\end{aligned}$$

Here, $C_i \in \mathbb{R}^{2nN^2 \times 2nN}$ is a linear transformation matrix, $\alpha_i \in \mathbb{R}^{2nN^2}$ is a lumped noise that has the characteristics of a white Gaussian distribution $\mathcal{N}_{2nN^2}(0, P_i)$, and the initial condition $e[0] = C_i[0]e_i[0]$.

Using Proposition 1 and Lemma 1, the stability of the proposed distributed estimation algorithm is shown below.

Theorem 1. Given the agent dynamics (1) and the control protocol (21), the equilibrium point $e_i = 0, \forall i \in \mathcal{V}$, in the proposed distributed state estimation algorithm is globally asymptotically stable if the system (43) is observable.

Proof. The Lyapunov function $V : \mathbb{R}^{2nN} \times \mathbb{N} \to \mathbb{R}$ is defined as:

$$\begin{aligned}
V(e_i[k], k) &:= e_i^T[k](\Sigma_i[k])^{-1}e_i[k], \\
\Delta V(k+1, k) &= V(\mathbb{E}[e_i[k+1]|e_i[k]], k+1) - V(e_i[k], k).
\end{aligned} \tag{45}$$

Because Σ_i is positive definite and bounded from Proposition 1, the quadratic form V satisfies the conditions of Eq. (41). To compute the conditional expectation of Eq. (45), we consider the estimation error dynamics. The error dynamics consist of two phases: prediction and update. The prediction of the estimation error of the i-th agent is computed by subtracting the predicted state of the i-th agent (14) from the MAS dynamics (28) with using Lemma 1 as:

$$\begin{aligned}
e_i^-[k+1] &= X[k+1] - \hat{X}_i^-[k+1] \\
&= F_1[k]e_i[k] + F_2[k](C_i[k]e_i[k] + \alpha_i[k]) + \omega[k] \\
&= \mathcal{F}_{i,1}[k]e_i[k] + F_2[k]\alpha_i[k] + \omega[k],
\end{aligned} \tag{46}$$

where $\mathcal{F}_{i,1}[k] \in \mathbb{R}^{2nN \times 2nN}$. The update step is done by subtracting the updated estimation (18) with Eq. (16) from $X[k+1]$ and substituting Eq. (46) for $e_i^-[k+1]$ followed as:

$$\begin{aligned}
e_i[k+1] &= X[k+1] - \hat{X}_i[k+1] \\
&= e_i^-[k+1] - G_i[k+1]H_i[k+1]e_i^-[k+1] \\
&\quad - G_i[k+1]H_i[k+1]\nu_i[k+1] \\
&= \mathcal{F}_{i,2}[k+1]\mathcal{F}_{i,1}[k]e_i[k] \\
&\quad + \mathcal{F}_{i,2}[k+1]F_2[k]\alpha_i[k] + \zeta_i[k+1],
\end{aligned} \tag{47}$$

where $\mathcal{F}_{i,2}[k+1] \in \mathbb{R}^{2nN \times 2nN}$, and $\zeta_i[k+1] \in \mathbb{R}^{2nN}$. From, using Eq. (47), the conditional expectation is obtained as:

$$\mathbb{E}[e_i[k+1]|e_i[k]] = \mathcal{F}_{i,2}[k+1]\mathcal{F}_{i,1}[k]e_i[k]. \tag{48}$$

Then, $\Delta V(k+1, k)$ can be written as:

$$\begin{aligned}
\Delta V(k+1, k) &= \mathbb{E}[e_i[k+1]|e_i[k]]^T(\Sigma_i[k+1])^{-1} \\
&\quad \times \mathbb{E}[e_i[k+1]|e_i[k]] - e_i^T[k](\Sigma_i[k])^{-1}e_i[k] \\
&= -e_i^T[k]\mathcal{M}_i[k+1]e_i[k],
\end{aligned} \tag{49}$$

where $\mathcal{M}_i[k+1] = (\Sigma_i[k])^{-1} - \mathcal{F}_{i,1}^T[k]\mathcal{F}_{i,2}^T[k+1](\Sigma_i[k+1])^{-1}\mathcal{F}_{i,2}[k+1]\mathcal{F}_{i,1}[k]$. To assure $\Delta V(k+1, k) < 0$, $\mathcal{M}_i[k+1]$ should be a positive definite matrix. From Lemma 1, the updated estimation error covariance Σ_i originally related to e can be derived in terms of e_i. The updated and predicted estimation error covariances of the i-th agent are derived using Eqs. (20) and (46), respectively, as:

$$\Sigma_i[k+1] = \mathcal{F}_{i,2}[k+1]\Sigma_i^-[k+1], \tag{50}$$

$$\begin{aligned}\Sigma_i^-[k+1] &= \mathbb{E}[e_i^-[k+1]e_i^{-T}[k+1]] \\ &= \mathcal{F}_{i,1}[k]\Sigma_i[k]\mathcal{F}_{i,1}^T[k] + F_2[k]P_i[k]F_2^T[k] + \mathcal{Q}.\end{aligned} \tag{51}$$

