Euleriana

Volume 2 | Issue 2 Article 3

2022

A Method for Calculating the Equation of Noon (an English translation of Methodus Computandi Aequationem Meridiei)

Patrick T. Headley
Gannon University, headley001@gannon.edu

A Method for Calculating the Equation of Noon (an English translation of *Methodus Computandi Aequationem Meridiei*[1])

Leonhard Euler

translator: Patrick Headley, Gannon University
109 University Square, Erie PA 16541
headley001@gannon.edu

Abstract

In this paper Euler presents a method for determining solar noon, the time at which the Sun crosses the meridian. The method requires the times of two observations of the Sun, one in the morning and one in the afternoon, at equal altitudes above the horizon. Solar noon is approximately the midpoint between two such observations, but, since the declination of the Sun will have changed during the day, a correction term, called the *equation of noon*, is required. Euler explains that this term is too large to ignore and discusses the table of values constructed by de la Hire; this table applies only at the latitude of Paris and relies on laborious calculations. For his own method, Euler describes the apparent motion of the Sun using spherical trigonometry and then uses differentials to complete the calculation with sufficient accuracy for his purposes. He provides examples and claims that his method makes it practical to construct a table at whatever latitude is required.

All footnotes are comments by the translator.

1. In order to find the time of solar noon[2], astronomers tend to use, among other methods, two observations of the Sun at equal altitude, one of which is made before noon, and one after. From such observations, the time of solar noon would seem to be easily found by taking the mean of the times at which these observations are made, just as the time of culmination for the fixed stars is determined correctly by this method from two observations at equal altitude. But, while this produces the proper result for fixed stars, it is inappropriate for the Sun, as its declination continually changes. If the Sun, like the fixed

[1] Euler, L.(1741)."Methodus computandi aequationem meridiei"(E50), *Commentarii Academiae Scientiarum Petropolitanae* 1741(8): 48-65. Reprinted in Opera Omnia: Series 2, Volume 30, pp.13-25. Original text available online at scholarlycommons.pacific.edu/euler/ (https://scholarlycommons.pacific.edu/euler/).

[2] In this translation, noon will always mean *solar noon*, the time at which the Sun crosses the meridian, while twelve o'clock will refer specifically to a time shown on a clock. Euler's aim is to reset the clock so that it would have shown twelve o'clock at solar noon. Note that, even when ignoring the issue of daylight savings, this is different from modern timekeeping in two ways. First, since the interval between consecutive solar noons varies due to Earth's elliptical orbit, a mean solar time is now used so that every day has the same length. Second, each location now uses a mean solar time fixed across its time zone, which is not necessarily its own mean solar time.

stars, always stayed in the same parallel, the mean time between these two observations would be the time of solar noon, and we would have no need for any correction.

2. However, as the Sun proceeds from one parallel to another, it is easily seen that the mean time between the times at which the Sun is observed at equal altitudes must be different from the time of solar noon. For if, as an example, the Sun's altitude is 30° at 9am while it ascends from Aries toward Cancer[3], then, at 3pm, with noon lying halfway between the two times, the Sun's altitude will be greater than 30°, since, in the meantime, the Sun will approach nearer the celestial pole, and thus nearer the zenith in our part of the world. On account of this, the Sun only arrives at altitude 30° after 3pm, from which it is seen that an error is made if the mean time between the times at which the Sun is observed at altitude 30° is taken for solar noon. The mean time in this case falls after noon; for this reason, it is necessary that something be subtracted in order to find the true time of noon, and this is what will be determined in this article.

3. By similar reasoning, if the Sun is descending from north to south, the opposite occurs, and something must be added to the time. This small amount of time that must be added or subtracted from the mean time between observations is called the equation of noon. A table of equations of noon contains these equations for every degree of solar declination and for various intervals of time between the two observations, as calculated for a given latitude. Such a table is of great necessity for an astronomer, since solar noon can be found from the table by a method that is simple, not affected by refraction, and capable of correcting clocks.

