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Suggestions on the teaching of atmospheric pressure at university and secondary school levels

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Abstract
The distinction between pressure in a liquid and in a gas is often treated in a cursory way, or not treated at all, even in university level textbooks. Most texts fail to point out the relation between pressure and density in a gas as compared to pressure in a—virtually incompressible—liquid. In many instances this also results in a dismissive treatment of atmospheric pressure. In this paper we suggest that in the physics curriculum of university and secondary school students, kinetic theory of gases be treated before fluid mechanics and thermodynamics. In this way, the definitions of pressure P and absolute temperature T in a gas can be derived consistently, with the remarkable advantage that the links between the macroscopic parameters P and T and the velocity of molecules—a microscopic parameter—are made clear at an early stage, as well as the relation between P and density ρ.

Keywords: thermodynamics, kinetic theory, atmosphere, atmospheric pressure

1. Introduction

Most textbooks, at both secondary school and university level, provide a general treatment of pressure in fluids under the influence of gravity, but make very little or no distinction between the frequent case of liquids, and the most relevant case in which gravity is not negligible in gases, i.e. atmospheric pressure. The situation is somewhat mirrored also in physics education, where notwithstanding a significant amount of research on buoyancy and pressure in fluids, there are only a few works [1, 2] in which the cases of liquids and air are differentiated in the investigation of students' learning, and even fewer [3, 4] are specifically centred on atmospheric pressure. Indeed, in [3], first year physics undergraduates state in clinical interviews that the kinetic theory of gases does not apply or has no connection to the concept of atmospheric pressure, which is not at all surprising since in practically all physics curricula, either in secondary school or at university, the former is treated at a later time than, and with no explicit connection to, the latter. However, in recent times the explosion of both highly sophisticated hand held devices (smartphones) which may be equipped with accurate barometers, and of publicly available

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open data on barometric measurements at meteorological stations has brought renewed attention on the experimental side of atmospheric pressure in education [5, 6]. This makes it even more important to help students construct consistent, integrated mental models of atmospheric pressure.

2. Discussion of the textbook treatment of atmospheric pressure

2.1. Microscopic treatment of atmospheric pressure

Only few contemporary university textbooks (e.g. [7]) introduce the equation:

$$\mathrm{d}P = -\rho g \mathrm{d}y \tag{1}$$

which allows to evaluate the dependence on the height y of the pressure P in a static fluid subject to gravity. In equation (1) ρ is the fluid density and g is the gravitational acceleration. The centrality of equation (1) has been highlighted in the educational literature by Besson [2] who remarked how, in the case of a single fluid, equation (1) prescribes that two points at the same elevation y must have equal pressure P, if a path within the fluid connecting the two points exists. This specification becomes necessary in cases such as the classical test item of the 'fish within an underwater cave' in which the naïve picture of fluid pressure as 'weight of the fluid column above' may be misleading for students. Equation (1) and its interpretation may be too abstract for secondary school teaching, but constitutes a good starting point for an undergraduate introduction. The derivation of the barometric equation can be obtained combining equation (1) with $\rho/\rho_0 = P/P_0$ (ρ_0 and P_0 are density and pressure at zero altitude, respectively) which descends from the equation of state for an ideal gas assuming the temperature T independent of y (not a very good approximation, but its merit will be discussed later):

$$P = P_0 \exp -gy\frac{\rho_0}{P_0}. \tag{2}$$

One also obtains an exponentially decreasing density:

$$\rho = \rho_0 \exp -gy\frac{\rho_0}{P_0}. \tag{3}$$

It is clear in this picture that the atmospheric pressure decreases with height because at constant T it is proportional to density, which decreases with height. In his famous lecture notes [8], Feynman used equation (1) to derive a slightly different exponential equation for the number of molecules per unit volume. In fact, since $N/V = n = P/kT$, again from the equation of state of an ideal gas, equation (3) can be rewritten as

$$n = n_0 \exp -\frac{mgy}{kT} \tag{4}$$

where m is the mass of a molecule of gas. Equation (4) is a form of the Boltzmann distribution and mgy represents the minimum mechanical energy that allows a molecule to rise to height y. An educational advantage of equation (4) is that it allows to discuss the variation of air composition at different heights, since molecules of different mass will decrease in concentration at different exponential rates.