The updated estimation error covariance can be rewritten using Eqs. (17), (19), and (20) as:

$$\begin{aligned}\Sigma_i[k+1] &= \Sigma_i^-[k+1] - \Sigma_i^-[k+1]H_i^T[k+1] \\ &\times (H_i[k+1]\Sigma_i^-[k+1]H_i^T[k+1] \\ &\quad + H_i[k+1]RH_i^T[k+1])^{-1}H_i[k+1]\Sigma_i^-[k+1].\end{aligned} \tag{52}$$

Using the matrix inversion lemma [36], Eq. (52) can be rewritten as:

$$\begin{aligned}\Sigma_i[k+1] &= \big((\Sigma_i^-[k+1])^{-1} + H_i^T[k+1] \\ &\quad \times (H_i[k+1]RH_i^T[k+1])^{-1}H_i[k+1]\big)^{-1}.\end{aligned} \tag{53}$$

After taking the inverse, multiplying $\Sigma_i[k+1]$ to the left-hand and the right-hand sides of $(\Sigma_i[k+1])^{-1}$ yields the following:

$$\begin{aligned}\Sigma_i[k+1] &= \Sigma_i[k+1](\Sigma_i^-[k+1])^{-1}\Sigma_i[k+1] \\ &\quad + \Sigma_i[k+1]H_i^T[k+1](H_i[k+1]R \\ &\quad \times H_i^T[k+1])^{-1}H_i[k+1]\Sigma_i[k+1].\end{aligned} \tag{54}$$

Substituting Eq. (50) into the right-hand side of Eq. (54) yields

$$\Sigma_i[k+1] = \mathcal{F}_{i,2}[k+1](\Sigma_i^-[k+1] + \mathcal{W}_i[k+1])\mathcal{F}_{i,2}^T[k+1], \tag{55}$$

where $\mathcal{W}_i[k+1] = \Sigma_i^-[k+1]H_i^T[k+1](H_i[k+1]RH_i^T[k+1])^{-1}H_i[k+1]\Sigma_i^-[k+1]$, which is positive definite. In addition, $(\Sigma_i[k+1])^{-1}$ can be expressed by the inverse of Eq. (55) after substituting Eq. (51) into $\Sigma_i^-[k+1]$ as:

$$\begin{aligned}(\Sigma_i[k+1])^{-1} &= (\mathcal{F}_{i,2}^T[k+1])^{-1}(\mathcal{F}_{i,1}[k]\Sigma_i[k]\mathcal{F}_{i,1}^T[k] \\ &+ F_2[k]P_i[k]F_2^T[k] + \mathcal{Q} + \mathcal{W}_i[k+1])^{-1}(\mathcal{F}_{i,2}[k+1])^{-1}.\end{aligned} \tag{56}$$

From this, $\mathcal{M}_i[k+1]$ can be rewritten by substituting Eq. (56) into $(\Sigma_i[k+1])^{-1}$ as:

$$\begin{aligned}\mathcal{M}_i[k+1] &= (\Sigma_i[k])^{-1} - \mathcal{F}_{i,1}^T[k](\mathcal{F}_{i,1}[k]\Sigma_i[k]\mathcal{F}_{i,1}^T[k] \\ &\quad + F_2[k]P_i[k]F_2^T[k] + \mathcal{Q} + \mathcal{W}_i[k+1])^{-1}\mathcal{F}_{i,1}[k].\end{aligned} \tag{57}$$

Multiplying $\Sigma_i[k]$ with the left-hand and the right-hand sides of Eq. (57) and then applying the matrix inversion lemma gives the following:

$$\begin{aligned}\Sigma_i[k]\mathcal{M}_i[k+1]\Sigma_i[k] &= \big((\Sigma_i[k])^{-1} + \mathcal{F}_{i,1}^T[k] \\ &\times (F_2[k]P_i[k]F_2^T[k] + \mathcal{Q} + \mathcal{W}_i[k+1])^{-1}\mathcal{F}_{i,1}[k]\big)^{-1}.\end{aligned} \tag{58}$$

Multiplying $(\Sigma_i[k])^{-1}$ to both sides of Eq. (58) gives the following:

$$\begin{aligned}\mathcal{M}_i[k+1] &= (\Sigma_i[k])^{-1}\big((\Sigma_i[k])^{-1} + \mathcal{F}_{i,1}^T[k](F_2[k]P_i[k] \\ &\times F_2^T[k] + \mathcal{Q} + \mathcal{W}_i[k+1])^{-1}\mathcal{F}_{i,1}[k]\big)^{-1}(\Sigma_i[k])^{-1}.\end{aligned} \tag{59}$$