4. However, the equation of noon depends first on the latitude of the observer, then also on the difference in time between the two observations, and finally on the motion of the Sun, from which the change in declination is determined. Thus, different latitudes require their own tables for the equation of noon; a table computed for this location[4] is not of use in other locations except for those lying in the same parallel. Furthermore, the table computed for a given latitude will not always be valid, since the motion of the Sun has a small annual variation in declination. However, this variation has such a small effect that the equation would rarely increase or decrease by a second because of it, at least in places within 60° of the equator. But differences of latitude lead to great differences in the equation of noon; the equations corresponding to this location are nearly twice as large as those for Paris. They are even larger for locations nearer the poles, and become infinite at the poles themselves; this is because noon is not defined there. Therefore, the closer the location of the

[3] Euler uses *ascending* or *descending* to mean increasing or decreasing in declination. *Aries* refers to that part, or *sign*, of the ecliptic between ecliptic longitudes 0° and 30°, and *Cancer* to the part between 90° and 120°. Due to precession of the equinoxes, this now-obsolete system had little connection with the actual positions of the constellations by Euler's time.

[4] *This location* is St. Petersburg, Russia.

DOI: 10.56031/2693-9908.1034

observer to one of the poles, the more this correction is necessary. In fact, for St. Petersburg the error in the determination of noon can exceed 50 seconds without this correction.

5. Mention of this correction is made in the second edition of the Astronomical Tables of Philippe de la Hire, in which a method for finding the equation for noon is also given. A table of equations of noon for the latitude of Paris is also provided in the same book[5], with the values calculated to the third[6]; this is necessary if we do not want an error in the number of seconds, even if we do not need to know the number of thirds. This table is computed for every degree of solar declination, but it only extends from four hours to ten for the interval between observations. Moreover, the table suffers from the error that the equations have the same values for ascending signs and descending signs. But the motion of the Sun in ascending and descending signs is not the same; this produces differences in the number of thirds, but from these can arise differences in the number of seconds. For instance, the difference between $20'', 35'''$ and $20'', 25'''$ is $10'''$, but astronomers tend to accept $21''$ for the former and $20''$ for the latter, and this difference is an entire second.

6. La Hire's method for computing the equation of noon is long and requires much time to implement for a single case; the most efficient calculator would have to work many months to construct a complete table of equations of noon. Moreover, since we want to have these equations in thirds, our tables of sines and tangents will not be sufficient; interpolation is always needed, generating an incredible amount of work. Therefore, as this kind of table has recently been sought for our observatory, I thought of another method for finding the equation that was brief, and for which the tables of sines and tangents would be satisfactory. To obtain such a method, I saw immediately that the equation should be determined not from the differences of angles whose sines and tangents enter the calculation, but from a certain angle or arc whose sine does not come into consideration. For the usual tables of sines will not distinguish between angles or arcs that produce differences in thirds of time. Interpolation, which I have chosen to avoid, would be required to find them instead.

7. In order to devise a method of this kind, I combined spherical trigonometry and infinitesimal calculus, from which I quickly obtained a method with the desired attributes. I made use of infinitesimal calculus in the following way: I regarded the daily variation in the declination of the Sun as an infinitely small quantity, since, at its maximum, it does not exceed $24'$. An arc of $24'$ can be taken to be a small line segment, and, thus, as infinitely small with respect to an arc of finite length, or to an entire circumference. With this assumption,

[5] de la Hire, P.(1727). *Tabulae Astronomicae Ludovici Magni*, 2nd edition. Paris, 1727. Original text available online at Google Books (books.google.com). The problem is discussed (in Latin) on pages 75-83 of the section Usus Tabularum, while the table itself is on page 81 of the tables that follow.

[6] A third is a sixtieth of a second for seconds of both time and arc. Thirds are indicated with the notation $'''$.

while working according to the rules of infinitesimal calculus, I obtained a brief and simple formula producing the equation of noon, as several terms that could be considered as differentials in seconds of arc dropped out of the calculation. From this, therefore, not only was it easy to compute a table for this location, but it will also be easy to calculate noon correctly without a table whenever and wherever two observations of the Sun are made at equal altitudes.