2.2. Discussion of the isothermal approximation

As previously anticipated, the assumption of constant temperature is not very good and has to be discussed. In figure 1 we show the dependence on height up to 120 km of thermodynamic quantities temperature, density, and pressure, normalized at unity at zero altitude, as computed from the empirical model of [9].

We note that, although temperature variations in the different strata of atmosphere are significant, temperature remains bounded within 25%–30% of an average value and thus a treatment at fixed temperature is much more appropriate than one at fixed density. Also, the behaviour of pressure is reasonably close to the exponential predicted by equation (2).

2.3. High school textbook treatment and the analogy with Stevin's law

In several high school textbooks only a very brief treatment of atmospheric pressure is given, which generally overlooks any differences with the case of liquids. In [10], it is stated that atmospheric pressure varies 'according to Stevin's law', although a formula is not given. In [11], a similar statement is accompanied by the information that

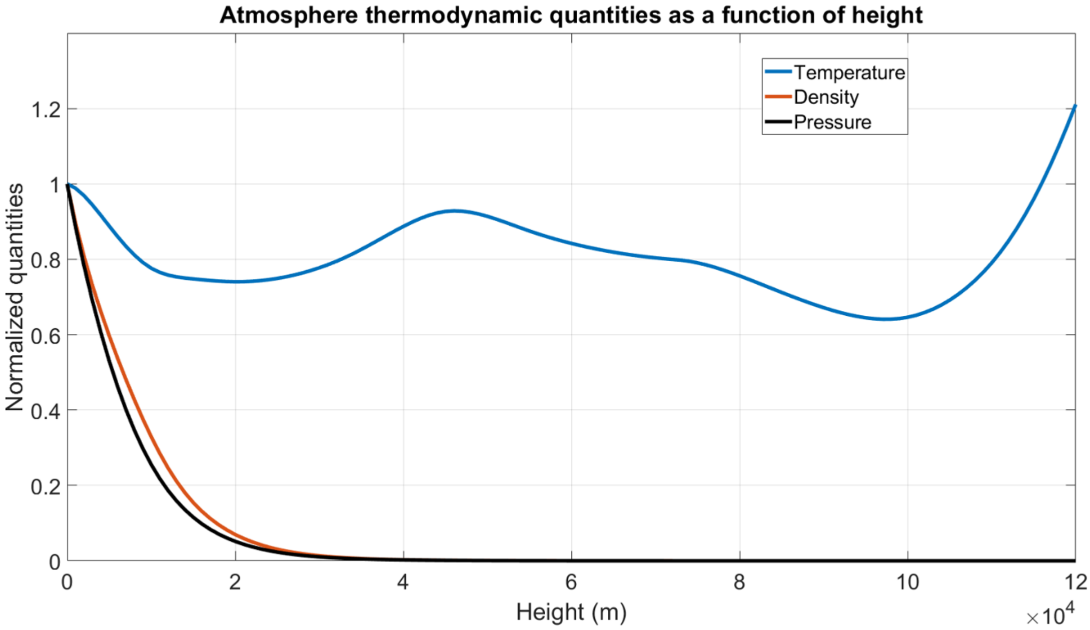

Figure 1. Graph of normalized values of temperature, density and pressure of the atmosphere as a function of height from 0 to 120 km. The value at 0 m is normalized at unity. Data are from the empirical model of [9], calculated at 45° latitude north, 0° longitude, in June. The general shape of these functions is similar for all zones of earth and periods of the year.

the variation of atmospheric pressure with altitude is about 1200 Pa for 100 m height. This is true within the first few kilometres of height from the earth surface [6] but if stated without explanation or warning may inevitably lead the student to thinking that Stevin's law $P_0 - P = \rho\, g\, y$ is valid for gases also, i.e. that there are no variations of density, or they are negligible. Instead, the reported linear variation of pressure in the first km of height can be obtained from a first order approximation of equation (2) which gives

$$P \approx P_0 \left(1 - gy\frac{\rho_0}{P_0}\right) = P_0 \left(1 - 1.16 \times 10^{-4} y\right)$$
$$= P_0 - \alpha y \qquad (5)$$

where $\alpha = 11.8$ Pa m^{-1} which of course reproduces the result reported above.