Because $(\Sigma_i[k])^{-1} \succ 0$ and $F_2[k]P_i[k]F_2^T[k] + \mathcal{Q} + \mathcal{W}_i[k+1] \succ 0$, $\mathcal{M}_i[k+1]$ should be positive definite. Therefore, the Lyapunov function satisfies Eqs. (41) and (42), meaning that the estimation error is globally asymptotically stable. ■

Remark 7. It is worth noting the proof of Theorem 1 requires that the entire MAS network topology is fixed and is known to individual agents. This is to show that the estimation error is theoretically asymptotically stable under the system satisfying the observability condition. In practice, however, the sensing-based estimation may not be able to access the network topology information, which is problematic especially for the time-varying network case. One way to circumvent this difficulty is employing the virtual fully-connected network topology in the estimation process. Then, although the estimation accuracy degrades due to the discrepancy between true network and virtual network, the estimation error covariance is still bounded and thus guaranteeing the estimation stability in stochastic sense as long as the observability condition is satisfied. A rigorous proof and analysis on this remain as future work. Furthermore, when communication is available, agents can exchange their local network information with neighbors, thereby estimating the true network topology in a distributed manner as in [37]. The proposed method can exploit this while retaining the advantage in communication overhead, i.e., estimating the other agents' state based on local observation and only communicating the network topology information.

Remark 8. For the distributed control side, even if the true network topology is unavailable, we could adopt the state estimate information based on the virtual fully-connected network. This is well demonstrated in the nonlinear flocking control scenario, where the individual agents only process the local observation as in Eqs. (16)–(20) without needing true network topology information, which is indeed time-varying.

Remark 9. Subject to nonlinear system dynamics, the stability analysis of the discrete-time extended Kalman filter (EKF) was carried out in [38]. In Theorem 3.1 of [38], the estimation error of a nonlinear system is exponentially bounded in mean square almost surely if the initial estimation error and noise terms are small enough. In this theorem, there are three assumptions: (i) the linearization matrix is bounded and nonsingular; (ii) the Riccati difference equation (which is the error covariance in the linear case) remains positive definite and bounded; and (iii) the high order term from the linearization is bounded. We argue that the stability of the proposed distributed estimator for the nonlinear MAS can be sketched in the similar fashion as the EKF case. First, by linearizing e_i (which is nonlinear function of the state) with respect to the estimated state, we can have a similar expression to Eq. (46) except that there is additional high-order terms from linearization. Then, the relationship between e_i and e can be expressed similarly as Eq. (44) where α_i now includes both the lumped noise and high-order terms. If the individual estimation error e_i is stable (which can be easily shown by Theorem 3.1 of [38]), there is a need that $C_i[k]$ of Eq. (44) is bounded for the augmented estimation error e to be bounded. This assumption can be satisfied if $C_i[0]$ is bounded. As a result, the distributed estimation error of the nonlinear system in which the estimated information is utilized in the control protocol (Eq. (39)) is exponentially bounded in mean square almost surely if the aforementioned assumptions hold. More rigorous stability analysis of the proposed distributed estimation for nonlinear systems remains as future study.

V. Numerical Simulation

In this section, the numerical simulation results of the proposed estimation-based distributed control are presented. The parameters used in the simulations are listed in Table I.