8. For the work that I will present it is necessary to prepare with the following[7]. For an arbitrary angle or arc AM let[8] sine PM equal A, and cosine CP equal $a = \sqrt{1 - A^2}$, taking the whole sine AC to be[9] 1. The arc AM is increased by the very small arc Mm, and the augmented arc has sine $A + a \cdot Mm$ and cosine $a - A \cdot Mm$. This is because the arc AMm has[10]sine $pm = PM + mN = A + mN$, and, since the triangles CPM and mNM are similar, $CM : CP = Mm : Nm$, or $1 : a = Mm : Nm$, and thus $Nm = a \cdot Mm$, and, equivalently, $pm = A + a \cdot Mm$. Similarly, $MN = Pp$ is found to be $A \cdot Mm$, and therefore the cosine of arc AMm is $Cp = CP - Pp = a - Pp$, so that the cosine of the augmented arc AMm is $a - A \cdot Mm$. It is clear that this is justified to the extent that arc Mm is small enough that it can be taken to be a small line segment. Therefore, in what follows, as the solar declination will change by less than half a degree between the two observations, and an arc of half a degree is not very different from a line segment, we can safely make this assumption in order to determine how the sine and cosine of the solar declination increases or decreases. The error that can arise from the curvature of such a small arc can be little more than a third at most; such an error, which cannot affect the number of seconds, can be ignored. Fig. 1

9. With this established, consider the hemisphere $APBE$, in which the point P represents the celestial pole and Z the zenith at the location where the equation of noon is sought. The circle AEB is the equator, CD and cd are parallels, and PZE the meridian at the proposed location. Furthermore, PZ is the complement of the latitude, and for this reason the arc ZE of the meridian is the latitude itself. If the Sun is observed before noon at S, the arc ZS is, of course, the complement of the solar altitude; PS is the complement of the solar declination, but this is an unknown, since the solar declination is found for the exact moment of noon, and is not known for observations away from the meridian. As the Sun ascends to the north from the equator, its declination continually increases, and it will not move along the parallel CD, but instead on an oblique path represented by the line SOT. Therefore, after noon, when the Sun is observed at the same altitude, it will not be in parallel CD again, but instead in another slightly higher parallel cd, and at point T, whose distance Fig. 2

[7]The figures for this article are on page 2 at eulerarchive.maa.org/docs/originals/E050.pdf and are also reproduced at the end of this translation.

[8]Note that Euler is using A in two different ways, both as a point in Figure 1 and as the length of the line segment PM in that figure.

[9]The *whole sine* is the radius, and what Euler calls the *sine* would now be considered the product of the radius and the sine. As will be seen later, the whole sine is not 1 in the tables that Euler uses for his calculations.

[10]The figure has H instead of N.

ZT to the zenith is equal to ZS, because it is arranged that the solar altitudes at S and T are equal, and ZS and ZT are the complements of these altitudes.

10. When these equal solar altitudes are observed, the times of observation should be recorded as carefully as possible using a good pendulum clock. A clock is called good if it moves uniformly and completes one period in twelve or twenty-four hours, even if 12 o'clock does not correspond to solar noon, since the difference between 12 o'clock and solar noon is investigated through these very observations, by which the clock is perfectly corrected and adjusted to the motion of the Sun. A good uncorrected clock allows us to find the interval of time between the two observations, and converting this time to equatorial degrees gives the angle SPT. However, the angle SPZ is desired, for converting this angle to time gives the interval between the morning observation and solar noon, from which the correction of the clock immediately follows. The meridian arc PO, which is the complement of the solar declination at the instant of noon, is known from ephemerides, but the arcs PS and PT are not since the angles SPZ and TPZ are unknown.

11. If the solar declination did not change, and the Sun always stayed in the same parallel CD, the afternoon observation would occur when the Sun was at t, from which point the distance Zt to Z would be the same as the distance SZ. For this case, in which likewise $PS = Pt$, the meridian PZE bisects the angle SPt, and when this angle is converted to the interval of time between the two observations, it is clear that dividing the interval into two parts will produce the time of solar noon. On the other hand, from this it is seen that, when the solar declination changes, the mean time between the two observations cannot be noon, since angle TPZ is larger than angle SPZ, the difference being angle TPt. Therefore, something must be subtracted from the mean time between the two observations in order to find the time of solar noon, and this small amount of time, converted to an equatorial arc, is half the angle TPt. Thus, it will be necessary for us to find the size of angle TPt; with this done we will have the equation of noon. Half of this angle, when converted to time, gives the amount of time to be subtracted from the mean between the observations in our case, which has the Sun placed in ascending signs. In descending signs the amount of time is found in the same way, but it should be added to the mean time between the observations.