As stated previously, however, equation (1) may be too abstract for secondary school, thus it may not be possible to obtain a rigorous derivation of equation (2), and the first order expansion in equation (5) may be far from students' possibilities at the time they are introduced to either the concept of pressure, or basic thermodynamics. In Italy, in the school organization of the curriculum of some schools, not even the exponential function is available to students at the time they begin to study the laws of gases. Thus, even moving the introduction of atmospheric pressure to the point in the curriculum at which kinetic theory and the gas laws are introduced may not be sufficient for a consistent introduction. Nevertheless, we believe that an attempt in such direction could be fruitful, for the following reasons:

(a) It should be made clear to students that the 'pressure' quantity which is discussed in the case of the atmosphere is the very same quantity which appears in the laws of gases, and originates from the same microscopic phenomena which are embodied in the classical introduction to kinetic theory, i.e. the rate of collisions of gas molecules on the container walls. What we call the 'weight' of the atmosphere is also an emergent property [12], originating from the force transferred by molecular collisions. These facts are by no means obvious to students in the present curriculum organization.

(b) Knowledge of the ideal gas alone is sufficient to determine that, considering the atmosphere in the isothermal approximation, the behaviour of pressure and density as a function of height in the isothermal approximation must have the same functional form.
(c) If necessary, a derivation along the lines of the one given in section 2.1 can be given with the help of a numerical simulation, relying on a typical finite difference representation of first order differential equations. This will be elaborated further in section 3.

Many secondary school textbooks [10, 11, 13], some for university [7] and other sources [14] report that the atmospheric pressure is due to the weight of the atmosphere, often with little or no further specification. Physics education researchers have pointed out several times that this kind of statement is not so trivial to interpret as it may seem, and it may produce various kinds of misunderstandings. As mentioned in section 2.1, students with a mental model of fluid pressure depending exclusively on the 'weight of the fluid above' may think that pressure at equal vertical coordinate is lower in semi-enclosed chambers; in the case of the atmosphere, they may think pressure is lower in a room than in the external environment [2]. Note that this is an 'educated' misconception, which appears with secondary instruction, persisting up to university level, and substitutes the naïve idea of pressure as confinement, which results in the answer (often given by primary school pupils) that pressure must be higher in a room with respect to the outside environment. Also, although the following misconception is often meaningful only in the case of liquids, students may think that pressure at equal height for filled containers of different capacity and equal base [2, 15] or communicating from the bottom [16] or even *independently* of the base surface [16] is higher for containers with higher capacity. These difficulties also appear very frequently at the level of higher education. In [17], clinical interviews with university students showing difficulties similar to those reported above have led the authors to warn about several cases of complete identification of the concepts of pressure and weight.

Although the statement that atmospheric pressure is due to the atmosphere weight is (a) misleading, and (b) gives no clue on how locally the pressure is produced, an argument for more precise statement with similar content can indeed be given at the level of secondary school, with some care. To be precise, the statement can be stated as follows: 'For a stationary atmosphere, pressure at a given height is equal to the weight of an air column, whose lateral surface is tangent to the gravity field lines and whose upper base is set in the thermosphere, divided by the lower base area'. If the solid angle spanned by the gravity field lines within the column is small, then both upper and lower equipotential surfaces (spherical caps) can be approximated by flat bases. If one takes a column of unitary lower base area with the bottom at, say, sea level, and the top at, say, an altitude of 240 km, then the pressure at sea level equates the weight of the air column, as the pressure at the altitude of 240 km is less than 10^{-7} Pa (in the high-vacuum range), i.e. virtually zero compared to $\approx 10^5$ Pa at sea level. The assumption that the atmosphere is a fluid at rest is crucial for this argument, and indeed, the proof gives absolutely no clue about why this should be true—the conditions which allow the atmosphere to be in a stationary condition are expressed by the equations in section 2.1. Nevertheless, in view of student difficulties discussed above, the argument has the merit of (a) clarifying the need of taking into account an upper surface subject to vanishing pressure: the argument will not work if the upper surface consists of a wall which can exert an unknown pressure on the gas; and (b) clarifying the need to consider the lateral surface of the column as tangent to the gravity field lines: the argument will not work for a distorted column of arbitrary shape. However, as previously stated, the assertion that pressure is due to the weight of the atmosphere is almost always accompanied by no precise proof or argument.