TABLE I. Simulation parameters

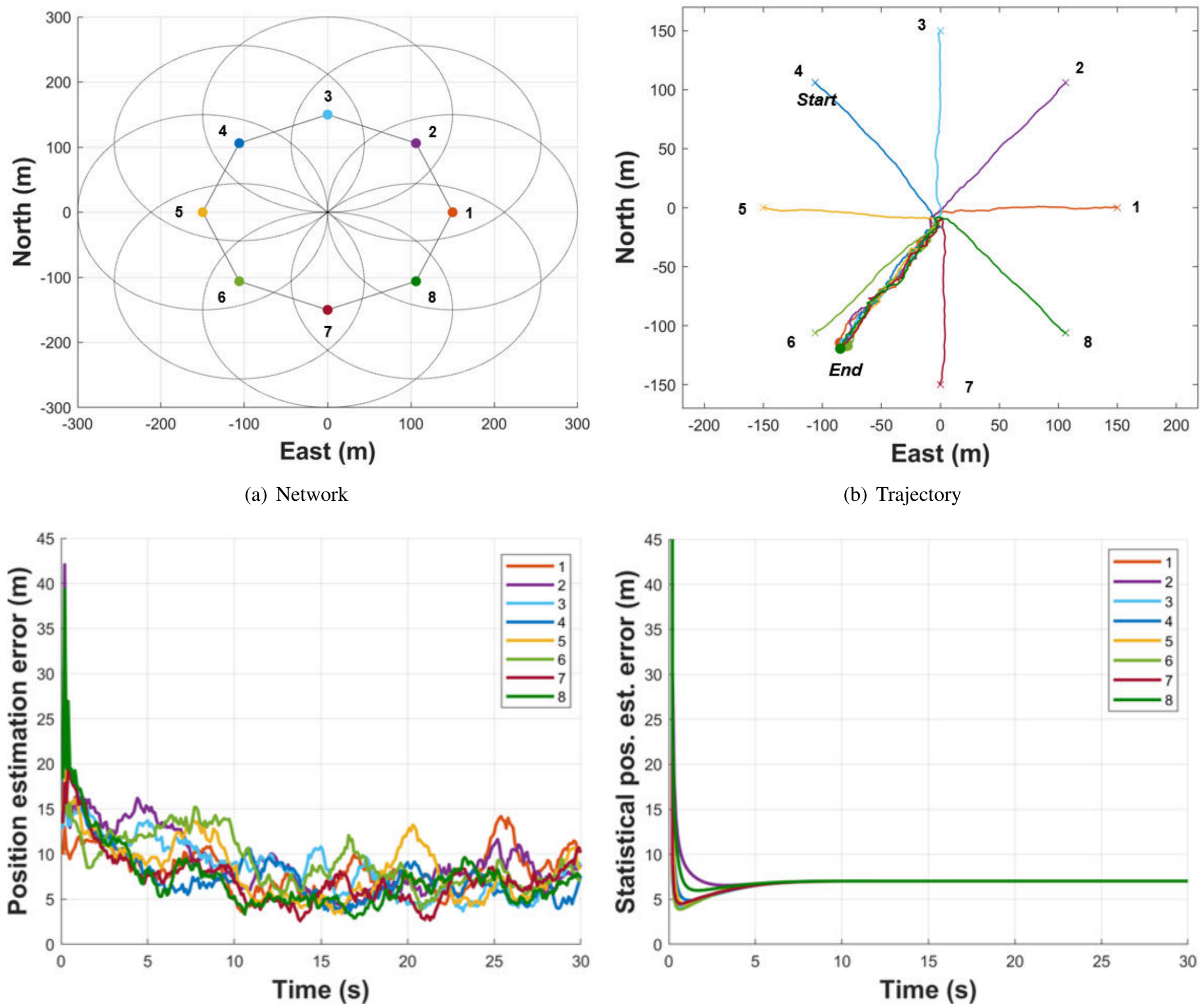
Parameter	Value	unit
Number of agents, N	8	
Dimension, n	2	
Sampling time, Δt	0.1	s
Sensing/Comm range, r	150	m
Rendezvous gains, (ρ_1, ρ_2)	(0.5, 1.5)	
Flocking gains, (ρ_3, ρ_4)	(3, 1)	
Function gains, (ρ_5, ρ_6)	(2, 1)	
Communication decay rate, β	0.33	
Desired boundary, $\mathcal{R}$	120	m

In general, although the larger rendezvous control gains (ρ_1 and ρ_2) achieve faster convergence to consensus, it must be set sensitively in consideration of the system's stability and other conditions. If the position feedback gain ρ_1 is too large compared to the velocity feedback gain ρ_2, for example, unnecessary fluctuations in the agent's movement may occur, resulting in poor control performance. Similarly, for stable flocking convergence, the gains ρ_3 for velocity consensus and ρ_4 for distance control should be appropriately set, and the gain ρ_5 should be greater than ρ_6 [19].

Besides, the performance of the proposed estimation-based control protocol highly depends on the estimation accuracy. In particular, as the improper setting of the initial state estimate and error covariance can cause disconnection of the network in the early configuration of the MAS, the initial state estimate and error covariance should be carefully selected.

A. Rendezvous control

Figure 2 shows the estimation errors and trajectories of the MAS with the proposed rendezvous control algorithm under the process noise covariance of I_{2n} and the measurement noise covariance of I_{2nN}. Considering the nature of a rendezvous, it is difficult to confirm the estimation performance of non-neighbors because all agents become neighbors of each other as they get closer. Thus, in this simulation, an estimation-based rendezvous control is conducted using a fixed network topology that is initially set. The color dots represent the individual agents in the subfigures. For example, the orange dot in Fig. 2(a) corresponds to the agent in Fig. 2(b), which has an orange-colored trajectory. Figure 2(a) depicts the MAS network topology with a circle representing each agent's sensing/communication range and solid lines connecting the neighbors. The initial positions of the agents are chosen to meet the connectivity and observability requirements. The trajectory of the agents when using the proposed rendezvous algorithm is depicted in Fig 2(b). Because rendezvous control aims to ensure that all agents have the same position and velocity, the MAS achieves the desired configuration, as shown in Fig. 2(b). Figure 2(c) shows the position estimation error of the MAS computed by each agent. It is clear that the estimation errors decrease initially and converge to certain bouded values. Figure 2(d) shows the statistical position estimation error which is the square root of the trace of the estimation error covariance. Figures 2 (c) and (d) show that the estimation error and the statistical estimation error are similar.