12. In order to calculate this value, let the sine of arc PZ equal[11] A , its cosine equal a, and let the radius be 1. Let the sine of the complement of the solar declination, or the sine of the arc PO, equal B, and its cosine equal b, so that $B^2 + b^2 = 1$. Furthermore, let angle SPT equal $2N$ degrees, and then half the angle SPT will be N degrees. Let the sine of this half-angle equal C, and its cosine equal c; then, likewise $C^2 + c^2 = 1$. Next, let dt be the amount by which the solar declination increases in one day; in descending signs

[11] Euler uses A as both a point and a length in Figure 2 just as in Figure 1, and now also does the same for B and C.

dt will represent the amount by which the declination decreases. This increase or decrease must be given in seconds as accurately as possible, if we want to find the equation of noon in thirds. However, since ephemerides usually only display the solar declination in minutes, I will later give a method based on the daily motion of the Sun, which is commonly given in seconds. The daily change in declination can then also be computed in seconds, so that it is not necessary to use interpolation in the customary tables of sines and tangents. I will let dx be half of the desired angle TPt; therefore, dx, converted to time, will give the desired equation of noon. I will take dt, the daily change in declination, and dx, half the angle TPt, to be differentials, since the quantities are so small that they can be taken as infinitely small with respect to the other arcs, and the equatorial arcs corresponding to dt and dx can be regarded as small line segments.

13. By estimating that the solar declination changes at an equal rate over one day, the change in declination during the interval between two observations can be found from the daily change in declination. Just as an interval of 24 hours is to the interval between the observations, so is 360 degrees to the angle SPT, or $2N$; for this reason, express the ratio 360 to $2N$ as dt to $\frac{N\,dt}{180}$, which will be the increase in declination as the Sun moves around the pole through angle SPT. Therefore, this quantity $\frac{N\,dt}{180}$ is equal to the difference between PS, or Pt, and PT, and hence $PS - PT = Pt - PT = \frac{N\,dt}{180}$. Next, the angle TPt has been set equal to $2\,dx$; thus, the angle SPt equals $2N - 2\,dx$, and the angle SPO or tPO, which is half of this, equals $N - dx$, and the angle TPO equals $N + dx$. Thus, reasoning as before, the change in declination while the sun moves through angle SPO is found from $360 : N - dx = dt : \frac{N\,dt}{360} - \frac{dt\,dx}{360}$, where the term $\frac{dt\,dx}{360}$ can be left out, since it corresponds to a differential in seconds of arc. Just as for $PS - PO$, the value $\frac{N\,dt}{360}$ is acceptable for $PO - PT$. But the formula[12] does not turn out to be more complicated if one wishes to use $\frac{N\,dt - dt\,dx}{360}$ for $PS - PO$ and $\frac{N\,dt + dt\,dx}{360}$ for $PO - PT$; in fact, it will not change.

14. Now, as the sine of arc PO equals B, and the cosine equals b, by the argument given earlier[13] the sine of arc PS, or $PO + \frac{N\,dt - dt\,dx}{360}$, equals $B + \frac{bN\,dt - b\,dt\,dx}{360}$, and the cosine equals[14] $b + \frac{-BN\,dt + B\,dt\,dx}{360}$, and these are also the sine and cosine of arc Pt. The sine of the arc PT, or $PO + \frac{-N\,dt - dt\,dx}{360}$, equals $B + \frac{-bN\,dt - b\,dt\,dx}{360}$, and the cosine is $b + \frac{BN\,dt + B\,dt\,dx}{360}$. Moreover, as the angle SPO, or tPO, equals $N - dx$, and N has sine C and cosine c, the sine of angle SPO or tPO equals $C - c\,dx$, and the cosine equals $c + C\,dx$. Similarly, the sine of angle TPO equals $C + c\,dx$, and the cosine equals $c - C\,dx$. Therefore, assuming dx is given, in the spherical triangle tPZ the sides PZ and Pt are given together with the angle ZPt, and, similarly, in the spherical triangle TPZ the sides PZ and PT are given together with the angle TPZ. Since three

[12]That is, the formula for dx, and thus for the equation of noon itself.

