We consider the approach to regularity structures introduced by Otto, Sauer, Smith and Weber to obtain a priori bounds for quasi-linear SPDEs. This approach replaces the index set of trees, used in the original constructions of Hairer et. al., by multi-indices describing products of derivatives of the corresponding nonlinearity. The two tasks of this thesis are:

- Construction and estimates of the model. We first provide the construction of a model in the regular, deterministic setting, where negative renormalization can be avoided. We later extend these ideas to the singular case, incorporating BPHZ-renormalization under spectral gap assumptions as a convenient input for an automated proof of the stochastic estimates of the singular model in the full subcritical regime.

- Characterization of the algebraic structures generated by the multi-index setting. We consider natural actions on functionals of the non-linearity and build a (pre-)Lie algebra from them. We use this as the starting point of an algebraic path towards the structure group, which as in the regularity structures literature is based on a Hopf algebra. This approach further allows us to explore the relation between multi-indices and trees, which we express through pre-Lie and Hopf algebra morphisms, in certain semi-linear equations.

All the results are based on a series of joint works with Otto, Tempelmayr and Tsatsoulis.