(a) Network

(b) Trajectory

(c) Position estimation error

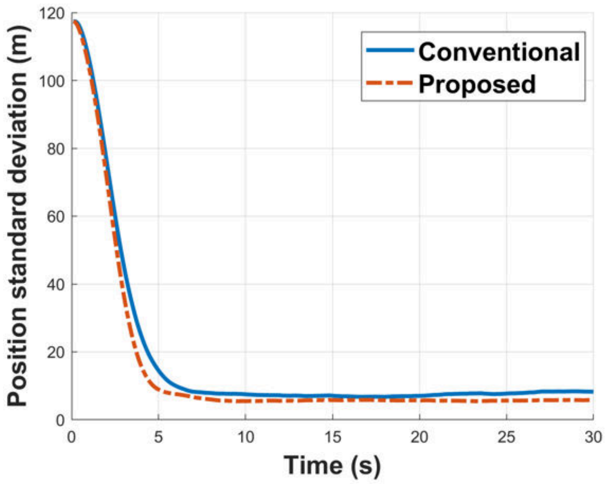
(d) Statistical position estimation error

Fig. 2. Performance of rendezvous control with the distributed estimation under a fixed network topology.

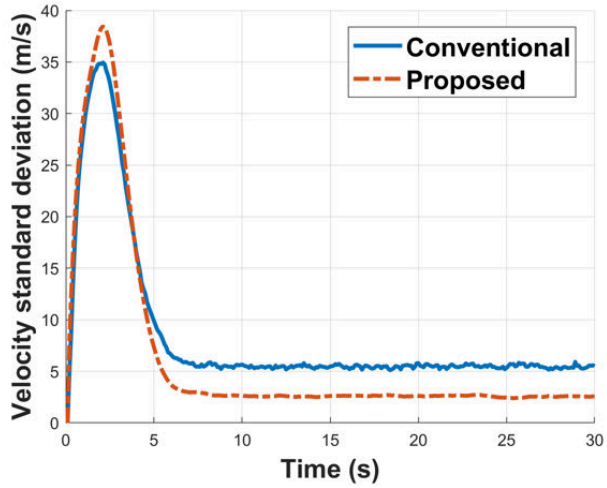
Figure 3 shows the root mean square standard deviation (STD) of the position and velocity of the MAS to verify the performance of the proposed rendezvous algorithm. The STD is averaged from Monte Carlo simulations of 50 runs with random initial positions. For comparing the conventional rendezvous algorithm using the measurement (i.e., Eq. (5)) and the proposed estimation-based rendezvous algorithm, process noise covariance is set to I_{2n} and the measurement noise covariance is set to $100I_{2nN}$. As shown in Fig. 3, the STD of the position and velocity of the estimation-based rendezvous control is smaller than that of the conventional method at the converged state. This result shows a clear advantage of using the proposed method to reach a second-order consensus for stochastic systems.

B. Flocking control

Next, the performance of the proposed estimation-based control algorithm is validated by applying it to the nonlinear flocking problem. The noise levels are the same as in the previous rendezvous problem. Figure 4(a) shows the initial network of agents and Fig. 4(b) shows the network of agents after applying the proposed flocking algorithm for a certain time. It is worth noting that the final network is not a fully-connected graph. Figure 4(c) shows the trajectory of agents using the proposed flocking algorithm. Figure 4(d) is the trajectory of all agents estimated by agent 1 ($\triangle$ symbol). The agents marked with a $\bigcirc$ symbol indicate the neighbors of agent 1, and the $\square$ agents indicate non-neighbors. This shows that the estimated trajectories of both neighboring and non-neighboring agents in Fig. 4(d) are close to the true trajectories in Fig. 4(c). The position estimation errors of the other agents from agent 1 are shown in Fig. 4(e). The estimation errors of agent 1's non-neighboring agents (i.e., agents 2 and 4) are greater than the neighboring agents, but they remain within a specific bound. Figure 4(f) depicts the statistical position estimation errors of the other agents from the perspective of agent 1. The error covariance of agents 2 and 4 are



(a) Position standard deviation



(b) Velocity standard deviation

Fig. 3. Rendezvous control performance comparison.