[13]This refers to Section 8.

[14]Euler actually writes this as $b - \frac{BN\,dt + B\,dt\,dx}{360}$. Similar expressions that would appear incorrect given the modern order of operations have been similarly altered.

values are given in these triangles, it will be possible to find the sides Zt and ZT. As these arcs are equal, they will give equations from which dx can be determined.

15. We thus have two spherical triangles to solve, for both of which two sides and the angle between them are given, and the third side is required. Before finding the third side from the given values, I will present a rule in which dropping a perpendicular is not necessary. If sides ZP and TP together with angle ZPT are given in spherical triangle ZPT, then[15] $\cos ZT = \cos ZPT \sin ZP \sin PT + \cos ZP \cos PT$. This rule is easy to derive from the results in trigonometry contributed by the late Professor Maier to Volume 2 of this Commentary[16]. Thus, by this rule, $\cos Zt = ABc + ab + ABC\,dx + \frac{ABcN\,dt - aBN\,dt + aB\,dt\,dx - Abc\,dt\,dx + AbCN\,dt\,dx}{360}$, and from the other triangle $\cos ZT = ABc + ab - ABC\,dx + \frac{-AbcN\,dt + aBN\,dt + aB\,dt\,dx - Abc\,dt\,dx + AbCN\,dt\,dx}{360}$. However, since $Zt = ZT$, the cosines are equal, and the following equation is obtained: $ABC\,dx + \frac{AbcN\,dt - aBN\,dt}{360} = 0$. From this, $dx = \frac{N\,dt}{360}\left(\frac{a}{AC} - \frac{bc}{BC}\right)$, and, if the arc dt is converted into thirds of time, then dx is immediately expressed in thirds, as is the equation of noon itself. Fig. 3

16. In order to examine this formula more clearly, we replace the symbols with the letters of the figure, and the equation of noon becomes[17]

$$\frac{\text{ang. } SPT \cdot dt}{720^\circ}\left(\frac{1}{\tan PZ \sin \frac{1}{2}SPT} - \frac{1}{\tan PO \tan \frac{1}{2}SPT}\right).$$

When doing the calculations for this rule, note that the whole sine must be set equal to 1, whereas in tables of sines and tangents it is 10000000. To adjust for this, the square of the whole sine should replace 1 in the numerator. However, I can change the formula so that the numerator and denominator have the same number of dimensions, and then this adjustment is not necessary. For, when the whole sine is 1, then $\frac{1}{\tan PZ} = \cot PZ = \tan ZE$, which is the tangent of the latitude, and $\frac{1}{\tan PO} = \cot PO = \tan OE$, which is the tangent of the solar declination. Making this substitution, the equation of noon will be

$$\frac{\text{ang. } SPT \cdot dt}{720^\circ}\left(\frac{\tan ZE}{\sin \frac{1}{2}SPT} - \frac{\tan OE}{\tan \frac{1}{2}SPT}\right).$$

From this formula it is immediately apparent that the equation becomes infinitely large at the pole, since ZE is 90°, for which the tangent is infinitely large. At the equator, however, $\tan ZE$ disappears, and the equation of noon is negative; it should be added, when otherwise it should be subtracted, unless OE

[15] Euler uses $\int$ for $\sin$ here and in other places. This notation seems too confusing to maintain in this translation.

[16] The citation is Maier, F.C.(1727). "Trigonometrica", *Commentarii Academiae Scientiarum Imperialis Petropolitanae* 1727)(2): 12-30. It can be found online at the pages beginning from https://www.biodiversitylibrary.org/item/38525#page/24/mode/1up .

