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Modelling the Dissipative Effect of Fisheries

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When managing marine fish resources in the present context of world generalized exploitation of resources, in which many species are overexploited, management processes have to consider research theoretical results. In this paper, a model is proposed for fisheries in the context of chaos. It is shown how the theory of chaos and a model based on chaos may explain and model the fisheries. This model shows that fisheries may dissipate marine species in certain circumstances. The model permits to show that overfishing may cause a problem of irreversibility in several species recovering, after certain stages for the stocks.

Keywords: chaos theory, dynamical systems, complex adaptive systems, fisheries

Introduction

In this study, some considerations about the status of fisheries are given and the general context of chaos is presented, as well.

Fisheries have been analysed in many contexts, considering several frameworks. The context of uncertainty is itself a very interest field to analyse the way fisheries can be organized, considering that marine resources are much exploited, often exploited either in the limits of sustainability or others beyond these limits.

Some works considering chaos in fisheries have been presented. This study intends to have in consideration chaos theory and complexity and intends to present a model to explain the dissipative effect of catches in fish marine systems.

Fisheries and Chaotic Systems

Many aspects of chaos are far-away from being understood or determined. It is possible to find chaos in many mathematical computer problems to be solved and in laboratory research. As soon as the idea of

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nonlinearity[1] is introduced into theoretical models, chaos gets obvious. A very complex structure is observed in field data.

The theory of chaos and complexity theory itself reflect the phenomena that in many activities (such as fisheries) are translated into dynamic forms of analysis and reflect a very complex and widespread reality, specific of complex systems. That reality falls within a range of situations integrated in a broader context, which is expressed in the theory itself but also in terms of their own realities (fisheries, for example), dynamic, complex and often chaotic features in their essence.

At present, with the help of computers, it is possible to make extremely complex calculations and to understand better the occurrence of chaos.

As Williams (1997) says, phenomena happen over time as at discrete (separate or distinct) intervals[2] or as continuously[3]. Discrete intervals can be spaced evenly in time or irregularly in time. Continuous phenomena might be measured continuously. However, we can measure them at discrete intervals[4]. Special types of equations apply to each of those two ways in which phenomena happen over time. Equations for discrete time changes are difference equations and are solved by iteration. In contrast, equations based on a continuous change (continuous measurements) are differential equations. The term “flow” often is associated to differential equations[5].

Differential equations are often the most accurate mathematical way to describe a smooth continuous evolution. However, some of these equations are difficult or impossible to solve. In contrast, difference equations usually can be solved right away. Furthermore, they are often acceptable approximations of differential equations. Olsen and Degn (1985) say that difference equations are the most powerful vehicle to the understanding of chaos.

Modelling Fisheries

In order to frame some methodological developments, it must be mentioned that some characteristics associated with some species support strategic survival features that are exploited by these species. Considering that, it is important to find the reasons and the way these strategies are developed and the resulting consequences. The species use their biological characteristics resulting from evolutionary ancient processes to establish defense strategies.

However, given the emergence of new forms of predation, species got weaker because they are not prepared with mechanisms for effective protection for such situations. In fisheries, there is a predator, man, with new fishing technologies who can completely destabilize the ecosystem. By using certain fisheries technologies, such as networks of siege, allowing the capture of all individuals of the population who are in a particular area of fishing, the fishers cause the breakdown of certain species, particularly the pelagic ones, normally designated by schooling species.

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[1] Nonlinear means that output isn’t directly proportional to input, or that a change in one variable doesn’t produce a proportional change or reaction in the related variable(s).
[2] Examples are the occurrence of earthquakes, rainstorms or volcanic eruptions.
[3] Examples are air temperature and humidity or the flow of water in perennial rivers.
[4] For example, we may measure air temperature only once per hour, over many days or years.
[5] To some authors (Bergé, Pomeau, & Vidal, 1984), a flow is a system of differential equations. To others (Rasband, 1990), a flow is the solution of differential equations.

To that extent, with small changes in ecosystems, this may cause the complete deterioration of stocks and the final collapse of ecosystems, which in extreme cases can lead to extinction. These species are concentrated on high density areas in small space. These are species that tend to live in large schools.

For the application of a mathematical model to fisheries, let's inspire in the model presented in Berliner (1992) related to dissipative systems in the presence of chaos.

In Berliner (1992), it is referred that noninvertibility is required to observe chaos for one-dimensional dynamic systems. Additionally it is said that "everywhere invertible maps in two or more dimensions can exhibit chaotic behavior". The study of strange attractors shows that in the long term, as time proceeds, the trajectories of systems may become trapped in certain bounded regions of the state space of the system.

