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Euler Archive Spotlight

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Since approximately 80% of the works of Euler were published in Latin, the Euler Archive relies on the time and translation skills of many people to provide translations into modern languages. This issue we spotlight the translations of Jordan Bell, one of the most prolific translators for the Euler Archive.

Jordan Bell, MS in Mathematics from University of Toronto and Graduate Certificate in Analytics for Business Decision Making, began translating Euler's articles while an undergraduate student in Honours Mathematics in the School of Mathematics and Statistics at Carleton University in Ottawa, Ontario, Canada. He completed three translations in his second year, six translations in his third year, and four translations in his fourth year. Mr. Bell has continued translating and currently has 43 translations appearing in the Euler Archive.

The earliest of Euler's work translated by Jordan Bell is "Observationes de theoremate quodam Fermatiano aliisque ad numeros primos spectantibus" (E26, "Observations on a theory of Fermat and others on looking at prime numbers"). Written in 1732 and published six years later, E26 was Euler's first number theory article. [2] He refuted Fermat's claim that numbers of the form $2^{2^n}+1$, now known as Fermat numbers, are prime, by observing that $2^{2^5}+1=4,294,967,297$ is divisible by 641. (See Figure 1.)

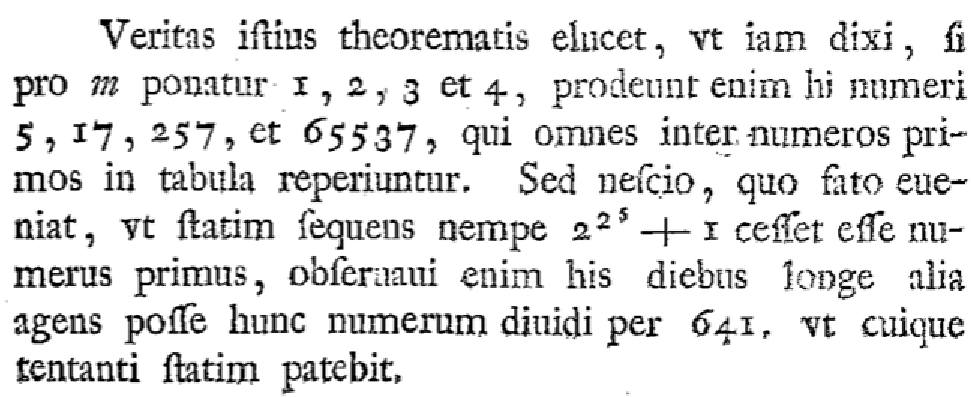
Veritas iſtius theorematis elucet, vt iam dixi, ſi pro m ponatur 1, 2, 3 et 4, prodeunt enim hi numeri 5, 17, 257, et 65537, qui omnes inter numeros primos in tabula reperiuntur. Sed neſcio, quo fato eueniat, vt ſtatim ſequens nempe $2^{2^5}+1$ ceſſet eſſe numerus primus, obſeruaui enim his diebus longe alia agens poſſe hunc numerum diuidi per 641, vt cuique tentanti ſtatim patebit.

Figure 1: *Euler's counterexample of Fermat's claim that numbers of the form* $2^{2^n}+1$ *are prime from E26.*

E26 also contains six conjectures, which Euler calls "theorems." Two of these conjectures are special cases of the Euler-Fermat Theorem, which Euler proved in E271 in 1758. The Euler-Fermat Theorem, a generalization of Fermat's Little Theorem, states that if *a* and *n* are relatively prime, then $a^{\theta(n)} \equiv 1 (\bmod n)$, where the Euler totient function $\theta(n)$ is the number of positive integers less than *n* and relatively prime to *n*. It is interesting to note that at this point in time, the totient was not yet viewed as a function. See [3] for more on the origins of the Euler totient function.

Jordan Bell also translated "Evolutio producti infiniti (1–x)(1–xx)(1–x3)(1–x4)(1–x5)(1–x6) etc. in seriem simplicem" (E541, "The expansion of the infinite product (1–x)(1–xx)(1–x^3)(1–x^4)(1–x^5)(1–x^6) etc. into a single series") and "De mirabilis proprietatibus numerorum pentagonalium," (E542, "On the remarkable properties of the pentagonal numbers"), a pair of number theory articles dealing with pentagonal numbers, which were both written in 1775 and published in 1783. The first six pentagonal numbers are pictured in Figure 2.