[17] Euler uses tang. or just t. for tangent.

is negative, or the sun is descending toward the south. The formula just found applies to northern declinations, but if the declination is southern on account of OE being negative, then the equation of noon is
$\dfrac{\text{ang. } SPT \cdot dt}{720^\circ}\left(\dfrac{\tan ZE}{\sin\frac{1}{2}SPT}+\dfrac{\tan OE}{\tan\frac{1}{2}SPT}\right)$; with this small change the calculation is done correctly.

17. The calculation of this formula is done most conveniently in the following way. The number of hours between the observations is multiplied by 15 in order to obtain the number of degrees in the angle SPT, and the logarithm of this number is taken. Then the daily change in the declination in seconds is found by the method to be described later. This number is multiplied by[18] 4, and the logarithm of the product is added to the prior logarithm. Next, the logarithm of 720 is subtracted from this sum so that the logarithm of $\dfrac{\text{ang. } SPT \cdot dt}{720^\circ}$ is obtained. After that, the logarithm of the sine of half the angle SPT is subtracted from the logarithm of the tangent of the latitude. The number corresponding to this difference in a table of logarithms will be $\dfrac{\tan ZE}{\sin\frac{1}{2}SPT}$. Similarly, the logarithm of the tangent of half the angle SPT is subtracted from the logarithm of the tangent of the solar declination, and the number corresponding to this difference in a table of logarithms will be $\dfrac{\tan OE}{\tan\frac{1}{2}SPT}$. In the case of a northern declination this number is subtracted from the one before; alternatively, in the case of a southern declination the numbers are added together. Again, the logarithm of the number produced is taken and added to the logarithm of $\dfrac{\text{ang. } SPT \cdot dt}{720^\circ}$ found previously. The number corresponding to the sum of these logarithms will be the equation of noon in thirds.

18. This procedure would seem to be lengthy enough to require a large amount of time for calculation of a table of the equation of noon. But anyone who has a little experience in calculating will immediately see that it is not necessary to repeat the whole operation for each equation; many numbers from one calculation are retained for other calculations. At any rate, I can claim to have devoted fewer than four whole days to the entire table for this location, even though this table is six times larger than the one for the latitude of Paris in la Hire's tables. For my own tables were prepared for both ascending and descending signs, doubling the amount of work. Furthermore, my table extends from an interval of one hour between observations to an interval of eighteen hours, while the table for Paris extends from four hours to ten.

19. Before I illustrate this rule with an example, it is necessary to present the method by which the daily increase or decrease of the Sun's declination may be found from its daily motion in the ecliptic. The change in declination must

[18] The 4 is needed to convert from arc to time, since there are 4 times as many thirds in a day as there are seconds in a circle.

be in seconds, so the motion of the Sun in the ecliptic must be in seconds as well. Let R be the pole and AB the equator in hemisphere $ARBA$, and let EC be the ecliptic, its angle ACE with the equator being $23°, 29'$. Let the Sun be at M and its declination the arc PM. In one day the Sun proceeds in the ecliptic through arc Mm, which we call dk. The increase in declination will be $mp - MP = mN = dt$. If, as before, we let the sine of arc PM equal b and the cosine equal B, the sine of arc mp will equal $b + Bdt$. Let the sine of angle ACE be e, so that[19] $e : 1 = \sin PM : \sin CM = b : \sin CM$, and therefore $\sin CM$ will be $\frac{b}{e}$. Let the cosine of arc CM equal f, and then $\sin Cm = \frac{b}{e} + fdk$. But then $\sin ACE$: whole sine $= e : 1 = \sin pm : \sin Cm = (b + Bdt) : \left(\frac{b}{e} + fdk\right)$, or $efdk = Bdt$. From this it is found that $dt = \frac{efdk}{B}$. Therefore, with $e = \sin 23°, 29'$, f equal to the cosine of the distance of the Sun from the equinox, and B equal to the cosine of the solar declination, dt is defined from dk. Since dk is known in seconds from ephemerides, dt can also be expressed in seconds. Fig. 4