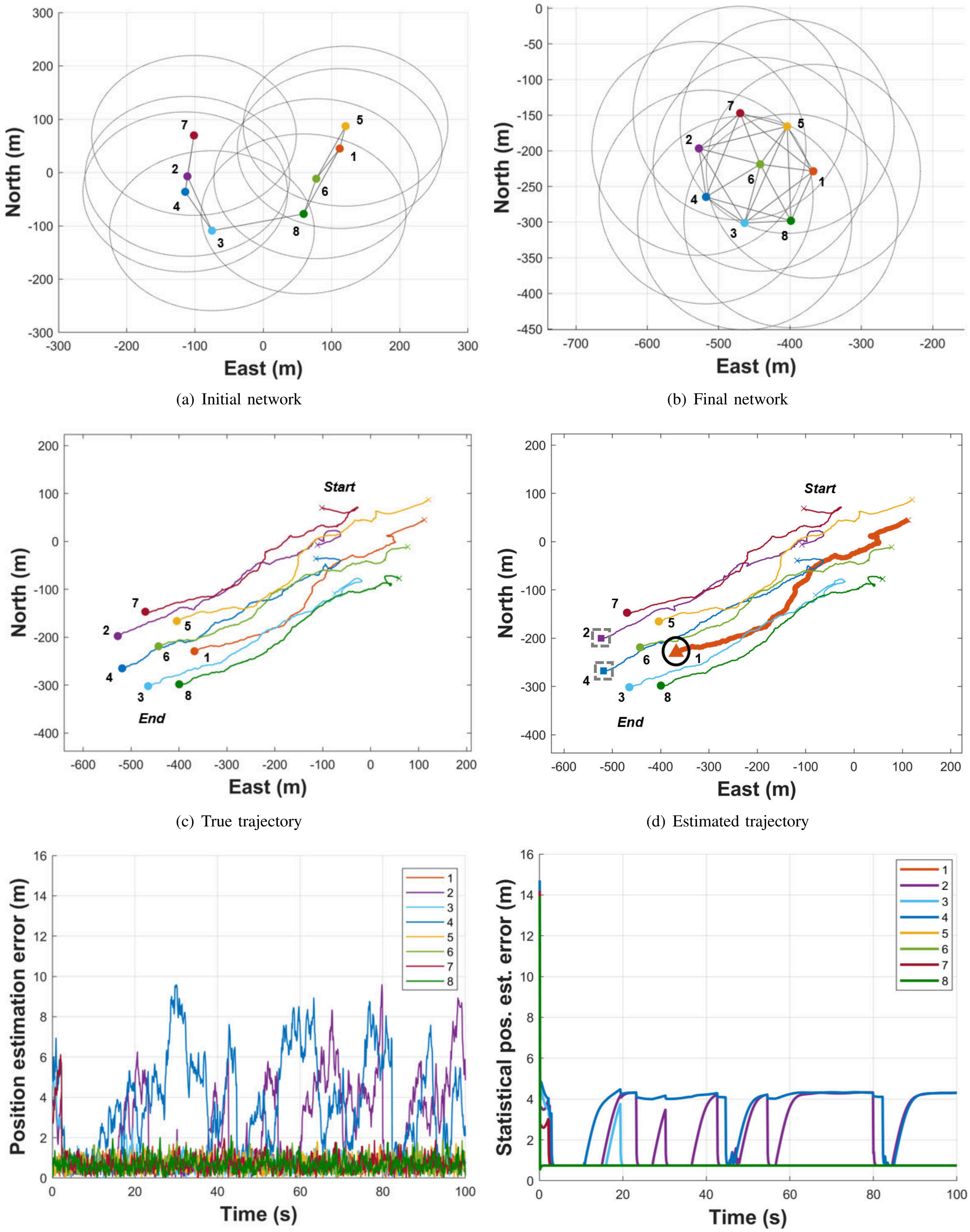
greater than those of the neighboring agents; however, when agents 2 and 4 temporarily come into the sensing/communication range of agent 1, the error covariances decrease sharply.

Figure 5 shows the root mean square STD of the position and velocity based on Monte Carlo simulations with 50 runs. The process noise covariance is set to I_{2n} and the measurement noise covariance is set to $10I_{2nN}$. The STD of the position and velocity are shown in Figs. 5(a) and (b), respectively, when comparing three cases: (i) a conventional flocking algorithm (i.e., Eq. (7)), (ii) a conventional flocking algorithm with a fixed fully-connected network topology at all times, and (iii) the proposed estimation-based flocking algorithm through a virtual fully-connected network. For cases (i) and (iii), the agent's neighbors are changed (i.e., a time-varying topology). From Figs. 5(a) and (b), one can easily see that agents are densely converged with smaller converged velocity STD using case (ii) as compared to case (i). Notably, the position and velocity of case (iii) STDs are similar to those from the ideal fully-connected case (ii), as shown in Fig. 5.