The model presented in Berliner (1992) is an example in two dimensions of the Hénon map (displaying the property of having a strange attractor).

The Hénon map appears represented by the equations:

$$x_{t+1} = 1 + y_t - ax_t^2$$

and

$$y_{t+1} = bx_t$$

for fixed values of a and b and $t = 0, 1, \ldots$

This invertible map possesses strange attractors and simultaneously has strong sensitivity to initial conditions.

The Hénon map, representing a transformation from $\Re^2$ to $\Re^2$, has Jacobian equal to $-b$.

If $0 < b < 1$, the Hénon map contracts the domains in which it is applied. These maps are said to be dissipative (on the contrary, maps that maintain the application domain are said to be conservative).

Presented the model, it is possible now to suggest a model on this basis for fisheries.

So, if a general situation is considered, the following equations may represent a system in which fish stocks, at time t, are given by x_t and catches by y_t. The model is as follows:

$$x_{t+1} = F(x_t) - y_t$$

and

$$y_{t+1} = bx_t$$

It is a generalization of Hénon model. The Jacobian is equal to *b*. As y_{t+1} is a portion of x_t, $0 < b < 1$. So, it is a dissipative model and the values of x_t are restricted to a bounded domain.

Considering the particular case below:

$$x_{t+1} = x_t - y_t$$

and

$$y_{t+1} = bx_t$$

so

$$x_{t+2} = x_{t+1} - y_{t+1}$$

and

$$x_{t+2} - x_{t+1} + bx_t = 0$$

Now, after solving the characteristic equation associated to the difference equation (Ferreira & Menezes, 1992), it is obtained:

$k = \frac{1+\sqrt{1-4b}}{2}$ or $k = \frac{1-\sqrt{1-4b}}{2}$; calling $\Delta = 1-4b$ and being $0 < b < 1$, comes that $-3 < \Delta < 1$.

So, $0 < \Delta < 1$ if $0 < b < \frac{1}{4}$ and $-3 < \Delta < 0$ if $\frac{1}{4} < b < 1$, being $\Delta = 0$ when $b = \frac{1}{4}$.

Consequently for, $0 < b < \frac{1}{4}$,

$$x_t = A_1\left(\frac{1+\sqrt{1-4b}}{2}\right)^t + A_2\left(\frac{1-\sqrt{1-4b}}{2}\right)^t$$

And for $b = \frac{1}{4}$,

$$x_t = \left(A_1 + A_2 t\right)\left(\frac{1}{2}\right)^t$$

Finally, for $\frac{1}{4} < b < 1$,

$$x_t = \left(\sqrt{b}\right)^t\left[A_1 \cos\left(\left(\arccos\frac{1}{2\sqrt{b}}\right)t\right) + A_2 sen\left(\left(\arccos\frac{1}{2\sqrt{b}}\right)t\right)\right]$$

In these solutions, A_1 and A_2 are real constants.

Note that the bases of t powers are always between 0 and 1. So, $\lim_{t\to\infty} x_t = 0$ and whatever the value of *b*, the dissipative effect is real, even leading to the extinction of the specie. Of course, this is evident according to the hypotheses of this particular situation of the model.

Concluding this approach, the general model does not allow obtaining in general such explicit solutions. But, of course, with simple computational tools it is possible to obtain recursively concrete time series solutions after establishing the initial value x_0 and to check the dissipative effect.

Concluding Remarks

Aspects of chaos are shown up everywhere around the world and chaos theory has changed the direction of science, studying chaotic systems and the way they work. The models of chaos are based on non-linear relationships and are very close to several disciplines, particularly in the branch of mathematics that study the invariant processes of scale or the fractals, for example.

Considering the fisheries in a broad context, the modeling of stocks of fish may be considered on the basis of an approach associated with the theory of chaos instead considering the usual prospect based on classical models.

On the fisheries analysis, it is interesting to see that overfishing may cause a problem of irreversibility in the recovering of several species, after certain stages for the stocks. Anyway, to analyze the specific situation of each case, it is necessary to obtain enough data to analyze the kind of function which is specific for that particular case, and it must be analyzed the situation for certain phases of fishing and it must be seen the consequences for these species.

In this paper, a model that shows the dissipative effect of catches on fisheries was presented. Additionally a particular model showed how stocks are dissipated and may tend to the extinction.

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