20. We will demonstrate the operations just described in the following example. At latitude $52°, 27'$ the Sun's altitude is observed at $8{:}21$am, and the sun returns to the same altitude at $3{:}49$pm, as noted using a good, but not corrected, clock. On that day the ephemerides show the Sun at[20] ♉ $16°, 35', 6''$, and its declination as $16°, 49'$. The question is to find the time at which solar noon occurred as indicated by the clock. From the ephemerides it is found that the daily motion of the sun is $57', 4'' = 3424''$, and therefore $dk = 3424''$. Furthermore, the distance from the Sun to the nearest equinox is $46°, 35'$, for which the cosine is equal to f. Also, $e = \sin 23°, 29'$ and $B = \cos 16°, 49'$. From this dt is found as follows[21]:

$$
\begin{array}{rclcr}
l\,e & = & l\,\sin 23°, 29' & = & 9.6004090 \\
l\,f & = & l\,\cos 46°, 35' & = & 9.8371456 \\
l\,dk & = & l\,3424 & = & \underline{\;\;3.5345338} \\
 & & l\,efdk & = & 22.9720884 \\
l\,B & = & l\,\cos 16°, 49' & = & \underline{\;\;9.9810187} \\
 & & & & 12.9910697
\end{array}
$$

For the purpose of consistency, the logarithm of the whole sine, 10, should be subtracted, as there are two sines in the numerator but only one in the denominator. This will leave $2.9910697 = l\,dt$.

21. It is not necessary to find the number corresponding to this logarithm, since it is the logarithm itself that is used in the other formula. Nevertheless, $dt = 979'' = 16', 19''$ can be found from tables, and this is the daily change in declination. If $979''$ is multiplied by 4, the number of thirds

[19] Euler is using the spherical law of sines: the ratio between the sines of two angles of a spherical triangle is equal to the ratio of the sines of the arcs opposite the angles.

[20] The symbol represents the sign Taurus, corresponding to ecliptic longitudes $30°$ through $60°$. Thus, the ecliptic longitude would now be given as $46°, 35', 6''$.

[21] Euler uses l in the following computations to represent the base-10 logarithm. Also, while the whole sine was 10^7 in the tables mentioned in Section 16, it is 10^{10} in the tables Euler uses here for logarithms of trigonometric functions.

of time corresponding to dt is obtained. Thus, with dt expressed in seconds, the other formula can be multiplied by 4, and the equation of noon becomes $\frac{\text{ang. } SPT \cdot dt}{180}\left(\frac{\tan ZE}{\sin\frac{1}{2}SPT} - \frac{\tan OE}{\tan\frac{1}{2}SPT}\right)$ in thirds of time. Now $ZE = 52°, 27'$ and $OE = 16°, 49'$ in the formula, and, since the interval between the two observations is 7 hours, 28 minutes, the angle SPT is $112°$, and $\frac{1}{2}SPT = 56°$. With these preparations made, the work is done as follows:

$$\begin{array}{rcccc} l.\text{ ang. } SPT & = & l112 & = & 2.0492180 \\ l.dt & & & = & \underline{2.9910697} \\ & & & & 5.0402877 \\ l.180 & & & & \underline{2.2552725} \\ l.\frac{\text{ang. } SPT \cdot dt}{180} & & & = & 2.7850152 \end{array}$$

The other part is found in this way:

$$\begin{array}{rcccc} l.\tan ZE & = & l.\tan 52°, 27' & = & 10.1142350 \\ l.\sin\frac{1}{2}SPT & = & l.\sin 56° & = & \underline{9.9185742} \\ l.\dfrac{\tan ZE}{\sin\frac{1}{2}SPT} & & & = & 0.1956608 \end{array}$$

Therefore,

$$\frac{\tan ZE}{\sin\frac{1}{2}SPT} = 1.569 \ .$$

Furthermore[22],

$$\begin{array}{rcccc} l.\tan OE & = & l.\tan 16°, 49' & = & 9.4803451 \\ l.\tan\frac{1}{2}SPT & = & l.t.56° & = & \underline{10.1210126} \\ & & l.\dfrac{\tan OE}{\tan\frac{1}{2}SPT} & = & -1.3593325 \end{array}$$

where the negative sign marks just the characteristic 1, and the rest of the digits are not affected [23] Therefore,

$$\frac{\tan OE}{\tan\frac{1}{2}SPT} = 0.229$$

and so

$$\begin{array}{rcr} \dfrac{\tan ZE}{\sin\frac{1}{2}SPT} - \dfrac{\tan OE}{\tan\frac{1}{2}SPT} & = & 1.340 \\ \text{which has logarithm} & = & 0.1271048 \\ \text{which, added to } l.\frac{\text{ang. } SPT \cdot dt}{180} & = & \underline{2.7850152} \\ \text{gives the logarithm of the equation of solar noon} & = & 2.9121200 \end{array}$$