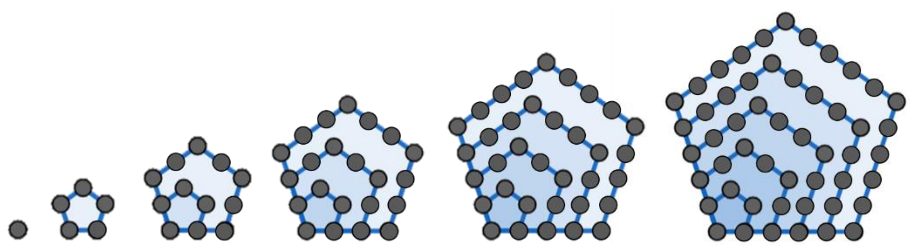

Figure 2: *The first six pentagonal numbers 1, 5, 12, 22, 35, and 51.*

In E541, Euler expanded the infinite product given in the title into a series in which some of the exponents are pentagonal numbers. The result is now known as the Pentagonal Number Theorem, which George E. Andrews [1] (a former president of the American Mathematical Society) calls, "one of Euler's most profound discoveries." In his next paper E542, Euler investigated properties of the pentagonal numbers $\frac{3nn-n}{2}$, along with numbers of the form $\frac{3nn+n}{2}$, and several related series.

The table below gives the current list of Jordan Bell's Euler translations, representing a substantial contribution to the Euler Archive. Such translations are vital to Eulerian scholarship. If you are interested in translating for the Euler Archive, please email the title and Eneström number of the work you propose to translate to Erik Tou (etou@uw.edu) or Chris Goff (cgoff@pacific.edu), with subject heading "Euler Translation Notice." Also, please consider submitting any completed translation projects to the journal *Euleriana*.

Euler Translations by Jordan Bell

Eneström number	Original Title	English Title
E26	Observationes de theoremate quodam Fermatiano aliisque ad numeros primos spectantibus	Observations on a theory of Fermat and others on looking at prime numbers
E30	De formis radicum aequationum cuiusque ordinis coniectatio	Inferences on the forms of roots of equations and of their orders
E41	De summis serierum reciprocarum	On the sums of series of reciprocals
E47	Inventio summae cuiusque seriei ex dato termino generali	Finding the sum of any series from a given general term
E100	De numeris amicabilibus	On amicable numbers
E153	Demonstratio gemina theorematis Neutoniani, quo traditur relatio inter coefficientes cuiusvis aequationis algebraicae et summas potestatum radicum eiusdem	A double demonstration of a theorem of Newton, which gives a relation between the coefficient of an algebraic equation and the sums of the powers of its roots
E158	Observationes analyticae variae de combinationibus	Various analytical observations about combinations
E164	Theoremata circa divisores numerorum in hac forma paa±qbb contentorum	Theorems about the divisors of numbers contained in the form paa±qbb
E243	Observatio de summis divisorum	An observation on the sums of divisors
E244	Demonstratio theorematis circa ordinem in summis divisorum observatum	A demonstration of a theorem on the order observed in the sums of divisors
E262	Theoremata circa residua ex divisione potestatum relicta	Theorems about the remainders left by division by powers
E275	Annotationes in locum quendam Cartesii ad circuli quadraturam spectantem	Annotations to a certain passage of Descartes for finding the quadrature of the circle
E335 (with Christian Léger)	Sur les rentes viagères	On life annuities
E394	De partitione numerorum in partes tam numero quam specie datas	On the partition of numbers into a number of parts of a given type