Figures 6(a) and (b) show the converged configuration reached by the conventional flocking algorithm and the proposed estimation-based flocking control with a virtual fully-connected network, respectively. As mentioned in Section II-D, at the converged configuration with the fully-connected network topology, the agent can be located within the radius of $\mathcal{R}$ from the center of all agent positions (i.e., the center of mass (CoM)). However, if the fully-connected network condition is not satisfied, the conventional flocking algorithm cannot achieve the desired flock configuration at a converged state, as shown in Fig. 6(a). In contrast, by the virtual fully-connected network, the proposed estimation-based flocking algorithm can form the desired configuration even within a limited sensing/communication range (i.e., not a fully-connected network topology), as shown in Fig. 6(b). From the results of Figs. 5 and 6, the proposed estimation-based flocking algorithm achieves a similar performance as a conventional flocking algorithm with a fully-connected network topology while overcoming the network topology constraints by creating a virtual network.

VI. Conclusions and Future Work

In this paper, an estimation-based distributed control protocol was proposed to improve the performance of the stochastic MAS. A distributed estimation algorithm was proposed to reduce communication overhead between inter-agents by using only local sensing information while estimating the states of other agents beyond the sensing range. Furthermore, the proposed distributed estimation algorithm allows for virtual interactions between non-neighbors as if all agents were fully-connected via a virtual network. The stability of the proposed distributed estimation algorithm is proved theoretically, and numerical simulations demonstrate that the estimation-based control protocol can significantly improve the performance of the cooperative control of the stochastic MAS. It is shown, in particular, that the estimation-based flocking control protocol with a virtual fully-connected network can achieve the desired flock configuration despite the limited sensing/communication range, which is not the case with a conventional flocking approach. This demonstrates the algorithm's utility, especially in communication-limited environments. Future studies will include a stability analysis of the proposed distributed estimation of the nonlinear system. In addition, since this study incorporates independent and identically distributed (i.i.d) white Gaussian random noise into the system dynamics as stochastic uncertainty, further stochastic peculiarity will be considered as future work. Besides, network connectivity preservation and obstacle avoidance will be considered for the rendezvous or flocking algorithm for obstacle-rich environments.



(a) Initial network

(b) Final network

(c) True trajectory

(d) Estimated trajectory

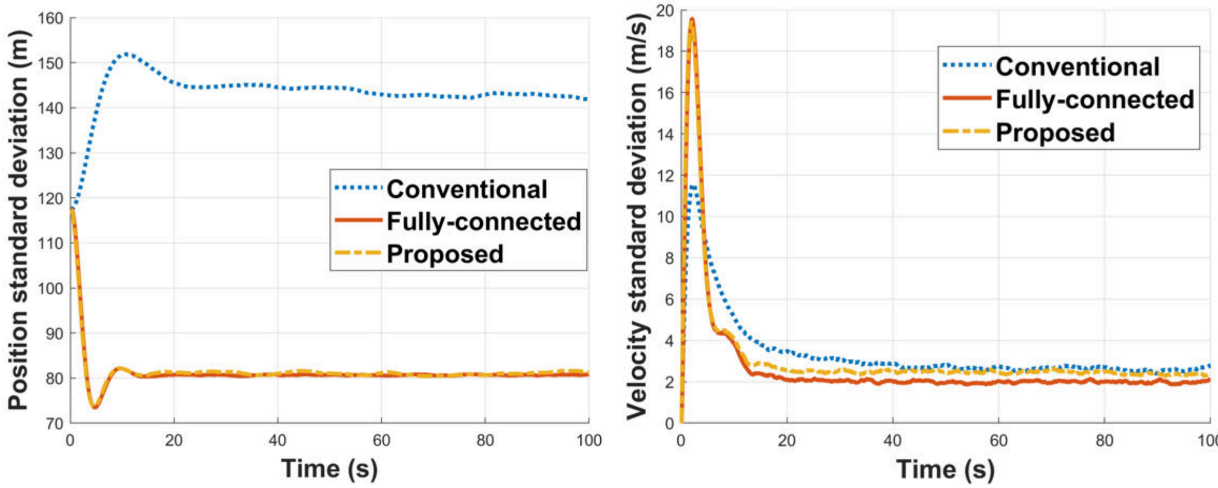
(e) Position estimation error for each agent from the first agent's perspective.

(f) Statistical position estimation error for each agent from the first agent's perspective.

Fig. 4. Performance of flocking control with the distributed estimation under a time-varying network topology.