Therefore, the equation will be 817 thirds. That is to say, the equation of noon corresponding to this case is $13'', 37'''$, where seconds and thirds of time are denoted.

[22] The value of $\log\tan 56°$ in the following computations is incorrect in the original text. There should be a 7 in place of the 2 in the hundredths place.

[23] What Euler means is that the number should be read as $-1 + 0.3593325$.

22. Since the Sun is in Taurus in this observation, its declination is increasing, and for this reason the equation that has been found should be subtracted from the mean time between the two observations to produce the time of solar noon. The mean time is found by adding the times of the observations

$$\begin{array}{rl} & 8^h.21' \\ & 3^h.49' \\ \text{together with 12 hours} & 12^h. \\ \hline & 24^h.10' \\ \hline \text{and taking half of this sum} & 12^h.5' \end{array}$$

If $13'', 37'''$ is subtracted from this time, the time of solar noon is found to be 12 hours, $4', 46'', 23'''$, from which a clock can be corrected as accurately as possible.

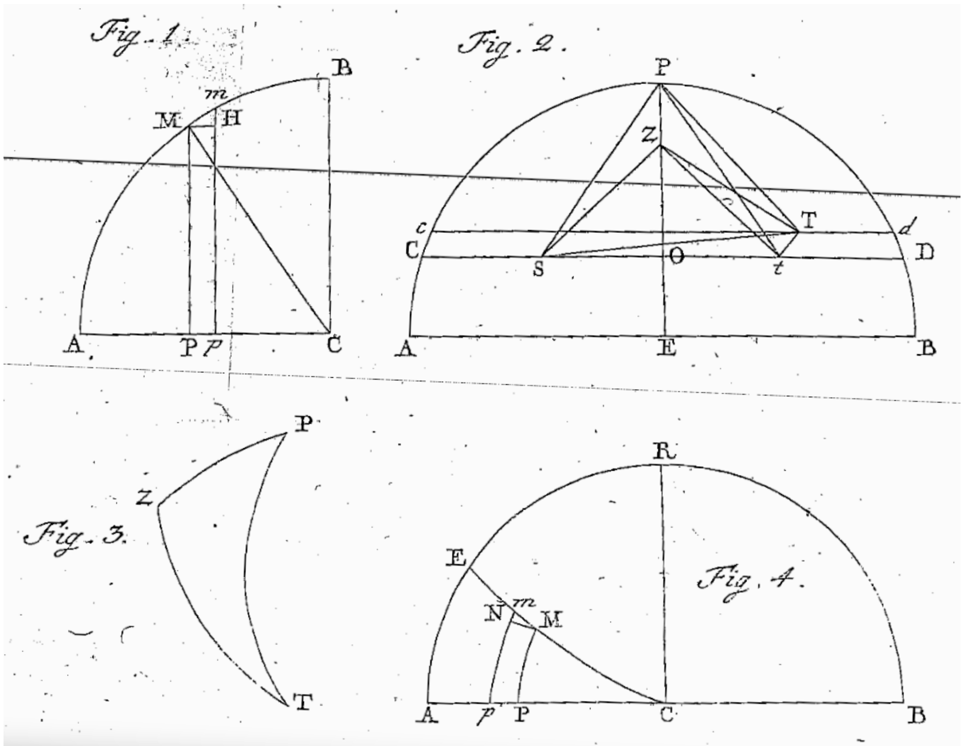
Fig. 1.
B
m
M
H
A
P
p
C
Fig. 2.
P
Z
T
c
d
C
S
O
t
D
A
E
B
P
Z
Fig. 3.
T
R
E
Fig. 4.
N
m
M
A
p
P
C
B