2.4. Possible consequences in the statement of problems and examples

In [7], the chapter on fluids contains an example (example 12) about the buoyant force acting on a helium-filled balloon immersed in the air. A similar example, referring to a hot air balloon, is found in [18]. In both cases it is assumed that the external air density has a constant value, independent of height. The solution proposed to the reader proceeds in the same way as if the air were an incompressible fluid. This is formally justified if the height Δy of the balloon is so small that the corresponding air density variation $\Delta\rho$ is a negligible fraction of the air density averaged over Δy. In other words, the solution proceeds as if the first order approximation provided by equation (5) were *implicitly* assumed. Nonetheless, no matter how small Δy is, if the air density ρ were independent of y, no buoyancy would be expected (at least in isothermal conditions, which in this case can be safely assumed). In other words, the air surrounding a balloon cannot have perfectly constant density and temperature, and still exert buoyancy (while a liquid surrounding an immersed object can). Without any caveat about this subtlety, the example hides any physical difference between liquids and gases and also downplays the origin of the buoyant force in the dependence of gas pressure on height. A discussion of the validity of the approximation used, performed through equations (2) and (3), in which the crucial role of the pressure gradient and its connection to the density gradient are also pinpointed, would be extremely beneficial to students. Note how the examples of [7, 18] somehow replicates, at a more advanced level, the famous high school example of the 'isobaric balloon', extensively used by Viennot and co-workers as a test of critical thinking for physics teachers [19–21]. In this kind of exercises, very common in secondary school textbooks, the student is required to find the buoyancy force on a gas filled balloon making use of the ideal gas law, and the assumption of a constant external pressure. In both this case and those of [7, 18], the problem is not that the assumptions (constant pressure or density on the balloon surface) cannot be justified, but that they are not discussed, encouraging 'plug-and-chug' solution strategies which elude critical thinking, and obscuring the connection between buoyancy and pressure in fluids.

2.5. On the evolution of textbooks in time

In our analysis of university level textbooks, we noted that while older versions of books more often (e.g. [22, 23]) provide a consistent treatment starting from equation (1) and derive the barometric equation, newer editions and new textbooks [24–26] provide simplified treatments, which either do not introduce equation (1) or do not use it for the case of atmospheric pressure, reverting to more elementary, and less consistent, expositions. With all the due distinctions, a similar pattern appears also in secondary school textbooks; for example, an older textbook such as [27] is the only one to our knowledge which shows in a figure the exponential behaviour of atmospheric pressure, although, because of the position of the topic within the high school curriculum, such observation cannot be connected to theoretical considerations. The tendency towards progressive hyper-simplification, with loss of *critical details* [28], which has been observed by other authors and for different topics [29] does not serve the best interests of physics education, leading to an impoverishment of the content of physical models, to a downplay of their mutual connections, and ultimately to the impossibility to actually understand, as opposed to memorize, them. In the case of atmospheric pressure, the most important critical detail lies in the connection between pressure and other thermodynamic parameters (usually density), mediated by the ideal gas law, which should differentiate student's mental models of liquids and gases, allowing true understanding (as opposed to memorizing formulas) of how pressure gradients due to gravity can be realized in a gas.

3. Recommendations for instruction

Our basic recommendation is that, in curricula at all levels, a basic introduction to the kinetic theory of gases, including the ideal gas law, is given before the treatment of pressure in fluids under the effect of gravity. This would allow instructors to differentiate the cases of liquids and gases, and students to build more consistent mental models of how pressure gradients due to gravity are formed in the two cases. Specifically, in the case of gases a pressure gradient requires a corresponding gradient in one (or both) other basic thermodynamic variables, density (proportional to the inverse of specific volume) and temperature, where a constant temperature treatment is usually more appropriate. In liquids, at least in the ideal incompressible case, on the other hand, the formation of pressure gradients can be consistently pictured in a similar way to the transmission of forces in solids, i.e. forces are transmitted essentially by direct contact [30][1]. Next we provide more specific indications for the two different levels of instruction.

3.1. Recommendations for university instruction

The derivation line given in equations (1)–(5) is, we believe, adequate for providing a consistent picture of atmospheric pressure for university instruction. What is required is that a basic introduction to the kinetic theory of gases (to clarify the microscopic meaning of gas pressure) and the ideal gas law are given as a prerequisite. Equation (4), apart from its usefulness specified in section 2.1, can be seen as a hint to a future more in depth introduction of the canonical ensemble and the Boltzmann distribution. Some caution may be in order also in the exercises and examples concerning buoyancy and Archimedes' law, with the explicit discussion of any constant density approximations, which are of course valid in the case of a small object such as a balloon. Paying attention to such details could help reinforce the connection between buoyancy and pressure gradients, which tends to be quite weak in students at all levels [2] and even teachers [31] presumably due to the habit of solving problems related to buoyancy only through a plug-and-chug approach using Archimedes' law.