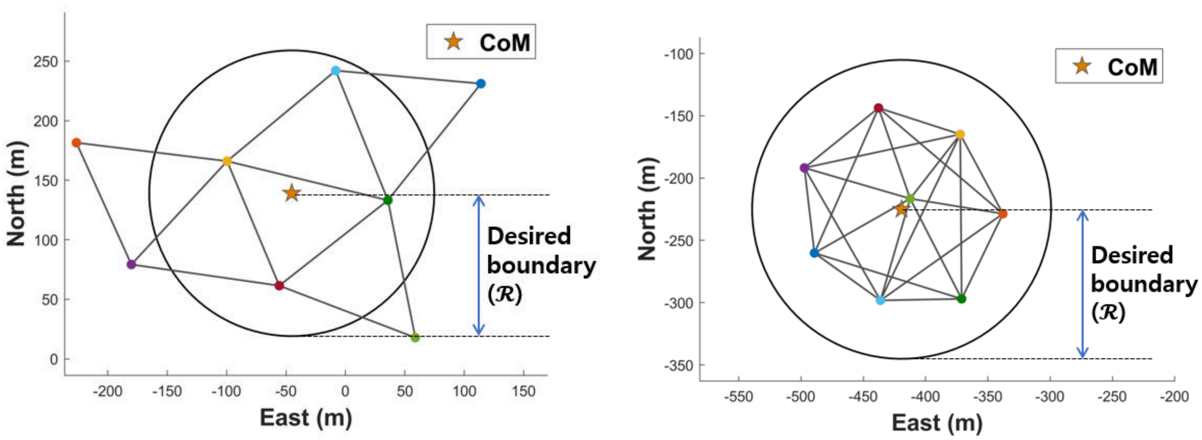
Appendix

Proof of Lemma 1. It can be shown through mathematical induction. Suppose the estimation error at time step k satisfies Eq. (44). To show Eq. (44) satisfied at time step $k+1$ with the definitions of $C_i[k+1]$ and $\alpha_i[k+1]$, augmented estimation



(a) Position standard deviation (b) Velocity standard deviation

Fig. 5. Flocking control performance comparison.



(a) Conventional flocking algorithm (b) Proposed flocking algorithm

Fig. 6. Converged flock configuration.

error, e, which concatenate Eq. (47) for all agents $i \in \mathcal{V}$ together can be represented using Eqs. (44) and $e_i[k]$ of (47) as:

$$\begin{aligned} e[k+1] &= \mathcal{F}[k+1]e[k] + \gamma[k+1] \\ &= \mathcal{F}[k+1]C_i[k]e_i[k] + \mathcal{F}[k+1]\alpha_i[k] + \gamma[k+1] \\ &= C_i[k+1]e_i[k+1] + \alpha_i[k+1], \end{aligned} \tag{60}$$

where $\mathcal{F}[k+1] \in \mathbb{R}^{2nN^2 \times 2nN^2}$, $\gamma[k+1] \in \mathbb{R}^{2nN^2}$. Note that $\mathcal{F}_{i,2}^{-1}$ exists by virtue of $\Sigma_i \succ 0$. Furthermore, considering $\lim_{\Delta t \to 0} \mathcal{F}_{i,1} = I_{2nN}$, there exists $\mathcal{F}_{i,1}^{-1}$ in general. Therefore, if the initial linear transformation matrix $C_i[0]$ for any $i \in \mathcal{V}$ exists, Lemma 1 holds. ■

It is worth noting that there exist $C_i[0]$ for all $i \in \mathcal{V}$ that satisfies $\Sigma[0] = C_i[0]\Sigma_i[0]C_i^T[0]$. In addition, since the lumped noise α_i in Eq. (44) is the weighted sum of the process and measurement noises, $P_i[k]$ with the initial $P_i[0] = 0_{2nN^2}$ can be updated using the following relationship:

$$\begin{aligned} &\mathbb{E}[\zeta_i[k]\zeta_j^T[k]] = \begin{cases} G_i[k]H_i[k]RH_i^T[k]G_i^T[k] \\ \quad +\mathcal{F}_{i,2}[k]\mathcal{Q}\mathcal{F}_{i,2}^T[k], & \text{if } i = j, \\ \mathcal{F}_{i,2}[k]\mathcal{Q}\mathcal{F}_{j,2}^T[k], & \text{otherwise}, \end{cases} \\ &\mathbb{E}[\zeta_i[k+1]\zeta_j^T[k]] = 0, \\ &\mathbb{E}[\zeta_i[k+1]\alpha_j^T[k]] = 0, \end{aligned}$$

where $\forall i, j \in \mathcal{V}$ and $\forall k$.

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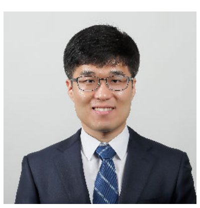

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