E428	Observationes circa bina biquadrata, quorum summam in duo alia biquadrata resolvere liceat	Observations about two biquadratics, of which the sum is able to be resolved into two other biquadratics
E445	Novae demonstrationes circa resolutionem numerorum in quadrata	New demonstrations about the resolution of numbers into squares
E507	De infinities infinitis gradibus tam infinite magnorum quam infinite parvorum	On the infinity of infinities of orders of the infinitely large and infinitely small
E522	De formatione fractionum continuarum	On the formation of continuous fractions
E541	Evolutio producti infiniti $(1–x)(1–xx)(1–x^3)(1–x^4)(1–x^5)(1–x^6)$ etc. in seriem simplicem	The expansion of the infinite product $(1–x)(1–xx)(1–x^3)(1–x^4)(1–x^5)(1–x^6)$ etc. into a single series
E542	De mirabilis proprietatibus numerorum pentagonalium	On the remarkable properties of the pentagonal numbers
E561	Variae observationes circa angulos in progressione geometrica progredientes	Various observations about angles proceeding in geometric progression
E564	Speculationes circa quasdam insignes proprietates numerorum	Speculations about certain outstanding properties of numbers
E565	De plurimis quantitatibus transcendentibus, quas nullo modo per formulas integrales exprimere licet	On highly transcendental quantities, which may not be expressed in any way by integral formulas
E596	De summa seriei ex numeris primis formatae 1/3 – 1/5 + 1/7 + 1/11 – 1/13 – 1/17 + 1/19 + 1/23 – 1/29 + 1/31 etc. ubi numeri primi formae $4n–1$ habent signum positivum, formae autem $4n+1$ signum negativum	On the sum of the series of numbers of the form 1/3 – 1/5 + 1/7 + 1/11 – 1/13 – 1/17 + 1/19 + 1/23 – 1/29 + 1/31 etc. in which the prime numbers of the form $4n–1$ have positive signs, and those of the form $4n+1$ have negative signs
E643	Methodus generalis investigandi radices omnium aequationum per approximationem	A general method for investigating all the roots of an equation by approximation
E655	Observationes generales circa series, quarum termini secundum sinus vel cosinus angulorum multiplorum progrediuntur	General observations about series, of which the terms arising for the sines or cosines of multiplied angles come forth
E661	Variae considerationes circa series hypergeometricas	Several considerations about hypergeometric series

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E664	Exercitatio analytica	Analytical exercises
E675	De valoribus integralium a termino variabilis x=0 usque ad x=∞ extensorum	On the values of integrals where the variable term is extended from x=0 all the way to x=∞
E699	Utrum hic numerus 100009 sit primus necne inquiritur	Inquiring on whether or not the number 100009 is prime
E706	De novo genere serierum rationalium et valde convergentium, quibus ratio peripheriae ad diametrum exprimi potest	On a new type of rational and highly convergent series, by which the ratio of the periphery to the diameter is able to be expressed
E709	De evolutione potestatis polynomialis cuiuscunque $(1 + x + x^2 + x^3 + x^4 + \text{etc.})^n$	On the expansion of the power of any polynomial $(1 + x + x^2 + x^3 + x^4 + \text{etc.})^n$
E718	Facillima methodus plurimos numeros primos praemagnos inveniendi	An easy method of finding several rather large prime numbers
E725	Illustratio paradoxi circa progressionem numerorum idoneorum sive congruorum	An illustration of a paradox about the idoneal, or suitable, numbers
E727	Accuratior evolutio problematis de linea brevissima in superficie quacunque ducenda	A more accurate treatment of the problem of drawing the shortest line on a surface
E731	Solutio problematis ob singularia calculi artivicia memorabilis	The solution of a memorable problem by a special artifice of calculation
E732	Solutio facilior problematis Diophantei circa triangulum, in quo rectae ex angulis latera opposita bisecantes rationaliter exprimantur	An easier solution of a Diophantine problem about triangles, in which those lines from the vertices which bisect the opposite sides may be expressed rationally
E746	Methodus succincta summas serierum infinitarum per formulas differentiales investigandi	A method for gathering the sums of infinite series by investigating differential formulas
E750	Commentatio in fractionem continuam, qua illustris La Grange potestates binomiales expressit	A commentary on the continued fraction by which the illustrious La Grange has expressed the binomial powers
E769	Solutio problematis Fermatiani de duobus numeris, quorum summa sit quadratum, quadratorum vero	A solution to a problem of Fermat, on two numbers of which the sum is a square and the sum of their squares

	summa biquadratum, ad mentem illustris La Grange adornata	is a biquadrate, inspired by the Illustrious La Grange
E794	Theorema arithmeticum eiusque demonstratio	A theorem of arithmetic and its proof
E795	De quadratis magicis	On magic squares
E808	Problema algebraicum de inveniendis quatuor numeris ex datis totidem productis uniuscuiusque horum numerorum in summas trium reliquorum	An algebraic problem of finding four numbers with the sum of the three others

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