3.2. Recommendations for secondary school instruction

As stated in section 2.3, conceptual clarity and the formation of consistent mental models are more important goals in secondary school instruction than developing fully detailed mathematical description. Despite scarce research in this respect [3] there can be few doubts that the current curriculum organization leads students to view atmospheric pressure and the gas pressure in kinetic theory as disconnected concepts. Our proposal goes in the direction of improving 'longitudinal' (within the physics curriculum) knowledge integration [32], by presenting these two concepts as strongly connected.

However, the progressive construction of mathematical models is also an important activity in secondary education, and an important route for learning, partly because it mirrors the activities and practices of scientific research [33]. Thus, it is also important to provide a viable route for constructing a consistent mathematical model of atmospheric pressure at the level of high school instruction which incorporates the above principles. We will do so through a standard strategy of modelling differential equations as finite difference equations which can be solved numerically using a software as simple as a spreadsheet [34, 35]. Starting from the argument of the air column discussed in section 2.3, it is easy to derive a discretized version of equation (1): it is sufficient to calculate the pressure P at two heights differing by a very small quantity Δy and compute the difference ΔP. Thus, the atmosphere can be divided in small sheets of approximately uniform density, for each of which the following equation holds:

$$\Delta P = -\rho(y)\, g \Delta y. \qquad (6a)$$

Proceeding as in section 2.1 with the introduction of the ideal gas law, one can obtain either

$$\Delta P = -P(y)\frac{\rho_0}{P_0} g \Delta y \qquad (6b)$$

or

$$\Delta P = -P(y)\frac{mg}{kT}\Delta y \qquad (6c)$$

which allow to write explicitly finite difference equations for the pressure of successive strata of progressively larger height which can be solved numerically using a spreadsheet software; for example from equation (6b) one obtains:

$$P_{n+1} = P_n\left(1 - \frac{g\rho_0}{P_0}\Delta y\right). \qquad (6d)$$

[1] Note that there has been considerable debate in the physics education community on whether, for educational purposes, it would be desirable to present liquids also as slightly compressible, see for example [2, 20]. Since such debate is outside the focus of our paper, we limit ourselves here to the standard educational view.

Thus, although students may not have been yet introduced even to the exponential function, a meaningful activity can be organized around a consistent mathematical model. While a textbook may only reproduce the previous argument and show a graph, performing the computer modelling activity with students is highly recommended, also in view of the mole of research demonstrating the effectiveness of such strategies [36] especially when the learning curve for both teachers and students is not steep. From our previous experience [37] understanding how to solve a first order finite difference equation using a spreadsheet is a task well within the reach of secondary school students.

4. Conclusions

In this paper we have proposed to introduce atmospheric pressure as a topic connected to thermodynamics and the kinetic theory of gases, taking inspiration from, and extending the presentation given by Feynman [8]. We propose that such a treatment will have positive reflexes in the direction of knowledge integration, helping students picture physics as a consistent whole, rather than as a fragmented set of models for different situations, as well as helping overcome some difficulties identified by physics education research. In particular, the treatment proposed emphasizes the fact that the ideal gas law is locally valid for the atmosphere at any given height, while in the typical introduction of atmospheric pressure it is hard for students to tell whether the ideal gas law or the kinetic theory have any connection at all with the concept of atmospheric pressure [3]. Finally, we have noted how both university and secondary school textbooks seem to display a tendency to progressive hyper-simplification of the treatment of atmospheric pressure, towards an almost complete identification with pressure due to gravity in liquids. We have argued that such tendency is harmful for physics education, as the loss of critical details and complexity can reflect in inconsistent or incomplete problems and examples (see section 2.4), induce fragmented ideas on the microscopic nature of gas pressure in different physical situations, and produce additional difficulties in connecting the phenomenon of buoyancy to the concept of pressure.

Data availability statement

No new data were created or analysed in this study.

Conflict of interest

The authors have no conflict of interest to report.

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